SERIES 1

Works for shielding against the oscillations of the sea

TRANSLATED VERSION OF THE ORIGINAL SPANISH TEXT

RECOMMENDATIONS FOR MARITIME WORKS


ROM 1.0-09

Recommendations for the Project Design and Construction of Breakwaters (Part 1: Calculation and Project Factors. Climate Agents)

GOBIERNO DE ESPAÑA
MINISTERIO DE FOMENTO
Puertos del Estado

RECOMENDATIONS FOR MARITIME WORKS **SERIES 1**

Works for shielding against the oscillations of the sea

ROM 1.0-09

Recommendations for the Project Design and Construction of Breakwaters (Part 1: Calculation and Project Factors. Climate Agents)

December 2010

INDEX

ISBN 978-84-88975-75-1

9 788488 975751

PVP: 50 €

GOBIERNO DE ESPAÑA

MINISTERIO DE FOMENTO

Puertos del Estado

ROM 1.0-09

Recommendations for the Project Design and Construction of Breakwaters (Part 1: Calculation and Project Factors. Climate Agents)

1st Edition
December 2010

EDITION:
Puertos del Estado

DRAFTING COMPOSITION:
Miguel Ángel Losada Rodríguez
Universidad de Granada

ENGLISH TRANSLATION:
Pamela Faber
Universidad de Granada

INTERIOR & COVER DESIGN:
Emilio Piñeiro Díaz
Puertos del Estado

PRINTER:
V.A. Impresores S.A.

I.S.B.N.:
978-84-88975-75-1

LEGAL DEPOSIT:
M-11320-2011

PRICE:
50 € (VAT included)

Prologue

The development of technology as well as the technical regulations and guidelines pertaining to harbor and maritime structures has a rich tradition in Spain. In contrast to other European countries, Spain has large number of ports located outside of the natural harbors offered by bays and estuaries. As a result, these ports are subject to the direct action of the sea. In such unsheltered areas, port activity is only possible because of infrastructures that provide protection from wave action. The financial investment in coastal defense structures is initially advanced by the Port Authority, and subsequently recovered year after year, thanks to the application of port taxes within the context of a policy of economic self-sufficiency for each port.

As an example of the importance of coastal access and protection works in Spanish ports, I would like to highlight the fact that in 2005-2020, the time period covered by the current *Plan Estratégico de Infraestructuras y Transportes* (PEIT) [Strategic Plan for Infrastructures and Transportation], projects were designed for more than 1700 hectares of new harbor areas along the Spanish coastline. Of this total amount, 750 hectares were generated in 2005.

Today in Spain, the experience accumulated over the years in the design and contruction of large port infrastructures has provided a solid foundation for the application of advanced methods of calculation and design. In fact, thanks to this wide experience, the complexity inherent in such methodologies has never overshadowed the realism that is such a necessary part of the construction and exploitation of port structures in general, and coastal access and protection works, in particular.

This harmonious balance between a technological ideal at the theoretical level and inevitable pragmatic constraints at the practical level is the criterion that defines Series 1 of the **ROM Program,** *Recommendations for Maritime and Harbor Structures*, pertaining to coastal access and protection works. Consequently, it is now a great pleasure for me to present the **ROM 1.0**, which is the first publication in this series.

The **ROM 1.0-09** provides a characterization of the climate agents that can have an impact on maritime and port structures. Moreover, it establishes the calculation methods for different types of breakwater (i.e. sloping breakwaters, vertical breakwaters, and composite breakwaters).

From a technical perspective, it should be underlined that the probability techniques described in the **ROM 0.0** can now be applied, thanks to the valuable data provided by the buoy networks created by *Puertos del Estado* for the measurement of climate parameters. These networks can be found all along the Spanish coastline, both in deep and shallow waters. They have significantly increased our knowledge of the physical environment where maritime and port structures are located. Furthermore, with the calculation techniques described in this text, there is now sufficient material to enable us to work towards an optimization of the reliability/cost relation in our port infrastructures.

I would like to point out that the effort made to develop the **ROM Program** should serve to advance our international projection in this field, of which there is already ample evidence. It should also enhance the quality of maritime infrastructures in the port and harbor system, which is undeniably an issue of great public interest. The project design of such structures should be an accurate and objective reference for the construction of maritime and port works. This signifies that projects should include predictions of potential problems as well as possible solutions for all adverse circumstances that might lead to modifications in the structure, which the public administration, as the principal investor, would regard as unacceptable.

I would like to reiterate my satisfaction at being able to launch Series 1 of the **ROM Program**, *Recommendations for Maritime and Harbor Structures*, pertaining to coastal access and protection works. It is my firm belief that it will contribute to making our ports safer and more efficient.

Fernando González Laxe
President of Puertos del Estado

Index

CHAPTER III. CLIMATE AGENTS AT THE PROJECT SITE

CHAPTER IV. ANNEX I. FOUNDATIONS AND JUSTIFICATION

Figures

CHAPTER II. GENERAL CALCULATION PROCEDURE

CHAPTER III. CLIMATE AGENTS AT THE PROJECT SITE

CHAPTER IV. ANNEX I. FOUNDATIONS AND JUSTIFICATION

Tables

CHAPTER II. GENERAL CALCULATION PROCEDURE

CHAPTER III. CLIMATE AGENTS AT THE PROJECT SITE

CHAPTER IV. ANNEX I. FOUNDATIONS AND JUSTIFICATION

Introduction

Breakwaters are one of the most characteristic harbor constructions, and can be regarded as basic infrastructures for the conceptualization of artificial maritime and land areas, such as ports. Ports can be defined as land and water surfaces sheltered from the action of atmospheric and marine dynamics, where vessels can be safely berthed or anchored, and which provide the necessary conditions for passenger and cargo handling operations.

Because of Spain's geographic situation and orographic conditions as well as the morphological and climate characteristics of its coastal zone, the experience with breakwater construction, which has been acquired over the years, is enormous. For many decades, Spain has been a pioneer in maritime engineering from a conceptual perspective as well as from the perspective of construction methods and techniques.

The contributions made by Jorge Juan in the 18^{th} Century, Agustín de Betancourt in the 19^{th} Century, and Ramón Iribarren in the 20^{th} Century have left a valuable and lasting legacy for engineering the world over, particularly in relation to breakwaters. For the first time, the methods of Prof. Iribarren provided engineers with tools to determine the way that waves propagate and to scientifically design coastal defense structures for ports. Particularly relevant is Iribarren's book, *Obras Marítimas. Oleaje y Diques* [Maritime Structures: Wave Action and Breakwaters], published in 1964, which for many years was one of the main references in the field. Iribarren's well-known formula for calculating sloping breakwaters was presented at the 21^{st} AIPCN-PIANC conference held in Stockholm in 1965. Until very recently, it was the formula most frequently used for dimensioning the main layer of breakwaters all over the world, and has been the basis for subsequent formulations. Iribarren was also the first to systematically analyze phenomena pertaining to the interrelation between wave action and maritime constructions as a determining factor for dimensioning port structures, particularly breakwaters. The majority of the maritime structures in Spanish ports and in many other parts of the world during the 1960s and 1970s were designed by engineers who used Iribarren's methods.

In the second half of the 19^{th} Century, Ramón Iribarren thus laid the foundations for maritime engineering in Spain. The prestige of this longstanding tradition is now internationally acknowledged. It has given rise to a long line of outstanding engineers who continue this tradition of excellence, such as Prof. Pedro Suárez Bores and Prof. Miguel Ángel Losada.

Spain's experience with coastal defense structures has increased even more in the first decade of the 21^{st} century. During this time period, the rise in investments in Spanish ports, both at a quantitative as well as a qualitative level, has been spectacular. No other country in the world has simultaneously built so many coastal defense structures of such importance. In certain cases, they have even been subjected to very extreme design and implementation conditions to the extent of breaking world records.

All of this technological experience accumulated over the years should be consolidated and disseminated. This will give added impetus to its future projection with the incorporation of the advances and scientific knowledge acquired with recently developed technical tools. This is indeed the best way of assuring a continuing high level of competence among professionals, companies, and maritime engineering in Spain. In this way, its influence and competitiveness will continue to grow in the future on an international scale.

With these objectives in mind and in consonance with the Spanish bibliographical tradition in the field of breakwaters, Series 1 of the ROM Program, *Recommendations for Maritime Structures,* on the design and construction of coastal defense works is now being written.

The ROM 1.0-09, the first document in this series, lays the foundations and develops the general criteria for coastal defense structures. It also provides a description and characterization of project design factors to be considered, with special emphasis on climate, atmospheric and marine agents whose actions can have a significant impact on maritime structures. .

This publication, which is the first in the ROM 1 Recommendations, is an important contribution to the very small set of official regulatory documents on coastal protection structures in the world, which have mainly been published in the USA, Japan, and other European countries. The ROM Program is thus in the process of establishing itself as one of the most solid and coherent references for guidelines in the domain of harbor and maritime structures on an international scale.

The ROM 1.0-09 is part of the second generation of documents of the ROM Program, which appeared after the publication of the ROM 0.0, *General Procedure and Requirements in the Design of Harbor and Maritime Structures*. This first ROM document signified the incorporation of new methodological procedures in the design and construction of maritime structures, based on the evaluation and verification of suitable safety and operational levels in each phase of their useful life. It also is based on the consequences that can thus be derived from their economic optimization.

The ROM 1.0-09 further specifies this methodological framework for coastal defense structures with verification procedures related to failure and operational stoppage modes with Level I semi-probabilistic methods. Accordingly, it describes and complements what had been generally outlined in the corresponding chapters of the ROM 2.0-08, *Recommendations for the Design and Construction of Docking and Mooring Structures. Part I: General Criteria and Project Design Factors*. In addition, it also develops procedures for the practical use of Level II and III probabilistic methods with a view to facilitating their application.

For this reason, the ROM 1.0-09 defines and characterizes project design factors in a way that is compatible with the application of these methodological procedures. It thus analyzes all aspects referring to atmospheric and marine climate agents, which as previously mentioned, are the source of the actions that have an impact on the project design of coastal defense structures. In this sense, these elements in the ROM 1.0-09 partly correspond to the contents of the ROM 0.3-91, *Climate Actions I: Waves and other Sea Oscillations*, a preview of which was published two decades ago as *Annex 1. Maritime Climate on the Spanish Coastline* as well as other subsequent publications such as the ROM 0.4-95, *Climate Actions II: Wind.*

In this sense, the second chapter of the ROM 1.0-09 provides a description of the procedures and calculation methods, previously reflected in the "pre-definitive" text published as a working paper in 2006 with the EROM 02. This ROM also includes a description or complete characterization of the climate agents at the project site, particularly in regards to the different types of sea oscillations. Project designers thus have at their disposal a wide range of elements that provide them with an in-depth knowledge of these agents, define them by analyzing their effects, and effectively manage them for the project design of harbor and maritime structures.

In conclusion, the ROM 1.0-09 is organized in four chapters. Chapter 1 is a general introduction that outlines the objectives, the scope of application of the Recommendations, and their relation to other regulations and guidelines currently in force. Chapter 2, "Procedures and project criteria" lays out the foundation for the design of coastal defense structures. Chapter 3 describes climate agents at the project site, and finally Chapter 4 provides an Annex of Foundations and Justifications for all of the contents in the ROM 1.0-09.

José Llorca Ortega
Director of the ROM Program

Chapter I
General

Table of contents

CHAPTER I. GENERAL

1.1. SCOPE OF APPLICATION

The objective of the ROM 1.0-09, Recommendations for the Project Design and Construction of Breakwaters (Part 1: Calculation and Project Factors. Climate Agents) is to provide a series of general regulations and technical criteria for the design, construction, exploitation, maintenance, repair, and dismantling of all types of breakwaters, regardless of the construction material, machinery, and elements used in these processes.

These regulations specifically apply to the following types of breakwater: sloping breakwaters with or without a crown wall, vertical breakwaters, composite breakwaters, berm breakwaters, and submerged breakwaters. However, these premises and the implementation proposed in these chapters expand the standard classification of maritime structures. More specifically, by analyzing the performance of each part of the type sections, these Recommendations make it possible to design breakwaters whose components are specifically geared to meeting project needs and requirements. The end result is the construction of breakwaters with an optimal performance under the action of sea oscillations, and furthermore, with the added value of being more cost-effective and better integrated in the marine ecosystem.

Note. *Docking, mooring, and anchoring structures for coastal management and defense, as well as offshore platforms are not the specific focus of this ROM even though many of its recommendations are also applicable to the design and construction of such structures. The underwater pipelines used mainly in the oil and natural gas industries, sea outfalls, and large networks of underwater cables for intercontinental communications are a different type of maritime structure. The ROM 1.1 offers criteria for the design and construction of structures to protect underwater pipelines, but not for their use, exploitation, and design.*

1.2. OTHER RECOMMENDATIONS

The ROM 1.1 is part of Series 1 of *Recommendations for Maritime and Harbor Structures*, and focuses on the construction of breakwaters. The objective of these recommendations, regulations, and guidelines is to improve breakwater design so that breakwaters are safer and better able to withstand sea oscillations. Their application also requires the consideration of the following recommendations, regulations, and guidelines, which are part of the ROM Program:

- **ROM 0.0.** General Procedure and Requirements in the Design of Harbor and Maritime Structures
- **ROM 0.3-91.** Climatic actions I: Waves
- **ROM 0.4-95.** Climatic actions II: Wind
- **ROM 0.5-05.** Geotechnical recommendations for Harbor and maritime structures.
- **ROM 2.0-08** [draft]. Design and execution of berthing, mooring, and anchoring structures. Part I: General criteria and project factors
- **ROM 3.1-99.** Design of the maritime configuration of ports, access channels and flotation areas
- **ROM 5.1-05.** The quality of coastal waters in harbor areas

Also to be considered are the other official guidelines and regulations for civil engineering in Spain and the European Union, pertaining to breakwaters.

1.3. WRITING THE ROM 1.0-09

The ROM 1.0-09, *Recommendations for the Project Design and Construction of Breakwaters (Part 1: Calculation and Project Factors. Climate Agents)* was written at the same time as the first part of Series 2, *General criteria and project factors for docking and mooring structures.* (ROM 2.0-08).

Draft versions of the first chapters of these publications were disseminated in 2006 as working documents in the E/ROM 02, *Studies and Scientific-Technical Analyses of the ROM Program*. In the near future, specific recommendations with be published for the ground and elevation plans as well as for the analysis of the performance and verification of: (i) types of breakwater (ROM 1.1. and following numbers); (ii) docking and mooring structures (ROM 2.0 and following numbers).

These Recommendations are of maximum priority for *Puertos del Estado,* since breakwaters are the most important and specific of harbor structures. In fact, they have been responsible for over 80% of harbor area investments in recent years. Furthermore, once this new stage of the ROM Program began with the publication of the ROM 0.0 (*General procedure and requirements in the design of harbor and maritime structures*), it was considered crucial to jointly develop Recommendations for breakwaters as well as those for docking and mooring structures with a view to clarifying contents and facilitating the practical application of methods and safety requirements to the most relevant harbor infrastructures, as well as their level of serviceability and operationality, included in the ROM 0.0.

Accordingly, to attain the objectives, it was decided to include some of the contents of the other ROMs (either in the process of being written or updated) in these Recommendations. This ROM can thus be read as an independent text, which reflects the entire process associated with the project design and construction of breakwaters as well as docking and mooring structures. For this purpose, the text was drafted by a panel of experts (ROM 1.0-09 and ROM 2.0-08) in order to assure maximum coherence of its criteria and contents.

The ROM 1.0-09 was written under the auspices of *Puertos del Estado,* the Spanish National Port Authority, subdivision of the Ministry of Public Works. The recommendations were elaborated by a permanent working group. All agreements and decisions were reached after extensive debate by a technical committee composed of experts.

- ***General Coordinator of the material included in the ROM 1.0-09:***
  Miguel Ángel Losada, *Universidad de Granada*

- ***President of the Committee and Director of the ROM Program:***
  José Llorca, *Puertos del Estado*

- ***Members of the working groups:***
  Sergi Ametllet, *SENER*
  Manuel Arana, *Puertos del Estado*
  José Mª Berenguer, *BERENGUER INGENIEROS*
  Antonio Berruguete, *IBERPORT*
  Pedro Canalejo, *ALATEC*
  Alfredo Carrasco, Autoridad Portuaria Bahía de Algeciras
  Juan Carlos Carretero, *Puertos del Estado*
  Beatriz Colunga, *Autoridad Portuaria de Vigo*
  Jesús Corral, *Universidad Politécnica de Cataluña*
  Antonio Corredos, *SATO*
  José Luis Díaz Rato, *Autoridad Portuaria de Gijón*
  Javier Escartín, *PROINTEC*
  Francisco Esteban, *FCC*
  Enrique de Faragó, *PROES*
  Jorge Flores, *KV CONSULTORES*
  Xavier Gesé, *Puertos del Estado*
  Gonzalo Gómez Barquín, *Puertos del Estado*
  Miguel Ángel Gómez Caldito, *ALATEC*
  Marta Gómez Lahoz, *Puertos del Estado*
  Gregorio Gómez Pina, *D.G. de Costas. Ministerio de Medio Ambiente*
  José Manuel González Herrero, *IBERINSA*
  Noelia González Patiño, *DRAGADOS-ACS*
  José Mª Grassa, *Centro de Estudios y Experimentación de O.P.*
  Juan Ignacio Grau, *Puertos del Estado*
  Gregorio Iglesias, *Universidad de Santiago de Compostela*
  José Ramón Iribarren, *SIPORT XXI*

Ana de Lope, *Puertos del Estado*
Cristina López Arias, *Autoridad Portuaria de Bilbao*
Enrique Maciñeira, *Autoridad Portuaria de A Coruña*
María Luisa Magallanes, *EGENOR*
Javier Martín Santo Domingo, *Puertos del Estado*
Mª Jesús Martín Soldevilla, *Centro de Estudios de Puertos y Costas*
J. Ramón Medina Folgado, *Universidad Politécnica de Valencia*
José Mª Medina Villaverde, *NAUTILUS INGENIERÍA MARÍTIMA*
Mario de Miguel, *Autoridad Portuaria de Gijón*
Rodrigo Molina, *TIPSA*
Pablo Molinero, *DRAGADOS ACS*
José Luis Monsó de Prat, *Instituto de Hidrodinámica Aplicada INHA*
Javier Mora, *Autoridad Portuaria de Tenerife*
José Moyano, *Autoridad Portuaria de Gijón*
Vicente Negro, *Universidad Politécnica de Madrid*
Enrique Peña, *Universidad de A Coruña*
Begoña Pérez Gómez, *Puertos del Estado*
Carlos Pérez Quintero, *Puertos de Andalucía*
Eloy Pita Olalla, *INCREA*
Jesús Poncela, *Puertos del Estado*
Ignacio Rodríguez Sánchez-Arévalo, *Puertos del Estado*
Antonio Marcos Ruiz Vega, *Autoridad Portuaria Bahía de Cádiz*
Mª Eugenia Sánchez Rivas, *Autoridad Portuaria de Avilés*
Carlos Sanchidrián, *PROES*
Francisco J. de los Santos, *Autoridad Portuaria Bahía de Algeciras*
Obdulio Serrano, *Puertos del Estado*
Antonio Soriano, *Universidad Politécnica de Madrid*
Javier Uzcanga, *Autoridad Portuaria de Barcelona*
José María Valdés, *EPTISA*
César Vidal Pascual, *Universidad de Cantabria*
José Luis Zataraín, *Autoridad Portuaria de Santander*

- ***General Coordination of the ROM Program:***
  Francisco J. González Portal, *Puertos del Estado*

1.4. DEFINITIONS

- **Abyssal plain.** Flat area of the ocean basin floor.

- **Accelerometer.** Instrument used to measure acceleration.

- **Advection fog.** Fog caused by the cooling of moist air to below its dew point by the process of heat radiation. It forms because of the slow passage of relatively warm, moist, stable air over a colder wet surface.

- **AEMET (Agencia Estatal de Meteorología).** Spanish National Meteorological Agency.

- **Aeroelastic model.** Model that reproduces the deformability of the structure, and thus the structure-flux nonlinear coupling.

- **Age of the sea waves.** Ratio of wave celerity to wind speed (C/U_{10}).

- **Age of the tide.** Time between new or full moon and spring-tides, which is generally from one to three days. Tidal amplitude movement depends on the relative position of the stars. Their alignment causes maximum tide ranges, given that the effects are in phase. These states are known as spring tides, and approximately coincide with the new or full moon.

- **Airy wave.** Nonlinear long wave that can be used to locally describe the astronomical and meteorological tide as well as wave breaking against a mild slope. Also known as *shallow-water wave*.

- **Alboran Sea.** Westernmost portion of the Mediterranean Sea, lying between Spain on the north and Morocco and Algeria on the south. At the west end of the Alboran Sean, is the Strait of Gibraltar, which connects the Mediterranean to the Atlantic Ocean. Its waters thus extend from the Strait of Gibraltar to Cabo de Gata (Almería) all the way to the city of Oran (Algeria).

- **Amplitude.** For oscillatory or wave movements and electromagnetic signals, maximum variation or displacement from a zero value or rest position. It also refers to any physical magnitude that varies periodically or quasi-periodically in respect to a given reference level.

- **Anabatic wind.** Wind created by warm air flowing up a steep slope or mountainside. Also known as *valley breeze*.

