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A review and assessment of longshore sediment transport equations for coarse-grained beaches

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Abstract

Previous assessments of analytical longshore sediment transport formulae have been heavily biased towards sand-sized sediment. All have noted the shortage of high quality field data from coarse-grained beaches against which to test predictions of longshore transport rates. In this paper, 12 existing formulae were identified as being potentially applicable for coarse-grained sediments and predictions from these formulae are compared using a measured annual transport rate from a shingle beach and a concurrent hindcast wave climate. Two new empirical equations are also derived, one from a numerical model calibrated against the same data set, the other derived from field experiments on coarse grained beaches. Energetics-based equations are found to give reasonable predictions of the shingle transport, despite being derived for sand beaches. In contrast, those dimensional analysis type equations which had been validated using laboratory data, grossly over-predicted the measured transport rates. The most accurate predictions were from formulae previously validated at sites similar to that used for this comparison and therefore require further testing against field data from dissimilar sites. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The current philosophy of the coastal engineering profession is to use soft engineering techniques where appropriate and, in the UK, increasing use is made of coarse-

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grained [1] sediment to replenish eroding beaches, often in conjunction with structures such as rock or wooden groynes or offshore breakwaters. For example, shingle replenishment schemes at Highcliffe and Hurst Spit in Dorset, Hayling Island, Hampshire, Elmer, West Sussex and Seaford, East Sussex have been implemented within the last decade, with others about to be initiated, e.g. at Hythe in Kent. This is because shingle beaches are known to be an efficient form of natural sea defence (Powell, 1990; Diserens and Coates, 1993). In the UK, one third of the coastline is protected by shingle beaches (Fuller and Randall, 1988), but they are widespread also in Canada, Japan, Argentina, New Zealand and Ireland and they occur along considerable stretches of the Pacific coast of the USA.

Most sediment transport research and the development of both analytical and numerical models of transport have concentrated on sand sized sediment. However, there is a growing realisation that the particular properties of a shingle beach have a number of consequences for the processes of sediment transport, which mean that the sediment transport characteristics of a shingle beach are very different from those of a sand beach (e.g. Quick and Dyksterhuis, 1994). Most notably, shingle can support a steep gradient (typically $\sim 1/8$) with a tendency to form a near vertical berm towards the high water mark. In the UK, it is usual to find a flat low tide sand terrace, although along micro-tidal coasts the shingle can extend well below the low water mark. The steep beach gradient means that waves can progress much closer inshore before breaking. For example, a 4 m breaking wave was measured on the beachface on a fetch-limited beach at Shoreham, UK during a southwesterly Storm Force 10 (Pope, pers. comm.). Energy dissipation through breaking is concentrated over a much narrower region than on a sand beach; a single line of plunging breakers is usual and even during storms a wide surf zone is rare. An important consequence of this narrow, energetic, unsaturated breaker zone is that the swash zone can be of similar width to the surf zone and, accordingly, the sediment transport within the swash zone is of more significance than on sand beaches.

A second effect of the steep beach gradient is that refraction processes are also confined to a narrower zone than is experienced on a sand beach, with the result that refraction is often observed to be incomplete and the waves arrive at the beachface with some considerable angle. A third distinguishing characteristic is the high hydraulic conductivity (permeability) of shingle which, compared to a sand beach, increases the potential for infiltration during the swash and is probably responsible for the berm found at the maximum swash runup. However, the specific retention of shingle is correspondingly low so that the potential for exfiltration during backwash is higher than on a sand beach. Both the hydraulic conductivity and specific retention of the sediment ultimately determine the vertical transmission of high frequency, wave-induced, sub-surface pore water pressure fluctuations which have recently been suggested as a mechanism for enhanced sand sediment transport under swash (Baird et al., 1996, 1997). The role of

[1] The terminology for coarse-grained beaches is fraught with ambiguities (Carter and Orford, 1993). In this paper, the term ''shingle'' is considered synonymous with ''coarse-grained'', given its widespread use in the coastal engineering literature; both terms are taken to refer to sediments with $D_{50} > 2$ mm, regardless of origin.

infiltration/exfiltration processes are, therefore, of potentially more importance than on sand beaches (due to the high potential for infiltration and since the swash zone comprises much of the surf zone as discussed above). However, incorporation of models of groundwater flow on shingle beaches is complicated by the likelihood of non-Darcian flow through the shingle and by the fact that many shingle beaches contain significant quantities of sand at some depth below the surface. Once the proportion of sand is about 30% by weight, the hydraulic conductivity of the bulk sediment reduces to that of a sand beach (Mason et al., 1997), as does the response of the beach profile (Quick and Dyksterhuis, 1994; Holmes et al., 1996).

Mean grain size of sediments on a typical shingle beach can range from 10 to 40 mm (Simm et al., 1996) yet despite the large grain size, the sediment is highly mobile; it is not unusual to experience 0.4 m of profile change during a single tide in only moderate wave conditions. The combination of harsh environmental conditions and sediment mobility is responsible for a marked lack of high frequency measurements of waves and currents over a shingle beach, since few instruments can withstand impact by shingle. No equipment exists as yet to monitor wave-by-wave transport of shingle[2] with the result that sediment transport rates can be measured, at best, at a resolution of an entire tidal cycle. A further handicap to the study of shingle transport processes is that the majority of transport takes place as bedload where, even for sand, measurement instrumentation lags that for suspended sediment. The results are that neither the forcing parameters nor the beach response are reliably documented for coarse-grained beaches and that little is known about the relative importance of the processes by which coarse-grained sediment is transported.

Clearly, understanding about the processes of sediment transport on coarse-grained beaches lags that for sand beaches. In the meantime, beach management considerations demand some quantification of shoreline evolution. The vast majority of coastal engineering projects still rely on one-line models since, as yet, 3D morphodynamic models are restricted to short- and medium-term predictions and none have been validated against coarse-grained field data. Most one-line models make use of empirical total longshore transport equations. Hence, the formulation of a reliable estimate of longshore sediment transport rate remains of considerable practical importance in coastal engineering applications such as feasibility studies of port extensions, derivation of sediment budgets for coastal areas and the appraisal of long term beach stability.

In this paper, a review of longshore sediment transport equations is presented, concentrating on formulae which can be applied to coarse-grained beaches, since no review of this nature has been published to date. The paper begins by reviewing existing field and laboratory experiments for coarse grained beaches, several of which form the basis of calibration and/or validation of the majority of existing longshore sediment transport equations. Subsequently, the transport equations themselves are discussed, including two newly derived equations, and finally their individual suitability for coarse-grained beaches are assessed.

[2] In situ passive measurements of shingle in transport have been made in deeper water (Thorne, 1986; Voulgaris et al., 1995) but at present these acoustic devices are unsuitable for the surf/swash zone.

2. Sediment transport rates from experiments on coarse-grained beaches

2.1. Field experiments

Schoonees and Theron (1993) extensive review of field data suitable for evaluating longshore sediment transport models identified a particular lack of information for grain sizes coarser than 0.6 mm, beach slopes steeper than 0.06 (1/17) and significant wave heights exceeding 1.8 m. Of the 42 experiments listed, only two were for coarse-grained beaches (Schoonees and Theron, 1993, Tables 1 and 2). The principal reasons for the lack of data are those referred to in the introduction — the lack of robust instrumentation to measure the hydrodynamics and inability to measure sediment whilst it is in transport. Consequently, sediment transport data from shingle beaches has traditionally relied on one of three methods; tracers, traps or profile/shoreline change.

The general problems with determining a reliable sediment transport rate from tracers are well known (see White, 1988) but there are several additional difficulties associated with shingle sized sediment, in particular the question of representativeness. The physical properties can be well mimicked by the new generation of aluminium (Wright et al., 1978) and electronic tracers (Workman et al., 1994; Van Wellen et al., 1997). These also go some way to addressing the problem of burial since they can be tracked at depths of about 0.4 and 0.8 m respectively (unlike painted pebbles, which can be recovered only from very near the surface). All types of tracers, however, are subject to concern about recovery rates. Whilst the aluminium and electronic tracers can be accounted for individually, even a 100% recovery rate over one tide represents only about 150 clasts at most, due to the labour-intensive recovery procedures. A further problem is the estimation of depth of the moving layer of sediment, which is much deeper than on a sand beach (e.g. Nicholls and Wright, 1991) and is spatially and temporally more variable (e.g. Stapleton et al., 1999). Similarly, the width of beach over which transport is occurring is narrower than on dissipative sand beaches but is also variable through the tide, particularly on a macro-tidal beach. There is some indication that transport rates derived from tracers consistently over-estimate the actual rate (Van Wellen et al., 1998). This may be due to the temporal and spatial variability of both the depth of disturbance and width of active beach which are important parameters in calculations to derive transport rates from tracers but their variability through the tide has not yet been taken into account.

Traps suffer a fundamental drawback on shingle beaches, in that much of the longshore transport occurs in the swash zone, rather than as quasi-unidirectional transport; thus, the net longshore component of the sediment in transport is much smaller in quantity than the volume of sediment being moved on- and offshore. Over the course of a tide, cross-shore transport of sediment lowers the longshore trap efficiency. In addition, traps are difficult to deploy on shingle beaches. Of necessity, they tend to be substantial structures and interfere with the current field in shallow water, which ultimately governs the sediment transport. Bray et al. (1996) compared sediment transport volumes measured by tracers and traps. The traps were difficult to secure in loose shingle and therefore few measurements could be made in the areas where sediment mobility was highest but, overall, they found that the trap volumes were several orders of magnitude lower than measured by tracers. They attributed this to poor trap efficiency on shingle beaches due to scouring, build up against the sides and loss of material on the ebb tide. They concluded that traps are unreliable in other than near-calm conditions. Another uncertainty with trap data, particularly for macro-tidal sites, is the conversion of trapped volumes to surf zone integrated transport rates, since traps generally sample at one location only. Therefore, an assumption of temporal stability is necessary.

