Evaluation of Wave Transmission Coefficient at Low-crested Structures in 3D Conditions Using Residual Analysis

RUNNING HEAD TITLE: Wave Transmission Coefficient Evaluation Using Residual Analysis

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Abstract Systems of Low Crested Structures (LCS) are widely used for the protection and restoration of eroded shoreline due to their advantages especially in environmental aspects. Functional design of LCS requires an accurate crest level, provided by the estimation of wave transmission in the sheltered area. The wave transmission, defined by the transmission coefficient K_t, is estimated by formulae based on 2D experiments. In 3D conditions, other phenomena affect K_t and the predicted values for K_t differ. In the present paper, experimental results from two 3D series dealing with emerged LCS were analyzed and a new formula for K_t prediction, based on the recalculation of d'Agremond et al. (1998) formula, is presented. Experimental results were compared to four existing formulae of transmission coefficient - including the new one - and are evaluated using residual analysis. The proposed formula has a very good agreement between measured and predicted values of K_t in 3D conditions, better than the predicted values by the other formulae based on 2D experiments, and can be used in the preliminary design of structures. The proposed formula (Giantsi and Moutzouris 2017) satisfies all the regression assumptions.

Keywords: physical model; wave transmission; low crested structures; residual analysis

1 Introduction

Eroded beaches and shorelines are very commonly protected by systems of Low Crested Structures (LCS) constructed by rubble mound. These structures provoke less environmental impacts than the conventional ones and are more eligible to be constructed. An essential parameter of the LCS is the crest level, which puts limitations on the structure performance. To obtain satisfactory results from the construction of LCS, an accurate estimation of wave transmission is required. Wave transmission is defined mainly by the transmission coefficient K_t, which is the ratio of the transmitted to the incident significant wave heights.

To investigate the performance of the LCS, many experiments, especially in 2D conditions, were undertaken. In 2D conditions, the main parameters influencing the phenomenon are the

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wave overtopping and the wave penetration through the porosity of the rubble mound. In 3D conditions, other parameters are also introduced, like diffraction, extra deformation of the wave spectrum in front of the structure due to the 3D bottom configuration, wave fluctuation, wave penetration through the openings etc. In the present work, an update of the Giantsi and Moutzouris (2016) formula is presented based on two datasets of 3D experiments, and, to evaluate the results, it is compared to other selected formulae using residual analysis. Residual analysis is needed to validate the regression analysis. The advantage of residual analysis is that it visually evaluates the regression assumptions, and it is an easy tool to determine whether the regression model that has been selected is appropriate (Berenson et al. 2012).

2 Wave Transmission

Wave transmission is defined by the transmission coefficient K_t, which is the ratio of the transmitted to the incident significant wave heights (e.g., H_t and H_i), and represents the transmitted energy from the open sea to the sheltered area between the breakwater and the shoreline, i.e., the square root of transmitted E_t to induced E_i time-averaged wave energy (Hughes 2011):

$$K_t = \frac{H_t}{H_i} = \left(\frac{E_t}{E_i} \right)^{1/2} \tag{1}$$

The main parameters affecting wave transmission at a Low Crested Structure, and used in the following analysis, are: H_i=the incident significant wave height, at the toe of the structure; H_t =the transmitted significant wave height, at the sheltered area; E_i=the time-averaged incident wave energy; E_t=the time averaged transmitted wave energy; T_p =the peak period; s_{op}=the wave steepness, given by $s_{op}=2\pi H_i/(gT_p^2)$; h_c =the structure's total height; h =the water depth at the seashore side; B =the crest width; R_c=the freeboard; D_{n50} =the mean diameter of the armor rock; $tan\ \alpha$ =the slope of the structure(seaward); ξ_{op}=the Iribarren parameter, given by $\xi_{op} = tan\ \alpha/(s_{op})^{0.5}$; L_0 =the wavelength in deep water, given by $L=gT_p^2/2\pi$, with T_p is measured at any location. A definition of the geometrical parameters is presented in Figure 1.

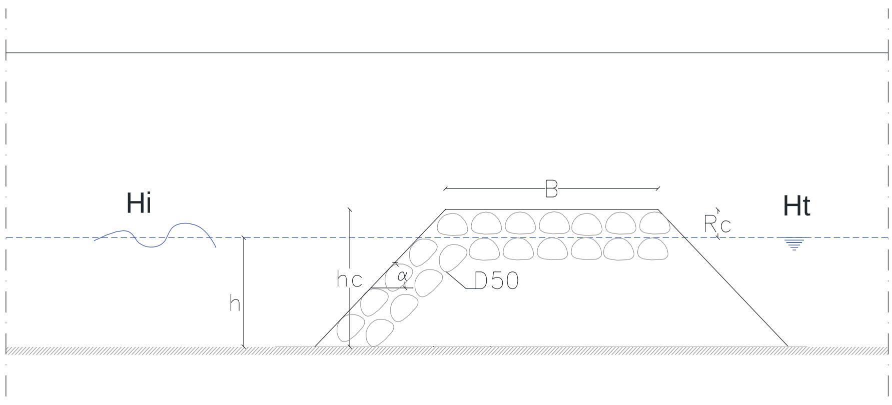

Fig.1 Definition of LCS geometrical parameters affecting wave transmission

To estimate the wave transmission at different conditions of breakwaters, Allsop (1983) and Powell and Allsop (1985) presented diagrams relating the transmission coefficient with the relative freeboard. A simple prediction formula is presented by CIRIA /CUR (1991). Losada et al. (1995) investigated experimentally the wave-induced flow on a porous structure. The relative width of the structure, B/L, associated with the formation of a standing wave and resonant conditions inside the structure, was found to be an important parameter to establish the location of the two regions.

Newer formulae have also been provided for the estimation of wave transmission by van der Meer and Daemen (1994) and d' Agremond et al. (1998) using regression analysis. The following equation has been proposed by van der Meer and Daemen (1994) for conventional breakwaters:

$$K_t = a\frac{R_c}{D_{n50}} + \beta \tag{2}$$

where: $a = 0.031\frac{H_i}{D_{n50}} - 0.24$

and

$$\beta = -5.42 s_{op} + 0.0323\frac{H_i}{D_{n50}} - 0.0017\left[\frac{B}{D_{n50}}\right]^{1.84} + 0.51$$

The following equation has been proposed by d' Agremond et al. (1998) for submerged and low-crested breakwaters:

$$K_t = -0.4\frac{R_c}{H_i} + 0.64\left(\frac{B}{H_i}\right)^{-0.31}(1 - e^{-0.5\xi_{op}}) \tag{3}$$

Both formulae have been limited to a maximum and minimum values for K_t, as follows:

$0.075 \leq K_t \leq 0.75$ and $0.075 \leq K_t \leq 0.8$ for Eqs (2) and (3), respectively.