- **Anchor.** To attach a vessel or object to the bottom of a body of water at a given point in the sea.

- **Anchoring area.** Area where vessels are secured, and their movements partially restricted by one or various anchors.

- **Angle of internal friction.** Engineering property of granular materials. The angle of internal friction has a simple physical interpretation, as it is related to the angle of repose (or maximum angle) of a slope of a small pile of granular material.

- **Angular frequency.** $\sigma = 2\pi/T$ [1/s]; also represented by ω.

- **Annual cost of the structure.** Sum of the estimated cost of annual damages and the equivalent annual cost of the investment.

- **Anticyclone.** Area of high atmospheric pressure where the atmospheric pressure (slightly different at the sea level) is higher than the surrounding air pressure. The air in an anticyclone is more stable than the surrounding air, and it sinks to ground level from the upper layers of the atmosphere, which causes a phenomenon called subsidence.

- **Antinode.** The points where $kx = 0, \pi, 2\pi \ldots$ etc and at which the free surface reaches its maximum value, whereas the horizontal velocity is null. In the vertical wall there is always an antinode.

- **Aphelion.** Point on the orbit of a celestial body which is most distant from the Sun, and the opposite point from the perihelion. In orbital elements it is represented by Q. If a is the mean distance, and e is the eccentricity, then $Q = a(1 + e)$.

- **Apogee.** Point farthest from the Earth in the elliptical orbit of a body around the Earth.

- **Ascending zero-crossing period.** Average time interval between upwards direction crossings of the mean water level in a wave record.

- **Astronomical tide.** Periodic rise and fall of the sea level, resulting from the gravitational attractions of the Moon, Sun, and other celestial bodies. Its intensity depends on the relative position of the Sun and Moon in respect to the Earth.

- **Backfill.** Inner surface (in contact with the retained ground) of a retaining structure.

- **Baric wind.** Wind generated by the pressure gradient in the air mass.

- **Barometric pulse.** Temporal and spatial variations of the vertical displacement of the free sea surface in regards to a reference level. These variations are produced by the spatial and temporal gradients of the atmospheric pressure.

- **Basic variable.** Variable that characterizes the agent or the action in a cycle, more specifically, the period (and length) and the amplitude.

- **Bathymetry.** Spatial variability of the sea bottom (depths).

- **Bearing capacity.** In reference to foundations, it is the capacity of soil to support the loads applied to the ground. Technically, the bearing capacity is the maximum average contact pressure between the foundation and the earth, which should not produce shear failure or an excessive differential settlement.

- **Bed roughness.** Irregularity of seabed material (gravel, small stones, sand, etc.) which affects the water flow. It is generally measured with the Manning or Chezy roughness coefficient.

- **Bedding layer.** Permeable coarse-grained fill material that facilitates load sharing and the release of interstitial pressure, thus providing a high resistance to shear strength and a low deformability.

- **Berm breakwater.** Breakwater made of loose materials, and which has a main layer with a berm. This berm is located slightly above mean sea level, and is made of light-weight materials to ensure its static stability. When a berm breakwater is subjected to design loading, the profile of its outer layer is modified until it attains a state of equilibrium.

- **Bora.** Strong, cold wind caused by a sudden temperature change between two layers of air (a warm layer on top of a cold one) circulating above an obstacle (e.g. a mountain).

- **Bore.** Wave front with an approximate celerity of $\sqrt{gh}$, usually caused by wave breaking.

- **Bound subharmonic wave.** Wave generated by the coupling (sum) of two low-frequency waves of similar periods.

- **Bound superharmonic wave.** Wave generated by the coupling (sum) of two high-frequency waves of similar periods.

- **Boundary layer.** Layer of fluid in the vicinity of a bounding surface whose velocity varies from zero (boundary at rest) to 90-95% of the free stream velocity.

- **Boundary layer wind tunnel.** Tunnel used to obtain time series of basic and instantaneous variables for the wind. The results depend on the accurate modeling of the turbulent boundary layer.

- **Boussinesq Regime.** Wave propagation conditions governed by Boussinesq-type equations and obtained by a series expansion of governing equations and boundary conditions of potential oscillatory flow with two small parameters: relative depth, $h/L \leq 1/20$ and relative wave amplitude a/h.

- **Breakwater.** Protective structure built to reduce wave action through a combination of reflection and dissipation of the incident wave energy.

- **Breakwater beginning.** Subset or section of a breakwater that connects it to the land or to another breakwater.

- **Buoy.** Floating object moored or anchored to the sea bottom to mark a channel, anchor, shoal, rock, etc.

- **Calm cycle.** Period of time during which the variable values that define the sea state uninterruptedly remain below the calm threshold.

- **Capillary waves.** Wave with a period $T < 1s$, and whose steepness and celerity is controlled by gravity as well as by the surface tension of the liquid in which the wave is traveling.

- **Celerity (propagation velocity).** Quotient between the wave length and the period of oscillatory motion. If it is progressive, then it is equivalent to the propagation velocity.

- **Central body.** Main load-bearing section of a breakwater against wave action, which can lead to its transformation by processes, such as wave breaking or reflection.

- **Centroid, geometric center, or barycenter** (of an object in *n*-dimensional space)**.** Intersection of all hyperplanes that divide the object into two parts of equal *n*-volume with regard to the hyperplane. Informally, it is the average of all points of X. In physics, it may be, under certain circumstances, the center of the mass or the center of gravity.

- **Chart datum.** Permanent reference level used for measuring soundings, tidal levels or water depths. The selection of a level lower than the mean sea level is regarded as a safety factor for navigation. The Hydrographic Institute of the Spanish Navy generally uses the highest local low tide as a chart datum for soundings.

- **Cliff.** Landform consisting of a very steep vertical slope or rock face that separates the sea from the land.

- **Closure depth.** Minimum depth at which the influence of sediment transport is not relevant in the behavior of the beach profile.

- **Cnoidal wave.** Theoretical long wave that propagates without changing the surface profile, and which is mathematically expressed in terms of the Jacobi elliptic function.

- **CNRT.** Public domain software program that permits the selection of the most suitable wave theory, based on adimensional monomial values.

- **Coastal zone.** Area that facilitates the sustainable use and exploitation of the littoral environment, such as the correction, protection or defense of the coastline, the generation, conservation and nourishment of beaches and swimming areas, as well as the exchange of land-sea transversal flows of a wide variety of substances.

- **Coastline.** Line on the Earth's surface that separates a dry land surface from an ocean or sea.

- **Coefficient of directionality.** Coefficient that gives the directional extreme regimes of the significant wave height, based on from the scale extreme regime corresponding to the area.

- **Coefficient of variation (Pearson coefficient).** Statistic that is a normalized measure of dispersion. It is useful for comparing dispersions on different scales since it is does not change with scale changes. However, it has certain disadvantages since it is sensitive to small changes in the mean, and unlike standard deviation, cannot be used to construct confidence intervals for the mean.

- **Cohesion.** Quality that makes certain soil particles hold together as a result of internal strengths, which depend, among other things, on the number of contact points that each particle has with adjacent particles.

- **Collapsing breaker.** Breaking wave in which the bottom face of the wave gets steeper, loses stability, and falls down $(2.5 < I_r < 3.5)$, resulting in foam.

- **Complementary event.** Event that contains none of the sample elements of another event.

- **Composite breakwater.** Breakwater consisting of a granular base which is the foundation for the reflecting structure. The performance of the breakwater as a wave-reflecting or wave-breaking structure depends on the foundation depth and on characteristics of the incident waves.

- **Compressibility.** Capacity of a fluid or solid to vary its volume as a response to a pressure or stress change, when compression loads are applied to its surface.

- **Conditioned function.** Function in which the density and distribution function of one of the variables is conditioned to the occurrence of a value or range of values of another variable

- **Confidence interval.** Statistical range with a specified probability that a given parameter lies within the range.

- **Conservation and exploitation costs.** All scheduled costs that are necessary for the subset to provide suitable use and exploitation conditions in the port area and its installations and at the same time meet all requirements for their optimal reliability, functionality and operationality.

- **Conservative substance.** Substance that does not undergo modification during natural or artificial processes of transportation or mixing (e.g. silica sand).

- **Continental shelf break.** Point where the continental shelf ends and drops abruptly into the continental slope.

- **Continental slope.** Seaward border of the continental shelf (depths of 200-400 m) and the abyssal zone (1000-2000 m). Generally speaking, the continental slope is on the order of ten degrees, and its horizontal extension is small in contrast to the length of the tide wave.

- **Control volume.** Fixed region in space for the thermodynamic study of mass and energy balances for flowing systems. The mass of the fluid remains constant. As fluid moves through the control volume, the mass entering the control volume is equal to the mass leaving the control volume. In the absence of work and heat transfer, the energy within the control volume also remains constant.

- **Copla.** Numerical model created by the University of Cantabria that consecutively (not simultaneously) resolves the evolution of the sea level and the circulation system thus generated.

- **Core.** Innermost part of a coastal defense structure, which underlies the outer layers. It is not subject to the direction action of the waves, and should prevent turbulence from affecting the inside of a port or harbor area.

- **Coriolis Effect.** Relative acceleration of an object that moves in a rotating (non-inertial) reference frame when its distance varies in respect to the turning axis. On Earth, the Coriolis Effect deflects moving bodies to the right in the northern hemisphere and to the left in the southern hemisphere.

- **Correlation coefficient.** Statistic that measures the degree of linear association between two variables, but which does not provide any information regarding their cause-effect relation.

- **Correlation parameter.** Indicator of the characteristic of the functional relation between data sets, which shows whether they are linear. If the correlation parameter value (r) is between -1 and $+1$, this signifies that the relation between the data sets is linear (i. e. a straight line).

- **COULWAVE Model.** Public domain Boussinesq-type model created at Cornell University (New York, USA), which effectively resolves equations for normal incidence as well as oblique incidence.

- **Covariance.** Statistical measure showing the degree to which two random variables U and V, $\mathrm{Cov}\,[U, V]$, vary or move together. It is the expected value of the product of their variations with respect their mean values.

- **Crest.** Highest part of a wave.

- **Crest amplitude.** Maximum positive vertical displacement in reference to the mean sea level (MW).

- **Crest berm.** Nearly horizontal section built on both sides of a sloping breakwater to reduce wave run-up and overtopping, as well as to provide access for maintenance work.

- **Crest period.** Time interval between two consecutive crests.

- **Crest wall.** Structure on top of a breakwater that protects it from overtopping. It also provides access to the structure, and when relevant, a mooring line downdrift from the breakwater. Also known as a *crown wall*.

- **Crown.** Highest point on a maritime structure. In some cases, it may also have a crown wall, located on top, to allow access to the breakwater as well as the partial reduction of overtopping and breakwater volume. Also known as a *crest*.

- **Current.** Portion of a stream of water which is moving with a velocity much greater than the average or in which the progress of the water is concentrated, and which can be the result of factors, such as tides, waves, river discharge, etc.

- **Cyclone.** Low atmospheric pressure area.

- **Damping**. The ability of a system or body to dissipate energy. It is also defined as the force exerted against the movement of two bodies in contact with solid or fluid substances, according to their velocity.

- **Dangerousness.** Exceedance probability of the threshold value of the agent in a given time period.

- **Darcy Regime.** As water drains, the solid skeleton must support an increasingly heavy load. The movement of water through the soil produces a variation in soil properties in respect to time. If the load applied to the ground is constant either by the structure or by a constant uniform flow, drainage will probably achieve a permanent regime when there is a well-established filtration network. Furthermore, if the medium has a low level of permeability, the hydraulic gradient will be proportional to the discharge velocity. This laminar regime is known as the Darcy filtration regime.

- **Declination.** Coordinate of a celestial body, which is comparable to latitude on Earth, and used to locate any position in the sky. It measures the angular distance of the celestial body from north (positive) or south (negative) of the celestial equator. It is one of two coordinates commonly used to define the position of an object in the sky, the other being right ascension.

- **Deep water.** Area in the sea or ocean where the wave does not touch the bottom, and ($h/L > 1/2$), where h is the depth and L is the wavelength.

- **Deformable soil.** Soil that loses its shape because of the action of an external stress.

- **Deformation.** The change of size or shape of a body due to internal stresses caused by one or more applied forces.

- **Density current.** Water movement due to differences in the density of the water from place to place usually caused by changes in the amount of material held in suspension or changes in temperature or variations in salinity.

- **Density function.** Function used to determine probabilities for a random variable X, which is continuous, $f(x)$, and measures the intensity or probability rate of the value x.

- **Deterministic formulation.** Mathematical expression in which the predominant and non-predominant agents and actions and of parameters take nominal or determined values, independently of their probability of exceedance.

- **Diffraction.** Oscillatory energy flux caused by the wave height gradient perpendicular to the propagation direction.

- **Diffraction parameter.** Dimensionless coefficient defined as the ratio between the diffracted wave height and the incident wave height.

- **Directional dispersion function.** Function that expresses the relative distribution of the energy in the wave direction domain.

- **Directional sea wave spectrum.** Function that expresses the wave energy in the frequency domain and the domain of the wave propagation direction. It is expressed as the product of the wave frequency spectrum and the directional dispersion function.

- **Directional wave tank.** Tank used in laboratories to obtain time series of instantaneous and basic variables of the wave action in port and shoreline areas. The results depend on the time series with which the paddles are activated as well as their response.

- **Dismantling.** Process of tearing down and removing a structure and, when necessary, restoring the location to its original conditions before the structure was there.

- **Dissipation.** Gradual loss of energy with the resulting decrease in wave height, due to wave breaking, turbulence and viscous effects, and in shallow water, due to the effects of bottom friction. In the case of breakwaters, dissipation is produced by the medium through which the wave propagates.

- **Dissipation coefficient.** Coefficient that quantifies the wave height variation, due to any process of wave energy dissipation.

- **Distribution function.** Function that that describes the probability that a system will take on a specific value or set of values. The distribution function of the probability of X (or accumulated distribution) is a function $F(x)$, which assigns each event its probability of occurrence.

- **Dock.** Structure built on the coastline or the shore of a navigable river where ships are loaded and unloaded or repaired.

- **Docking and mooring area.** Area with the necessary conditions for vessels to be safely berthed in a harbor area and/or to carry out passenger and cargo handling operations. This includes all activities related to the loading, unloading, embarking, and disembarking of passengers, vehicles, and cargo, which allow their transit between vessels or between vessels and other means of transportation.

- **Dolphin.** Fixed mooring structure composed of a cluster of timber or steel piles driven into the seabed, and which is designed to resist horizontal stresses generated by mooring or docking operations. It can also protect other structures from the impact of vessels.

- **Domain I.** Ocean and open sea.

- **Domain II.** Outer continental shelf.

- **Domain III.** Inner continental shelf and shoreline areas, including the surf zone.

- **Domain IV.** Port areas.

- **Domain V.** Confined seas of relatively shallow depth.

- **Downdrift.** Side of the structure that is sheltered from the waves.

- **Draught.** Water depth required for a ship to float. As a safety measure, there should always be sufficient room under the keel to allow for a full cargo. Also *draft*.

- **Dredging.** Process involving the removal of solid materials from water courses, lakes, bays, and port accesses to increase the depth of navigation channels or a river in order to increase its water transport capacity to prevent flooding upstream and increases its depth to facilitate maritime traffic, thus reducing the risk of vessels running aground.

- **Duration of the state.** State that quantifies the time that should pass before a significant change in the process, and consequently, the time during which it is admitted that the underlying hypotheses are fulfilled.

- **Dynamic viscosity.** Property of a liquid resulting from internal flow resistance opposing the relative movement of adjacent layers. It is usually represented by $\mu = \rho\nu$, where ρ is the fluid density and ν is the kinematic viscosity.

- **ECAWOM Model.** Model that combines the physics of the atmosphere (resolved by the HIRHAM model), sea waves (WAM model), and ocean circulation (HAMSOM model), and also takes their interactions into account.

- **Edge wave.** Oscillation that propagates parallel to the coast. Its amplitude oscillates and diminishes rapidly seaward, and is negligible at a distance of one wavelength offshore until its total extinction.

- **Effective stress.** Mean normal force per unit area transmitted directly from particle to particle of a rock or soil mass. Also called *intergranular pressure* and *effective pressure*.

- **Elastic modulus.** Mathematical description of an object or substance's tendency to be non-permanently deformed when a force is applied to it. The elastic modulus of an object is defined as the slope of its stress-strain curve in the elastic deformation region. Also called *Young's modulus*.

- **Elastic soil.** Soil that returns to or is capable of returning to its initial shape after deformation. If the relation between the stress and deformation is linear, it is known as *linear elastic soil.*

- **Emergence line or level.** Maximum water height reached on the crown of the structure at which the structure is still visible.

- **Empty event.** Event that cannot exist or which is impossible.

- **Epicenter.** Point on the Earth's surface vertically above the hypocenter (or focus), point in the crust where a seismic rupture, such as an earthquake or an explosion, originates.

- **Equator.** Plane perpendicular to the axis of rotation, which passes through the Earth's center, and which divides the surface of the Earth into two hemispheres, the North Hemisphere and the South Hemisphere. It is the line circling the Earth at 0 degrees latitude, and measures $40.075{,}004$ km.

- **Equilibrium tide.** Instantaneous response of the ocean due to the action of the force of gravity, assuming that it covers the entire surface of the Earth.

- **Equinoctial spring tide.** Spring tide that occurs at the time of either the summer or fall equinox.

- **Equivalent return period.** Rough indicator of the confidence in the structure in relation to climate agents. It does not provide any information regarding the reliability of the structure when various different agents intervene in its failure modes.

- **Ergodic.** Of or relating to a process in which every sequence or sizable sample is the same statistically, and therefore, equally representative of the whole.

- **ERI (Economic Repercussion Index).** Index that quantitatively evaluates the economic repercussions because of the reconstruction of the structure or because of the foreseeable cessation or modification of its activities in the event of its destruction or loss of total operationality.

- **Erosion.** Process involving the physical breakdown, chemical solution and transportation of surface material (rocks, cover soil, cliffs, etc.) resulting in the morphological modification of the Earth's surface by exogenous geological agents, such as surface water, waves and other sea oscillations, wind, temperature changes, and human action.

- **Estimate.** Value that the estimator gives when it is applied to a concrete case.

- **Estimator.** Method or procedure of obtaining an estimate from a random sample of data (e.g. the mean sample is a point estimator of the mean population).

- **Evaporation fog.** Fog caused by the advection of cold air over warm water or warm or moist land. This air is not saturated and when it mixes with cold dry air, it becomes saturated and causes the appearance of fog.

- **Event.** Outcome formed by a sample element or a combination of sample elements, which represents a manifestation or project state. The sample elements are the simplest events that permit the description of the set of possible events.

- **Exceedance duration.** Time interval (e.g. meteorological year or useful life) during which the state descriptor remains higher than the selected value.

- **Exceptional work conditions.** Set of project states associated with certain project factor values that have a very low probability of exceedance, and which have a probability of occurrence much smaller than the predominant project factors that define extreme work conditions. Their occurrence can be unexpected and accidental or they can occur for foreseeable use and exploitation reasons. These conditions can be either predictable or unexpected. When they are unforeseen, they can be caused by environmental agents or by accidents.

- **Extratropical cyclone.** Low-pressure meteorological weather system at a synoptic scale in the middle latitudes of the Earth. Also known as a *mid-latitude cyclone*.

- **Extreme value.** Largest value that a random variable can take in a given number of observations.

- **Extreme work conditions.** Conditions defined by foreseeable maximum values of the variable in a period of time.

- **Failure domain.** Domain defined by the set of state descriptor values or variables that defines the failure for which the safety margin S is less than or equal to zero.

- **Failure mode.** Geometric, physical, mechanical, chemical or biological form or mechanism that causes the structure or one of its components to go out of service because of structural reasons. A failure mode is assigned to an ultimate or serviceability limit state for its verification.

- **Fetch.** Length of unobstructed open sea surface across which the wind can generate seas.

- **Filler.** Natural materials from the Earth's crust (e. g. soil, rocks) or special artificial elements (e.g. stone column, tetrapod) or industrial or urban waste material.

- **Fitness for service/Functionality.** Complementary value of the joint probability of failure in a given project phase or sub-phase as opposed to the failure modes assigned to serviceability limit states.

- **Floating area.** Area where vessels are berthed.

- **Floating breakwater.** Breakwater in a harbor area to protect a zone of short-period oscillations. It is fastened to the ground by anchors and cables, and oscillates under the action of the sea waves.

- **Flood line.** Maximum height attained by the water because of the joint action of the astronomical tide, meteorological tide, and the impinging sea waves on a beach.

- **Foehn effect.** Warm dry wind on the lee side of a mountain range, whose temperature is increased as the wind descends down the slope. It is created when air flows downhill from a high elevation, raising the temperature by adiabatic compression. The term was originally applied to a wind of the Alps but is now used as a generic term for all winds of this type. Also *Föhn effect*.

- **Fog.** Cloud of small water droplets near ground level and sufficiently dense to reduce horizontal visibility to less than 1.000 m.

- **Forcing agent** (in atmospheric and marine environments)**.** Agent causing sea level oscillations.

- **Forchheimer Regime.** Hydraulic interstitial flow regime of permeable soils, fillers, and layers. During the semi-cycles of this regime, the flow is turbulent.

- **Foundation.** Section of a breakwater in contact with the land surface, and thus, the point where stress is transferred to the ground and soil.

- **Free modes.** Oscillations in water bodies. Their celerity of propagation is $\sqrt{gh}$.