In summary, tracers, rather than traps, are the preferred technique for obtaining short-term sediment transport rates on coarse-grained beaches, because they are non-invasive and can be used in high energy conditions, but the input parameters for calculation of transport rates remain uncertain and require further evaluation.

For engineering purposes, the volumetric changes from survey data are of most use for longer-term patterns of beach response, with all the strengths and weaknesses of smoothing out short-term fluctuations. Reliable estimates of transport rates require the presence of a shore-normal barrier, e.g. a harbour wall or an extended groyne, which will trap all the longshore sediment transport. Ideally, transport should also be constrained some distance updrift of the barrier, so that all the sediment passing through the area of interest can be accounted for. Nicholls and Wright (1991) observed that loss of shingle offshore is normally negligible. Therefore, at suitable sites, the impoundment technique is particularly well suited for obtaining long-term longshore transport rates on shingle beaches. However, the precision of some of the measurement techniques can limit the usefulness of some of the data for less than half yearly transport trends, unless the confidence limits on the measurements exceed those of the survey errors (Van Wellen et al., 1998).

Only two field experiments on coarse-grained beaches satisfied Schoonees and Theron (1993) criteria, which essentially required measurements of wave conditions (height, period, angle), transport rate, beach gradient and grain size; the two experiments were by Nicholls and Wright (1991) and Chadwick (1989) and are discussed below. Most other published sources of shingle beach data failed on the lack of concurrent wave measurements (even visual records were acceptable, though incurred a low weighting). For example, Chesil Beach has a long history of transport experiments, mainly in the attempt to explain the remarkable lateral sorting along its 28 km length (a summary is given in Bird, 1996) but few with detailed wave measurements. Other, detailed tracer experiments elsewhere suffer the same handicap, e.g. Jolliffe (1964).

In contrast, Hattori and Suzuki (1978) experiment recorded offshore wave data, including two typhoons, but analysis of the results concentrated on the (surface) velocity of the tracers and did not include calculations of transport rates. The experiment deployed 7000 non-indigenous (dacite) tracers on a micro-tidal, sand and gravel beach in Suruga Bay, Japan. Recovery rates were low, only 2–3% for each survey (but which still involved up to 350 tracers). The results are probably representative of the upper portion of the beach, since tracers are more likely to become buried below the mean water line. The mean longshore velocity of the tracers ranged from 2 m/day in typical wave conditions, to 50–60 m/day in storms, with up to 1 km/day during the typhoon. Hattori and Suzuki (op cit.) found a reasonably linear relationship between longshore tracer velocity and the longshore component of the wave energy flux and concluded that the threshold wave height for tracer movement was about 0.2 m.

Nicholls and Webber (1987) reported two long-term experiments on Hurst Castle Spit, in May 1981 and March 1982 using 99 and 759 aluminium tracers respectively. The equivalent D_{50} grain size was between 34 and 55 mm for the first experiment and 27 and 49 mm for the second, which represented the coarser part of the indigenous shingle population. Cumulative recovery rate of the tracers reached 62% after 80 and 48 days from the start of each experiment. Two further experiments near Hengistbury Head were described by Nicholls and Wright (1991) with 75 and 460 aluminium tracers of approximately 40 mm D_{50}. Both series of experiments included some wave observations and therefore could be used to calibrate sediment transport equations for shingle beaches. Their results are discussed further below. Chadwick (1989) experiments are described below with other experiments from Shoreham Beach.

Since Schoonees and Theron's review in 1993, several other field data experiments have been conducted although again, in some cases, insufficient information has been included to make them useful for further evaluation. For example, Bray (1997) deployed aluminium tracers at several locations along the eastern part of Lyme Bay. Some 40 tracer searches were conducted over a year, covering a variety of wave conditions, with reported transport rates of between 2 m^3/day to 22 m^3/day in low energy conditions and a maximum of 168 m^3/day during high energy conditions. Bray stated that the results of the tracer experiments were used to calibrate sediment transport equations after the manner of Nicholls and Wright (i.e. to derive a value for K in the CERC equation, described in Section 3.1), but no details were given in Bray (1997) or any readily available publication.

Several shingle beach field experiments have taken place at Shoreham-by-Sea, West Sussex, on the south coast of the UK, including the most recent deployments, e.g. Workman et al. (1994); Bray et al. (1996); Van Wellen et al. (1997, 1998). The prevailing wave direction is from the southwest and south–southwest and the site is fully exposed to storm waves generated within the English Channel (Fig. 1). The macro-tidal beach is open towards the west and is in a natural state over an alongshore distance of 2 km. To the east, the beach is confined by a long harbour breakwater, which extends seawards approximately 200 m (Plate 1). The toe of the shingle bank extends only halfway along the breakwater and therefore it can be assumed that no longshore transport of sediment occurs past the breakwater. A more detailed description of the field site is given in Van Wellen et al. (1997). Annual aerial surveys of this site have been carried out since 1973. From these, Chadwick (1989) estimated a mean sediment volume accretion of 14,500 $m^3/annum$, based on statistical analysis of the trends of beach line movement and the changes in cross-sectional area. This figure was independently corroborated by the volume obtained from a recent sediment bypassing scheme around the harbour breakwater which suggested that sediment accumulates against the breakwater at a mean rate of 15,000–20,000 $m^3/annum$ (Wilson, 1996). The establishment of a net annual transport rate by two independent methods makes this a particularly valuable long term figure against which coarse-grained sediment transport equations can be tested; it will be referred to henceforth as the Shoreham long term transport rate.

Short-term transport rates at Shoreham were also derived from traps placed at about mid-tide level and orientated alongshore facing the expected direction of wave approach (Chadwick, 1989). Seven successful trapping experiments were conducted. Concurrent

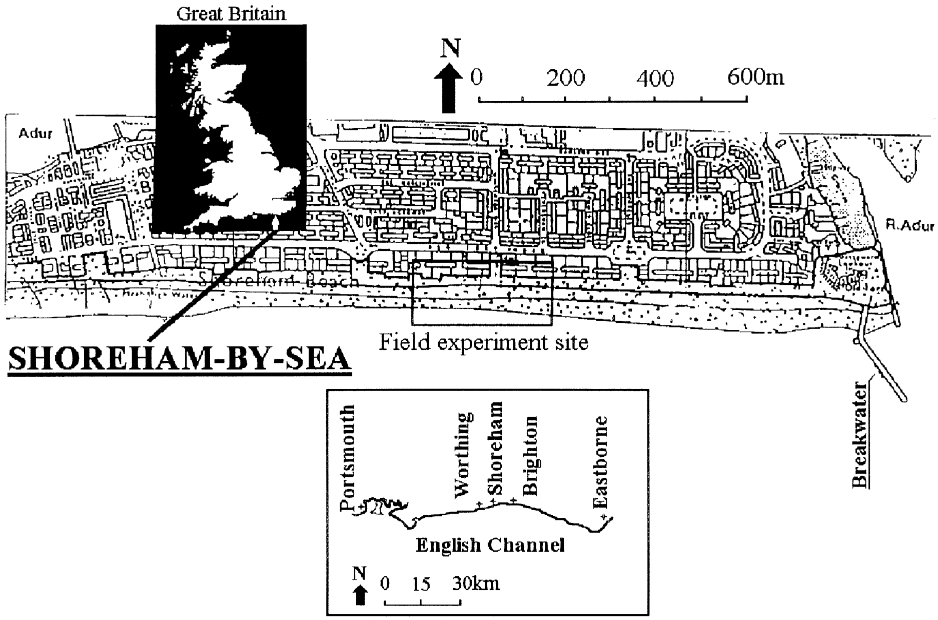

Fig. 1. Location of the Shoreham field site.

measurements of waves were made by a resistance staff and wave angle was estimated by timing the progression of wave fronts through an array of wave poles. Longshore

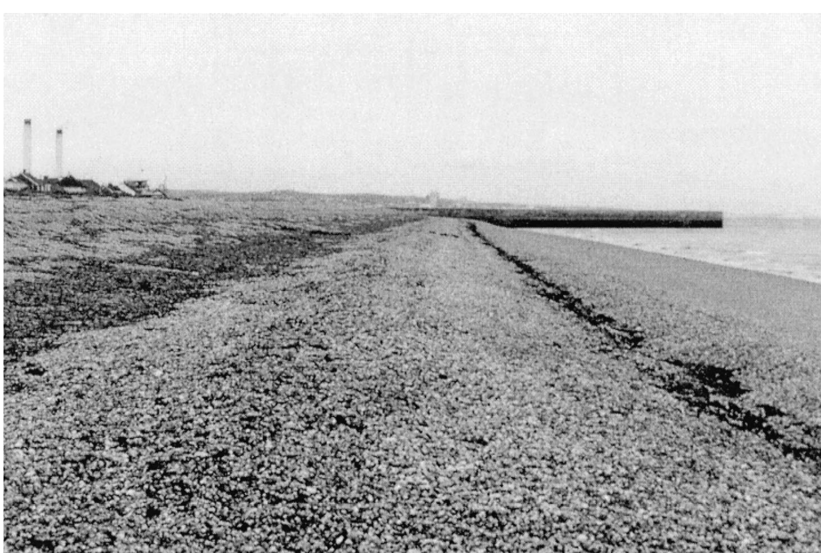

Plate 1. Shoreham beach, looking east towards the harbour wall.

transport rates varied from 4 to 32 m^3/day for waves of 0.23 and 0.48 m H_{rms}, respectively.