An improved formula has been proposed for $B/H_i > 10$, for smooth structures (van der Meer et al. 2004, 2005), as follows:

$$K_t = -0.35\frac{R_c}{H_i} + 0.51\left(\frac{B}{H_i}\right)^{-0.65}(1 - e^{-0.41\xi_{op}}) \tag{4}$$

Seabrook and Hall (1998) proposed a formula for submerged breakwaters only. Pinto (2002) also analyzed data from submerged breakwaters. Further analysis has been performed by Shiladarma and Hall (2003) by introducing a diffraction coefficient. van der Meer et al. (2005) proposed an improved formula for smooth slopes introducing the wave incidence. A lot of laboratory experiments on LCS have been performed within the DELOS project (Kramer et al. 2005; van der Meer et al. 2005). During the DELOS project, there were also investigated the waves and the currents around LCS (Johnson et al. 2005; Cáceres et al. 2005; Losada et al. 2005). Most of these experiments were undertaken in 2D conditions and some in 3D conditions but always at constant depth. An attempt to modify the Shiladarma and Hall formula (2003) was made by Giantsi (2006). Lamberti et al. (2006) and Zanuttighi et al. (2008), investigated the wave overtopping, the wave transmission, and the deformation of the spectrum shape on LCS. Panizzo and Briganti (2007) analyzed the wave transmission behind low crested structures using a neural network. van der Meer et al. (2004) and Wang et al. (2007) investigated the wave transmission under oblique wave incidence.

A new formula to predict K_t has been proposed by Goda and Ahrens (2008) considering the wave energy transmitted over and through an LCS. According to Goda and Ahrens (2008), the wave transmission coefficient at LCS is formulated as the summation of wave energy transmitted over and through LCS by referring to the approach by Wamsley and Ahrens (2003). The resultant formula reads as follows:

$$(K_t)_{all} = \min\left\{1.0, \sqrt{(K_t)^2_{over} + K_h^2 (K_t)^2_{thru}}\right\} \quad (5)$$

where: $$K_h = \min\left\{1.0, \left(\frac{h_c}{h+H_i}\right)\right\} \quad (6)$$

The portion of the wave transmission by overtopping has been proposed to be calculated by the following formula:

$$(K_t)_{over} = max\left\{0, \left(1 - exp\left[\alpha\left(\frac{R_c}{H_i - R_0}\right)\right]\right)\right\} \quad (7)$$

$$\alpha = 0.248exp\left[-0.384\ln\left(\frac{B_{eff}}{L_0}\right)\right] \quad (8)$$

$$\text{where } R_0 = \begin{cases} 1.0 & : D_{eff} = 0 \\ max\left\{0.5,\ min\left(1.0, \frac{H_i}{D_{eff}}\right)\right\} & : D_{eff} > 0 \end{cases} \quad (9)$$

where: B_{eff} and D_{eff} are the effective crest width and effective diameter, respectively, as they are defined later in the text.

In the proposed formula $(K_t)_{thru}$is given by the following equation proposed by Numata (1975) for wave passing through the sloped mound made of deformed concrete blocks, such as tetrapods:

$$(K_t)_{thru} = \frac{1}{\left[1 + C\left(\frac{H_i}{L}\right)^{0.5}\right]^2} \quad (10)$$

where: $$C = 1.135\left(\frac{B_{eff}}{L_0}\right)^{0.65} \quad (11)$$

In the data analysis, the effective width is proposed to be calculated as follows:

- emerged breakwaters: B_{eff}= width at still water level;
- zero freeboard: B_{eff}= (9 × crest width + bottom width)/10;
- submerged breakwaters: B_{eff} = (4 × crest width + bottom width)/5.

The effective diameter D_{eff} for conventional breakwaters is calculated as the weighted mean diameter of the armor and core units when the cross section is known. If unknown, it is taken as $D_{eff} = (M/\rho)^{1/3}$, where M the mass and ρ the specific mass of the armor.

An energetic wave propagation model that reproduces shoaling, refraction, diffraction, wave-current interaction, bottom friction and wave breaking was modified to simulate also the processes of overtopping and wave transmission over and through permeable coastal structures by Sierra et al. (2010). Sierra et al. (2011) analyzed experimental data, investigating the spectral changes in wave transmission and reflection in LCS. According to this and to older research on the spectral modification, they found a serious modification of the spectral shape, transposing the peak period T_p to lower values. Formentin and Zanuttigh (2013), predicted the wave transmission using an artificial neural network developed for wave reflection. The model essentially works with 13 input parameters, which describe the wave attack conditions and the main feature of the structures. In this work, the errors between measured and calculated values (residuals) were also analyzed. Zhang and Li (2014) proposed new formulae based on numerical flume results by solving the modified Boussinesq-type wave equations (MBEs). Giantsi and Moutzouris (2016) proposed a modified version of d' Agremond et al. (1998) formula based on 3D experimental data, which reads:

$$K_t = -0.0391\frac{R_c}{H_i} + 0.64\left(\frac{B}{H_i}\right)^{-0.31}(1 - e^{-0.5\xi_{op}}) + 0.1072 \qquad (12)$$

3 Residual Analysis

To evaluate a regression, the coefficient of determination R^2 is an important parameter, but not sufficient. Residual analysis is a tool to evaluate the regression analysis. The biggest advantage of the method is the evaluation of the regression analysis by satisfying visually the regression assumptions and simultaneously the trends of the residuals leading to the correction of the initial model.

Residual analysis can also be used in multiple regression models with more than two independent variables. Trends in the scatterplots of the residuals versus an independent variable may indicate the existence of a non-linear effect. In this case, the introduction of a non-linear independent variable can be the solution.