- **Free surface.** Surface of contact between the air (atmosphere) and a fluid (generally water).

- **Freeboard.** Vertical distance between the still water level and the upper part of a structure.

- **Frequency.** Inverse of the period $f = 1/T$.

- **Friction.** Resisting force between two surfaces that are sliding against each other, and where one opposes the movement of the other. It arises because of imperfections, even microscopic ones, between the surfaces in contact.

- **Frontal fog.** Fog formed by frontal precipitation falling into the colder air ahead of the warm front, which causes the air to become saturated through evaporation.

- **Froude number.** Dimensionless number that represents the ration between inertial and gravitational forces that act on water, $Fr = u/\sqrt{gh}$, where v is the velocity of the flow; h is the depth; and g is the acceleration due to gravity.

- **Fully developed sea (FDS).** Ocean waves or sea state that have the maximum height possible for a given wind speed, fetch, and duration of wind.

- ***Galerna*.** Sudden violent northwest wind that blows in the Cantabrian Sea and its coastline, generally in the spring and fall. It can be classified as a coastally-trapped disturbance.

- **GD** (Geodetic Datum)**.** Reference from which geographic measurements are made.

- **General Algebraic Modeling System (GAMS).** Modeling system for mathematical programming and optimization with a high number of variables and restrictions.

- **General filler.** Filler located in zones far from the structure and/or without any structural role.

- **General nature of the subset.** Indicator for measuring the relevance of a given subset in terms of its economic, social, and environmental impacts when it is destroyed or when its functionality is irreversibly damaged. It thus reflects the magnitude of the consequences derived from the failure of the maritime structure after it has begun to operate.

- **Geographic North.** Point in the northern hemisphere where the Earth's axis of rotation meets the Earth's surface, and where all lines of longitude converge.

- **Geological strength index (GSI).** Index that estimates the rock mass strength and rock mass deformation parameters. The guidelines given by the GSI system are for the estimation of the peak strength parameters of jointed rock masses.

- **Geostrophic wind.** Theoretical wind that would result from an exact balance between the Coriolis force and pressure gradient force (geostrophic approximation or equilibrium). To simplify the problem, centripetal acceleration and friction are eliminated from the equations.

- **Geotechnical state.** State that describes and characterizes stresses, interstitial pressures, and strain rates of the soil.

- **Global circulation.** Large-scale movement of air, and the means (along with the smaller ocean circulation) by which heat is distributed on the surface of the Earth. Since solar radiation causes the warming of the tropics and the cooling of the poles, global circulation compensates this temperature difference.

- **Granular filler.** Filler composed of gravel and/or sand extracted from the ground, and which has very little fine-grained material.

- **Graphical Method.** Method of applying a probability model to a data series of one random variable by means of probability paper.

- **Green's Law.** Energy flux conservation equation applied to long waves.

- **Groin.** Linear structure (length is predominant over the other dimensions), composed of large rocks or prefabricated concrete slabs, located in the water, rivers, streams, or near the coastline with a view to directing the water flow in a given direction, reducing the wave action, or favoring the sedimentation of the sand.

- **Ground surface.** Upper part of the ground on which the fluid boundary layer is located, and which is subject to the direct action of acceleration and velocity fields, due to the oscillatory motion of the water. This action can result in soil erosion, transport, and accumulation.

- **Group celerity.** Propagation velocity of the energy of oscillatory motion. In deep water it is equal to half the phase celerity. Also known as *group velocity*.

- **Gust.** Strong, sudden wind of short duration.

- **HAMSOM.** Tridimensional model of ocean circulation developed by the IFM (Institute für Meereskunde, Hamburg) and by *Clima Marítimo* (*Puertos del Estado*, Madrid). The HAMSOM model is based on a set of seven differential equations in partial derivatives. The unknowns are the three components of velocity, pressure, water density, salinity, and temperature.

- **Harbor mouth.** Entrance or exit of a port or dock area.

- **Head.** End of a breakwater, which is generally the part most exposed to the action of sea or ocean waves.

- **Hemisphere.** Half of the terrestrial globe, or a projection of the same in a map or picture. The Earth is considered to be divided into two hemispheres by the Equator: the Northern Hemisphere and the Southern Hemisphere.

- **High tide (high water).** Maximum sea level (due to the tide wave) between two upwards crossings of the mean sea level.

- **High water equinoctial spring tide (HWES).** Maximum high sea level of the spring tide during periods near the equinox.

- **Histogram.** Multiple-bar graphical representation, used in statistics to represent the frequency distribution of a group as a function of some variable. The frequency of each class is proportional to the length of its associated bar. The vertical axis represents the frequencies, and the horizontal axis represents the values of the variables.

- **Hooke's Law.** Law that states that the extension of an elastic material is in direct proportion to the force *F* applied to it.

- **Hopper barge (dump scow).** Flat vessel with one or more compartments between the fore and aft bulkhead, designed to carry granular materials (rocks, sand, soil, rubbish, etc.) for dumping into a water body. The barge is formed by a peripheral steel pontoon, and the bottom of its hull is made up of two doors that open downwards to deposit the material onto the sea, ocean, or lake bottom.

- **Huygens Principle.** Method of analysis for analyzing the single slit diffraction of waves.

- **Hydraulic filler.** Filler that is hydraulically deposited by means of the sedimentation of solid particles in an effluent (e.g. material obtained from dredging).

- **Hydraulic jump.** Sudden change in water level, frequently seen in open channel flow because of a change in hydraulic flow regime, which goes from slow to fast, or vice versa.

- **Immersion or submersion line or level.** Water height at which the crown of the structure is underwater.

- **Incident energy** or **energy flux.** Rate of energy flow through a reference or control volume surface.

- **Infragravity wave.** Gravity wave whose period range is 50-500 seconds.

- **Initial breakdown state.** State of the structure in which there is an incipient breakdown in one of its sections caused by one of the principal failure modes.

- **Inland waters.** Water bodies such as tidal inlets and estuaries that long waves can only access through a channel whose width is much smaller than the wavelength.

- **Inner continental shelf.** Shelf that begins at a depth of approximately 50 meters and extends to the coastline, and where shoaling and wave refraction are important. Furthermore, it is the area where sediments actively participate in the formation of the shoreline. Its width is on the order of hundreds of meters.

- **Instantaneous variable.** Variable that describes the instantaneous movements of a fluid, vessel, particles, etc. and which can be kinematic (velocity and acceleration of the fluid particle), or dynamic (pressures and tangential stresses on the surface of the particle, and forces per volume unit).

- ***Instituto Geográfico Nacional* (IGN).** National Geographic Institute of Spain.

- ***Instituto Hidrográfico de la Marina* (IHM).** Hydrographic Institute of the Spanish Navy.

- **Intrados.** Outer surface of a soil containment structure.

- **Inverse shoaling.** Propagation of a wave train through deep waters, which causes wave steepness to decrease (opposite of shoaling).

- **Iribarren number.** Number or parameter used to calculate the stability of sloping breakwaters, and when sea waves will break on or be reflected by the slope. It is represented by

$$I_r = \frac{\tan(\alpha)}{\sqrt{\frac{H}{L_o}}}$$

where α is the slope gradient; H is the wave height in deep water; and L_o is the wavelength in deep water.

- **Irregular wave train.** Series of waves with different periods, heights, and directions.

- **Isobar.** Contour line drawn on a weather map connecting points of equal barometric pressure on the surface of the Earth.

- **JONSWAP (Joint North Sea Wave Project) Spectrum.** Multiparametric theoretical frequency spectrum developed by Hasselman and based on the Pierson-Moskowitz spectrum, fit to real spectra measured in the North Sea. It defines partially developed seas, in other words, those situations in which the sea waves are limited by the fetch and the duration of the wind.

- **Katabatic wind.** Wind that is created by cool air flowing down the slope of a mountain or glacier.

- **Keulegan-Carpenter Number.** Dimensionless quantity describing the relation between the displacement of the water particle in the wave cycle and the dimension of the obstacle. It is represented by $KC = U\,T/D$, where U is the maximum horizontal velocity of the water particle due to oscillatory motion; T is the period; and D is the principal dimension or obstacle to the movement of the wave.

- **Kinematic viscosity.** Measure used in fluid flow studies, usually expressed as the absolute viscosity divided by the density of the fluid. Its unit is the stoke or centistoke (cm^2/seg). It is usually represented by v.

- **Laminar regime.** Slow, smooth, stratified, non-turbulent flow in which the fluid travels in parallel streamlines if the stream is between two parallel planes, or in coaxial cylindrical layers.

- **Latent heat.** Amount of energy absorbed by a chemical substance during a change of state, from solid to liquid (latent heat of fusion) and from liquid to gas (latent heat of vaporization). When gas becomes liquid or when liquid becomes solid, the same amount of energy is released.

- **Leeward.** Side of the structure that is sheltered from the wind.

- **Leeward side.** Side of the structure from which the waves come.

- **Leibniz Rule.** In mathematical analysis, formula used to find the derivatives of products of function. Also known as *product rule*.

- **Level I Method.** Verification method of failure modes in which project design factors and term values are determined generally with deterministic criteria. Examples of this type of method include the global and partial safety coefficient methods.

- **Level II Method.** Verification method in which the verification equation is defined according to first-order statistical moments, and in which, thanks to functional transformations, it is expressed in terms of reduced and independent Gaussian variables. This method relates the failure probability to the minimum distance from the origin of the coordinates to the surface of the failure ($G = 0$), which is a verification equation in safety margin format. In the time interval, the distribution functions and covariance of the project factors should be known.

- **Level III Method.** Verification method used to obtain the solution of the verification equation by integrating a multidimensional function in the failure domain. This integration is complex, and generally, the failure domain and project factor values can be obtained by numerical simulation techniques (e.g. Monte Carlo simulation). In the time interval, the joint distribution of the project factors intervening in the verification equation should be known.

- **Limit states method.** Calculations to verify that for each failure or stoppage mode, the project requirements are met in regards to reliability, functionality, and operationality in all project phases and states, assigned to limit states of safety, serviceability, and use and exploitation.

- **Liquefaction.** Process by which a mass of soil or sediment is temporarily transformed into a fluid mass, and thus loses its load-bearing capacity.

- **Lithology of the rock mass.** Characteristics and properties of a rock, such as its structure, color, mineral composition, grain size, and arrangement of its component parts.

- **Littoral.** Zone on a shoreline between mean high tide and mean low tide. It includes the inner continental shelf and the surf zone. As such, it is where maritime structures and port installations are generally built.

- **Littoral current.** Current in the nearshore region, generally caused by the action of wind, astronomical tide, and global circulation.

- **Loading cycle.** Sequence of meteorological states that begins when certain statistical descriptors that define the state exceed a given threshold, and which ends when they fall below it again. Sea loading cycles are also known as *storms*.

- **Local datum** or **port zero.** Arbitrary datum defined by the local port authorities as a reference for sounding and topography in the Port.

- **Local resonance.** Resonance produced when the oscillation that reaches the port or shoreline area has periods similar to those of the area or of one of its wharfs or littoral zones.

- **Long wave.** Wave whose relative depth satisfies the following condition: $h/L \leq 1/20$.

- **Long-crested wave.** Sea wave whose height is constant or varies only slightly.

- **Long-period oscillation.** Oscillation with a period band ($T > 3$ *hours*), and which principally includes the meteorological tide related to the passage of a storm and astronomical tide.

- **Longshore current.** Littoral current that moves parallel to the shore, usually generated by waves breaking at an oblique angle to the shoreline.

- **Low water (low tide).** Lowest level of the sea surface (due to tide waves), which occurs between two flood tides.

- **Low water equinoctial spring tide (LWES).** Exceptionally low water spring tide that occurs at the time of the equinox, when night and day are approximately equal.

- **Lunar semidiurnal tide.** Tide having two high waters and two low waters during a lunar day.

- **Main alignment.** Breakwater section that controls and shelters the land from sea oscillations.

- **Main layer.** Outermost layer and most resistant element of a sloping breakwater, against which wave energy is dissipated.

- **Main propagation direction.** Direction with the maximum energy flux.

- **Maneuvering area.** Area in or bordering the harbor area where vessels may stop or start to navigate as well as change direction.

- **Marginal function.** Density and distribution functions of one of the variables, independently of the other.

- **Maritime climate.** Characterization of the sea over long periods of time or the statistical description of temporal variation in terms of sea states at a given location. It can be defined according to one-dimensional and bidimensional statistics of geometric-statistical and spectral parameters, representative of the sea state in the area under study.

- **Mass concrete.** Concrete that has no structural reinforcement.

- **Maximum annual or extreme regime.** Distribution function of the largest value of the loading cycle peaks in each meteorological year.

- **Maximum duration of a stoppage.** Maximum expected time of an operational stoppage, expressed in hours.

- **Maximum likelihood method.** Statistical method in which the likelihood distribution is maximized so as to produce an estimate to the random variables involved.

- **Maximum peak.** Maximum state of all peaks in a given time interval (usually one year).

- **Maximum wave height.** Height of the highest wave in a time record or in a wave train in a given sea state.

- **Mean.** Total quantity of the variable, equally distributed among each observation.

- **Mean energy flux per unit of crest width.** Value that defines the energy that passes through a section from the seabed to the mean sea level (in linear theory).

- **Mean local sea level (MW).** Mean water level based on periodic measurements taken at a specific location.

- **Mean period.** Average period of the waves observed, weighted by wave energy.

- **Mean solar day.** Average of the true solar day, corresponding to Coordinated Universal Time or Greenwich Mean Time. The mean solar day is equivalent to $86{,}400\ seconds$, a unit based on very precise atomic properties, which makes it possible to measure the time difference between it and the true solar day.

- **Mean square velocity.** Average value of the square of the velocities of a group of particles.

- **Mean still water level (MSWL).** Reference level in the absence of any oscillation or current.

- **Meander.** Extreme U-bend in the course of a river or stream with a sinuosity of over 1.5, and usually occurring in a series and in streams flowing over a very shallow elevation grade.

- **Mechanism.** Way in which a failure or stoppage occurs.

- **Medium depths.** Area in the sea or ocean where the wave touches bottom, and $(1/20 < h/L < 1/2)$, where h is the depth and L is the wavelength.

- **Medium-period oscillation.** Oscillation composed of barometric pulses, meteo-tsunamis, and tsunamis, which cause oscillations on the free surface with a period of $5\ min < T < 4\ hours$.

- **MELWES.** Mean extreme low water equinoctial springs, which often corresponds to the mean value of the “lowest low waters” in a given area.

- **Mesoscale.** Scale of meteorological phenomena that range in size from several kilometers to approximately 100 kilometers. Smaller phenomena are classified as microscale. The storm scale of cumulus cloud systems, though larger, is classified as synoptic-scale. Examples of mesoscale systems are sea breezes.

- **Meteo-tsunami.** Rapid pressure pulses and medium-intensity winds that produce oscillations on the free sea surface, formed by a small number of wave groups in respect to a reference level.

- **Meteorological chart.** Weather map showing the spatial distribution in an instant of time of high and low pressure frontal systems and other phenomena. Furthermore, the wind can be approximately obtained for a given area by using the geostrophic wind scale. This is done by calculating the distance separating the isobars in degrees and the latitude of the location.

- **Meteorological state.** State that describes and characterizes the simultaneous manifestation of atmospheric agents (wind speed and direction, precipitation, fog, etc.) and marine agents (sea waves, meteorological and astronomical tide, other long-period oscillations and currents).

- **Meteorological tide.** Variations in the sea level produced by variations in the atmospheric pressure and the wind on the surface of the water.

- **Meteorological year.** Time period from 1 October until 30 September of the following year, which is regarded as the meteorological pulse of the planet.

- **Minimax approach.** Criterion that minimizes the maximum failure of a given number of terms.

- **Moments method.** Method of estimating the parameters of a frequency distribution by first computing as many moments of the distribution as there are parameters to be estimated and then using a function that relates the parameters to moments.

- **Monochromatic wave.** Oscillation on the free sea surface that is identically repeated in time and space.

- **Monochromatic wave train.** Series of waves of equal period, height, and direction. Also known as a *regular wave train.*

- **Monsoon.** Seasonal wind caused by the deflection of permanent winds from their normal path through the heating of a neighboring land mass, especially in the Indian Ocean and southern Asia.

- **Monte Carlo Simulation.** Analytical technique for modeling a process or system subjected to random forcings, and calculating the probability distribution of possible outcomes. It involves performing a large number of trial runs called simulations, and inferring a solution from the collective results of the trial runs. The objective is to understand the behavior of the system or to evaluate various strategies that can be used to operate the system. Also called *Monte Carlo Method.*

- **Mountain waves.** Wave in the atmosphere that occurs when in potential temperature conditions that grow steadily with height, air is forced to circulate over a geographic landform. Sometimes it is marked by lenticular clouds to the lee side of mountain barriers.

- **MSPE-UGR software program.** Public domain software program of the University of Granada that resolves the elliptic approximation of the mild slope equation (MSPE), including second-order terms in seabed variation, and can be used to model diffraction, reflection, shoaling, and refraction effects. The MSPE is resolved by means of finite elements in a closed domain, once the reflection and dissipation boundary conditions are specified that define and close the domain. The model includes dissipation effects because of friction and breaking by means of different empirical models.

- **Mutually exclusive modes.** Modes in which the occurrence of one mode excludes the occurrence of the others.

- **Nautical chart.** Graphical representation of navigable waters and adjacent land areas. It usually indicates water depth and land height, as well as the natural features of the seabed, details of the coastline, navigational hazards, locations of lighthouses, details of tides and currents, as well as natural and man-made aids to navigation.

- **Navigation area.** Area for vessel traffic.

- **Navigation channel.** Long, narrow, natural or artificial channel in surface waters for the one-way traffic of ships and other vessels. It may be bounded by natural landforms, including those forming separation zones.

- **Neap tide.** Tide that coincides with the first and last quarter of the moon, and which has a small tidal range because the gravitational forces of the Moon and Sun are perpendicular to each other.

- **Nivmar.** Set of applications that provide a short-term prediction of the sea level (48 hours) in terms of the meteorological and astronomical tide. Nivmar is based on the harmonic prediction of the sea level, and on the use of the HAMSOM and HIRLAM numerical models. Additionally, Nivmar uses tide gauge data provided by REDMAR to correct systematic deviations that cannot be resolved by the circulation model.

- ***NMMA*** (*Nivel Medio del Mar en Alicante*) [Mean Sea Level in Alicante]**.** Reference level for the geometric altitudes of the geodesic signals distributed throughout Spain.

- **Nodal tide.** Tide that occurs because of the backwards movement along the ecliptic of the ascending node of the Moon, defined as the point at which the Moon crosses the ecliptic from south to north. This backwards movement takes 81.61 years to complete one revolution. It causes the declination of the Moon to vary between 18.5 and 28.5 degrees during that period, and this variation produces the nodal tide.

- **Node.** Points at which $kx = \pi/2, 3\pi/2 \ldots$ At these points the free surface is never separated from the mean surface; horizontal velocity is maximal; and vertical velocity is zero.

- **Non-conservative substance.** Substance that is modified during natural or artificial processes of transportation or mixing (e.g. carbon or nitrogen).

- **Non-principal mode.** Failure or stoppage mode for which small increases in the total costs of the structure significantly improve the reliability, functionality or operationality of the structure when affected by the mode.

- **Normal work conditions.** Conditions in which an installation can operate without any constraints or limitations, and in which its exploitation or operationality is not affected by environmental elements.

- **Null event.** Event that does not contain any sample elements.

- **Oblique incidence.** Angle at which wave trains impinge on a breakwater or coastline.

- **Obukhov Length.** Parameter that quantifies the proportion of turbulence kinetic energy to the sensible heat flux on the surface.

- **Ocean resonance.** Resonance produced when the propagation speed of the barometric pulse is equal to the celerity of the wave.

- **OERI (Operational economic repercussion index).** Index that quantitatively evaluates the costs of the operational stoppage of a subset of the structure.

- **Open sea (high seas).** Continuous body of salt water, which is navigable and lies outside territorial waters of any country, and thus is far from geographic landforms or maritime structures.

- **Operational cycle.** Time during which the value of certain descriptors that define the state continuously remains above a threshold value.

- **Operational limit state.** State in which the exploitation of the port area is temporarily reduced or suspended because of causes external to the structure without any structural damage.

- **Operational nature.** Indicator of the economic, social, and environmental impacts that may occur when operationality conditions in breakwaters or sheltered areas are reduced or not attained. It thus reflects the magnitude of the consequences of the operational stoppages in the serviceability phase.

- **Operational stoppage.** Period of time between two continuous operational cycles.

- **Operationality.** Complementary value of the stoppage probability in a project phase or subphase, as considered in relation to the stoppage modes assigned to the operational stoppage limit states.

- **Orthogonal line.** Tangent lines to the wave propagation direction at each point.

- **Oscillatory regime.** Regime of the set of sea oscillations.

- **OSERI (Operational social and environmental repercussion index).** Index that qualitatively estimates the social and environmental impact of an operational stoppage mode of the maritime structure. For this purpose, it values the possibility and consequences of the loss of human life, damage to the environment, loss of historical and cultural heritage, and social disruption.

- **Outer continental shelf.** Shelf that has a slope of approximately 1 degree, a lower order of magnitude than the continental shelf, and a horizontal extension on the order of tens of kilometers. It is generally characterized by intermediate depths $[1/10 < h/L < 1/2]$ for wave action.