Traps and tracers (electronic and aluminium) were deployed concurrently at Shoreham in autumn 1995 (Bray et al., 1996). Three deployment periods covered low, intermediate and high energy conditions of 11, 3 and 10 tides, respectively. Between 54 and 102 aluminium and 30 to 60 electronic tracers were used for each experiment. The nearshore wave climate and direction were measured using an array of resistance staffs (Chadwick et al., 1995). The traps were steel cages ($1.2 \times 0.5 \times 0.5$ m) with the open section oriented in the direction of the expected longshore transport. Recovery rates were higher for the electronic tracers than for the aluminium, due mainly to their deeper detection depth. Transport rates were derived for a succession of tides, although the time scale over which each transport rate was averaged is not clear from the data given in Bray et al. (1996). Transport rates measured by the electronic pebbles and aluminium tracers were broadly similar in low and intermediate energy conditions. However, during high energy conditions, the transport rate measured by the electronic tracers was over one-third higher than for the aluminium (although recovery rates were low for both types of tracers during this experiment).

Following the 1995 trials of the electronic pebbles, the UK Ministry of Agriculture, Fisheries and Food funded two major field deployments. The experiments took place in the autumn of 1996 at Shoreham and at an adjacent groyned and replenished beach at Lancing in autumn 1997. Some preliminary results of the transport rates and survey data have been published (Van Wellen et al., 1997, 1998), but analysis of the extensive database is ongoing.

2.2. Laboratory experiments

Laboratory experiments for coarse-grained sediment are hampered by the impossibility of representing the hydraulic characteristics of the individual coarse-grained particles and bulk properties of the sediment. Whilst Kamphuis' (1991a) experiments established that the particle size and associated beach slope can be adequately scaled, the scale effects of hydraulic conductivity have not yet been quantified, which introduces an unknown scale effect when predicting prototype transport rates from laboratory models for coarse-grained sediment. The result is that alongshore transport observed in a laboratory model using scaled sediment is often very different from that which would occur in nature (Brampton and Motyka, 1987). A further problem with laboratory experiments is that a large basin is needed to undertake research into longshore transport under oblique waves and, in general, experiments have tended to concentrate on the profile response of coarse-grained beaches under wave attack (e.g. Powell, 1988, 1990). Despite the scaling problems, data from laboratory experiments have been used to calibrate and validate a number of longshore sediment transport models, as discussed later.

In Van Hijum and Pilarczyk's (1982) laboratory investigation of gravel sized material, longshore transport was measured from beach profile surveys using the principle of continuity of sediment in the longshore direction. Brampton and Motyka (1984) pointed out that the random wave tests were limited in number, using small wave heights and apparently with only one sediment size. Van Hijum and Pilarczyk's analytical equation subsequently became known as the Delft equation and is given later (Eq. (30)).

In the wave basin test programme reported by Coates (1994), crushed and graded anthracite was used to represent shingle. Longshore and cross-shore transport was measured using traps and from changes in beach profiles. The results indicated that a larger proportion of suspended transport occurred in the model tests than might be anticipated on a prototype beach. No quantitative transport rates were included in the report (the primary aim of the study was to examine the response of shingle beaches in the presence of groynes and detached breakwaters) but the results were later used to test the force-balance equation of Damgaard and Soulsby (see below, Eq. (22)).

3. Categorisation of longshore sediment transport equations

The equations reviewed in this paper have been divided into three groups, two based on the forcing mechanism behind the sediment transport and one based on the method of derivation of the analytical equations. The first category is the application of energetics methods, which comprises two sub-categories, the energy flux approach and the stream power approach. The energy flux method was developed specifically for coastal sediment transport, while the stream power approach is of more general application to any sediment transport situation. The second category is that of force-balance which, classically, is the alternative method to the stream power approach in predicting sediment transport. The third grouping comprises equations which were derived by various forms of multi-regression analysis between laboratory or field experimental results, and groups of non-dimensional parameters which are thought to be of importance for sediment transport. Several of these equations appear to be similar in form to the energetics-based equations, but they were derived experimentally, not from a theoretical energetics approach. All equations are presented in standard SI units unless otherwise stated.

3.1. Energetics methods

3.1.1. Energy flux approach

This approach is based on the principal that the longshore immersed weight sediment transport rate, I_{ls}, is proportional to longshore wave power per unit length of beach, P_{ls}. The most widely used formula in this category is commonly known as the CERC equation (US Army Corps of Engineers, 1984). The equation was derived from sand beaches and has been developed over a number of years, latterly by Inman and Bagnold (1963), Komar (1969) and Komar and Inman (1970). The formula is intended to include both bedload and suspended load and is usually given in the form of:

$$I_{ls} = KP_{ls} \qquad (1)$$

where

$$P_{ls} = \left(EC_g\right)_b \sin\theta_b \cos\theta_b \tag{2}$$

and where K is a dimensionless empirically derived coefficient. It should be noted that in this paper P_{ls} has been calculated using H_{rms} unless otherwise stated. The volumetric transport rate, Q_{ls}, is related to I_{ls} by:

$$Q_{ls} = \frac{I_{ls}}{\Gamma} \tag{3}$$

where

$$\Gamma = \frac{(\rho_s - \rho)\,g}{1+e} \tag{4}$$

There is no direct inclusion of the influence of grain size in Eq. (1), other than via the coefficient K, which has been found to be quite variable even for sand beaches (Dean, 1987; Komar, 1988). Komar (1988) noted that the limited ability to correlate the K coefficient to environmental parameters (such as grain size, beach slope and wave breaker heights) indicated a lack in data quality. It should be noted that for random waves, the choice of wave height used in the CERC equation (H_s or H_{rms}) must be correlated with the K value. Much confusion can arise, as some authors have used H_s and others H_{rms} without explicitly stating which one. For Rayleigh distributed waves, the K value using H_{rms} is twice that using H_s. A suggested value for K using H_{rms} is 0.77 for sand sized sediments (US Army Corps of Engineers, 1984), although Greer and Madsen (1978) pointed out that a number of the data points used in the regression analysis, notably those of Watts (1953) and Caldwell (1956), are of questionable quality.

More recently, Schoonees and Theron (1993, 1994) fitted an energy flux expression (using H_s) to the 46 data points which best satisfied their selection criteria. The best fit relationship for $D_{50} < 1$ mm was:

$$I_{ls} = 0.41\,P_{ls}\,(R^2 = 0.77) \tag{5}$$

This is equivalent to a $K = 0.82$, if H_{rms} is used.

The full 206 data points available included a wider range of wave energy flux values, but the coefficient K was half that obtained using the higher quality data set, despite a similar R^2 value. For $D_{50} > 1$ mm:

$$I_{ls} = 0.01\,P_{ls}\,(R^2 = 0.011) \tag{6}$$

The correlation coefficient in Eq. (6) is extremely poor. However, only 13 of the 34 data points were from coarse-grained sediments; six from Nicholls and Wright (1991) using sediment with $D_{50} \sim 50$ mm and seven from Chadwick's (1989) trap data, where the D_{50} grain size was 20 mm. The remaining 21 data points were for coarse sand. The results reported by Nicholls and Wright (1991) and Chadwick (1989) are combined with those derived recently using electronic and aluminium tracers (Bray et al., 1996) in Fig. 2, to give 27 data points, from which:

$$I_{ls} = 0.22\,P_{ls} - 36\,(R^2 = 0.62) \tag{7}$$

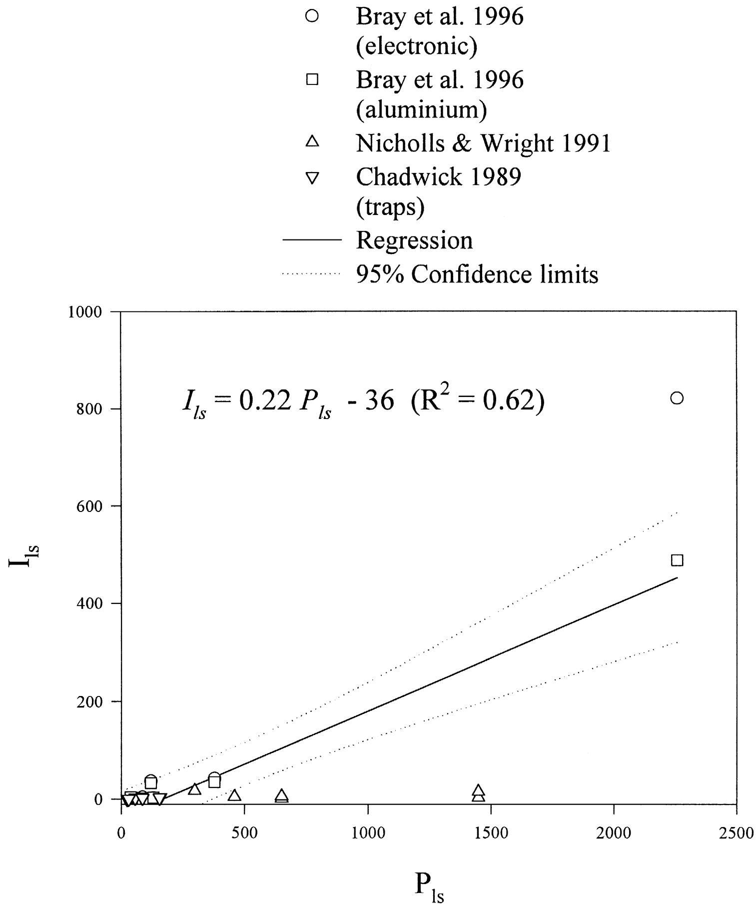

Fig. 2. K derived from coarse-grained tracer and trap data.