We consider the measured value as Y_i, and the estimated or predicted or calculated value as $\widehat{Y_i}$[A1][A2]. The residual or estimated error value, e_i, is equal to the difference between the measured value of Y_i, and the predicted value of $\widehat{Y_i}$, the dependent variable for a given value of X (the chosen independent parameter):

$$e_i = Y_i - \widehat{Y_i} \qquad (13)$$

The four assumptions of linear regression are: 1) linearity; 2) independence; 3) normality; and 4) equal variance. Linearity means that the relationship between variables is linear. Violation of this assumption is the most common. Transformation of the independent variable can solve the problem. Independence of errors means that the errors are independent of one another. Normality means that the errors are normally distributed at each value of X (residuals). If the distribution of the errors at each level of X is not extremely different from a normal distribution, the results can be accepted. Equal variance, or homoscedasticity, means that the variance of the errors is constant for all values of X.

To evaluate the four assumptions the scatterplots of the residuals versus the values of the independent variable X and the histogram or normal probability plot of the residuals are needed. The 1st, 2nd, and 4rth assumptions are visualized by the scatterplots of residuals versus the values of X. The 3rd assumption is visualized by the histogram or normal probability plot of the residuals. In the case of the multiple regression models with more than one independent variables, the following residual plots need to be constructed and analyzed: 1) residuals versus $\widehat{Y}_i$ (estimated values); 2) residuals versus X_{1i}, X_{2i}, X_{ni} (independent variables); and 3) histogram and/or normal probability plot.

Any observed pattern in the scatterplots shows a possible violation of the assumptions. A pattern between the residuals and the predicted values shows a possible non-linear effect in at least one independent variable, a possible violation of the 4^{th} assumption (equal variance) and a possible need to transform the Y variable.

In our case, the evaluation of the selected formula for the estimation of the transmission coefficients was handled using multiple nonlinear independent variables.

4 Experimental Datasets

For the evaluation of the presented formulae, two datasets (A and B) of experimental results were used. Both experimental procedures were undertaken at the wave basins of the Laboratory of Harbour Works, National Technical University of Athens (NTUA), Greece.

The first dataset (A) consists of wave measurements obtained from a 3D physical model (M1) with two low-crested breakwaters (AB1 and AB2). The model was constructed in the above-mentioned facility to investigate the protection and restoration of a beach under a geometrical scale of 1:40. An absorbing rip-rap was constructed to isolate the response of the beach. Each breakwater was 2.50 m long. The azimuth of the breakwater's axis was 8^o and 350^o for AB1 and AB2, respectively. Then wave measurements were conducted under oblique wave attack to calculate the transmission and reflection coefficients at the two breakwaters (Giantsi and Moutzouris 2016). The angle of the wave incidence had a 300^o azimuth. For the needs of this study, 15 wave gauges were used. View of the M1 is presented in Figure 2a and a detail of the layout in Figure 2b. The transmission coefficient was calculated at the middle of each breakwater, where H_i and H_t are the measured wave heights seaward and shoreward of the breakwater, respectively (i.e., at wave gauges G10/G4 and G14/G8; Figure 2b).

The second dataset (B) consists of wave measurements obtained by a 3D physical model of a system of LCS (M2). A system of 7 (seven) detached breakwaters (BB1 to BB7) has been designed to protect an eroded shoreline and the axis of the breakwaters is almost parallel to the shoreline. To examine the performance of the proposed system, a physical model was built on a geometrical scale of 1:100, in a wave basin of the Laboratory of Harbour Works NTUA. The length of each breakwater was 0.80 m, the gap between them was 0.40 m in model scale, and the azimuth of the axis of the breakwaters was 120°. Two directions of wave incidence were tested, 330° and 0° azimuths, respectively. Wave measurements were undertaken seaward and shoreward of the two breakwaters. View of the M2 is presented in Figure 3a and a detail of the layout in Figure 3b. The transmission coefficient was calculated at the middle of the breakwaters BB2 and BB3 (i.e, at wave gauges G2/G1 and G6/G5; Figure 3b). A typical cross section of the experimental set-up in the wave basin is presented in Figure 4.

Geometrical parameters of the tested structures are presented in Table 1. We can distinguish 3 datasets according to the angle between the axis of the breakwater and the wave incidence. Datasets A1, A2 and B. The datasets A1 and A2 are subsets from the same physical model.

For the reproduction of the waves, a 3 paddles wave generator was used, of piston type, for both data series, producing JONSWAP type spectra with a peak enhancement factor of 3.3. Resistive type wave gauges collected wave data under a 50 Hz sampling rate throughout the entire 600 s duration of each test. Absorbing rip-rap was placed all around the wave basins to eliminate the wave reflection.

Regarding A data series, measurements were carried out for 8 different wave peak periods, from T_p= 0.48 s to T_p= 1.52 s (model scale), at three different water depths. For each wave period from 1 to 3 wave heights were tested. Waves from the 3 higher periods were broken seaward the breakwaters. Finally, 58 tests for AB1 and 52 tests for AB2 were evaluated.

a) b)

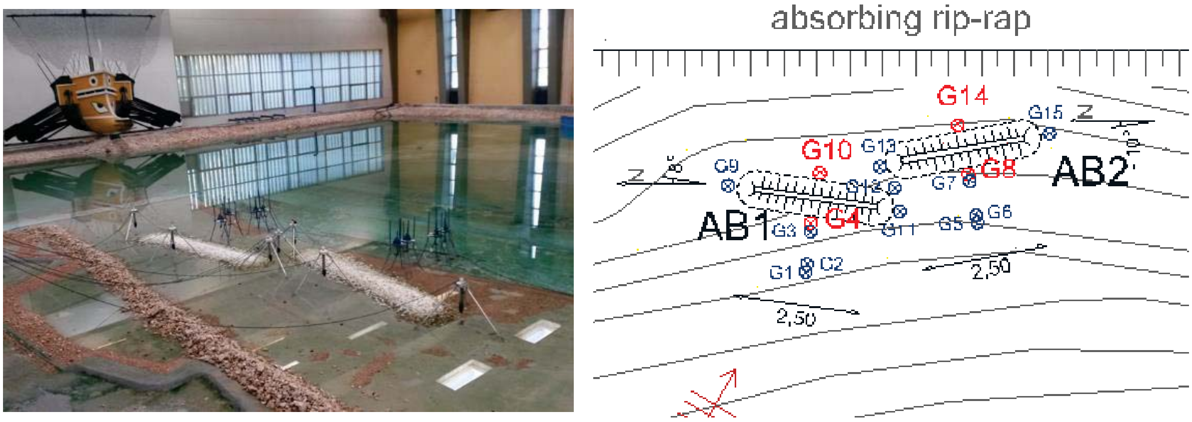

Fig. 2 a)View of the physical model M1; b) Detail of Layout M1

a) b)