- **Oversplash.** Water reflected by a breakwater or seawall after a wave impacts against it.

- **Overtopping.** Part of the water carried over the top or crest of a coastal defense structure as a consequence of wave action, and which does not return directly to the sea.

- **Parallelepiped method.** The propagation of the wave train depends on the averaged performance of a set of points, specifically, the vertices and center of the parallelepiped. Once the dimension of the side of the parallelepiped (on the order of $L/4$) is selected, the method avoids the problem of the caustics or ray crossovers, and the instantaneous response of the wave train because of sudden changes in depth.

- **Parapet.** Low solid wall built along the crown or crest of a maritime structure.

- **Parasitic wave.** Secondary crest that propagates in the channel, and which distorts the wave profile.

- **Partial operation.** State during the construction phase during which the breakwater or one of its subsets provisionally begins to function. The admissible failure probability during this transitory phase should be specified in the project design. Accordingly, all of the social and environmental consequences of the failure will be considered in this type of situation, including the conditions related to the construction process.

- **Partially developed sea (PDS).** Sea that represents the sea states in which either because of limitations related to time or generation surface, some of the spectral components of the frequency band, related to the generating wind do not have the energy of a fully developed sea.

- **Partially stationary motion.** Motion profile that slowly advances, slightly oscillating around semi-fixed points (quasi-nodes), between which there are other points of maximum oscillation (quasi antinodes).

- **Partially stationary wave.** Oscillatory motion of the water surface that propagates in such a way that it exhibits small-amplitude oscillation points (quasi-nodes) and other points that oscillate with a maximum amplitude (quasi-antinodes).

- **Peak.** Maximum state in the loading cycle.

- **Peak period.** Period with the maximum wave energy, as determined from the wave spectrum.

- **Peak-over-threshold Regime (POT).** Distribution function of the maximum states (those above the threshold value) in each time interval (usually, a year).

- **Penetration depth.** Distance at which the wave train reduces its energy to $1 = e$ (number e).

- **Perigee.** Point in the elliptic orbit of a celestial body (whether natural or artificial) around the Earth where it is at the least distance from the Earth's center.

- **Perihelion.** Point in the orbit of a celestial body that is nearest to the Sun. It is represented by q. If a is the mean distance, and e is the eccentricity, then $q = a\,(1 - e)$.

- **Permeability.** Property of material (sand, crushed rock, soft rock in situ) which permits movement of a fluid through its pores without any change in its internal structure. A material is permeable if a significant quantity of fluid can pass through it in a given time interval, and is regarded as impermeable if the quantity of fluid is negligible.

- **Persistence of the loading cycle.** Time interval during which a state descriptor uninterruptedly remains above the threshold value. Like exceedance, persistence is also a random value.

- **Persistence of the operationality cycle.** Uninterrupted time interval between a downwards crossing and the next upwards crossing of the threshold value of the state descriptor.

- **Phase-averaged method.** Method that resolves wave train propagation by means of a phase-averaged magnitude.

- **Phase-resolving method.** Method that resolves the instantaneous performance of the free surface

- **Piece of the main layer.** Large quarry rock or prefabricated concrete block in a shape, such that it can be used as the principal protection against wave action in the main layer of a sloping breakwater.

- **Pierson-Moskowitz spectrum.** Wave frequency spectrum that permits the definition of fully developed seas in deep waters.

- **Pile.** Construction element that is driven or otherwise embedded in the ground as a support, protection, or foundation of a structure, and which is capable of transferring loads to a more resistant ground layer, when a more conventional foundation of blocks or slabs is not viable.

- **Plastic soil.** Soil in which the deformation remains after the stress disappears.

- **Plunging breaker.** Breaker in which the crest becomes almost vertical, then curls over an air pocket, and collapses with great violence. It is associated with beaches with steeper gradients, where wave energy is released suddenly $(0.5 < I_r < 2.5)$.

- **Point function.** Local function.

- **Poisson coefficient.** Ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative.

- **Polar stereographic projection.** Mapping that considers that the source of light is in the antipodes. The surface that can be represented is larger than a hemisphere. Its most characteristic feature is that the scale increases the farther away that it gets from the center.

- **Population.** Set of all possible values of a given project factor.

- **Pore index.** Volumetric ratio of gaps or holes in a solid mass.

- **Port area.** Area where port and logistic operations are performed, pertaining to the integral management of vessels, and all aspects of maritime transportation as well as its connections with land and air traffic. Sport, nautical, industrial and military operations are also carried out in this area.

- **Principal mode.** Failure or stoppage mode for which the improvement of the reliability, functionality or operationality of a subset of a structure is difficult, or can only be achieved with significant increases in the costs of the structure.

- **Probabilistic deterministic formulation.** Mathematical expression in which some of the predominant agents and actions are assigned, depending on their probability of exceedance, whereas others take nominal or determined values, independently of probability of exceedance.

- **Probabilistic formulation.** Formulation in which the values of the equation terms are determined, based on their respective probability models in the phase analyzed. They are calculated from probability models of parameters and agents.

- **Probability of the occurrence of the failure/stoppage mode in the time interval.** Product of the potential danger to a structure and its vulnerability.

- **Probability paper.** Graph paper with one axis specially ruled to transform the distribution function of a specified function to a straight line when it is plotted against the variate as the abscissa.

- **Progressive oscillatory motion.** Motion profile that advances in only one direction with celerity C, and without modifying its shape.

- **Progressive wave.** Oscillatory motion of the water surface that propagates while maintaining the same profile.

- **Project.** Set of activities that includes the preliminary study and drafting of the project, construction, exploitation, conservation, repairs (if necessary) and the dismantling of a maritime structure.

- **Project phase.** Temporal sequence of project states during which the subset of a structure maintains the same main activity although it can have other secondary activities. Project phases include the following: preliminary studies and project design, construction, service, conservation, repairs, and dismantling. The duration of the serviceability phase is the useful life of the subset.

- **Propagation direction.** Direction in which a wave travels.

- **Protective layer.** One of the layers that make up the sections of a sloping breakwater.

- **Quay.** Long, narrow artificial dock that projects into the sea, and which allows the mooring of small vessels and the unloading of light cargos.

- **Radiance.** Radiant flux per unit solid angle and per unit projected area of a source in a stated angular direction from the surface.

- **Radiation.** Propagation energy in the form of electromagnetic waves or subatomic particles through a vacuum or material medium.

- **Radiation fog.** Fog caused by the cooling of moist air to below its dew point by the process of heat radiation. It is formed by near surface cooling by radiation loss during the evening hours. Also called *ground fog*.

- **Random value.** Measurable function that assigns real values to the elements of the sample space.

- **Random variable regime.** Conditioned or marginal joint distribution function of representative variables, once the time interval is selected for the description of an agent or agents, and the random variables characterizing the process are defined.

- **RANS (Reynolds-averaged Navier-Stokes equations).** Time-averaged equations of motion for fluid flow, which are primarily used while dealing with turbulent flows, and are generally resolved with the finite volume technique.

- **Rare event.** Event that almost never occurs, and which can be described by a Poisson process.

- **Rate of change.** Evaluation time of a given state.

- **Ray.** Lines that run parallel to the direction of the energy propagation.

- **Re-adaptation of the breakwater.** Modification of an existing structure, adjusting it to another use and exploitation for the rest of its useful life or for a new useful life.

- **REDCOS Database.** Database of measurements from the Coastal Buoy Network of *Puertos del Estado*.

- **REDEXT Database.** Database of measurements from the Deep Water Buoy Network (Outer Network).

- **REDMAR.** Set of measurements from the tide gauge network of *Puertos del Estado*. This network monitors the sea level along the coastline in Spain as it generates historical data series on which to base future simulations, evaluations, and predictions.

- **Reflection.** Process by which the incident energy produced when a wave train impinges on a breakwater is reflected and returned seaward. This process occurs whenever there is a sudden change in the geometric properties of the environment.

- **Reflection coefficient.** Ratio of the wave height reflected against an obstacle to the incident wave height.

- **Refraction.** Process by which the direction of a wave changes when it moves from one water depth to another or when it obliquely impinges on a current.

- **Refraction and shoaling coefficient.** Coefficient that quantifies the wave height variation due to seabottom effects, based on the wave height in deep waters.

- **Regime.** Distribution of one or various state variables in a given time interval. In hydrodynamics, this refers to specific flow conditions (laminar, turbulent, or oscillation, and Stokes or Boussinesq).

- **Regular wave train.** See *monochromatic wave train*.

- **Rehabilitation of a breakwater.** Modification of the structure to fit project requirements either because of the extension of the structure's useful life, the application of new legal regulations, the possible modification of infrastructures, or the improvement of use and exploitation resources.

- **Relative wave amplitude.** Ratio between wave amplitude (or height) and water depth.

- **Reliability.** Complement of the joint failure probability in the project phase or subphase against the failure modes assigned to the ultimate limit states.

- **Resonance**. Process in which there is an indefinite increase in the amplitude of oscillation since the oscillatory system is forced by its own oscillation period.

- **Return period.** Mean time interval (usually in years) that should pass before a given state descriptor threshold value is repeated or exceeded. If the value has a real return period of t_p years, the probability P that this value will occur be exceeded in a given year is: $P = 1/t_p$.

- **Reynolds number.** Dimensionless quantity that represents the ration between inertial and viscous forces in a fluid, as defined by $Re = UD/\nu$, where U is the maximum horizontal velocity of the water particle due to the oscillatory motion; D is the principal dimension opposing the passage of the wave; and ν is the kinematic viscosity.

- **Rigid soil.** Soil that does not experience deformation when a stress is applied.

- **Root mean square wave height.** Quadratic mean, which is a statistical measure of the wave height.

- **Rubble mound breakwater.** Breakwater whose core is made of quarry run and rock fill.

- **Run up / Run down.** Maximum and minimum levels, in respect to the still water level, reached by a wave on a beach face or coastal structure.

- **Safety margin.** $S = x_1 - x_2$, where x_1, x_2 stand for the actions that oppose and favor the occurrence of the failure mode.

- **Safety threshold state.** State that describes and characterizes the beginning and end of a storm or loading cycle. It is usually related to extreme work conditions, in other words, the occurrence of the most severe meteorological states.

- **Salinity.** Dissolved salt content in a body of water, usually expressed in *ppm* or in *mg/l*.

- **Sample.** In statistics, part of a population.

- **Sample space.** Set of all possible values of the population.

- **SAPO (Sistema de Predicción de Oleaje).** Local wave forecasting system at Spanish ports.

- **Sea.** Alterations of the sea surface caused by the wind on a surface during a given period of time. This phenomenon produces a set of more or less irregular random waves with different propagation directions and with periods ranging from one to thirty seconds.

- **Sea reference level (SRL).** Reference level for the representation of the local bathymetry. It should be indicated on each bathymetric chart and related to the topographic reference level.

- **Sea state.** Time interval in which any sea wave manifestation can be considered to be statistically stationary. It can be characterized by representative wave height and period (e. g. significant wave height, mean period or peak period).

- **Sea state curve.** Curve that represents the sequence of sea states that occur over time. It represents the evolution of a given statistical parameter in the sea state, such as the significant wave height.

- **Sea state regime.** Distribution function of sea states in a mean year in which the significant wave height is used as the statistical descriptor. Also known as *mean regime*.

- **Sea wave spectrum.** Function that expresses wave energy in the frequency domain.

- **Secondary alignment.** Breakwater section that joins other breakwater sections.

- **Secondary layer.** Inner layers of a sloping breakwater, which support the main layer, and are the transition to the core.

- **Seiche.** Short-period oscillation in an enclosed or semi-enclosed body of water, such as that found in a port area.

- **Seism.** Tremor of the earth's surface, usually triggered by the release of underground stress along fault lines within the Earth. The released energy passes through the Earth as seismic waves (low-frequency sound waves), which cause the shaking. The point at which the earthquake originates is called its hypocenter. When earthquakes occur under water, they can produce a tsunami.

- **Senoidal wave.** Oscillatory motion described by the mathematical sine function.

- **Sensible heat.** The heat absorbed or released when a substance undergoes a change of temperature, but not a change of state.

- **SERI (Social and environmental repercussion index).** Index that qualitatively evaluates the expected social and environmental impact in the event of the destruction or total loss of operationality of the maritime structure. It values the possibility and consequences of the loss of human life, damage to the environment, loss of historical and cultural heritage, and social disruption. The failure in the structure is always considered to have occurred when the economic activities directly related to the structure have been consolidated.

- **Serviceability limit state.** Project state in which the structure as a whole or one of its subsets or elements becomes unusable or out of service because the service requirements specified in the project are not met. In this state, there is a reversible or irreversible loss of functionality in the structure or one of its parts, due to environmental, aesthetic or structural failure or because of a legal determinant. This state considers all those failure modes that reduce or condition the use and exploitation of the structure, and which can signify a reduction in the useful life or the probability of its survival because of the deterioration of the properties of construction materials or soil, of strains or excessive vibrations in the structure for the use and exploitation of the structure or accumulative geometric alterations.

- **Shallow depths.** Area in the sea or ocean at which the entire water column is subjected approximately to the same horizontal velocity field, and $(h/L < 1/20)$, where h is the depth and L is the wavelength.

- **Shear strength.** Internal strength per unit of area of soil mass against a fault or landslide along the plane of one of its cross-sections.

- **Shear wave.** Instability of the longshore current generated by the oblique incidence of waves. The period of this wave is usually in the range $(100 < T\,(s) < 600)$.

- **Shelf wave resonance.** Resonance produced when the barometric pulse and the corresponding ocean wave have periods that coincide with the periods of the continental shelf.

- **Sheltered area.** Water and land surface sheltered from the action of atmospheric and marine dynamics.

- **Shields parameter**. Dimensionless number that relates the tangential stress in the seabed to the stress that the initiation of motion of sedimentation in the flowing water would produce.

- **Shoaling.** Propagation of a wave train from deep to shallow waters, which causes wave height to increase. Both wave celerity and length decrease, whereas the wave amplitude varies as a consequence of the reduction in the energy propagation velocity. These two modifications result in a change of the wave steepness value H/L.

- **Short wave breaking.** Process that occurs when short-period waves in the sea state break because of wind action or because they are overtaken by waves with a longer period. They are easily detectable because of the whitecaps on the sea surface produced by this process.

- **Short-crested wave train.** Ocean wave whose crest is of finite length. It is applied to the encounter of waves coming from different directions, and to seas, representative of generation states. Its height varies along the crest in smaller distances than the wavelength.

- **Short-period oscillation.** Oscillation with a period in the interval $[3 < T(s) < 300]$, which is very irregular, and has at least three different temporal (period) and spatial (wavelength) scales: capillary waves, sea waves, and wave groups.

- **Sidereal day.** Period of time that it takes to complete one Earth rotation relative to the position of a fixed star, or between two successive crossings of the first point of Aries (vernal equinox). It can also be defined in respect to the first point of Libra (autumnal equinox). The sidereal day is approximately four minutes shorter than the mean solar day.

- **Sidereal year.** Time required for the Earth to cross the same point of its orbit twice, relative to the fixed stars. Generally used by astronomers, it is the most accurate measure of a year, and has a duration of 366.56436918716 sidereal days, which corresponds to 365.256363 mean solar days (365 days, 6 hours, 9 minutes and 9.7632 seconds).

- **Significance test.** Statistical test aimed at demonstrating the probability that observed patterns cannot be explained by chance. It is performed to determine if an observed value of a statistic differs enough from a hypothesized value of a parameter to draw the inference that the hypothesized value of the parameter is not the true value, and if the data significantly differs from the distribution tails.

- **Significant wave height.** Mean height of the one-third highest waves, recorded during a given time interval.

- **Significant wave period.** Mean period of the one-third highest waves recorded during a given time interval.

- **Sill.** Secondary breakwater that provides protection from less severe waves and marks the boundaries of the harbor entrance and the port area.

- **Simar-44 Database.** Database of time series of atmospheric and oceanographic parameters coming from the high resolution numerical modeling of the atmosphere, sea level and waves affecting the Spanish coastline and coastal zone.

- **Simple compressive strength.** Resistance of a deformable solid or continuous medium to loading or pushing forces, which is characterized by a reduction in volume or in a given direction.

- **Simple harmonic motion.** Repetitive back-and-forth movement through a central, or equilibrium, position in which the restoring force is proportional to the displacement.

- ***Sistema de modelado costero* (SMC) [Coastal Modeling System].** Public domain software program that applies a parabolic propagation model coupled to another vertically integrated model that calculates the equations of the current and free surface oscillations in the surf zone (*www.smc.unican.es*).

- **Sloping submerged breakwater.** Breakwater whose section geometry is similar to that of the emerged breakwater without a parapet. However, its crest level is underneath the water.

- **Snell's Law.** Formula used to calculate the relationship between the angles of incidence and refraction, when referring to ondulatory motion (e.g. light or sea waves) passing through a boundary between two different isotropic media, which changes the propagation conditions (density of the fluid or water depth).

- **Solitary wave.** Long wave that propagates without changing shape, and which consists of a single elevation above the water surface. It is obtained as a solution of the Kortwev-de Vries equation.

- **Soliton.** Solitary wave that retains its shape as it travels in a nonlinear system. These waves arise as the solutions of nonlinear differential equations, describing physical systems.

- **Spatial domain.** Surface on which an activity takes place, whether physical or mathematical. In the case of oscillatory motion, this domain is defined according to the wavelength L and the relative depth, h/L, where *h* is the water depth.

- **Spectral peak period.** Period in which the spectrum has its maximum energy content.

- **Spectral width.** Standard deviation radius of the spectrum surrounding its mean frequency.

- **Spilling breaker.** Breaker whose crest collapses gradually over a nearly flat bottom. The energy dissipates slowly as the water spills down continuously over the advancing wave front ($I_r < 0.5$).

- **Spring tide.** Tide of maximal range, which occurs at or near the time of the new or full moon, when the Sun and the Moon are aligned with the Earth. As a result, it shows the most extreme values in respect to the mean sea level.

- **Spurious reflection.** Wave reflection that is not genuine.

- **State curve.** Curve that represents the temporal evolution of a state descriptor at a given point in the sea.

- **State descriptor.** Descriptor that represents the statistical or frequency variability in a given state.

- **Stationary motion.** Motion that does not depend on time.

- **Stationary oscillatory motion.** Motion profile that does not advance, but only oscillates without changing its shape around fixed points (nodes), between which are other points of maximum oscillation (antinodes).

- **Stationary regime.** Hydraulic conditions in which the movement of the fluid does not depend on time. Also known as *permanent regime*.

- **Stationary wave.** Surface wave that does not propagate, but which has fixed points (nodes) that do not oscillate and other points (antinodes) that only oscillate vertically. Also called *standing wave*.

- **Statistic.** Function derived from a set of data containing one or more random variables, which is quantifiable, and which does not depend upon any unknown parameter.

- **Statistical inference.** Process of reaching conclusions concerning a population upon the basis of random samplings.

- **Stokes conjecture.** Conditions for wave breaking, namely, limit wave steepness, horizontal velocity or vertical acceleration of water particles in the wave crest.

- **Stokes regime.** Wave propagation conditions that are governed by Stokes-type equations, obtained by means of a series expansion of governing equations and boundary conditions of potential oscillatory flow with a small parameter, wave steepness, ak.

- **Stokes theory.** Wave description by means of the series expansion with a small parameter (wave steepness, ak) of the governing equations and boundary conditions of the potential oscillatory flow.

- **Stokes wave.** Wave that is described mathematically by a series expansion of its variables, according to a small number related to wave steepness. Its field of theoretical validity is in the domain, $h/L \geq 1/10$.

- **Storm.** Strong low pressure center or meteorological disturbance, which is associated with severe weather, such as strong winds and heavy precipitation.

- **Storm conditions.** Sea states that exceed a given significant wave height threshold. This threshold is different for each area, depending on its climate characteristics. It defines the loading cycle, generally associated with extreme work conditions.

- **Storm regime.** Distribution function of the annual maximum state in the year, in which the significant wave height is used as the statistical descriptor.

- **Structural concrete.** Concrete that is reinforced with iron bars, and which is capable of carrying a structural load.

- **Structural filler.** Filler located near a structure, which plays an important role in its stability or deformation.

- **Subduction.** Process describing when one tectonic plate collides with and is overridden by, or slides beneath an adjacent plate. Subduction zones involve an oceanic plate sliding beneath either a continental plate or another oceanic plate.

- **Submerged breakwater (reef breakwater).** Breakwater, generally parallel to the coastline, with a crest below low tide in order to reduce wave action by forcing the waves to break on the reef.

- **Submersion or immersion depth.** Water depth of a submerged structure. In a submerged breakwater, the water depth over the breakwater crest.

- **Sub-pressure.** Upwards force produced by the water on a structure, retaining element or submerged foundation.

- **Subset of a structure.** Continuous set of sections (or breakwater alignments) that fulfills a specific function in line with the objectives and exploitation requirements of the structure. This set is subject to the same action levels of all agents, particularly the predominant agents, and are part of the same formal and structural typology.

- **Summer solstice.** Point on the ecliptic, midway between the equinoxes at which the Sun reaches its greatest northern declination. This means that on June 21 in the northern hemisphere and December 22 in the southern hemisphere, the Sun appears to be directly overhead at the Tropic of Cancer.

- **Sunspot.** Dark colored region on the Sun that represents an area of cooler temperatures and extremely high magnetic fields

- **Surf zone.** Nearshore zone along which the waves become breakers as they approach the shore. Its width can range from tens to hundreds of meters.