The two extreme data points were obtained concurrently during high energy conditions at Shoreham (Bray et al., 1996); the higher transport rate was predicted using electronic pebbles, the lower from aluminium tracers. These results confirm the well-recognised reduction in K for coarse-grained sediment and indicate that a certain minimum value of P_{ls} must be exceeded before transport takes place. However, this value of K is around 30% of the value of K from a sand beach, whilst other values of K from field experiments of coarse-grained beaches have been much lower, e.g. Nicholls and Wright (1991) found K to be between 1% and 15% of that for sand (Nicholls and Wright, 1991) whilst Chadwick's (1989) trap data suggested a K value 7% of that for sand. This raises some doubts about the traditional method of deriving transport rates on shingle beaches. On-going research from a recent field project at Shoreham (see below) suggests that the depth of disturbance, as measured by tracer columns (see Nicholls, 1989) is not a realistic representation of the depth of sediment actually being transported (Stapleton et al., 1999). If, in fact, the depth of sediment layer in transport is much less than the depth to which sediment is disturbed at some point during the tide, this needs to be taken into account when deriving transport rates, to avoid over-estimation. In addition, an over-estimate of the sediment transport rate can result if the tracers are larger than the natural beach material, since there is some evidence that larger pebbles travel faster (Jolliffe, 1964). On the other hand, low recovery rates, even for aluminium tracers, could lead to gross under-estimation of transport rates, by not accounting for scattered or deeply buried tracers. If the two high energy data points on Fig. 2 were omitted, the K value would become 0.034 (with an extremely low correlation coefficient), which is only 4% that of sand. It may be, then, that the lack of transport data from high energy conditions has led to an under-estimation of the efficiency of shingle transport.

An alternative explanation is the possibility that, after a certain threshold energy level is exceeded, shingle transport becomes more efficient. As yet, no reliable relationship has been established between K and D_{50} (Schoonees and Theron, 1994). Swart (1976) suggested the introduction of a variable K as a function of the median grain size, giving:

$$Q_{ls} = 0.116\log_{10}\left(\frac{0.0146}{D_{50}}\right)P_{ls} \tag{8}$$

However, using a more extensive data set, Schoonees and Theron (1994) found that Eq. (8) was unable to give a reliable K even within the range 0.1 mm $< D_{50} <$ 1 mm, despite its dependence on the grain diameter.

A further modification of the CERC equation was the introduction of a threshold term by Brampton and Motyka (1984), who also included a dimensionless particle size:

$$Q_{ls} = \frac{KP_{ls}}{\Gamma}\left(\frac{L}{D_{90}}\right)^{\varepsilon_1}\left[1 - \frac{8.1D_{90}}{H}\right]^{\varepsilon_2} \tag{9}$$

This transport formula contains three parameters (including the new parameters, ε_1 and ε_2) which may need to be evaluated for each new site. Their more pragmatic approach is to neglect waves with a significant wave height at breaking of 0.5 m or less, to consider ε_1 and ε_2 as zero and use a greatly reduced value of K (Brampton and Motyka, 1987). The only effective difference then, compared to the original CERC equation, is the much lower value of K. This is borne out by the values of K derived from field experiments on shingle, as discussed above.

Chadwick (1989) applied Eq. (9) in a slightly different manner by omitting the dimensionless particle size and defining ε_2 as 1, resulting in:

$$Q_{ls} = \frac{KP_{ls}}{\Gamma}\left[1 - \frac{8.1D_{90}}{H}\right] \tag{10}$$

When applying Eq. (10) to the Shoreham long-term field data, the resulting K value of 0.07 (using H_{rms} for wave height) was 9% of the K value for sand, whereas using the standard CERC equation, i.e. without the threshold and particle size terms in Eq. (9), the K value obtained was 7% that of sand.

Another way of implementing a threshold of motion term with a CERC type formula is to have a wave power below which no significant sediment transport takes place, P_{ls0} (Chadwick, 1989):

$$Q_{ls} = \frac{K}{\Gamma}(P_{ls} - P_{ls0}) \tag{11}$$

Chadwick derived a value for P_{ls0} of 13.9 W/m using the Shoreham trap data with $K = 0.0366$, $\Gamma = 10{,}807$ kg/(m^2 s^2) and P_{ls} calculated using H_{rms}. However, the value of P_{ls0} will depend on other parameters such as the grain size. As yet, there is no deterministic relationship between P_{ls0} and other parameters such as D_{90} and, therefore, Eq. (11) remains site specific.

3.1.2. Stream power approach

A more physically realistic approach to energetics based equations was developed by Bagnold and extended later by Bailard. For stream flow, Bagnold (1963, 1966) introduced the concept of stream power, arguing that a proportion of stream power is consumed in transporting sediment as bed and suspended load. For oscillatory flows, Bagnold (1963) argued that the oscillatory wave motion acts to move the sediment back and forth in amounts proportional to the local rate of energy dissipation. Although no net transport results from this motion, a steady current of arbitrary strength, when superimposed on the wave induced oscillatory motion, is capable of transporting the sediment particles in the direction of the current.

Bailard (1981) generalised Bagnold's energetics based total load transport equation and derived an expression for the total load transport which included the local time-averaged longshore sediment transport rate. In a later paper, Bailard (1984) integrated the local time-averaged longshore transport rate and introduced the following equations which produce a K value (to be used in conjunction with H_{rms}) which can be used in the CERC formula:

$$K = \varepsilon_b K_1 + \varepsilon_s K_2 + \varepsilon_s^2 K_3 \tag{12}$$

Bailard found ε_b and ε_s to be equal to 0.13 and 0.032, respectively and that:

$$K = \varepsilon_b K_1 + \varepsilon_s K_2 + \varepsilon_s^2 K_3 \tag{13}$$

$$K_2 = 0.228 u_{mb}/w_s \tag{14}$$

$$K_3 = 0.123 \tan\alpha (u_{mb}/w_s)^2 \tag{15}$$

Substituting these terms in Eq. (12) Bailard gives the following formula:

$$K = 0.05 + 2.6 \sin^2 2\theta_b + 0.007 u_{mb}/w_s \tag{16}$$

in which the term $\varepsilon_s^2 K_3$ (representing suspended load) and the contribution to the K_1 term from beach slope were omitted, due to their negligible influence. Bailard concluded that this modification of the K coefficient extended the range of application of the CERC equation, which can also be applied to a range of sediment sizes (with grain size represented through its fall velocity, w_s).

Bailard's expression, as given here by Eq. (16), takes both bedload and suspended load into account. Since suspended sediment transport is unlikely to occur on coarse-grained beaches, Eq. (16) can be reduced to:

$$K = 0.05 + 2.6\sin^2 2\theta_b \tag{17}$$

However, this removes the dependency of K on grain size. Additionally, Bailard explained that the effect of the threshold of motion had not been accounted for in his derivation and hence its application to coarse-grained beaches is likely to result in over-estimation of the total longshore transport rate.

Finally, Morfett (1988) argued that the wave energy dissipation rate (D_d), rather than stream power, could be used as the driving mechanism for longshore transport. His equation is given in this section, since the underlying principle is effectively energetics-based. He derived the following longshore transport equation:

$$Q_{ls} = K_M \frac{\left(\rho u_+^3 - \rho u_{+\,cr}^3\right)^{3/2} (\sin\theta)^{3/4}}{g(\rho_s - \rho) D_{90}^2} \tag{18}$$

where K_M is a calibration coefficient of the order of 2.84×10^{-5} (Morfett, 1989a,b). Morfett termed the expression $pu_+^3 - pu_{+\,cr}^3$ as the virtual wave power P_+. This is an expression analogous to that of excess stream power. The dissipation velocity (u_+) is given as:

$$u_+ = \left(\frac{D_d}{\rho}\right)^{1/3} \tag{19}$$

and the dissipation rate by:

$$D_d = \frac{\rho g^{3/2} H^3}{4 H^{1/2} L} \tag{20}$$

The effects of sediment grain size, wave breaking angle, wave height etc. on the rate of shingle transport were evaluated by applying multiple regression analysis to the laboratory data of van Hijum and Pilarczyk (1982) and the Shoreham trap data (Chadwick, 1989). The critical or threshold wave conditions for sediment movement were derived from regression analysis performed on the van Hijum and Pilarczyk (1982) data, which yielded the following expression:

$$H_{cr} = 2D_{50} + \left(0.087 d_{cr} \log(1000 D_{50})\right) \tag{21}$$

Morfett used his sediment transport formula (Eq. (18)) in a one-line shoreline evolution model for two shingle beach sites on the south coast of the UK (Brighton and Shoreham). The model predictions for Shoreham showed good agreement with measured beach evolution, but for the Brighton site they turned out to be rather poor. Subsequent developments of the formula did, however, maintain the dependency of the longshore transport on $P_+^{3/2}$ and $(\sin\theta)^{3/4}$ (Morfett, 1990, 1991).

3.2. Force-balance methods

The second group of predictive longshore transport equations is usually known as force-balance formulae, where the sediment transport is related to the bed shear stresses. This method requires an appropriate hydrodynamic model to determine the wave induced currents from the radiation stresses and is therefore more complex than the energetics or dimensional analysis based methods.

One of the earliest sediment transport formulae which considered the shear stresses rather than the energy input into the system by waves is the Kalinski–Frijlink formula (originally suggested by Frijlink, 1952). The formula consists of two parts, one representing the stirring up of sediment by the waves and the other representing the sediment in transport. This concept was subsequently adapted and extended by Bijker (1967, 1992) for use in hydrodynamic models, where local transport rates were integrated numerically across the surf zone. However, an analytical total longshore transport formula was not derived.