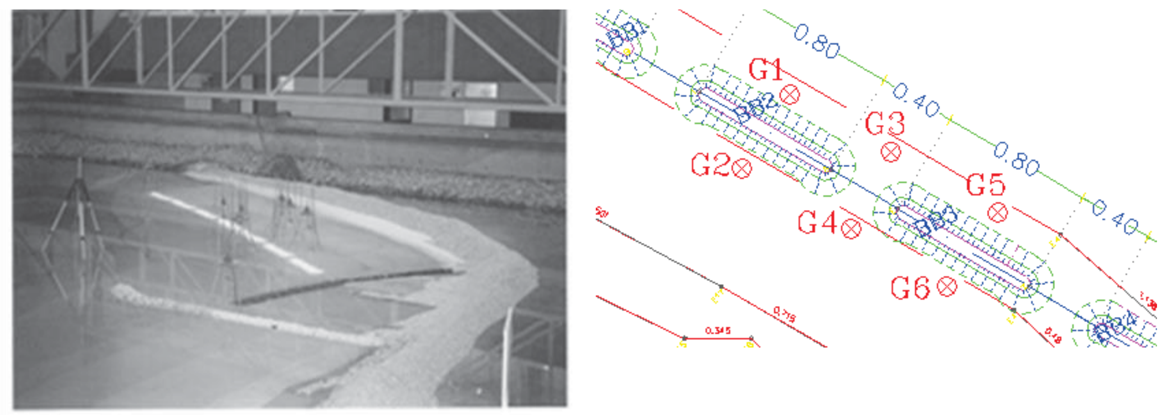

Fig. 3 a)View of the physical model M2; b) Detail of Layout M2

Regarding B data series, measurements were carried out for 4 different wave peak periods, from T_p=0.51 s to T_p=0.79 s (model scale), at one water depth. The transmission coefficient was measured in front of the two breakwaters (BB2 and BB3). In total, 58 tests were evaluated. For both datasets the distance between the wave gauges and the toe of the breakwaters was ~10 cm. The water depths in each case, for both locations of measurement, had no significant difference.

As both datasets are the result of a 3D physical models, a lot of other parameters influence the results and are not fully comparable with datasets from 2D experiments. In spite of these uncertainties, a very good correlation with the existing formulae was achieved.

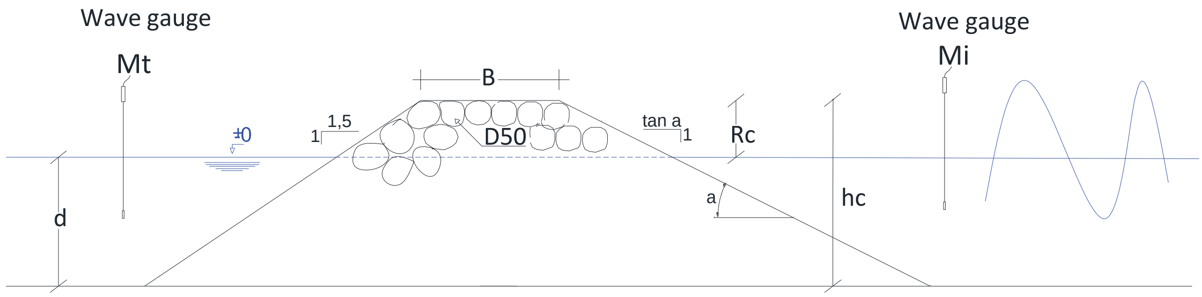

Fig. 4 Typical cross section of the experimental setup

Table 1. Geometrical parameters of the models

Dataset	Wave direction (°)	B (mm)	hc (mm)	Rc (mm)	d (mm)	tan α	D_{n50} (mm)
A1.1	22	112.5	153	55	98	2.5	30
A1.2	22	112.5	153	35	118	2.5	30
A1.3	22	112.5	153	15	138	2.5	30
A2.1	40	112.5	138	50	88	2.5	30
A2.2	40	112.5	138	30	108	2.5	30
A3.3	40	112.5	138	10	128	2.5	30
B1	30	72.0	60.0	20.0	40.0	2.0	15
B2	60	72.0	60.0	20.0	40.0	2.0	15

5 Analysis of the Results

5.1 Modification of the d'Agremond et al. formula

Wave measurements were analyzed and the transmission coefficient was estimated. For the same conditions, the transmission coefficient according d'Agremont et al. (1998) formula was calculated. Many of K_t calculated values according to this formula, were negative or very small, so we used the limitation K_t=0.075. Finally, we had not calculated values by a relationship but many of the calculated values had the same constant value.

To modify the d'Agremond et al. (1998) formula, the residual analysis was used. The target of residual analysis was to evaluate the four assumptions for each regression and for each independent variable. We considered as the residuals of transmission coefficient e_{ki} , the difference between the measured value of transmission coefficient K_t minus the calculated value of K_t for a given value X:

$$e_{ki} = K_{t(measured)} - K_{t(calculated)}$$

In our case, we had multiple nonlinear regression models with more than one independent values. Therefore, we needed to construct and analyze the following residual plots: 1) Residuals versus $K_{t(calculated)}$; 2) Residuals versus X_{1i}, X_{2i}, X_{ni} (independent variables); and 3) Histogram and/or normal probability plot. Regarding the plots of the residuals versus the independent variables, we chose to present here only one independent variable, the most critical. The selected variable must be dimensionless, including the incident wave height H_i, which is the independent variable according to the definition of K_t. For the d' Agremond et al. (1998), formula the most critical parameter, which leads to negative values, is the relative freeboard R_c/H_i . Our basic assumption was to accept the calculated values without any limitation, even negative ones. In Figure 5a and 5b are presented the residuals versus K_t, calculated according to the d' Agremond et al. formulation wihout limitations. In Figure 6a are presented the K_t calculated, measured and the residuals versus the independent variable (relative freeboard) and finally at Figure 6b is presented the Histogram and normal probability plot of the residuals.

a) b)