- **Surging breaker.** Breaker in which the bottom of the wave catches up with the crest, and the breaker surges up the beach face as a wall of water (with the wave crest and base traveling at the same speed). The flow is similar to that of a stationary wave ($I_r > 3.5$).

- **SWAN.** Numerical model that quantifies wave propagation in medium and shallow water depths, and which maintains the open sea source terms and adds those for friction-induced dissipation and for breaking due to wave-bottom interaction.

- **Swell.** Ocean waves, generated by the action of the wind, which have traveled out of their generating area and exhibit a more regular and longer period.

- **SWL** (Still Water Level). Sea level without movement.

- **Synoptic chart/map.** Graphical representation of an isobar field (pressure variations) over a large area of the Earth at a given moment in time. It is an essential source of data concerning the mean atmospheric pressure and its spatial distribution during the passage of storm or when there is an anticyclone.

- **Temporal domain.** Time interval in which an activity is carried out, whether physical or mathematical. In the case of oscillatory motion, it is defined in terms of the wave period T and the number of waves.

- **Thermal wind.** Mean wind-shear vector in geostrophic balance with the gradient of mean temperature of a layer bounded by two isobaric surfaces.

- **Threshold state of use and exploitation.** State that describes and characterizes the beginning and end of an interval of calms or an operationality cycle, during which the structure and its installations are operational, and consequently, its use and exploitation are possible. It is usually associated with normal operational work conditions.

- **Threshold value.** Value below which no effect is expected to appear.

- **Tidal current.** Alternating horizontal movement of water associated with the rise and fall of the astronomical part of the cycle being.

- **Tidal period.** Time interval between the two upwards crossings by the tide wave of the mean sea level.

- **Tidal platform.** Coastal rocky platform at the low tide level, which appears at the foot of a cliff.

- **Tidal prism.** Total amount of water that flows into a tide channel or out again with movement of the tide, excluding any fresh water flow.

- **Tidal range.** Vertical distance between a consecutive high tide and low tide.

- **Tide coefficient.** Ratio of the tide level at a specific point to the tide level at the mean sea level during the equinoctial spring tides (when the Moon's declination is zero and the Moon and the Sun are at their mean distances from the Earth).

- **Tide gauge (marigraph).** Instrument for measuring or recording tides, which is generally located at the entrance of ports to orient and inform vessels of the water depth. It is part of the meteorological and oceanographic networks to aid navigation.

- **TMA** or **FRF spectrum.** Experimental-theoretical spectrum in the open sea that permits the determination of the spectral form, based on specific measurements at the project site, when no data is available and depending on the age of the wave and the relative depth. This spectrum is used for partially developed seas and medium-depth and deep waters. It can also be applied to the waves generated at medium-depth and shallow waters.

- **Toe berm.** Protective element generally composed of granular material located at the breakwater toe, and which is the foundation of the protective layers.

- **Toe depth.** Water height measured at the vertical face of the breakwater.

- **Topographic reference level (TRL).** Reference level for the physical features of a given geographic surface area. It should be indicated on each topographic map and related to the sea reference level.

- **Tornado.** Meteorological event consisting of a violently rotating column of air in context with and extending between a convective cloud and the surface of the Earth.

- **Total investment.** Sum of the estimated cost of annual damages produced by each of the principal failure and stoppage modes and the cost of the investment.

- **Trail.** Cloud formed behind a solid body moving through the air.

- **Transition section.** Subset of the breakwater between two alignments or typologies. This section can be a structurally weak part of the breakwater.

- **Transmission** (of the energy of sea oscillations). Process by which part of the incident energy produced when a wave train interacts with the breakwater is transmitted through or over the breakwater section.

- **Transmission coefficient.** Ratio of the transmitted wave height behind an obstacle and the incident wave height.

- **Tropic.** Either of two latitudes on Earth at which the Sun appears directly overhead at mid-summer and mid-winter. The Earth moves in an elliptic orbit around the Sun. This horizontal is known as the plane of the ecliptic. Since the Earth's axis is not perpendicular to this plane, the intersection of this plane with the sphere does not coincide with a parallel plane. The maximum latitude at which this plane cuts through the Earth is 23º:26:22N and 23º:26:22S (23Gr 26Min 22Sec). For this reason the parallels that pass through these latitudes are important, and are known as the Tropic of Cancer (Northern Hemisphere) and the Tropic of Capricorn (Southern Hemisphere).

- **Tropical cyclone.** Large area of low atmospheric pressure, characterized by inward-spiraling winds and generally formed in warm (tropical) waters, from where they get their energy as a result of evaporation and condensation. Also known as *tropical storm*, *hurricane* (Atlantic Ocean), or *typhoon* (Pacific Ocean).

- **Tropical year.** Period of time taken for the Sun's tropical longitude (longitudinal position along the ecliptic relative to its position at the vernal equinox) to increase by 360 degrees (i.e. to complete one full seasonal circuit). It has a duration of 365.424198 days of mean solar time (365 days, 5 hours, 48 minutes and 45.9 seconds).

- **Troposphere.** Lowest layer of the atmosphere from the Earth's surface to approximately 17 km where it meets the tropopause. It contains the vast majority of weather phenomena that affect human life, such as the clouds, wind, rain, and hurricanes. Its temperature generally decreases with increasing altitude.

- **Trough amplitude.** Maximum negative vertical displacement of a wave with reference to the mean sea level.

- **Tsunami.** Long wave with a period between fifteen minutes and an hour and a height of over 15 meters when it reaches land. It is due to the sudden displacement of water by submarine earthquakes, landslides, volcanic eruptions, submarine slumps or other causes.

- **Turbulence regime.** Flow regime in which particles move irregularly and chaotically, and tend to form a number of small eddies of different spatial and temporal scales.

- **Turning area.** Area where vessels change their course without advancing significantly in any direction.

- **Ultimate limit state.** Project state in which the structure as a whole or one of its subsets or elements becomes unusable or out of service because the safety requirements specified in the project are not fulfilled. This state produces the deterioration, breaking, or collapse of all or part of the structure. It considers all those failure modes due to the loss of equilibrium of all or part of the structure as a rigid solid or due to the excessive plastic strains, breaking, loss of stability, the accumulation of strains, progressive cracking, or fatigue under repeated loads.

- **Undercutting.** Water erosion of a material at the foot of a cliff or bank. It can be caused by waves breaking against a cliff formation or eddies, especially where there is an obstacle to the water. It can also be due to the action of water against river banks on the outside of a meander.

- **Unidirectional TMA spectrum.** Unidirectional, frequency spectrum representative of the sea state at medium-depth and shallow waters if there is no wave breaking because of bottom effects.

- **Ursell number.** Dimensionless parameter of sea waves that relate H, L and h, (wave height, wavelength, and depth), $U_R = H\,L^2/h^3$. If U_R has small values, the waves are Stokes waves, and linear wave theory is applicable. If U_R has large values, the waves can be described by cnoidal theory.

- **Use and exploitation cycle.** Sequence of meteorological states in which the probability of a stoppage mode assigned to the operational limit states is not significant either in the calculation of the joint probability of operational stoppage in the useful life of the structure or in the number of operational stoppages or in the duration of the operational stoppage.

- **Useful life.** Time interval during the serviceability phase of a structure or one of its subsets. Generally speaking, this corresponds to the time during which the structure or one of its subsets fulfills the main function for which it was conceived.

- **Variance.** Measure of the dispersion of a random variable in respect to its mathematical expectation.

- **Verification equation.** Functional relation between project factors that describe the occurrence of each of the failure modes. This equation can have different formats: global safety coefficient, safety margin, etc

- **Vertical breakwater.** Breakwater with a vertical face, in which the central part and the superstructure are composed of a sole structural element, which reflects the incident energy flux.

- **Vessel.** Watercraft with a deck, which because of its size and strength is suitable for navigation and water transportation.

- **Viscoelastic soil.** Soil that experiences temporary deformation when stress is applied, but which afterwards totally recovers its shape when there is no more stress.

- **Viscoplastic soil.** Soil that experiences temporary deformation when stress is applied, but which afterwards does not totally recover its shape when there is no more stress.

- **Visibility.** Quality or fact or degree of being perceptible with the eye, which permits object to be seen at a given distance. Low visibility can be caused by weather phenomenon.

- **Vortex.** Rapid spiraling motion of air or liquid around a center of rotation, the trajectories of which are curves that encircle a single line (axis). The speed and rate of rotation of the fluid are greatest at the center, and decrease progressively with distance from the center.

- **Vulnerability.** Probability of exceedance of the morphological or structural response in the face of the action of one or various agents.

- **Wake.** Region of turbulence behind a solid body moving through the water.

- **WAM.** Wave action model that uses HIRLAM wind field models to calculate the evolution of the spectral density function without previous assumptions regarding the shape of the function.

- **WANA data network.** Time series (state curves of the significant wave height, peak and mean spectral periods, and mean direction of origin) obtained by *Puertos del Estado* in cooperation with the Spanish National Institute of Meteorology by applying wind-field generation models (HIRLAM) and wave generation models (WAM). The WANA database can generally be applied to all of the Spanish coastlines and coastal zones. Nevertheless, precautions should be taking when applying this information to the Strait of Gibraltar, the Catalonian Sea, and the Canary Islands.

- **Wave breaking.** Energy dissipation process that occurs when waves reach shallow water areas. The surface profile of the wave goes from being continuous to discontinuous.
- **Wave breaking index.** Monomial value indicating how close the wave is to its relative limit height.
- **Wave breaking regime.** Hydraulic conditions of wave moment, where there is a transition from a continuous sea surface profile to a discontinuous one. It propagates as a bore in the surf zone.
- **Wave group.** Sea at a certain location, usually in the form of temporal sequences of high waves, followed by others of a lesser altitude.
- **Wave number.** Number of wave crests per unit distance, which is represented by. $k = 2\pi/L$ where k is the wave length.
- **Wave period.** Time interval between two successive crests or between two successive zero crossings in the same direction.
- **Wave profile.** Profile with vertical and horizontal symmetry, which is the linear solution to the problem of the wave propagating over a horizontal bed.
- **Wave propagation.** Transmission of waves through space. During this process, the waves undergo a series of transformation processes that affect their height, period, direction, and spectrum. The propagation processes most frequently used in the evaluation of this transformation are shoaling, refraction, diffraction, and the transmission and breaking of waves.
- **Wave return wall.** Projection located at the crest or highest part of a vertical breakwater parapet to deflect waves seaward.
- **Wave steepness.** Ratio of wave height to wavelength.
- **Wave train.** Indefinite repetition of the wave cycle.
- **Wavelength.** Horizontal distance between two points of the same phase of a wave train, such as two consecutive crests or two consecutive troughs.
- **WAVEWATCH.** WAM-type model with a more efficient computational algorithm that reduces numerical diffusion. It includes local variation terms (non-stationary terms) of water depth and current, and source terms of bottom-induced friction and wavebreaking.
- **WFD.** Water Framework Directive.
- **Wharf.** Sheltered area in a port equipped with anchoring and mooring facilities for ships and machinery for cargo loading or unloading operations.
- **Wind sea or sea.** Wind waves that are generated and affected by the local winds within the generating area, and which are fairly irregular.
- **Window** (of the tide). In navigation channels where there is a tide, time interval in which the water depth uninterruptedly exceeds a certain threshold value.
- **Windward side**. Side of the structure from which the wind blows.

- **Winter solstice.** Point on the ecliptic, midway between the equinoxes, at which the Sun reaches its greatest southern declination. This means that on December 22 in the northern hemisphere and June 21 in the southern hemisphere, the Sun appears to be directly overhead at the Tropic of Capricorn.

- **Zero-crossing wave height**. Sum of the crest amplitude and the trough amplitude between two successive upwards or downwards crossings of the mean water level.

1.5. SYMBOLS

These Recommendations define each symbol as it appears in the text. Given the length of this ROM and the wide variety of topics considered, this is evidently a way of avoiding confusion and enhancing clarity. The symbols are the following:

a = amplitude (m).
C = wave celerity (m/s).
C_g = energy propagation velocity or group celerity (m/s).
C_D = drag coefficient.
d = depth at the toe of the berm.
D_* = energy dissipation per surface unit.
D_w = directional spectrum of the waves.
E = mean energy per horizontal surface unit of the wave train.
f = frequency ($1/s$).
F_r = Froude number.
F = fetch (m).
g = gravity acceleration (m/s^2).
h = depth of the terrain (m).
H = wave height (m).
I = hydraulic gradient in a porous medium.
I_r = Iribarren number.
k = wave number in water.
K = wave number in a dissipative porous medium.
K_R = reflection coefficient.
K_T = transmission coefficient.
L = wavelength (m).
m = bottom slope or spectral moments.
p = pressure (N/m^2).
p_f = failure probability.
R = reflection coefficient.
S = spectral density function (m^2/s).
T = period of oscillatory motion.
t = time.
U,V,W = velocity field components, according to the axes in Cartesian coordinates.
V = useful life of the subset of the structure.
x,y,z = coordinates on the Cartesian axes.
α = slope angle or spectral coefficient.
γ = relative specific weight or wavebreaking index.
δ = boundary layer thickness (m).
ζ = vertical displacement of the free surface with respect to the still water level (m).
η = vertical displacement of the free surface with respect to the sea with motions (m).
θ = incidence angle of the wave train, measured in respect to the normal of the wavefront.
μ = dynamic viscosity.
ν = kinematic viscosity (m^2/s), spectral parameter, or Poisson parameter.
ρ = density (t/m^3) or correlation coefficient.
σ = angular frequency (rad/s), normal stress, variance of the random process, or spectral coefficient.

τ = tangential stress (N/m^2).
ϕ = potencial velocity function or angle of internal friction.
Φ = potential velocity function.
Ψ = depth function, stability function, or contingency function.
φ = phase of oscillatory motion.
ω = angular frequency (rad/s).

1.6. CLASSIFICATION OF MARITIME STRUCTURES IN THE FACE OF SEA OSCILLATIONS

A maritime structure is has at least one of its boundaries in the sea, and permits the exploitation and management of the sea. This annex gives a classification of maritime structures, depending on the service that they provide with a view to facilitating the organization of the ROM documents. As a result, the classification is partial and incomplete.

There are two large groups of maritime structures:

- Main maritime structures
- Auxiliary maritime structures

Main maritime structures can be classified in the following groups according to their function:

- Breakwaters against wave action
- Structures for shoreline management and protection
- Docking structures: Docks
- Mooring and berthing structures
- Offshore platforms
- Underwater pipelines

Auxiliary maritime structures include the following:

- Dredging and filling structures
- Structures for the construction and repair of vessels and floating structures

Main maritime structures in the face of sea oscillations

This section organizes and classifies breakwaters in the face of sea oscillations, which are generally short-period waves $[3 < T(s) < 30]$. All maritime structures interfere with sea oscillations, and furthermore, are useful for the management and exploitation of port and shoreline areas. They are initially divided into fixed structures and floating structures, depending on their response to sea oscillations. Accordingly, marine structures are then subdivided into gravity types and structural types, depending on the way that they resist the action of the predominant agent. Each group can then be classified in terms of function and the main purpose of the structure.

Note. *Berthing structures, which include docks as well as mooring and anchoring structures, are generally an exception. Since they are located in harbors or sheltered areas, the predominant load is produced either by vessel operations or by the installations and the soil. This is also applicable to certain shoreline protection and management structures, such as seafront promenades, where soil action is the predominant load.*

Fixed maritime structures

Fixed maritime structures do not move, and remain in the same position in respect to the land. Except in the case of breakdown or deformation, their shape does not change over time. If the main structural stability agent is gravity, and they thus rely solely on their own weight to maintain stability, they are classified as gravity types. If the actions or loads that the structure must bear are transmitted to the earth or soil by a small number of structural elements, then they are classified as structural types. In certain circumstances, it is possible to design structures that are both gravity and structural. According to the ROM 1.1, these structures are composite fixed maritime structures.

- ***Gravity-based fixed maritime structures.*** As their name indicates, the predominant agent in the stability of this type of structure is gravity. They include, among others, the following types:

  - **Breakwaters**
    - Breakwaters of rubble stone or precast sections.
    - Vertical breakwaters of floating caissons, concrete blocks, and mass concrete.
    - Special composite breakwaters.

  - **Shoreline protection and management structures**
    - Walls, revetments, and slopes made of granular material or precast sections.
    - Groins perpendicular to the coast with no granular elements.
    - Seafront promenades.
    - Underwater pipeline protection made from granular material.

  - **Berthing structures**
    - Docks of floating caissons, concrete blocks, or mass concrete.

  - **Mooring and anchoring structures**
    - Mooring dolphin on a floating caisson.

  - **Offshore platforms**
    - Ekofisk platform.

- ***Structural-type fixed marine structures.*** The maritime structures in this group are constructed with elements that have a structural behavior. This means that they are elements that experience deformation in order to resist and transfer loads to the soil. Such structures include piles, continuous, slotted, or multiple screens, sheet pile areas, and sea walls. They include, among others, the following types:

  - **Breakwaters**
    - Narrow screen that can be permeable or impermeable, vertical or sloping.
    - Multiple screens.
    - Sheet-pile breakwater.

  - **Shoreline protection and management areas**
    - L-shaped walls, anchored bulkheads.
    - Wooden or metal groins.
    - Seafront promenades as a structural wall.

  - **Berthing structures**
    - Docks of anchored bulkheads, L-shaped dock.
    - Pile-supported jetty.
    - Sheet-pile dock, area surrounded by sheet piles or precast elements.

  - **Mooring and anchoring structures**
    - Dolphin: one mooring pile or a group of piles.

  - **Offshore platforms**
    - Raised platform.
    - Pile-supported platform.

Floating maritime structures

Floating maritime structures are those that are waterborne or anchored to the seabed. They include the following types:

- ***Breakwaters***
  - Group of floating tires.
  - Pontoons.
  - Buoy fields.
  - Floating bulkhead.

- ***Shoreline management and protection structures***
  - Pontoons.
  - Buoy fields.

- ***Berthing structures***
  - Floating pontoons and jetties.

- ***Mooring and anchoring structures***
  - Monobuoy.
  - Chain system.

- ***Offshore platforms***
  - Ballasted platform.
  - Cable-stayed platform.
  - Anchored platform.
  - Composite platform.

Chapter II

General calculation procedure

Table of contents

CHAPTER II. GENERAL CALCULATION PROCEDURE

In maritime contexts a sheltered area is a water and land surface that is protected from sea and weather actions. There are two types of sheltered area, harbor and coastal. The difference between these two areas resides in the degree of protection offered and the characteristics of their respective installations. A harbor area is where port activities take place, whereas a coastal area, as its name implies, is directly related to the use and management of the coastline. This includes any focal point of land-sea interaction.

Within the scope of application of the ROM documents, *project* in its most general sense is defined as the set of activities related to the preliminary study and design of a building project, as well as the construction, exploitation, conservation, repair, and dismantling of a maritime structure. When the construction project of any sheltered area is drafted, the structure should be designed in such a way so that it meets optimal standards of functional, economic, and environmental operation. Such criteria should apply to the design and construction of maritime structures, as well as their use, and exploitation. Accordingly, each subset of the structure and the elements in each subset should satisfy all requirements pertaining to reliability, serviceability or functionality.

The objective of this ROM is to provide the methodology and requirements to design, build, exploit, maintain, repair, and dismantle breakwaters that are necessary for the creation of a sheltered harbor or coastal area.

Breakwater design

To control sea oscillations, particularly wave action, it is often necessary to build maritime structures and particularly breakwaters, whose presence may affect and even interfere with sea oscillations. The superposition of incident oscillations such as those generated and transformed by the presence of breakwaters constitutes the set of oscillations that affect the sheltered area and constrain its levels of use and exploitation, safety, and service.

Designing a sheltered area its structures comprises the following basic sequence of actions:

1. Specification of the general criteria that define the purpose of the construction, its functional constraints, timeline, spatial units (subsets) for each design and construction phase, general and operational characteristics of the structure and of each subset, and finally the project design requirements;
2. On-site description and characterization of the sheltered area;
3. Description and characterization of the project factors at the site that define its geometry, physical environment, soil, and construction materials. Such a characterization should identify and evaluate the agents and actions as well as their temporal and spatial scales, specifying, if necessary, meteorological years, solicitation cycles, and operationality cycles.

On the basis of this information, the following steps are recommended:

1. Preliminary studies should be carried out with a view to defining alternatives for the ground plan of the sheltered area and the selection of breakwater type according to use and exploitation requirements, the morphological, climatic, and environmental characteristics of the soil, construction materials and methods, local conservation and maintenance, as well as dismantling capacity.
2. The structure should be pre-dimensioned in plan and elevation, and its spatial scales or subsets should be specified.
3. A study should be made of the hydrodynamic, geotechnical, and structural performance of the construction and its subsets in relation to the project factors as well as its interaction with the coastal environment. Failure modes should be identified in relation to safety and serviceability, and stoppage modes in relation to use and exploitation.
4. It is necessary to verify that during construction, the breakwater subsets and their elements comply with the project requirements in each of its phases for all failure and stoppage modes. For this purpose, the verification equation for each mode is formulated and solved with the calculation procedures recommended in this ROM.
5. The sheltered area and breakwaters should be functionally, economically, and environmentally optimized by taking into account initial building costs, conservation costs, and eventually maintenance costs during the structure's useful life as well as its future dismantling. Alternatives should also be proposed.