Damgaard and Soulsby (1996) used the force-balance method specifically for predicting total longshore bedload transport of shingle. The derivation of the formula is based on a bedload transport formula for combined waves and currents developed by Soulsby (1994), which relates the non-dimensional transport rate vector $\vec{\Phi}$ to the non-dimensional Shields parameter $\vec{\theta}$. The second key element of the formula is that the shear stress vector is split up into a mean, $\vec{\theta}_m$, and an oscillatory part, $\vec{\theta}_w$, resulting from the incoming waves. Cross-shore integration of the volumetric sediment transport rate produces the total longshore transport rate, Q_{ls}.

In order to perform this integration and produce an analytical expression, Damgaard and Soulsby made a number of simplifying assumptions. These included uniform beach conditions, shallow water waves, constant breaking index, no further refraction in the surf zone and radiation stress gradient balanced by bottom shear stress. The resulting analytical expression for Q_{ls} is a combination of current dominated transport, Q_{x1} and wave dominated transport, Q_{x2}:

$$Q_{ls} = \mathrm{sign}\{\theta_b\}\max\{|Q_{x1}|,|Q_{x2}|\} \tag{22}$$

The threshold condition is: $Q_{ls} = 0$ for $\theta_{max} \leq \theta_{cr}$, where:

$$\theta_{max} = \sqrt{(\theta_m + \theta_w \cos\varphi)^2 (\theta_w \sin\varphi)^2} \tag{23}$$

The current- and wave-dominated parts of the transport are expressed as:

$$Q_{x1} = \begin{cases} 0.21\dfrac{\sqrt{g\gamma_b \tan\alpha}\, H_b^{5/2}}{s-1}\left(\sin 2\theta_b - 5/3\theta_{cr}^*\right)\sqrt{|\sin 2\theta_b|} \\ \quad \text{for} \sin 2\theta_b > 5/3\theta_{cr}^* \\ 0 \\ \quad \text{for} \sin 2\theta_b > 5/3\theta_{cr}^* \end{cases} \tag{24}$$

and

$$Q_{x2} = \begin{cases} (0.25 + 0.051\cos 2\varphi) \dfrac{g^{3/8} D^{1/4} \gamma_b^{3/8} H_b^{19/8}}{T^{1/4}(s-1)} \sin 2\theta_b \\ \qquad \text{for} \quad f_{w,r}/f_{w,sf} > 1 \\ (0.050 + 0.010\cos 2\varphi) \dfrac{g^{2/5} \gamma_b^{3/5} H_b^{13/5}}{(\pi T)^{1/5}(s-1)^{6/5}} \sin 2\theta_b \\ \qquad \text{for} \quad f_{w,r}/f_{w,sf} \le 1 \end{cases} \tag{25}$$

where

$$\theta_{cr}^{*} = \theta_{cr} \frac{8(s-1)D}{\gamma_b H \tan\alpha} \tag{26}$$

and

$$\varphi = \frac{\pi}{2} - \theta_b \tag{27}$$

The friction factor for rough turbulent flows, $f_{w,r}$, is based on the analysis of a large data set (Soulsby, 1994) and is approximated by:

$$f_{w,r} = (g\gamma, H)^{-1/4} \sqrt{\frac{2D}{T}} \tag{28}$$

For mobile beds, where sheet flow conditions may occur, the friction coefficient derived by Wilson (1989) was used:

$$f_{w,sf} = 0.0655 \left(\frac{\gamma_b H}{g} \right)^{1/5} (\pi (s-1) T)^{-2/5} \tag{29}$$

Damgaard and Soulsby's formula (Eq. (22)) predictions were compared with the transport rates calculated from beach profile data at Seaford, E. Sussex. They were found to over-predict transport by a factor of 12. Subsequent comparison against Chadwick's (1989) trap data and the laboratory data of Coates (1994) and van Hijum and Pilarczyk (1982) suggested that the results of Eq. (22), when divided by 12, produced reliable predictions of bedload transport on coarse-grained beaches. It should be noted that those results were produced using H_s in their equations.

3.3. Dimensional analysis methods

These formulae were developed primarily from laboratory experiments and relate measured environmental parameters to volumetric transport rates. The resulting expressions bear a close resemblance to the energetics based equations, but they were derived from mathematical relationships between groups of dimensionless variables, rather than from physical principles. The earliest of these formulae aimed specifically at coarse-grained transport rates is often termed the Delft longshore transport equation for random waves (van Hijum and Pilarczyk, 1982) and was derived from the laboratory experiments of van Hijum (1976) and van Hijum and Pilarczyk (1982):

$$\frac{Q_{\mathrm{ls}}}{gD_{90}^2 T_{\mathrm{s}}} = 7.1210^{-4}\frac{H_{\mathrm{sd}}(\cos\theta)^{1/2}}{D_{90}}\left[\frac{H_{\mathrm{sd}}(\cos\theta)^{1/2}}{D_{90}} - 8.3\right]\frac{\sin\theta}{\tanh\left(\frac{2\pi d}{L}\right)} \tag{30}$$

The expression between the square brackets indicates that sediment motion is initiated when $H_{\mathrm{sd}}\sqrt{\cos\theta} > 8.3D_{90}$, although Brampton and Motyka (1984) later argued that this threshold term needed to be raised to some higher power in order to reproduce sediment movement at high levels of wave energy.

Eq. (30) introduces unwanted complications by using wave parameters measured at an offshore location and at the toe of their model beach. Chadwick (1989) recast the Delft experimental data in terms of conditions at the breaking position to give:

$$Q_{\mathrm{ls}} = 0.0013(gD_{90}^2 T_{\mathrm{s}})W(W-8.3)\sin\theta_{\mathrm{b}} \tag{31}$$

where

$$W = \frac{H_{\mathrm{sb}}\sqrt{\cos\theta}}{D_{90}} \tag{32}$$

van der Meer (1990) also recognised the difficulty of obtaining the required parameters for the original Delft equation and re-analysed the data of Van Hijum and Pilarczyk (1982) to produce his own longshore transport equation:

$$Q_{\mathrm{ls}} = 0.0012\, gD_{n50}T_{\mathrm{p}}H_{\mathrm{s}}\sqrt{\cos\theta_{\mathrm{b}}}\left(\frac{H_{\mathrm{s}}\sqrt{\cos\theta_{\mathrm{b}}}}{D_{n50}} - 11\right)\sin\theta_{\mathrm{b}} \quad [\mathrm{kg/s}] \tag{33}$$

where

$$D_{n50} = (M_{50}/\rho_{\mathrm{s}})^{1/3} \tag{34}$$

This equation is very similar to Eq. (31) differing mainly by a slight modification of the threshold term and the value of the constant.

In a later paper, van der Meer and Veldman (1992) specified that Eq. (33) should only be applied within the limits $10 < H_{\mathrm{s}}/\Delta D_{n50}$ (for rock/gravel beaches) and that Eq. (33) should be simplified to

$$Q_{\mathrm{ls}} = 0.0012\pi H_{\mathrm{s}}C_{\mathrm{op}}\sin 2\theta_{\mathrm{b}} \quad (\text{gravel/sand beaches}) \tag{33a}$$

For $H_{\mathrm{s}}/\Delta D_{n50} < 10$ (berm breakwater) and angles of 15–40°

$$Q_{\mathrm{ls}} = 0 \text{ for } H_0T_{\mathrm{op}} < 105 \tag{33b}$$

$$Q_{\mathrm{ls}} = 0.00005(H_0T_{\mathrm{op}} - 105)^2$$

Kamphuis et al.'s (1986) formula for longshore transport was developed for use on sand beaches but is included in this review since it includes the effect of both beach slope and grain size and, therefore, could be applicable for coarse-grained sediments. It was derived from an extensive series of laboratory tests and a broad set of field data. The formula is given by:

$$Q_{\mathrm{K}} = 1.28 \frac{\tan\alpha H_{\mathrm{sb}}^{7/2}}{D} \sin 2\theta_{\mathrm{b}} \quad [\mathrm{kg/s}] \tag{35}$$

where Q_{K} is the *immersed mass* transport.

The expression was refined later using a further series of hydraulic model tests (Kamphuis, 1991a):

$$Q_{\mathrm{K}} = 2.27 H_{\mathrm{sb}}^{2} T_{\mathrm{p}}^{1.5} (\tan\alpha)^{0.75} D_{50}^{-0.25} \sin^{0.6} 2\theta_{\mathrm{b}} \quad [\mathrm{kg/s}] \tag{36}$$

Eq. (36) was found to be valid for both laboratory and field sand transport rates. Kamphuis also investigated whether Eq. (36) was applicable to coarse grained beaches by comparing its predictions with the experimental results of Van Hijum and Pilarczyk (1982). He found that it over-predicted these results by a factor of 2 to 5, concluding that this was to be expected, since gravel beaches will absorb substantial wave energy by percolation and the motion of the larger grains is much closer to the critical mobility numbers. Neither of these factors was included in his dimensional analysis leading to Eq. (36). Although Eq. (36) includes both grain size and beach slope, these two parameters tend to cancel each other as beach slope reduces with grain size. Schoonees and Theron (1996) re-calibrated Eq. (36) using 123 data points from their field data sets, to give:

$$Q_{\mathrm{ls}} = 63{,}433\, x_{\mathrm{Kamphuis}} [\mathrm{m^3/annum}] \tag{37}$$

where

$$x_{\mathrm{Kamphuis}} = \frac{1}{(1-p)\rho_{\mathrm{s}}} \frac{\rho}{T_{\mathrm{p}}} L_0^{1.25} H_{\mathrm{sb}}^{2} (\tan\alpha)^{0.75} \left(\frac{1}{D_{50}}\right)^{0.25} (\sin 2\theta_{\mathrm{b}})^{0.6} \tag{38}$$

Taking into account the accuracy of predicted transport rates during storms, an alternative version was given as:

$$Q_{\mathrm{ls}} = 50{,}000\, x_{\mathrm{Kamphuis}} \quad [\mathrm{m^3/annum}] \tag{39}$$

Schoonees and Theron (1996) recommended the use of Eq. (37) where the significant wave height normally exceeds 0.3 m and where the sediment is of a finer nature (usually < 1 mm). At sites where calm conditions prevail and/or where the sediment is coarser, Eq. (39) should be used.