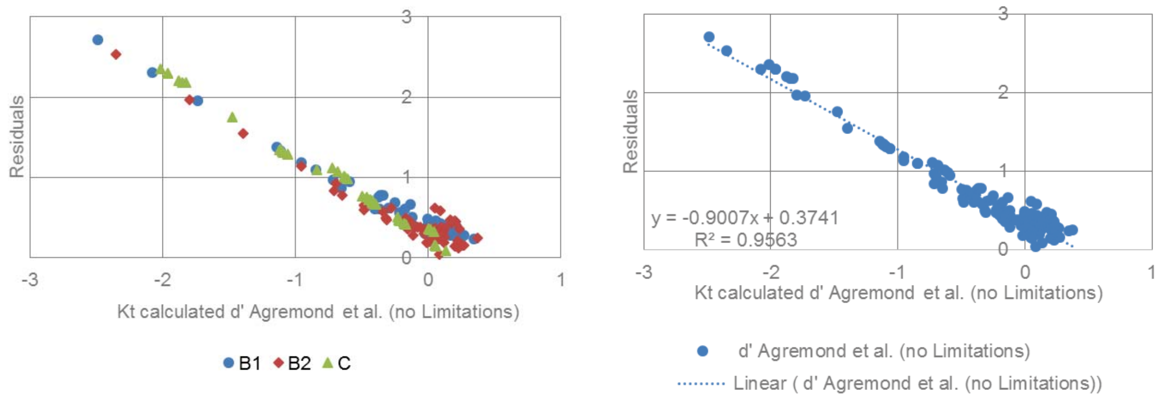

Fig. 5 Residuals versus K_t calculated for: a) ~~for~~ each dataset; b) ~~for~~ all the data

a) b)

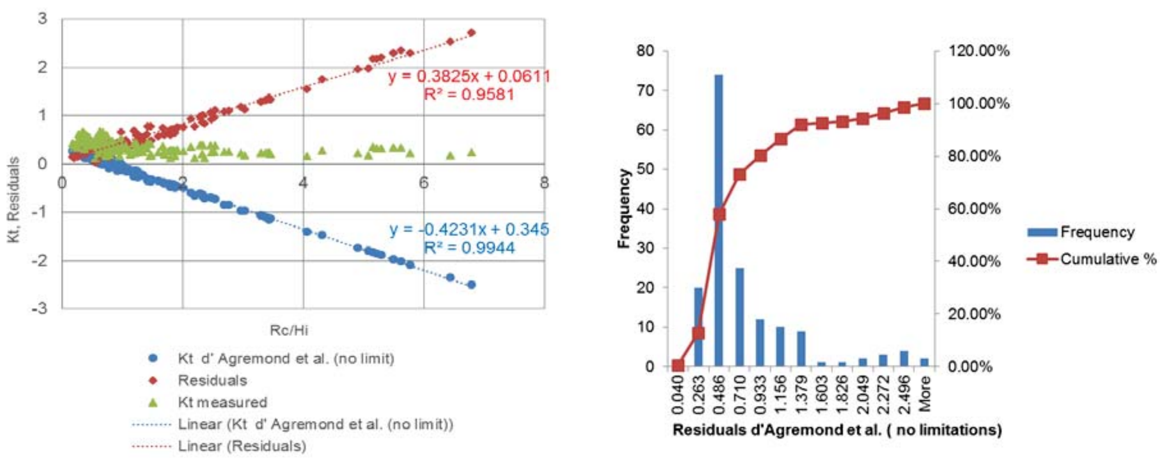

Fig. 6 a) K_t calculated, measured and residuals versus the independent variable relative freeboard; b) Histogram and normal probability plots of residuals

In Figure 5, a linear systematic distribution of the residuals versus the calculated values of K_t is observed, the same for all datasets. From Figure 6 a), a systematic linear error for the d'

Agremond et al. (1998) formula (no limitations) versus the independent variable (relative freeboard) is observed, with the parameter R^2=0.9581. All the mentioned assumptions of the residual analysis were violated. The relative freeboard factor of the formula was corrected by introducing the function of the residuals and the re-estimation of the modified by Giantsi and Moutzouris (2016) d' Agremond et al. (1998) formula, reads:

$$K_t = -0.0175\frac{R_c}{H_i} + 0.64\left(\frac{B}{H_i}\right)^{-0.31}(1-e^{-0.5\xi_{op}}) + 0.0611 \quad (14)$$

Equation (14) is replacing Eq. (12)

5.2 Transmission Coefficient

Wave measurements were analyzed and the transmission coefficient was estimated for all datasets. For each dataset the transmission coefficient K_t is calculated and plotted versus the relative freeboard R_c/H_i for datasets A1, A2 and B (Figure 7). In Figure 7, it is observed that for R_c/H_i>2.5 the transmission coefficient is almost stable.

The transmission coefficient K_t was then calculated using the four formulae proposed by van der Meer and Daemen (F1) using Eq. (2), by d' Agremond et al. (F2) using Eq. (3), by Goda and Ahrens (F3) using Eq. (6), and finally, by Giantsi and Moutzouris (F4) using Eq. (14) with the following limitation. The F1 formula is proposed for conventional breakwaters, not for low crested structures, so a deviation between calculated and measured values of K_t is expected. In the present analysis, we used the initial F2 formula, not the revised for the smooth breakwater (monolithic material) due to the permeability of the structures. Many calculated values of K_t according to the F2 formula, were under the lower limit of 0.075, some of them with a negative sign, which is not acceptable. The F2 formula seems to be appropriate for submerged breakwaters. The calculated values of K_t versus the measured values are presented in Figure 8.

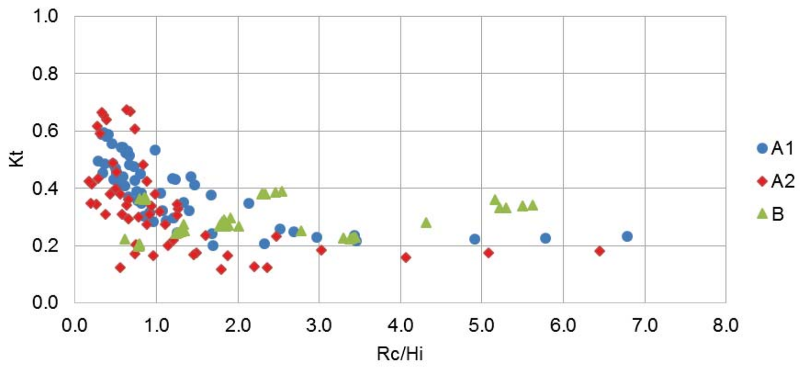

Fig. 7 Transmission coefficients K_t versus R_c/H_i

In Figure 8 it is observed that F1 and F2 formulae underestimate, generally, the calculated transmission coefficients. Transmission coefficients obtained by F3 and F4 formulae are better correlated with the measured ones. The most appropriate formula shall be chosen by the residual analysis.