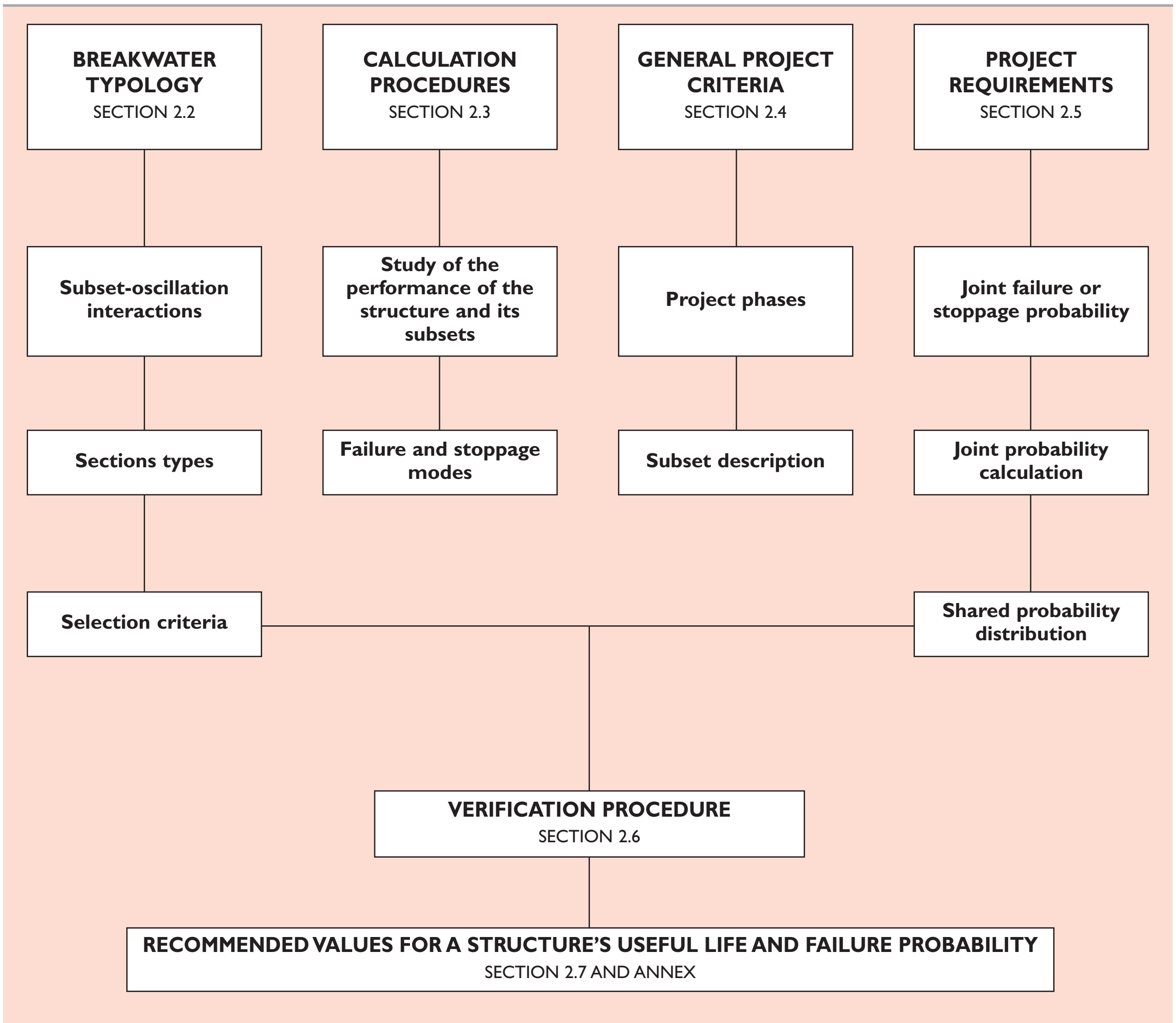
6. A preliminary study as well as the actual construction project should be drafted. They should include a sequence of activities expressing the geometric configuration of the solution, and also describe its construction. Conservation tactics should be specified, maintenance strategies identified, and eventually repair actions as well during the useful life of the structure. Other elements to be included are an assessment of the dismantling capacity of the structure and the possible restoration of environmental quality after it has been dismantled.

This ROM explains how these procedures can be successfully carried out. This section specifies basic project criteria and requirements. Chapters 3 and 4 describe the sheltered area and its location, and also explain how to pre-dimension it. Chapters 5, 6, 7 and 8 identify and characterize the most frequent types of breakwater: sloping breakwaters, vertical breakwaters, berm breakwaters, composite breakwaters, and floating breakwaters.

Organization of Chapter 2

Chapter 2 discusses general calculation procedures in the project design of breakwaters. A graphic representation of its organization is shown in figure 2.2.1.

Figure 2.2.1. ***Organization of Chapter 2***



Sections 2.1 and 2.2 of this ROM define sheltered areas and the most representative types of breakwaters as well as the selection criteria to be used in the selection of the most effective type of breakwater. Section 2.3 describes the calculation procedures for these structures, and section 2.4, the general project criteria for breakwater design. These project criteria specify the spatial and temporal organization of the project and the intrinsic nature of each of the subsets and phases of the structure.

Based on the organizational schema in the ROM 0.0, sections 2.5 and 2.6 of this ROM analyze project requirements for breakwaters, and discuss the distribution of the joint probability in the principal modes and verification procedure. Section 2.7 includes a table of the recommended project requirements for breakwaters along with the general and operational nature of the most representative maritime and coastal defense structures in harbor and coastal areas. Chapter 2 concludes with an example of a joint probability calculation based on the possible failure modes of a vertical breakwater, and discusses the joint probability distribution.

2.1 SHELTERED AREAS AND MARITIME STRUCTURES

These recommendations refer to both harbor and coastal sheltered areas.

2.1.1 Harbor area

Generally speaking, harbor areas are designed to facilitate the port and logistic operations associated with maritime transportation and its relation to other transportation modes and the integral management of vessels. This includes all operations pertaining to nautical sports as well as to industrial, and naval activities. Among other things, a harbor area has the following infrastructures that are directly related to the following:

- safety, use, and exploitation of vessels: minimum water surface and dock space required for each vessel, and if necessary, anchoring area and other spaces such as dry docks, etc;
- wave oscillation control: breakwaters and maritime structures;
- land use and exploitation of the surrounding area: minimum land surface and operational surfaces, parking and storage, shipping movements and merchandise, including cargo handling systems;
- vehicle access and transit (road and railroad traffic).

In regards to the necessary infrastructures for the safety, use, and exploitation of vessels, the following sub-areas must be envisaged: access canal, entrance, maneuvering and anchoring zones, docking and mooring areas such as quays, wharfs, piers, etc. The dimensions of the maritime structure in question depend on the general and operational characteristics of the sheltered area, the characteristics and stopover frequency of the design fleet, admissible service quality levels and the local climate conditions. The predominant climate factor is usually the wave action, but in certain cases there may be other local constraints as well (see section 3.3).

2.1.2 Coastal area

Generally speaking, a coastal area is designed to facilitate the orderly and sustainable use and exploitation of the coastal environment. This includes *inter alia* the correction, protection and defense of the shoreline, the generation, conservation and nourishment of beaches and bathing areas, as well as the interchange of transversal land-sea flows of various substances (1).

A coastal area includes infrastructures related to:

(1) Such substances are either conservative or non-conservative. Conservative substances are those that do not change during the transport and mixing processes (e.g. silica sand). Non-conservative substances are those that can undergo changes during the transport and mixing processes (e.g. carbon, nitrogen).

- the safety and use of the area by the users: perimeter of the area, including the dry land boundary (defined by the flood level) and the sea boundary (defined by the closure depth [2]), and the natural or artificial, permeable or impermeable lateral boundaries of the area.
- the safety, use and exploitation of the dry land area: the dry land surface, which should specify usable surface area, parking and storage areas, pedestrian and vehicle traffic areas, and consumer services areas.
- sea oscillation control: breakwaters and other maritime structures.
- land and sea accessibility to the area according to its use and exploitation.

The dimensions of the infrastructures depend on the general and operational characteristics of the sheltered area, the use and admissible exploitation conditions for each of its components, and the local climate and circulation conditions usually associated with wave action. Nevertheless, in certain cases it may be necessary to consider other forcing conditions.

2.1.3 Breakwater construction

When atmosphere and sea dynamics do not allow project design requirements to be satisfied in the area or its infrastructures, the construction of one or more breakwaters may be necessary. In this case, and according to the ROM 0.0, the breakwater project design should ensure the achievement of a set of objectives that are conducive to the satisfaction of the requirements and criteria discussed in this section. For this purpose, a distinction should be made between newly constructed breakwaters, and the restoration or renovation of already existing breakwaters.

2.1.3.1 Newly constructed breakwaters

It is the task of the project developer to define the site where the breakwater is to be constructed, and also to specify the use and exploitation needs of the sheltered area. In some cases, the breakwater location will have to be selected from various alternatives after a detailed operational, social, environmental, and economic analysis.

General criteria

For the breakwater in its totality and for each of its subsets, the developer should define the following: (1) the useful life of the structure or the its projected duration when it is temporary; (2) specification of the start-up date of the breakwater components; (3) the operationality and general nature of each subset, which naturally constrain all other specifications detailed in the subsequent items; (4) duration of each project phase; (5) reliability and functionality requirements for each subset; (6) operational level, average number of operational stoppages, and maximum duration of an operational stoppage in the specified time period; (7) dismantling plan and, where applicable, the restoration of the coastline and surrounding environment to its original state.

In those cases in which the developer of the construction has not defined some or any of the general criteria, or when the general and operational nature proposed are unjustifiably divergent from what is generally acceptable for such projects (see section 2.7), the project designer must ascertain the general and operational nature of each subset of the structure, and on the basis of these results, he/she must also determine the other project requirements, following the guidelines set out in the ROM 0.0.

Time period for the operational analysis

The developer should define the time periods for the verification of the safety, serviceability, and use and exploitation requirements pertaining to the structure and its subsets, based on studies of economic and operational

(2) The closure depth identifies the minimum depth beyond which the influence of sediment transport is not relevant to beach profile performance.

performance. Since the time period for this verification is generally measured in years, the useful life of a structure is also calculated in this time unit.

PRELIMINARY ESTIMATE OF THE CHARACTERISTICS OF THE BREAKWATER

When the nature of the breakwater has not been previously defined by the project developer, it must be determined by the method described in the ROM 0.0. Firstly, the sheltered area should be pre-dimensioned (3). The next step is to divide the structure into subsets, according to the use and exploitation of each one and also according to the type of breakwater being considered. Finally, the general and operational nature of each subset is determined and specified.

VERIFICATION OF THE PROJECT REQUIREMENTS

The design of a totally new structure should verify all project requirements, whether they are structural, formal, of use and exploitation, or environmental and/or legal. After this verification is made, the performance of the breakwater and its interaction with the prevailing agents is analyzed.

SAFETY, SERVICEABILITY, AND USE AND EXPLOITATION

These requirements are verified in both normal and extreme work and operating conditions, and when relevant, in exceptional work and operating conditions as well.

ENVIRONMENTAL REQUIREMENTS

Environmental requirements for maritime structures and breakwaters are described in the applicable environmental regulations and, more specifically, in the ROM 5.0. The requirements pertaining to water quality and coastal morphodynamics should be in consonance with the specifications in the ROM 5.1-05 and 5.2, respectively.

SPECIFIC LEGAL REQUIREMENTS

There are also legal requirements to be complied with depending on the location of the sheltered area and the administrative context of the location. Consequently, such requirements are also regarded as project factors, and should be considered in each project phase.

2.1.3.2 Breakwater readaptation and rehabilitation

According to the ROM 0.0, all structures should be subject to periodic inspections, monitoring, and instrument checks in order to assure that they meet project requirements during the construction and design phase, and particularly during their useful life. When such requirements are not satisfied, repairs should be immediately carried out.

BREAKWATER REHABILITATION

It is often necessary to adjust the structure to fit project requirements. This may be done to lengthen its useful life or to comply with new regulations, either by modifying its infrastructures or improving its means of use and

(3) In the context of this document, *to dimension* means to set out, determine or resolve the dimensions of a structure and its properties. *Pre-dimension* means to carry out this action beforehand.

exploitation. According to this ROM, the actions to be taken in such cases are known as the rehabilitation of the structure.

Breakwater readaptation

Readaptation is the adaptation of an existing structure for a different use and exploitation for the rest of its useful life or for a new useful life.

General Criteria

The developer of the structure should be the one to specify general criteria for repair and renovation. This depends on the general and operational nature of the structure, as stipulated in section 2.1.3.1 of this ROM. If no such criteria have been specified, then it is necessary to follow the guidelines in that section for this eventuality.

2.2 BREAKWATER TYPES

This section describes the types of breakwater generally used to protect harbor and coastal areas, and outlines their main components and elements. In fact, breakwater design, construction, conservation, and dismantling are the focus of this ROM. The subsequent chapters give a detailed characterization of each breakwater type. Breakwaters can be differentiated from each other in the way that they deal with climate agents. They also differ in the way that they transmit forces to the soil. In this respect, a relevance difference between breakwaters lies in the vertical dimension of their elevations, which can vary significantly.

This section is organized as follows: Firstly the topmost components of the breakwater are defined. This is followed by a review of the premises underlying the interaction of these breakwater sections with sea oscillations, mostly in the form of the wave action striking perpendicularly against the section. This type of analysis should be complemented by the study ground plan of the entire structure, which assesses the effect of the energy radiation on each subset and any changes in the shape and alignment of the structure, especially the head (see Chapter 4). The final part of the section presents a set of selection criteria for breakwater types.

2.2.1 Breakwater sections

The section type of a breakwater can be described as followed (see figure 2.2.2.):

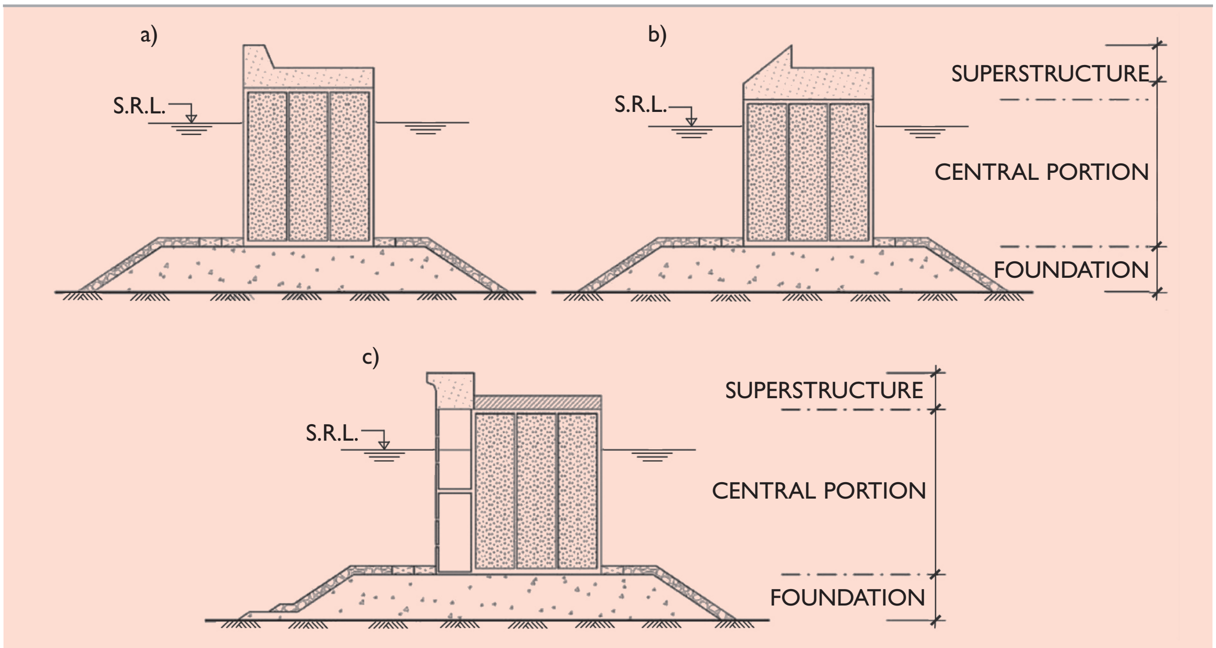
- Foundation, which determines the way in which the structure transmits the forces to the soil.
- Central body that controls the transformation of the energy flow of the incident wave action, and transmits the result of these actions to the foundation.
- Superstructure, which controls wave overtopping at the crest, and if necessary, offers an access path.

Note. *The organization of this section and the differential criteria provided facilitate the description and comparison of breakwater types. The dividing line between the different breakwater components is far from precise, but broadly speaking, each part can be defined in terms of its primary function. Thus, the superstructure offers protection against wave overtopping as well as providing an access and sometimes a mooring line landward from the breakwater. The central portion is the main part that resists the wave action, thus causing wave breaking, reflection, etc. The foundation is the part of the breakwater in contact with the soil, and thus transmits its forces to the ground.*
Furthermore, this organization facilitates the description of breakwater components and subcomponents. For example, the main layer is generally a structural element that has two levels: the natural or artificial part of the main layer is a structural subcomponent of the central portion of the breakwater.

2.2.2 Interaction of the breakwater section and sea oscillations

The presence of a breakwater transforms the energy of sea oscillations, altering the distribution of the components of the frequency and directional spectrums (see Chapter 3). The breakwater reflects, dissipates, transmits and radiates incident energy in proportions that vary according to its type, ground plan, design, and characteristics of the oscillation. This section describes the interaction of the breakwater section and sea oscillations as represented by a wave train with waves of specific heights and periods that normally affect the section. Since the majority of sea oscillations can be mathematically represented by a wave train, the results of this analysis can be applied to wave action against the breakwater or even to a tsunami or astronomical and meteorological tides. Finally, reference is made to the effect of oblique incidence and irregular wave action on transformation processes.

Figure 2.2.2. ***Breakwater components***



2.2.2.1 Wave energy conservation equation

The incident energy is distributed when the breakwater section interacts with the (a) energy reflected and returned to the sea; (b) energy that is transmitted through or over the section, and which is propagated landwards from the breakwater; (c) energy that is dissipated and thus extinguished; (d) energy that is transferred to other oscillatory modes or which generates other circulatory movements. This energy transformation can be analyzed by defining a unit width control volume, containing the section (see figure 2.2.3), and in which the incoming and outgoing energy flows are evaluated as well as the dissipation processes inside it. The wave energy conservation equation in the control volume can be written as:

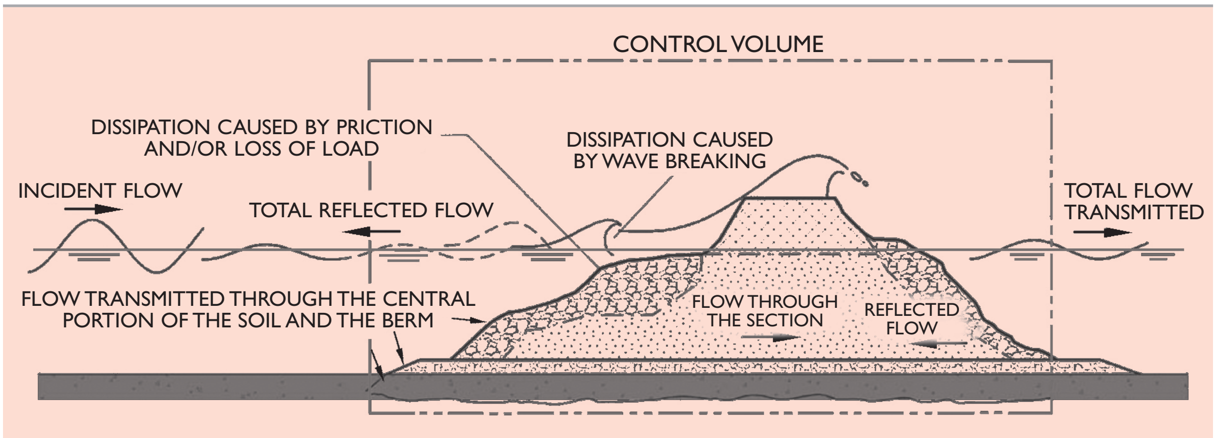
$$F_I - F_R - F_T - D'_* = 0 \qquad (2.1)$$

where $F_{I,R,T}$ represent the average incident energy flows (4) reflected and transmitted by the seaward and landward sections, respectively. In this same equation, D'_* evaluates the dissipation per time unit inside the

(4) Energy flow is the quantity of energy that passes through a volume control surface in the specified time unit. Normal and tangential stresses can act on this surface, which in the time unit can contribute (i.e. add or extract energy) with work.

control volume, which is caused by the presence of the breakwater and the soil. The minus sign represents energy extraction or loss from the control volume. The plus sign expresses the contribution or entry of energy in the control volume.