It is useful to note here that Eq. (38) as given by Schoonees and Theron (1996) is in fact incorrect. To convert from *immersed mass* transport to volumetric transport requires division by $(\rho_{\mathrm{s}} - \rho)$ not ρ_{s} as given in Eq. (38). In consequence, Schoones and

Theron's Eq. (37) is almost identical to Kamphuis's Eq. (36) after the latter is converted to m^3/annum. This may be shown as follows:

$$\frac{L_0^{1.25}}{T} = \frac{g^{1.25}T^{2.5}}{(2\pi)^{1.25}T} = 1.745T^{1.5} \tag{40}$$

Substituting into Eqs. (38) and (37) yields:

$$Q_{\mathrm{ls}} = 63{,}269 H_{\mathrm{sb}}^2 T_{\mathrm{p}}^{1.5} \tan\alpha^{0.75} D^{-0.25} \sin^{0.6} 2\theta_{\mathrm{b}} \quad [\mathrm{m}^3/\mathrm{annum}] \tag{41}$$

for $p = 0.32$, $\rho = 1030$ kg/m^3, $\rho_s = 2650$ kg/m^3.

Converting Eq. (36) similarly yields:

$$Q_{\mathrm{ls}} = 65{,}029 H_{\mathrm{sb}}^2 T_{\mathrm{p}}^{1.5} \tan\alpha^{0.75} D^{-0.25} \sin^{0.6} 2\theta_{\mathrm{b}} \quad [\mathrm{m}^3/\mathrm{annum}] \tag{42}$$

4. Numerical model for longshore transport of shingle

A recent numerical model for longshore shingle transport prediction is the BORESED model (Chadwick, 1991a,b). It comprises a hydrodynamic, phase-resolving model, coupled with a bedload transport formula. The hydrodynamic module uses the non-linear shallow water wave equations (based on Hibberd and Peregrine, 1979; Packwood, 1980; Ryrie, 1981, 1983). These are combined with a sediment transport module based on Bagnold's stream power concept, as extended by McDowell (1989). Instantaneous transport rates across the surf and swash zones are subsequently summed in space and time to determine the total longshore transport rate. Thus, this model specifically includes a sediment threshold term and transport in the swash zone, both of which are of importance on shingle beaches. The model required calibration of only the friction coefficient, which was determined from the Shoreham long-term transport rate.

Although the development of a numerical model is of interest for process-based research into transport of coarse-grained sediments, it is computationally too intensive for engineering purposes, given the current state of knowledge of shingle transport. Accordingly, an algebraic formula has been derived from the input parameters and numerical model results shown in Table 1.

The proposed form of the equation is:

$$Q_{\mathrm{new}} = c0\frac{(1+e)}{(\rho-\rho_{\mathrm{s}})} H_{\mathrm{sb}}^{c1} T_{\mathrm{z}}^{c2} \tan\alpha^{c3} D_{50}^{c4} \sin 2\theta_{\mathrm{b}}^{c5} \tag{43}$$

in which Q_{new} is the volumetric transport rate as predicted by the new equation in terms of [m^3/s], $c0$, $c1$, $c2$, $c3$, $c4$ and $c5$ are the coefficients that need to be determined by fitting the new equation to the data in Table 1. In essence, the procedure consisted of iteratively guessing values for the six c-coefficients and seeing how well the analytically

Table 1
Results of the numerical simulations using the BORESED model

Test	Q_{BORESED} [m^3/s]	H_{sb} [m]	T_z [s]	tan α	D_{50} [m]	θ_b [°]
Basic set		1.25	4.85	0.10	0.02	10.0
Variation of Q with H_{sb}	0.00009	0.25	4.85	0.10	0.02	10.0
	0.00098	0.75	4.85	0.10	0.02	10.0
	0.00353	1.25	4.85	0.10	0.02	10.0
	0.00859	1.75	4.85	0.10	0.02	10.0
	0.01520	2.25	4.85	0.10	0.02	10.0
	0.02470	2.75	4.85	0.10	0.02	10.0
Variation of Q with T_z	0.00218	1.25	2.91	0.10	0.02	10.0
	0.00289	1.25	3.88	0.10	0.02	10.0
	0.00353	1.25	4.85	0.10	0.02	10.0
	0.00431	1.25	5.83	0.10	0.02	10.0
	0.00600	1.25	7.12	0.10	0.02	10.0
Variation of Q with tan α	0.00182	1.25	4.85	0.04	0.02	10.0
	0.00221	1.25	4.85	0.06	0.02	10.0
	0.00278	1.25	4.85	0.08	0.02	10.0
	0.00353	1.25	4.85	0.10	0.02	10.0
	0.00450	1.25	4.85	0.12	0.02	10.0
Variation of Q with D_{50}	0.00420	1.25	4.85	0.10	0.01	10.0
	0.00353	1.25	4.85	0.10	0.02	10.0
	0.00218	1.25	4.85	0.10	0.04	10.0
	0.00140	1.25	4.85	0.10	0.06	10.0
	0.00052	1.25	4.85	0.10	0.10	10.0
	0.00016	1.25	4.85	0.10	0.14	10.0
Variation of Q with θ_b	0.00000	1.25	4.85	0.10	0.02	0.0
	0.00074	1.25	4.85	0.10	0.02	2.5
	0.00153	1.25	4.85	0.10	0.02	5.0
	0.00244	1.25	4.85	0.10	0.02	7.5
	0.00353	1.25	4.85	0.10	0.02	10.0
	0.00488	1.25	4.85	0.10	0.02	12.5
	0.00656	1.25	4.85	0.10	0.02	15.0
	0.01120	1.25	4.85	0.10	0.02	20.0

predicted transport rates matched those obtained from the numerical model. In order to do this most efficiently, two matrices were constructed:

$$\mathrm{MQ}_{\mathrm{BORESED}} = \begin{bmatrix} q_{\mathrm{BORESED}1} \\ q_{\mathrm{BORESED}2} \\ \vdots \\ q_{\mathrm{BORESED}n} \end{bmatrix} \text{and} \quad \mathrm{MQ}_{\mathrm{new}} = \begin{bmatrix} q_{\mathrm{new}\,1} \\ q_{\mathrm{new}\,2} \\ \vdots \\ q_{\mathrm{new}\,n} \end{bmatrix} \tag{44}$$

of which matrix $\mathrm{MQ}_{\mathrm{BORESED}}$ contains all the transport rates as predicted by the numerical model BORESED for the varying input conditions from Table 1 and $\mathrm{MQ}_{\mathrm{new}}$ obtains all the analytically predicted transport rates using one set of guessed c-coefficients.

A computational program was written to optimise values for the c-coefficients using the simplex method in order to attempt to minimise the Euclidean norm l given by:

$$l = \|\mathrm{MQ}_{\mathrm{BORESED}} - \mathrm{MQ}_{\mathrm{new}}\|_2 = \sqrt{\sum_{i=1}^{n} (q_{\mathrm{BORESED}\,i} - q_{\mathrm{new}\,i})^2} \tag{45}$$

Inserting the optimised values for the c-coefficients in Eq. (43) results in:

$$Q = 1.34 \frac{(1+e)}{(\rho_s - \rho)} H_{sb}^{2.49} T_z^{1.29} \tan\alpha^{0.88} D_{50}^{-0.62} \sin 2\theta_b^{1.81} \tag{46}$$

Fig. 3 shows a comparison between the results obtained from Eq. (46) and the numerical model BORESED. To evaluate the goodness of fit, the correlation coefficient,

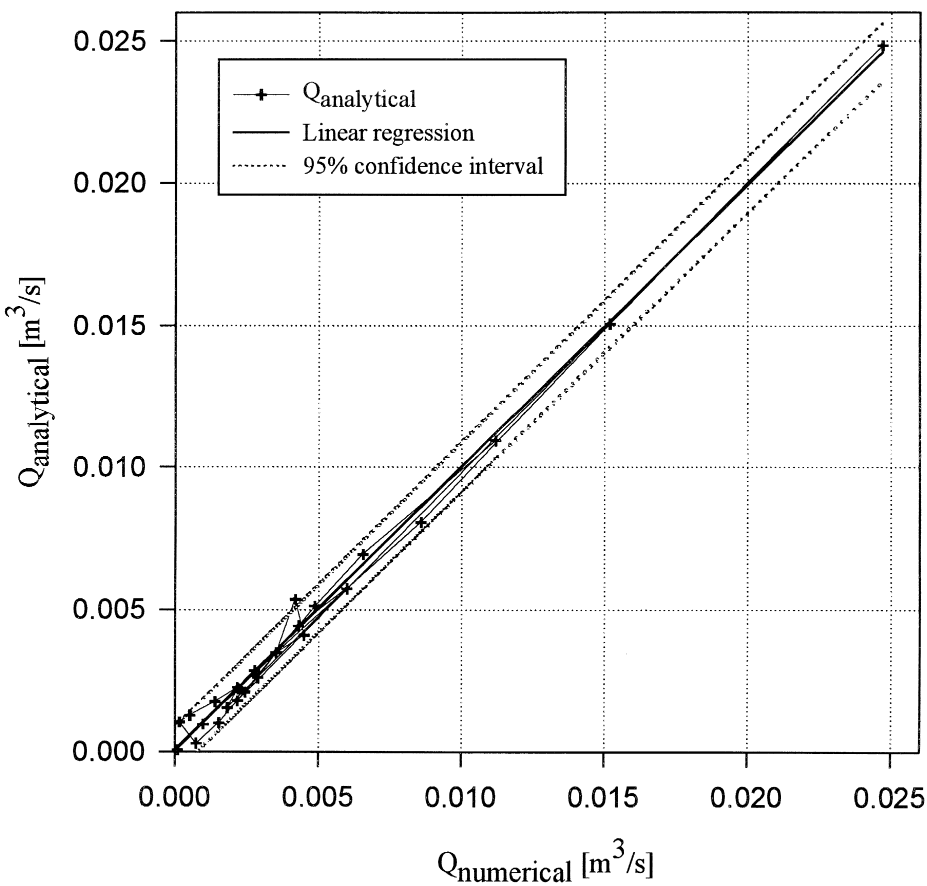

Fig. 3. Comparison between the analytically predicted transport rates using CVW and the numerical model BORESED.