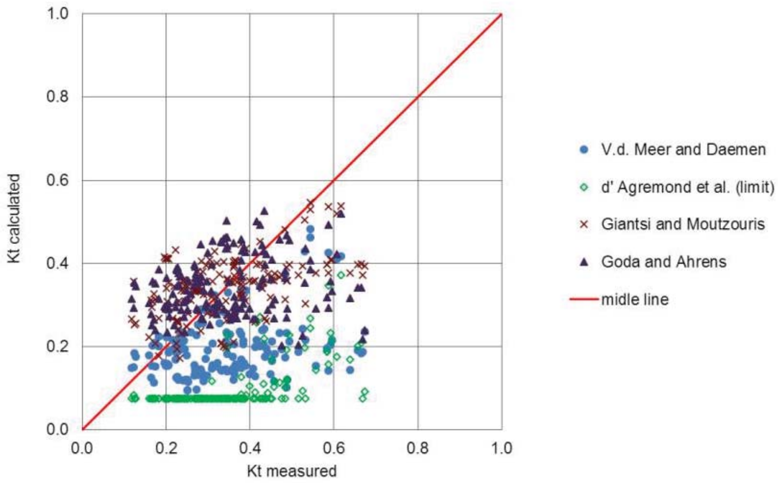

Fig. 8 Transmission coefficient K_t *calculated* versus K_t *measured* for the selected formulae.

5.3 Evaluation of the Formulae

To evaluate the formulae for the estimation of wave transmission in the sheltered area of a detached breakwater, the residual analysis was used.

In our cases, we have multiple nonlinear regression models with more than one independent variables. Therefore, we need to construct and analyze the residual plots mentioned previously. For the plots of the residuals versus the independent variables, we chose to present here only one independent variable for each formula, the most critical.

For formula F1, we considered as independent variable the ratio H_i/D_{n50}, which is the main parameter on both parts of Eq. (2), and for all the others formulae, the relative freeboard R_c/H_i.

The scatterplots of the residuals versus the calculated values of K_t, respectively for each formula, are plotted in Figures 9a to 9d. The residuals, the calculated and the measured values versus the selected independent variables are plotted in Figures 10a to 10d. The histogram with the normal probability are presented in Figures 11a to 11d. Finally, statistical parameters from the histogram and the probability distribution of the residuals, for the four formulae, are summarized in Table 2.

As it is observed in Figure 9a and 9b, the residuals are not well distributed to the calculated transmission coefficients for formulae F1 and F2. The estimated transmission coefficients from formulae F1 and F2 are low, especially for the B dataset. The plots show a violation of assumptions 1 and 4 (linearity of one or more independent variables, and equal variance). The most important violation of the two is this of equal variance. Two patterns are observed at each plot. In the first pattern, the residuals are distributed perpendicular to the x-axis at a specific value of $K_{t(\text{calculated})}$, and in the other pattern, they are distributed parallel to the x-axis. The second trend shows that an area of validity exists, for K_t >0.2 and K_t >0.075 for F1 and F2 formulae, respectively.

A possible violation of the first assumption (linearity) for one or more independent variables in shown by the clouds in Figures 9c and 9d (formulae F3 and F4), and a possible violation of assumptions 2 and 4 (independence and equal variance) is shown in Figure 9c (formula F3) as a negative linear trend is observed in both datasets.

Regarding the F1 formula, the scatter plot of the residuals versus the ratio H_i/D_{n50} (Figure 10a) shows that for H_i/D_{n50} <2 the assumptions of residual analysis 1, 2 and 4 are satisfied for the selected parameter. Maybe a dummy parameter is needed to improve this formula for H_i/D_{n50} >2, because two trendlines are observed, or a constant value should be added. For H_i/D_{n50} >2, a linear modification is needed to improve it.