Figure 2.2.3. ***Energy flux at the breakwater***



Applying linear wave theory and considering an incident wave train of wave height H_I and wave period $T_{z,I}$, the average incident energy flux (i.e. the average energy that perpendicularly impinges on the seaward face of the control volume in the time unit) of velocity $C_{g,I}$ is:

$$F_I = E_I C_{g,I} = \frac{1}{8}\rho_w g H_I^2 C_{g,I} \tag{2.2}$$

where E_I represents the total average incident kinetic and potential energy flux [(5)] per horizontal surface unit of the incident wave train, which is a quadratic function of the wave height

$$E_I = \frac{1}{8}\rho_w g H_I^2 \tag{2.3}$$

The relations between the phase velocity (phase celerity) C and the energy propagation velocity C_g of the wave movement and the wave number k are related in the following equation:

$$C_{g,I} = \frac{1}{2} C_I \left(1 + \frac{2k_I h}{\sinh 2k_I h}\right) \tag{2.4}$$

$$k_I = \frac{2\pi}{L_I}, \quad C_I = \frac{L_I}{T_{z,I}} = \frac{\sigma_I}{k_I} \tag{2.5}$$

where h stands for the water depth in the control volume. Accordingly, h and k are related to the angular frequency $\sigma_I = 2\pi/T_z$ by means of the dispersion equation

$$\sigma_I^2 = g k_I \tanh k_I h \tag{2.6}$$

(5) The energy flux is the quantity of energy that passes through a control volume surface in a specific time unit. Shear stresses (normal and tangential) can act on this surface, and in the stated time unit they can contribute (add or extract energy) with work.

Given the water depth h and the wave period T_x, the wave number k is the real root of the equation. When this wave number is known, the celerity or velocity of the energy propagation is immediately obtainable. And when the height H is known, this gives the incident energy E_I and the energy flux through section F_I.

Similarly, both the reflected and transmitted outgoing energy fluxes of the volume control are defined as

$$F_R = \frac{1}{8}\rho_w g H_R^2 C_{g,R} = E_R C_{g,R} \tag{2.7}$$

$$F_T = \frac{1}{8}\rho_w g H_T^2 C_{g,T} = E_T C_{g,T} \tag{2.8}$$

where H_R is the reflected wave train height; H_T is the transmitted wave train height; and $C_{g,R,T}$ are the energy propagation velocities or group celerities, defined on the basis of the representative reflected and transmitted wave train periods.

$$C_{g,R} = \frac{1}{2}C_R\left(1+\frac{2k_R h}{\sinh 2k_R h}\right) \tag{2.9}$$

$$C_{g,T} = \frac{1}{2}C_T\left(1+\frac{2k_T h}{\sinh 2k_T h}\right) \tag{2.10}$$

E_R and E_T are, respectively, the total average kinetic and potential energy of the reflected and transmitted wave trains per unit of horizontal surface.

When the transformation processes of the wave train are described by linear theory and in the absence of a current, there is no change of period or angular frequency σ and if there is also no change in the water depth on both sides of the breakwater, the wave number and wave train celerity do not change when they are reflected or transmitted. Consequently, they can be written $k_I = k_R = k_T = k$. The three phase celerities and the three group celerities can also be expressed in the same way.

If the following transformation coefficients of the wave train are defined as

- Reflection coefficient, $K_R = \dfrac{H_R}{H_I}$
- Transmission coefficient, $K_T = \dfrac{H_T}{H_I}$
- Dissipation coefficient (per unit of incident energy) $D_* = \dfrac{D_*^{'}}{\frac{1}{8}\rho_w g C_g H_I^2}$

then the energy conservation equation in the control volume (in which the transformation processes are produced because of the presence of the breakwater section) can be written as:

$$K_R^2 = K_T^2 + D_* = 1 \tag{2.11}$$

These coefficients depend on the breakwater section type and ground plan of the structure. According to linear theory, the incident wave height is the scale factor of the transformations.

Note. This way of expressing the energy conservation equation allows us to evaluate the efficiency of the breakwater to control incident wave train energy. If the reflection coefficient $K_R = 1$*, the breakwater is totally reflective, whereas if* $D_* = 1$*, the breakwater is totally dissipative. However, in the strictest sense, no breakwater can be said to be totally reflective or dissipative. Depending on the dimensions and the hydraulic characteristics of each breakwater component and of the soil as well as the kinematic characteristics of the wave train, the breakwater acts as though it is essentially reflective or dissipative. The Annex at the end of Chapter 4 deals with wave train transformation in the presence of a maritime structure.*

One of the objectives of the preliminary design project is to choose the type of breakwater that can best control the wave energy flux. For this reason, it is essential to know the values of K_R, K_P, and D_* for each breakwater type, depending on the characteristics of incident wave action. These coefficients can be obtained by analytical, numerical or experimental methods, and are stated in this ROM in the sections describing breakwater types.

Within the sphere of applied engineering, the distribution of incident energy based on linear theory is usually sufficient for most purposes even though this distribution omits energy transfer processes to other frequencies by the non-linear interaction between components or other non-linear processes. Nevertheless, it should be stressed that such processes can be important in the dimensioning of underwater breakwaters on top of which wave-breaking occurs. They can also be significant in the dimensioning of floating breakwaters, whose amplitude of oscillation is of the same order as the amplitude of the incident wave train.

2.2.2.2 Breakwater element types and energy efficiency against wave movement

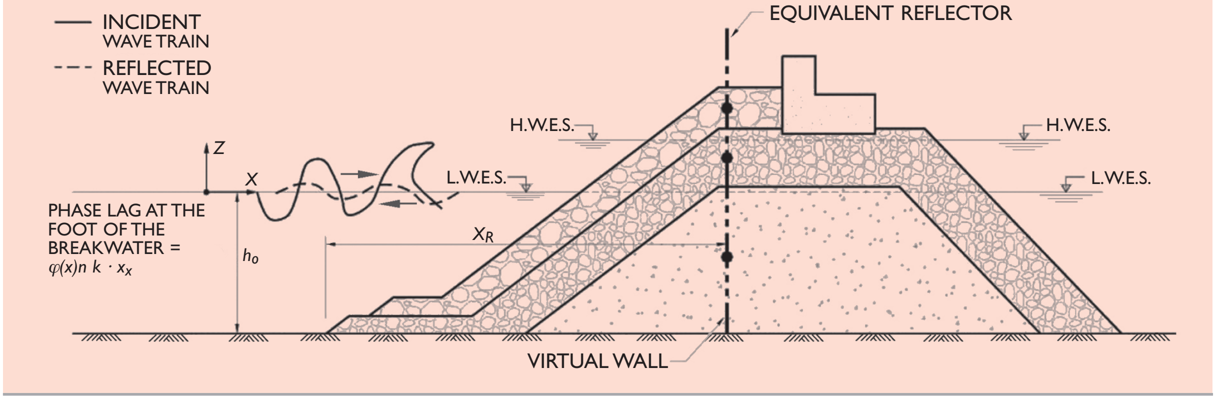
Depending on the geometry and configuration of the elements of the breakwater section, certain wave transformation processes can be favored over others. The following subsections briefly analyze these processes and their dependence on individual breakwater components.

Reflection

Wave energy reflection occurs whenever there is a sudden change in the geometrical properties of the medium where the wave train is propagated. This modifies the celerity of the wave train phase, and thus, of the wave number and direction of propagation. Consequently, sudden changes in the water depth of a toe berm or in the hydraulic characteristics of the central portion of a rubble-mound breakwater, or the presence of an impermeable wall in a vertical breakwater can cause the seaward reflection of part of the incident energy. Similarly, when the wave train is transmitted through a breakwater, moves away from it or is propagated along a navigation channel, part of the energy is reflected in the upstream section as well as the downstream section.

Generally speaking, reflection at breakwaters does not occur at a fixed point or surface. Rather, there are numerous simultaneous contributions throughout the propagation process. The last of these occurs when the wave train moves away from the breakwater section and is propagated landward from the breakwater. However, whether wave reflection happens suddenly (e.g. at the outer wall of a vertical breakwater, locally over a slope) or whether it happens gradually when the wave train is propagated into and through a rubble mound breakwater, it is essentially a linear process. Wave reflection thus depends on the geometrical characteristics (change of depth h_2 / h_1 or slope incline tan α) and hydraulic characteristics (porosity n, permeability and equivalent friction C_f and seabed texture K_s) of the breakwater section and the surrounding area. It also depends on the characteristics of the incident wave action, more specifically on the wave period T_z, expressed in terms of relative depth kh.

Figure 2.2.4. Diagram of the equivalent reflector and the phase lag associated with a sloping breakwater



According to linear theory, the reflected energy can be evaluated with a coefficient or module K_R, and a phase ϕ, which represents the adimensional distance (equivalent to kx) to a point of reflection (measurement on axes of reference) at which the reflection of the wave train is supposed to take place. Consequently, the evaluation of the outgoing reflected energy of the control volume allows us to obtain a reflection coefficient whose module globally quantifies all the specific reflection processes. Moreover, the phase of this coefficient helps to define a point in the inner part of the structure where an equivalent reflector would be found, which is capable of generating the same reflection effect as the total effect produced by all of the individual reflectors at their respective locations.

Transmission

The transmission of wave energy landward from the breakwater can occur because of wave overtopping at the crest, because of propagation through the central portion (as in the case of rubble mound breakwaters), as well as through the soil and foundation when they are permeable. In the first case, the magnitude of the energy transmitted depends on the relation between the height of the crest or freeboard F_c, and the height of the sheet of water that reaches the crest (expressed in terms of wave height (6) at the toe of the breakwater, and in its presence, H_*). In other words, it depends on the relative freeboard F_c / H_*.

In the second case, the magnitude of the energy transmitted either through the central portion of the breakwater or the foundation depends on its hydraulic properties and on the length and width of propagation B, expressed in terms of the wave length or its equivalent wave number, F_B or B/L. During its propagation through the breakwater or its foundation, part of the wave energy is consumed. If the breakwater is sufficiently wide, all of the energy may be dissipated. In such circumstances the wave energy transmitted through the breakwater section is negligible. In wave terms, the distance at which the wave train reduces its energy to $1/e$ (number e) is known as the skin depth.

According to linear theory, the energy flux transmitted through the section can be evaluated by a coefficient or module K_T, and a phase ϕ, which represents the adimensional distance (equivalent to kx) to a generation point, measured on reference axes, at which the wave train is presumably generated. In the case of energy transmission by overtopping and when the relative freeboard is low, the wave movement can pass over the crest, and continue its propagation landward with the corresponding reduction in height expressed by the transmission coefficient. If the relative freeboard is high, energy transmission occurs when the water mass associated with the passage of the wave hits the body of water on the leeward side of the breakwater. This generates a wave movement that propagates outwards from the impact zone. In both cases, the phase generally shows that the transmitted wave train is generated in the backfill of the breakwater.

Note. Breakwaters built with a quarry run core and layers of rubble have a porosity within range $n = 0.35 - 0.45$, and hydraulic characteristics, $K_p = 10^{-3}$ cm/s and $C_f \approx 1$. A breakwater can be regarded as impermeable to wave energy flux if $n \Rightarrow 0$ and $K_p < 10^{-8}$ cm/s.

The permeability and impermeability of the structure under the effect of wave transmission can be achieved by a combination of certain materials and the geometric dimensions of the section. Thus, for example, a slender steel screen (significantly thinner than the length of the wave), which extends vertically to the bottom of the breakwater is impermeable to wave propagation. However, this perforated screen is permeable from a hydraulic viewpoint since energy transmission occurs through tiny perforations or slits.

A porous vertical breakwater with quarry mount and rubble layers is permeable to wave propagation if its width is $B/L < 0.25$. If the breakwater has a relative width of $B/L > 1$, and $K_T \Rightarrow 0$, landward of the breakwater, no wave movement occurs, and for all practical purposes, the breakwater acts as though its width were infinite.

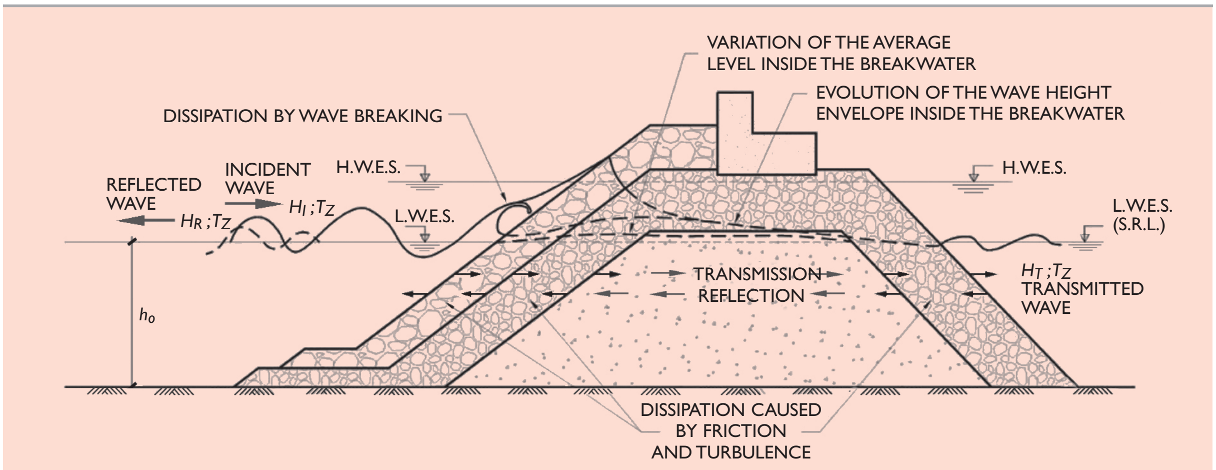
A screen of piles in front of the breakwater permits the transmission of wave energy through the structural elements as long as the flow is continuous. The hydraulic characteristics of the space between the piles govern the load losses in the energy transmission process. The radius of the piles, as compared to the wavelength, number of rows, and ground plan of the breakwater, controls the quantity of energy transmitted landward with regard to the incident energy, that is, K_T.

(6) See the subsection on wave height in the same section of this ROM.

Dissipation

The dissipation of wave energy is largely produced by two mechanisms: wave breaking and friction. Such friction occurs at the surface and bed boundaries as well as within the wave propagation medium. The most efficient type of dissipation is that of a spilling and plunging breaker in which more than 90% of the incident energy is dissipated. In contrast, the collapsing breakers are less efficient since generally only about 60% of the energy is dissipated. The remaining energy is reflected, dissipated internally by means of friction, or transmitted landward (see figure 2.2.5).

Figure 2.2.5. ***Transformation of incident wave energy at a traditional rubble mound breakwater***



Although this is not the only possible way, wave breaking can occur because of wave steepening at the breakwater slope. The breaker at the slope can be identified by the Iribarren number, defined as the ratio of the breakwater slope to the wave steepness at the toe of the slope:

$$I_r = \frac{\tan(\alpha)}{\sqrt{\frac{H}{L}}} \tag{2.12}$$

The following table shows an interval of values I_r with their corresponding breaker type:

Breaker type	Ir
Spilling breaker	< 0.1
Plunging breaker	0.1 – 2.3
Collapsing breaker	2.3 – 3.5
Surging breaker	> 3.5

These intervals are approximate since the wave steepness on the slope depends on the characteristics of the incident and reflected wave trains as described in the following section. For this reason, if the wave steepness value is unknown, it is advisable to evaluate it at the toe of the slope. Reflection can only be regarded as negligible in the case of spilling breakers and plunging breakers (e.g. on dissipative beaches).

This number also provides qualitative information about the intensity and rate of energy dissipation in the breaking process. The on-slope dissipation of a spilling breaker is "uniform" in the surf zone. A surging breaker reflects the greatest amount of energy, and there is practically no surf zone. On a scale of intensity, plunging breakers and collapsing breakers, which are more local and less dissipative, fall somewhere between spilling and surging breakers. It is unusual to find spilling breakers at a breakwater except in the case of very broad sloping breakwaters.

The energy dissipated by means of friction depends on its hydraulic regime. If it is laminar, then the dissipation is proportional to the square root of the water particle velocity in regards to the boundary or the elements that cause the friction. If the regime is turbulent, the dissipation is proportional to the cube root of the velocity, and varies considerably, depending on the Reynolds number since a substantial amount of the dissipation is mainly associated with convergence and divergence flow processes, flow separation around obstacles, vortex shedding, the development of swirls and wakes, and the transmission of momentum related to turbulent cascades that eventually is transformed into heat. Generally speaking, the energy dissipated by friction processes depends on the Reynolds number, the porosity of the structure, and the size and shape of its components (grain size, hole size, pile size, etc.). The evaluation of the dissipation of a breaking wave can be sufficiently determined by likening it to the dissipation that occurs at a hydraulic jump. However, if the wave is a plunging, collapsing or surging breaker impacting against a steep slope, the evaluation of the dissipation rate of the wave movement $D_*^{'}$ during the process is extremely complex and uncertain. An indirect way of calculating the dissipated energy, which is applicable to breakwaters, is to resolve the wave energy conservation equation, once the reflected and transmitted energy flux radiated from the control volume have been evaluated:

$$D_* = 1 - K_R^2 - K_T^2 \qquad (2.13)$$

$$D_* = \frac{D_*^{'}}{\frac{1}{8}\rho g C_g H_I} \qquad (2.14)$$

Note. Generally speaking, a rubble mound breakwater is the type of breakwater that causes the greatest dissipation because of wave breaking and friction in the central portion of the breakwater. The Iribarren number was first presented in 1949 in the article "Talud límite de rotura y reflexión" by R. Iribarren and C. Nogales. Its purpose is to delimit when waves break freely on the slope and when the reflection process is dominant.

Wave height at the toe of the breakwater and in its presence

Generally speaking, it can be said that the presence of a breakwater causes the reflection of part of the energy of an incident wave train of wave height H_I and wave period T_z. At the toe of the breakwater, due to the interference of incident and reflected wave trains, the oscillatory motion is partially standing. According to linear theory the period of the incident, reflected, and partially standing wave train is the same (T_z). However, the wave height H_* of this wave train depends on the geometry of the front of the breakwater and the phase lag between both wave trains. H_* is a wave height at the toe of the breakwater and in its presence. Generally, this wave height can be expressed by $H_* = \mu H_I$ where μ is a coefficient (7) quantifying the magnitude of the linear interference of the incident and reflected wave train.

Note. Let us consider a vertical breakwater with a smooth, impermeable face, built on a horizontal bed, and whose foundation is of negligible thickness as compared to its depth. If this breakwater cannot be overtopped, $K_R \simeq 1$ and the coefficient at the toe of the breakwater is $\mu \simeq 2 = (1 + K_R)$. However, if it can be overtopped and the breakwater wall is rough, then $\mu < 2$. But if the thickness of the breakwater and its length seaward from the foundation berm are not negligible in comparison to the wave depth and length, respectively, and the wave can break against the berm, then it is possible that $\mu > 2$. For an Iribarren-type sloping breakwater (see fig. 2.11) on a horizontal bed, the values of μ can be found in the interval $0 \leq \mu < 1.8$.

2.2.2.3 Slope roughness and porosity

When the wave train propagates on a slope, a rough bottom, or through a porous medium, part of its energy is dissipated. It is important to remember that reflection occurs at the same time as dissipation. This includes a

(7) As explained in Chapter 4, the value of μ not only depends on the typology, but also on the ground plan of the structure and its immediate surroundings.

change of phase in the reflected wave train because when the wave train is propagated in all three cases, there is a change in the propagation conditions. The residual energy is transmitted.

When the wave train is propagated on an impermeable slope with a smooth bed and gentle slope such that $I_R < 0.1$, the celerity of the wave train gradually adapts to the change of depth without any significant reflection process. The shoaling process and amplitude dispersion cause the wave to become steeper, sometimes transforming it into a spilling or plunging breaker.

When the wave train propagates over a horizontal bed of depth h, and there is a sudden change of depth (8) from h to $d < h$, the wave train continues its propagation over depth d, and part of the energy is reflected at the step, and another part is transmitted over depth d. The adaptation of the wave to the new conditions is not immediate. It needs to propagate a certain distance over the new depth d before the amplitude and frequency dispersion mechanisms begin to work, and the wave eventually breaks. Generally speaking, when the depth h and wave height H_* are of the same order, the wave can break directly on the edge of the step.

Finally if slope steepness is such that $I_r \geq 2.3$, the wave train must rapidly adjust to the change in depth as the slope becomes progressively steeper, and the incident wave energy is reflected. The interaction of the incident and reflected wave trains, and the decrease in depth all combine to cause the wave to collapse at its base. It thus either becomes a collapsing breaker or else surges on the slope with a turbulent wave front to become a surging breaker. When these conditions occur, the presence of the breakwater causes the reflection of 35% to 85% of the incident wave height, depending on the breaker type, as evaluated by the I_r, porosity of the layers and central part of the breakwater.

One possible way to stimulate energy dissipation at the boundaries of the breakwater is to place rough-textured elements there at irregular intervals, for example, on the crest of the breakwater or at its seaward base. Similarly, the dissipation within the central portion or the superstructure of the breakwater can be intensified by the use of porous walls or walls with holes or slits to increase the number of contractions, expansions and separations of the lamina, which should exceed the wave propagation movement. This is the process that occurs naturally countless times in a permeable granular medium, for example, in the layers and central part of a rubble-mound breakwater.

2.2.2.4 Spatial evolution of the breakwater section

Breakwaters can be designed with different elements and configurations in order to control and transform incident energy in different ways, and also to favor certain transformation properties over others. Fig. 2.2.6 shows the spatial evolution of a breakwater depending on the way that it transforms such energy. A coastal area can thus be protected by different types of breakwaters.