R, and the relative standard error of estimate, σ, between the two results were calculated:

$$\sigma = \left[\frac{\sum_{i=1}^{n} \left(\log Q_{\mathrm{p},i} - \log Q_{\mathrm{m},i} \right)^2}{n-1} \right]^{0.5} \tag{47}$$

in which n is the sample size, Q_p is the predicted transport rate (in this case the analytically predicted transport rate) and Q_m is the measured transport rate or in this case the numerically predicted transport rate. The lower σ is, the closer the agreement between the predicted values and the measured values. The values of 99.69% for R and 0.20 for σ indicate that the sediment transport predictions made by this formula do not differ significantly from the numerical results. Eq. (46) contains an implicit threshold term, since the numerical model from which it was derived contained a threshold term.

5. Assessment of longshore sediment transport equations

In this section, an attempt is made to assess which analytical longshore transport equations are most suited to coarse-grained beaches. The 14 equations identified in the

Table 2
Longshore transport equations

Reference	Equation number	Abbreviation	Comments
Energetics			
Chadwick (1989)	10	CERCt	Brampton and Motyka (1984), with threshold term only
Chadwick (1989)	11	CERCf	Field data
Bailard (1984)	3 and 16	BAILa	Including suspended transport
Bailard (1984)	3 and 17	BAILb	Disregarding suspended transport
Morfett (1988)	18	MORF	
This paper	46	CVW	Chadwick and Van Wellen equation
This paper	7	MASON	Mason equation
Force balance			
Damgaard and Soulsby (1996)	22	DS96	
Dimensional analysis			
Chadwick (1989)	31	MDELFTa	Re − cast 30
van der Meer (1990)	33	MDELFTb	Re − cast 30
Kamphuis et al. (1986)	35	KAM86	
Kamphuis (1991a,b)	36	KAM91	
Schoonees and Theron (1996)	37	SCH96a	Modified KAM 91 for exposed sites
Schoonees and Theron (1996)	39	SCH96b	for protected sites

Table 3
Mean annual wave climate at Shoreham-By-Sea

Frequency [%]	T_z [s]	H_{sb} [m]	θ_b [°]
2.71	3.37	0.14	−10.09
1.96	3.83	0.37	−14.53
0.24	4.29	0.60	−16.93
0.02	4.75	0.86	−17.71
3.25	3.37	0.20	−9.74
2.38	3.83	0.54	−13.92
0.82	4.29	0.88	−16.14
0.24	4.75	1.21	−17.24
0.04	5.22	1.55	−18.36
0.04	5.68	1.90	−19.19
1.85	3.37	0.25	−7.23
0.92	3.83	0.67	−10.30
0.92	4.29	1.09	−11.89
0.25	4.75	1.48	−12.89
0.12	5.22	1.87	−13.66
0.03	5.68	2.24	−14.35
0.03	6.14	2.65	−15.25
2.07	3.37	0.27	−1.38
0.96	3.83	0.71	−1.96
1.07	4.29	1.15	−2.26
0.80	4.75	1.50	−3.57
0.77	5.22	1.88	−4.43
0.55	5.68	2.27	−5.32
0.13	6.14	2.68	−5.66
0.02	6.60	3.09	−6.60
3.62	3.37	0.25	4.25
1.84	3.83	0.65	6.03
2.03	4.29	1.05	6.98
2.10	4.75	1.43	6.38
2.51	5.22	1.79	6.19
1.42	5.68	2.12	5.17
0.83	6.14	2.44	4.75
0.30	6.60	2.77	3.52
0.01	7.07	3.17	3.69
3.63	3.37	0.19	6.77
5.44	3.83	0.49	9.67
6.61	4.29	0.80	11.20
3.47	4.75	1.11	11.28
1.81	5.22	1.41	11.02
1.00	5.68	1.71	10.44
0.27	6.14	1.98	10.45
0.01	6.60	2.29	9.62
5.87	3.37	0.13	6.68
4.85	3.83	0.33	9.60
1.28	4.29	0.54	11.08
0.55	4.75	0.76	11.49
0.06	5.22	0.96	11.34

review as being potentially applicable to coarse-grained beaches are summarised in Table 2, together with an abbreviation by which they will be referred subsequently. The equations are compared using the long-term Shoreham transport rate because, in common with Schoonees and Theron (1993), it is considered that field data are preferable to laboratory data. The long-term Shoreham field data has been chosen because it has been independently corroborated by two different methods, as explained earlier. In addition, a previous comparison of transport rates obtained from short-term point measurements and volumetric surveys found that the short-term measurements produced grossly over-estimated net annual transport predictions (Van Wellen et al., 1998). It is also felt that annual predictions of transport are a rigorous test for the equations and are of most importance for coastal engineering applications.

To predict the long-term Shoreham transport rate using the longshore transport equations requires the specification of the wave climate at the breakpoint, for the period concurrent with the aerial survey data. In the absence of such wave climate measurements, recourse had to be made to determining them from wind records. As previously reported by Chadwick (1989), such estimates were obtained from a study originally carried out by HR Wallingford. In this study, a four year record of wind data (1980–1984) had been used to hindcast the offshore wave climate. This was transformed to the breakpoint using a simple refraction and shoaling model. Table 3 presents the results as a mean annual wave climate for this four year period. A mean climate has been used in preference to a chronological record as the long-term Shoreham transport rate was derived using the long-term accretion rates determined from data covering the same period as the wind records. A final critical factor in using this data is the assumption that the beach orientation remains constant and knowing its value. In the case of Shoreham Beach, this assumption was investigated by Chadwick (1989). The mean beach line accretion rate was found to be 1.06 m/annum (varying from 0 to 2.12 m/annum over a 2 km length). As the active beach height was found to be 6.8 m, then the change in beach orientation is 0.06°/annum (i.e. very nearly constant). Secondly, the correct value for this constant beach orientation was also established by Chadwick (1989). This was achieved by calibrating the sediment transport equation in a one line model and subsequently determining a mean beach orientation which, in combination

Table 4
Basic environmental parameters for the calculation of the mean annual transport rate

Parameter	Value
D_{50}	0.02 m
D_{90}	0.04 m
e	0.47
ρ	1030 kg/m^3
ρ_s	2650 kg/m^3
Mean beach	
Orthogonal orientation	181°
tan α	1/8.8
Water temperature	10°

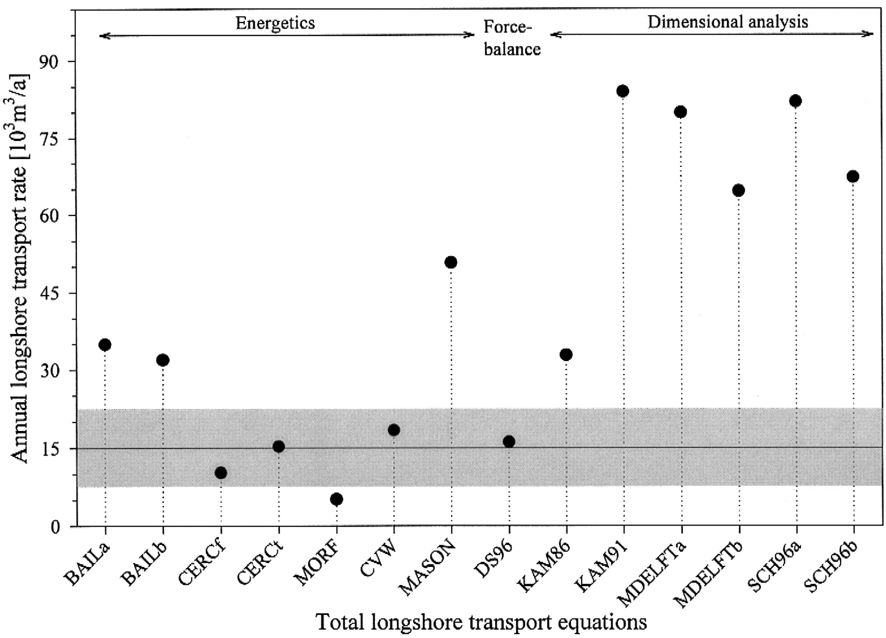

Fig. 4. Net annual Total Longshore Transport predictions based on the mean annual wave climate.

with the mean wave climate, resulted in the same long-term transport rate using the calibrated sediment transport equation from the one line model.

The net annual total longshore transport rate was predicted for each equation, using input parameters from Tables 3 and 4. It should be noted that the following relationships have been applied where necessary: $T_p = 1.05\ T_s$, $T_p = (24/19)T_z$ and $H_{rms} = 0.707\ H_s$. The results of the predictions are shown in Fig. 4, where the grey band depicts a margin of 7500 m^3/annum imposed on the expected annual longshore transport of 15000 m^3/annum. This margin of error represents a simple approximation to the errors which may be expected to result from using the hindcast data.