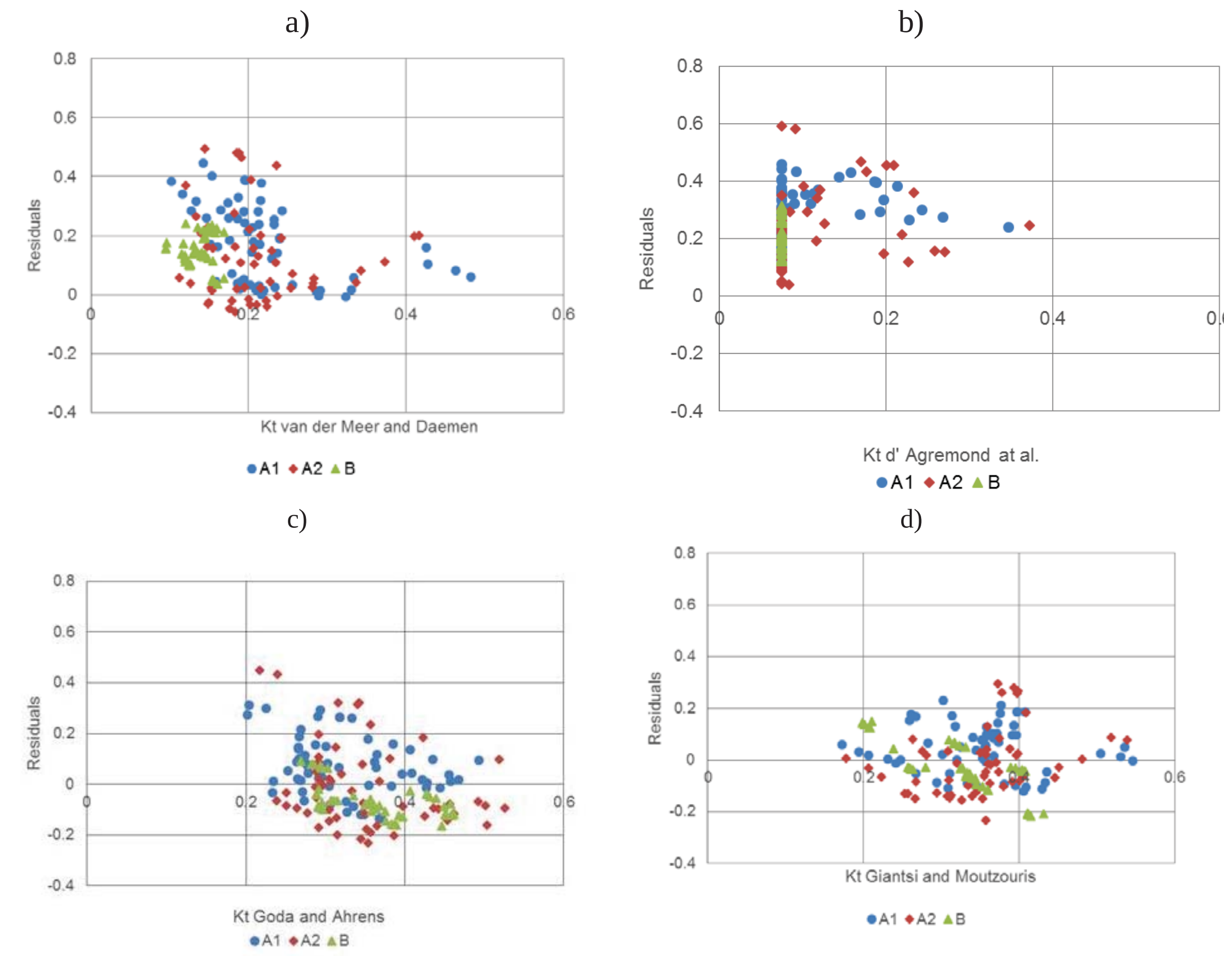

Fig. 9 Residuals versus K_t calculated by formulae: a) van der Meer and Daemen; b) d' Agremond et al.; c) Goda and Ahrens; and d) Giantsi and Moutzouris

The residuals of F2 formula versus the relative freeboard (Figure 9b) show a significant dispersion, high values, the highest between the four formulae, and a linear modification is needed to improve the formula. The assumptions of linearity, independence and equal variance are violated for R_c/H_i<2.

A linear or curvilinear modification can improve the formula F3 for R_c/H_i <4 (Figure 10c). Possible violation of assumptions 1, 2 and 4 are observed in the plot of residuals versus the independent variable (relative freeboard). An underestimation is observed for R_c/H_i >2.

A linear or curvilinear modification can improve the formula F4 for R_c/H_i <1.5 (Figure 9d). Except of assumption 1 (linearity), assumptions 2 (independence) and 4 (equal variance) are satisfied for the relative freeboard, the main independent variable of the above-mentioned formula.

The F1 and F2 formulae show significant differences between the measured and calculated values of transmission coefficients plotted versus the independent variables. The F4 formula shows the best correlation.

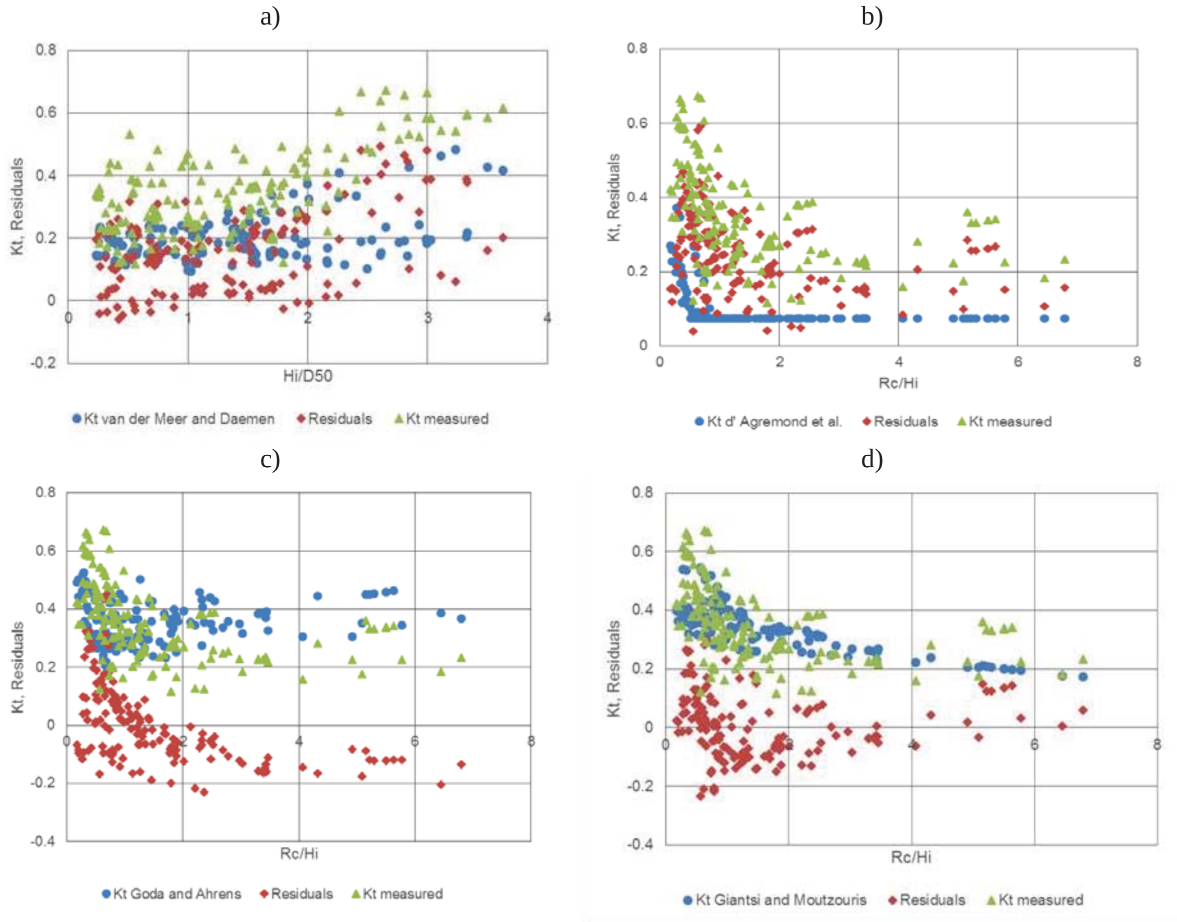

Fig. 10 K_t calculated, measured and residuals versus the independent variable for a) van der Meer and Daemen, b) d' Agremond et al., c) Goda and Ahrens and d) Giantsi and Moutzouris formulae

The 3rd assumption (normality) states that the errors are normally distributed at each value of *x* (residuals). From the histograms (Figures 11 a, b, c, d) and the statistical parameters in Table 2, it is obvious that only the residuals obtained by the formula F4 are close to normal distribution, with the mean value, peak value, and cumulative probability 50% almost equal to zero. F3 formula also has a mean value of the residuals almost equal to zero, while the peak value is negative. F1 and F2 formulae have positive mean values showing an underestimation of the residuals.