6. Results and discussion

It can be argued that the long-term Shoreham transport rate and the associated mean annual wave climate constitute only a single field data point. Although this is strictly correct, it has been argued in this paper that is should be considerably more reliable for calibration purposes than short-term single point field measurments. In recognition of the fact that the mean annual wave climte was derived from hindcasting, the simple expedient of introducing an error margin of $\pm 50\%$ in the mean annual transport rate due to errors in the wave data allows reasonable conclusions to be drawn concerning any prediction outside this band.

With the exception of those equations which were previously calibrated using data from Shoreham or sites nearby (CERCf, CERCt, DS96 and MORF), all equations over-predicted the measured total annual longshore transport rate by varying factors, all of which fell outside the suggested margin of error.

The BAILa and BAILb equations gave relatively good predictions for the net annual total longshore transport rate (over-predicting by factors of 2.3 and 2.1, respectively). These equations were not developed specifically for coarse-grained sediment, neither have they been calibrated against longshore sediment transport rates for sediment sizes comparable to those used in this study. The dependency on grain size influences only the suspended sediment component, which is relatively small for shingle-sized sediment and which is why the predictions of BAILa are only marginally higher than BAILb (BAILb neglects suspended load). However, the lack of some form of grain size parameter for bedload implies constant K, which will result in a predicted minimum transport rate, regardless of a change in grain size.

Only two equations under-predicted the annual transport rate. Of these, CERCf was calibrated against the Shoreham trap data, which has subsequently been shown to under-predict transport rates in other than relatively calm conditions (see Section 2.1). Accordingly, when scaled up to annual predictions, it is likely to under-estimate the total transport. The lowest prediction was produced by MORF (under-predicting by a factor of 0.3) despite being developed for sediment sizes comparable to those at Shoreham. This under-prediction is more difficult to account for since, although MORF was partly calibrated against the Shoreham trap data, it was also calibrated against the Delft Equation laboratory data, which over-predicts the transport rates, as shown later.

The results predicted by the newly derived equation (CVW) are included on Fig. 4 for the sake of completeness; by definition the predictions will be good, since the original numerical model was calibrated from the same long-term data used for this comparison. Hence, the equation requires testing against field data from a different site but, given the inclusion of a representative grain size and the implicit incorporation of the effects of critical mobility, should be capable of more universal application. CERCt was also calibrated originally using the long-term data at Shoreham and requires validation from other field sites.

The MASON equation overpredicts by a factor of 3.4. The low correlation coefficient, shown in Fig. 2, together with the arguments concerning the current accuracy of predictions using tracers, given in Section 3.1, serves to highlight the need for further field measurements on coarse grained beaches.

The sole force-balance equation (DS96) had previously been calibrated for shingle of similar size to that found on Shoreham beach and produced a good estimate of the average annual total longshore transport rate in the study area, over-predicting only by a factor of 1.1. This equation shows promise in that it uses shear stresses at the seabed and takes waves and currents into account. Although the resultant set of equations are significantly more complex than most other formulae, the input parameters required for them are just as easily obtainable from field measurements as those needed for the energetics and dimensional analysis based equations.

KAM86, produced the most accurate prediction from the dimensional analysis group of formulae, over-predicting by a factor of about 2.2. This over-prediction is perhaps not surprising since the equations were developed essentially for sand beaches, rather than being specifically aimed at coarse-grained beaches.

The poor performance of the KAM91 equation, and the associated SCH96a and SCH96b was expected for the reasons given by Kamphuis in Section 3.3, over-predicting the expected annual transport rate by a factor of about 5.6, 5.5 and 4.3, respectively. The two versions of the Delft Equation (MDELFTa and MDELFTb) performed equally as poorly as KAM91 against the long-term transport measurements, over-predicting by factors of 5.3 and 4.5, respectively, despite being specifically developed for shingle beaches.

Notwithstanding the poor predictions against the long term Shoreham data, the fact remains that KAM91, MDELFTa and MDELFTb respectively had been found previously to be the most accurate of 52 transport equations tested against a wide range of field data (Schoonees and Theron, 1996). However, the field database listed by Schoonees and Theron (1993, 1994, 1996) is biased towards a certain group of environmental parameters. The range of values for wave period, wave height and angle of breaking is not dissimilar to those found on coarse-grained beaches in general and the Shoreham long-term wave climate database in particular. If a comparison is made between the annual transport rates, as included in the Schoonees and Theron database, and the mean annual longshore transport rate at Shoreham, it is clear that apparently similar hydrodynamic input produces a large discrepancy in predicted transport rate. Most of the measured annual longshore transport rates given by Schoonees and Theron (1993, 1994) are around the 100,000 m^3/annum with a significant number higher than that. This compares with 14,500 m^3/annum measured at Shoreham. It is not surprising, therefore, that equations which over-predict the sediment transport on Shoreham beach might fare much better when compared against the Schoonees and Theron database. This essentially indicates that equations which give good predictions for sand beaches are unlikely to give good predictions for coarse-grained beaches.

In the case of MDELFTa and MDELFTb, their poor performances against the Shoreham long-term data are further evidence that coarse-grained laboratory experiments over-estimate considerably prototype transport rates, as suggested by Brampton and Motyka (1987).

7. Conclusions

Sediment transport on coarse-grained beaches is influenced strongly by factors such as the steep beach gradient, high hydraulic conductivity of the sediment and swash transport. These factors can lead to a different mode of energy dissipation compared to sand beaches, which may partially invalidate methods developed previously for sand beaches. For example, researchers such as Kamphuis (1991b) have presented data which clearly show that, under certain conditions, a significant proportion of the total longshore transport takes place in the swash zone rather than exclusively in the surf zone. Ideally therefore, surf and swash zone transport should be either de-coupled or consciously combined, rather than by default as at present, when equations are calibrated against field data. The review confirms that only a few equations have been developed specifically for longshore sediment transport on coarse-grained beaches, most of which have involved some form of calibration from a very limited dataset (either the experiments carried out at Delft Hydraulics or Chadwick's field data at Shoreham).

The energetics based equations as a group produced reasonable predictions of mean annual transport rate. These include the Bailard equations, which were derived for sand, and do not include the effect of grain size, thus the equation needs to be applied with care. The remaining CERC-based equations (CERCt, CERCf) had been calibrated using data at the field site. More field data from different sites are needed if they are to be evaluated more thoroughly and applied to other sites with a suitable degree of confidence. All equations in this group have the benefit of simplicity in use, with a minimum number of input variables.

Of the dimensional analysis group of equations, those derived from coarse-grained laboratory experiments appear to over-estimate considerably the prototype transport rates (MDELFTa, MDELFTb) as do the remaining equations, with the exception of KAM86. Again, the input parameters required are straightforward and readily available from typical field measurements.

The force-balance equation (DS96) was the most accurate of those formulae which had not previously been tested against data from Shoreham. The equations are the most complex of those reviewed and do not include swash transport, but again the parameters required are readily obtainable.

The newly derived equation (CVW) includes potential swash zone transport and implicitly accounts for critical mobility, but needs to be evaluated more fully against other field data from coarse-grained beaches.

Notation

C_g	group wave celerity (m/s)
d	water depth (m)
$D_{(50/90)}$	(50%/90%) representative grain diameter (m)
D_d	wave energy dissipation rate (W/m^2)
e	void ratio
E	wave energy density (kg/s^2)
$f_{w,r}$	wave induced friction factor for rough turbulent flow
$f_{w,sf}$	wave induced friction factor for sheet flow conditions
g	gravitational acceleration (m/s^2)
H	representative wave height (m)
H_{rms}	root mean square wave height (m)
H_s	significant wave height (m)
I_{ls}	total immersed weight longshore transport rate (N/s)
$K_{1,2,3}$	proportionality coefficient
L	wave length (m)
L_0	deep water wave length (m)
M_{50}	median mass of one unit given by 50% on the mass distribution curve (kg)

n	number of observations
p	porosity
P_+	virtual wave power (kg/s^3)
P_{ls}	longshore wave energy flux (W/m)
P_{ls0}	threshold value of the longshore wave energy flux (W/m)
Q_{ls}	total volumetric longshore transport rate (m^3/s)
Q_{x1}	cross-shore integrated longshore transport rate in current dominated conditions (m^3/annum, s)
Q_{x2}	cross-shore integrated longshore transport rate in wave dominated conditions (m^3/s)
q	volumetric bedload transport rate (m^3/m s)
R	correlation coefficient
s	relative density
T	representative wave period (s)
T_p	peak wave period (s)
T_s	significant wave period (s)
T_z	zero crossing wave period (s)
u_+	dissipation velocity (m/s)
u_{mb}	maximum orbital velocity at bottom (m/s)
w_s	fall velocity of the sediment (m/s)
α	beach slope (rad)
$\varepsilon_b, \varepsilon_s$	efficiency terms for bedload and suspended load
$\varepsilon_1, \varepsilon_2$	calibration coefficients
γ_b	wave breaking index angle between the wave direction and the beach contours (°)
Φ	non-dimensional transport rate
Γ	conversion factor between mass and volumetric transport rates ($kg/m^2\ s^2$)
θ	wave angle (°)
θ	Shields parameter
ρ	density of the fluid (kg/m^3)
ρ_s	density of the sediment (kg/m^3)
σ	relative standard error of estimate
b	subscript denoting values sampled at the point of wave breaking
cr	subscript denoting critical values
d	subscript denoting values sampled at the point where the water depth is equal to d
m	subscript denoting mean values
max	subscript denoting maximum values
w	subscript denoting wave induced values

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