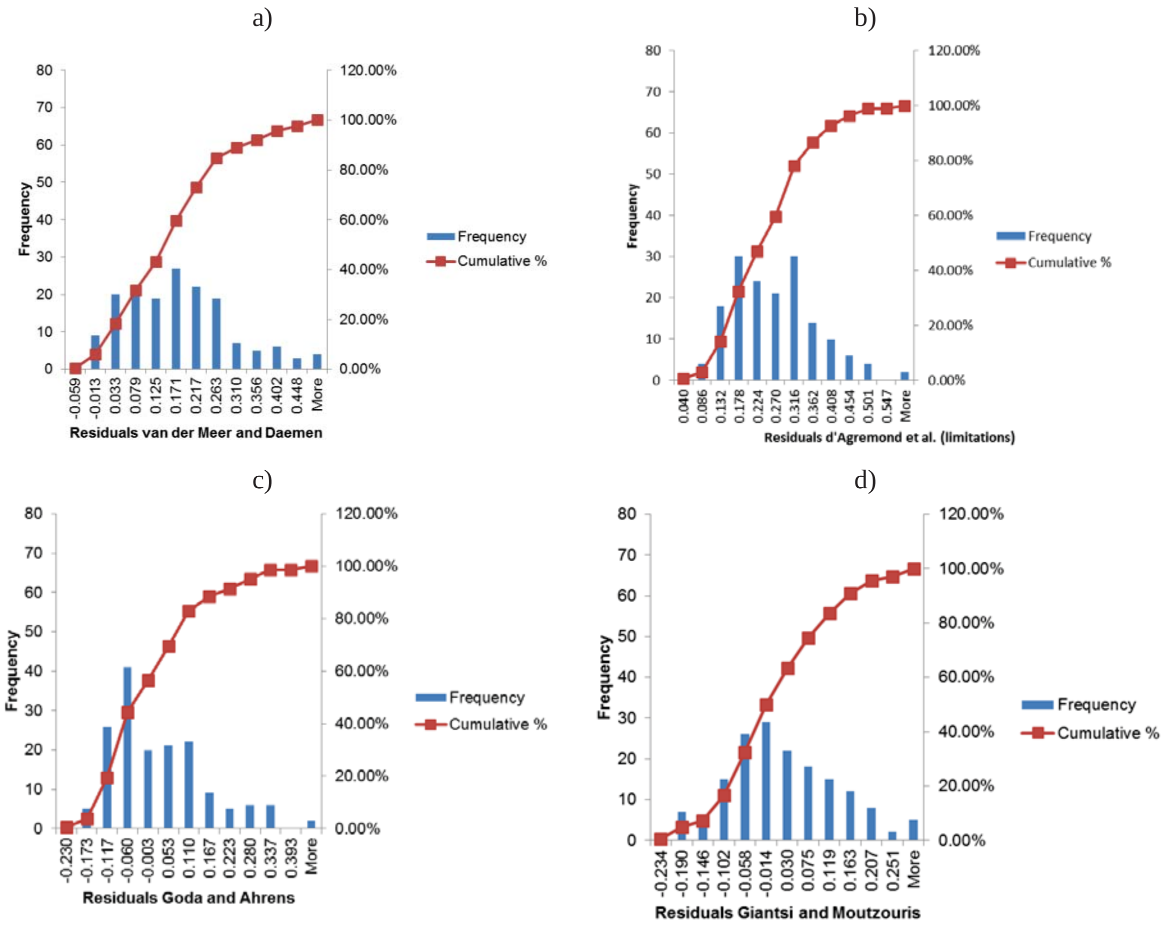

Fig.11 Histogram and normal probability plots of residuals for formulae: a) van der Meer and Daemen; b) d' Agremond et al.; c) Goda and Ahrens; and d) Giantsi and Moutzouris

Table 2. Statistical parameters from the probability distribution of the residuals

Residuals	van der Meer and Damen	d' Agremond et al.	Goda and Ahrens	Giantsi and Moutzouris
Mean	0.152	0.245	-1.78E-04	1.37E-05
Peak	0.171	0.132/0.316	-0.117	-0.014
Cumulative 50%	0.144	0.235	-0.034	-0.014
Standard deviation	0.122	0.105	0.136	0.111
Max. value meas.	0.494	0.593	0.450	0.295
Min. value meas.	-0.059	0.040	-0.230	-0.234
Max. value estim.	0.396	0.455	0.272	0.222
Min. value estim.	-0.092	0.035	-0.272	-0.222

6 Conclusions

Two datasets of wave measurements from two different 3D physical models of Low Crested Structures were analyzed and the transmission coefficients were measured behind of them. Then, using four different formulae, the measured and the calculated values of transmission coefficient were evaluated by residual analysis, which is an easy method to evaluate the regression models visually by satisfying four regression assumptions.

The four formulae evaluated were: 1) the van der Meer and Daemen; 2) the d' Agremond et al.; 3) the Goda and Ahrens; and 4) the modified d' Agremond et al. by Giantsi and Moutzouris.

According to the residual analysis, the formula that satisfied most assumptions of the regression analysis was the modified d' Agremond et al. by Giantsi and Moutzouris using the residual analysis. The second formula was the one by Goda and Ahrens, third in rank was the van der Meer and Daemen, and last was initial formula proposed by d' Agremond et al.

The formula proposed by Giantsi and Moutzouris can be used for the preliminary design of Low Crested Structures, providing an acceptable estimation for the wave conditions in the sheltered area of the structures.

The transmission coefficient seems to be better estimated when the relative freeboard is greater than 2. Improvements can be achieved at the existing formulae using results from residual analysis.

Testing in 3D conditions, due to the existence of many parameters, leads to more uncertainties than testing in 2D conditions. Considering the circumstances, the proposed formula shows a very good agreement with the experimental data.

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