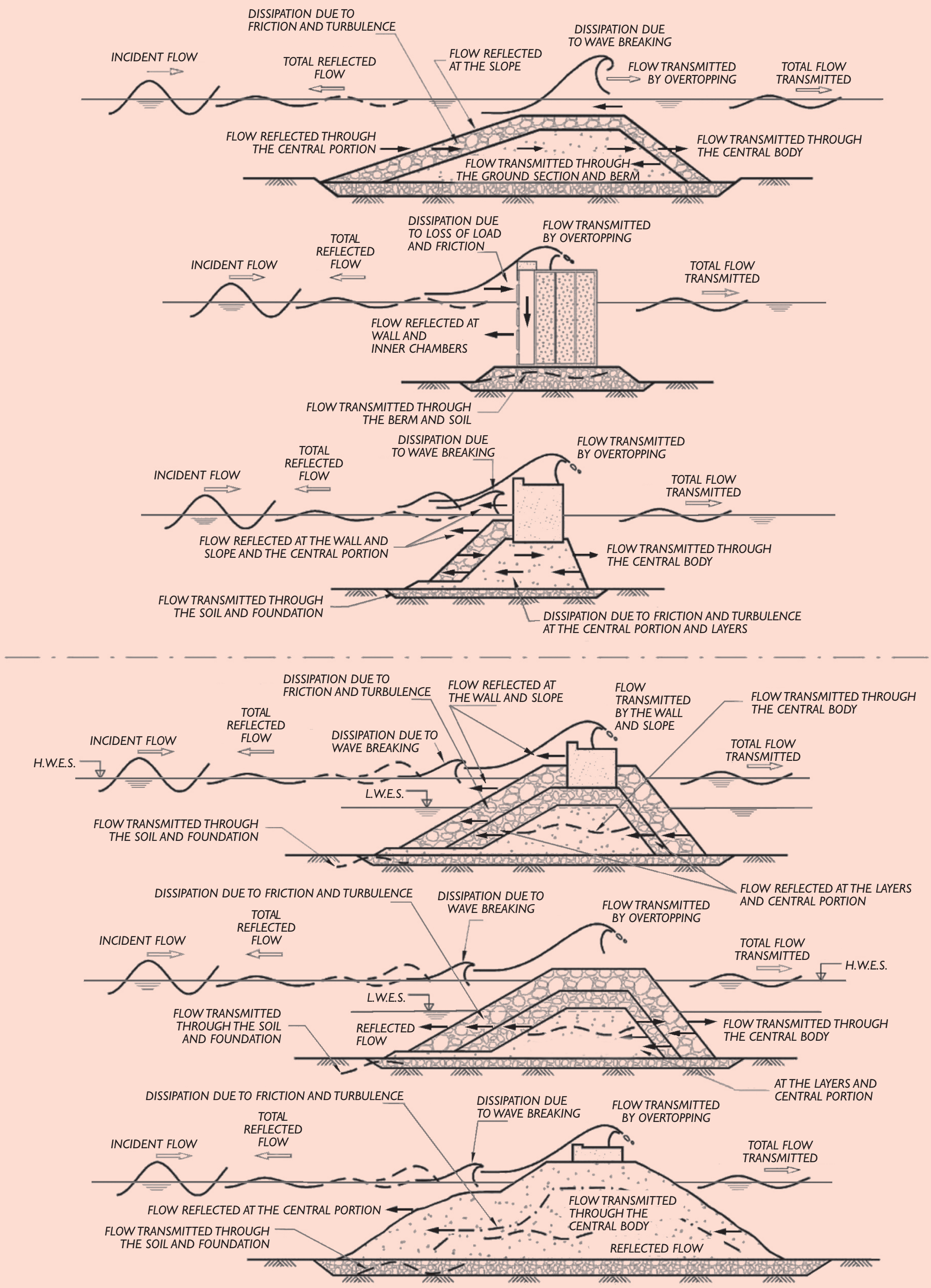
For example, it is possible to construct a submerged breakwater whose crest is located at depth d, and which has a width B. The structure can be made of natural material (e.g. rubble) or artificial material (e.g. concrete). The division of incident energy depends principally on d/h, B/L_d, H_*/h. Moreover, if the breakwater is composed of granular material, energy transformation also depends on the porosity and grain-size of this material. L_d is the length of the wave over the breakwater.

Depending on its dimensions, a wave can break at the toe of the breakwater, on the breakwater, or propagate itself landwards without being significantly affected by the breakwater or with its height reduced because of the friction. H_* is the height of the wave at the toe of the submerged breakwater, but in its presence.

Note. The value of the coefficient can be within the interval $1 \leq \mu \leq 1.8$, depending on d/h, H_I/h, the geometry of the breakwater and the hydraulic characteristics of its layers and central portion.

(8) This sudden change of depth can be a vertical step, but generally it is sufficient that the horizontal length of the slope l_t is much less than the wavelength, in other words $l_t / L \ll 1$, where $l_t = h\text{–}d/\tan\beta$ and β is the angle of the slope.

Figure 2.2.6. **Spatial evolution of the parts of the section and the transformation of incident wave energy**



If the protection layer is not sufficient as a foundation, it can be reinforced by a prefabricated caisson with a slightly emergent impermeable vertical wall. This wall acts in the same way as the central portion of the breakwater, thus increasing the control of incident energy though at the risk of increasing the reflected energy. This is the usual configuration of a composite breakwater. If wave overtopping at the crest is excessive, a possible solution is to build a superstructure on the central portion, whose freeboard is of the same order as the wave height at the toe of the breakwater and in the presence of it H_*. The breakwater can now no longer be overtopped and energy transmission only occurs below the caisson through the porous foundation. This signifies a new increase in reflected energy.

If the energy reflected from the breakwater creates conditions that are unacceptable in its immediate surroundings or for other zones or infrastructures in the harbor area, then the seaward face of the caisson must be modified in order to increase wave dissipation. Possible ways to achieve this goal are: (1) to increase the roughness of the seaward wall of the breakwater, thus increasing the roughness of the boundary; (2) to give the seaward wall a greater slope so as to produce wave breaking; (3) to make the wall permeable, thus creating an inner chamber that will increase the friction losses and give the reflection a phase lag; (4) to build an armor layer on the seaward side of the breakwater.

If granular material is not readily available or the effects produced by the wave energy reflected from the breakwater are of little importance, the caisson can be extended down to the bed, supported by a bottom berm. This transforms it into a vertical breakwater, which for all practical purposes is totally reflective. When the reflected energy must be controlled to a certain extent, the same procedure can be used as for a composite breakwater. In that case, the probability of wave overtopping at the breakwater depends on the relative freeboard, in other words F_c/H_*.

Note. If the wave does not break at the toe of the breakwater, when the freeboard is sufficiently high, the reflection coefficient exceeds 0.7. Then $1.7 < \mu = H_*/H_I \leq 2$. In such conditions if there is no probability of wave overtopping at the vertical breakwater, $F_c/H_* \gtrsim 0.5$ over depth [9] h at the toe of the breakwater.

If sufficient granular material is readily available and it is necessary to control the reflected energy, instead of reinforcing the caisson, the berm can be raised until it emerges; a superstructure can then be built on top of the breakwater. The quarry run material is protected by a succession of sloping layers that significantly increase energy dissipation due to wave breaking, the roughness of the boundary, and the internal friction. The size and phase lag of the reflected energy depend on the angle of the slope, the width and roughness of the armor layers, and the width and porosity of the central portion of the breakwater. This is the usual composition of a traditional rubble mound breakwater. The magnitude of wave overtopping depends on the wave regime at the slope and the relative freeboard, in other words, F_c/H_*.

Note. For an "Iribarren" sloping breakwater, $1.1 < \mu = H_*/H_I < 1.7$, depending on the type of unit pieces in the main layer and on the I_r. For such a breakwater to be impervious to overtopping, the following must be true: $F_c/H_* \gtrsim 0.75$.

In some cases it is not strictly necessary to build a traditional breakwater. Rather, a new breakwater type can be envisaged that is conducive to a different type of energy distribution. For example, a row of piles with diameter D and a separation between the axes of the piles $s/D \approx 2$ causes the radiation (reflection) of part of the incident energy at the outer surface of the piles, the dissipation of energy due to friction and the head loss associated with the passage of the wave through the piles. If there are two rows of piles, this effect becomes even greater; (b) a perforated vertical or slightly sloping screen, extending downwards to the bed or to a certain depth has similar effects. Depending on the project requirements, it may be possible to combine two types of breakwater (e.g., a set of quarry blocks that cause dissipation due to wave breaking or friction, and rows of piles or perforated screens or a caisson facing seaward, which in its backfill has a dock, etc.

The selection of a certain breakwater type and dimensioning should be in consonance with certain requirements, such as the structure's capacity to control incident energy. In the following section other aspects

(9) This depth at the toe of the breakwater should be simultaneous and compatible with the height H_* of the wave train at that point.

are analyzed. Despite the fact that this is not general practice, the optimization of the section should be performed for each subset of the structure in medium and high-level natural construction projects. All intervening factors (e.g. wave energy) should be taken into account.

2.2.2.5 Effects of oblique incidence and irregular wave action

The analysis in the previous section assumes that the wave movement is a monochromatic wave train that impinges perpendicularly against the breakwater section. Nevertheless, especially at great depths, the wave train generally strikes the breakwater obliquely. Its structure can thus be represented by the linear superposition of a high number of random-phase components of various frequencies, directions, and energy content. As a rule, the magnitude of the transformation processes related to incident wave energy, such as reflection, transmission, wave run-up on the slope, overtopping, etc. are reduced when the angle of incidence is increased [(10)].

STATE DESCRIPTORS WITH OBLIQUE INCIDENCE

The representation of the transformation phenomena of an irregular wave train by means of a statistical descriptor is a synthesis of the set of transformations experienced by each wave in the train. However, it does not give sufficient information regarding the behavior of each wave, particularly the highest ones. It should be stressed that in safety-related questions, the highest waves are those that should be considered. For this reason, the spatial results of the transformation processes obtained with statistical methods or spectral wave descriptions are usually more "benign" than those obtained with the application of a regular wave train.

SLOPING BREAKWATER STABILITY

Sloping breakwaters built with slender unit pieces (*piezas esbeltas*), such as tetrapods, dolos, and other similar elements, are maximally stable when they form a specific slope, known as the maximally stable slope for this type of unit piece. The majority of experimental results pertaining to the stability of these blocks have been obtained with normal incidence.

In the case of oblique incidence, the mechanical performance of these unit pieces is as though they were working against a force that impinges at an angle $\tan \alpha \leq \cos \theta$ where α is the angle of the slope and θ, the angle between the normal to the breakwater and the crest of the wave. If this correction is applied to the Iribarren number, it is possible to conclude that that when the oblique incidence increases, other changes produced are a change in the breaker type, a reduction of flow on the slope and eventually a reduction in wave overtopping. These results are valid for moderate angles of oblique incidence, and always in the interval $\theta < 50^o$. The angle of Wrebster falls within the interval $50^o < \theta < 65^o$. Also known as the angle of minimum reflection, this angle depends on the roughness and permeability of the slope, and the flow change on the slope is produced within its boundaries. For values $\theta < 65^o$, the flow is associated with the propagation of the wave train on the slope (shoaling, refraction, transmission, and eventually breaking), while for greater values, the flow is practically longitudinal to the slope.

When the angle of incidence $\theta > 65^o$ is exceeded, the flow due to wave breaking is produced longitudinally to the slope. The wave breaks against the breakwater at mid-level in the form of a stream; reflection is negligible; and wave steepening is mostly produced by diffraction or and "along-crest" transmission of energy. In such conditions, none of the usual stability calculation formulas can be applied, and, generally speaking, any failure in the layer and its components is caused by the desegregation of the layer and the loss of contact between unit pieces.

The interlocking artificial unit pieces are more vulnerable to oblique incidence than cubic or *parallelepiped* artificial blocks and natural rubble material. In those cases in which the design sea states have oblique incidence, oblique, cubic and parallelepiped blocks should be used in the main layer.

(10) The incidence is measured by angle formed by the normal angle to the wave crest and the normal angle to the obstacle.

FORCES IN VERTICAL BREAKWATERS

The magnitude of the dynamic pressure produced by waves impinging on the vertical wall and their distribution along its surface depend on the angle of incidence. For angles such that $\theta < 50º$ and depending on the permeability of the wall, the interference of the incident and reflected wave trains produces a partially standing short-crested train. For small angles of incidence and perfect reflection, the pressure can be greater with small angles of incidence $\theta < 15º$ than with normal incidence.

For angles of incidence falling within the interval $15º < \theta < 50º$, available data seem to confirm that the pressure decreases with the cosine of the angle of incidence.

When the angle of incidence exceeds $\theta > 65º$ (the Wrebster angle) this causes the longitudinal propagation of the wave crest against the wall and the dynamic pressure on the wall decreases considerably. The wall thus acts more like a flow channeler than a flow reflector or deflector. Wave steepening on the wall can thus be significant [(11)], and wave breaking can even occur. This can have undesirable effects such as an increase in wave overtopping and the possible impact that the crest can have on structures or elements transversal to the wall. Consequently, these two effects should be analyzed, always bearing in mind the non-linear nature of the process (in other words, applying higher-order wave theory).

There is little information available concerning the behavior of wave trains impinging within the interval $50º < \theta < 65º$, since the wall can act either as a reflector or a channeler, depending on slight modifications in local conditions. In these circumstances, both processes should be considered and the worst case scenario, selected.

2.2.3 Type sections

As mentioned in the previous section, the different type sections of the breakwater, which are the focus of this ROM, can be obtained by the evolution of the dimensions of each of the parts, the transition mode between them, and their connection to the bed, apart from their various structural and formal elements and sub-elements. The following sections briefly describe the type section of vertical breakwaters, composite breakwaters, sloping breakwaters, berm breakwaters, screen breakwaters, and floating breakwaters.

2.2.3.1 Vertical breakwater

Figure 2.2.7 shows a vertical breakwater section, whose central portion and superstructure are composed of a single structural element. Traditionally, the seaward face of the breakwater is vertical (thus, its name). It can be constructed with prefabricated caissons, massive concrete blocks, sheet piles, etc. The central portion of the breakwater is usually built on a foundation berm made of granular material, adequately protected so as to guarantee its stability against sea oscillations. At great depths, the dimensions of this foundation can be a relevant factor. Such a foundation generally consists of a quarry run base, leveled at a depth that permits the establishment of the central portion (e.g. the caisson anchoring structure) so that its stability is not affected by sea oscillations.

In areas of medium or shallow depth, unless there are complications regarding the carrying capacity of the soil, the foundation can be composed of a filter layer of quarry run material and the self same level berm, all of which are usually significantly less thick than the central part of the breakwater. The thickness of each of these elements and the grain-size of the materials should be in consonance with geotechnical and hydraulic needs. To protect the foundation and the natural seabed against possible erosion, a toe berm should be built, consisting of the extension of the quarry run central portion and the necessary number of armor layers. A guard block is also frequently placed on the berm and attached to the central portion of the breakwater with a view to reducing or producing a phase lag in the subpressure peak on the seaward edge of the foundation in relation to the pressure peak on the breakwater wall.

(11) In technical literature this process is called *Mach-stem*.

The superstructure is usually crowned with a parapet that has a seawards curve in order to facilitate flow return. This structure is called a wave screen.

The breakwater essentially behaves like a reflector of the incident wave energy, and landwards energy transmission only occurs because of wave overtopping or in very small quantities because of the permeability of the foundation.

Generally speaking, vertical breakwaters are more apt for locations where wave breaking against the breakwater is fairly improbable. This only happens when the following conditions hold:

$$\frac{d}{h} \geq 0.85 \tag{2.15}$$

$$\frac{l_t}{L} < \frac{1}{20} \tag{2.16}$$

$$\frac{H_I}{L} \leq \left(0.11 + 0.03\frac{1-K_R}{1+K_R}\right)\tanh\frac{2\pi h}{L} \tag{2.17}$$

$$H_* \simeq \left(1+K_R\right)H_I \tag{2.18}$$

where H_* is a wave height at the toe of the breakwater, but in its presence (12), and is representative or characterizes a meteorological state of extreme work and operating conditions; h is the water depth at the toe of the breakwater (13); and d is the water height at the berm. H_I is the incident wave train height and should be representative of the greatest possible heights that occur in the meteorological state.

The following equation gives an initial estimate of the freeboard with regard to sea level associated with depth h so that there is no probability of wave overtopping at the breakwater:

$$\frac{F_c}{H_*} > 0.50 \tag{2.19}$$

For breakwaters with an impermeable vertical wall which is impervious to wave overtopping, $H_* \approx 2H_I$. If these conditions are satisfied and the breakwater is designed so that wave overtopping cannot occur, the pre-dimensioning of the breakwater should begin with a minimum width B because of wave action

$$\frac{B}{H_*} \sim 1.0 \tag{2.20}$$

When the probability of wave breaking against the breakwater wall is not negligible, its pre-dimensioning should begin with a minimum width B because of wave action,

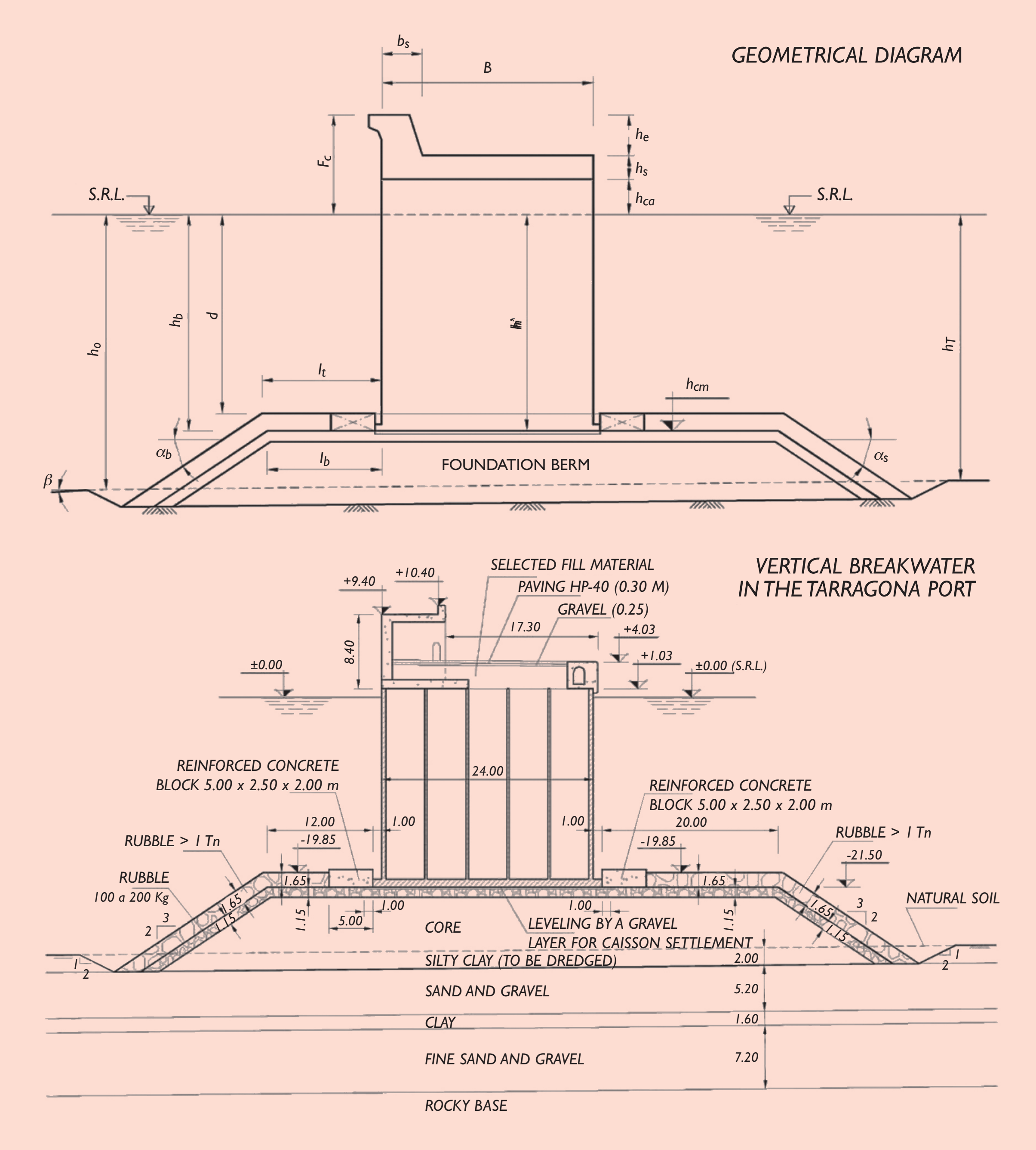
$$\frac{B}{H_*} \sim 1.5 \tag{2.21}$$

Note. It is customary to consider that $H_I \approx (1.6 \text{ to } 1.8)H_{s,I}$, depending on the duration of the sea state, and where $H_{s,I}$ is the significant wave height of this state. For a breakwater with a smooth, impermeable wall impervious to overtopping, reflection is perfect. Thus, $H_* \simeq 2H_I$, $H_* \approx (3.2 \text{ to } 3.6)H_{s,I}$.

(12) In other words, taking into account the modification of the sea oscillation by the harbor or coastal area, breakwaters, river or estuary mouths, changes of alignment, etc. (see Chapter 3).

(13) This depth is simultaneous and compatible with the sea state in which wave height H_* can occur. In this case it is advisable to analyze more than one depth.

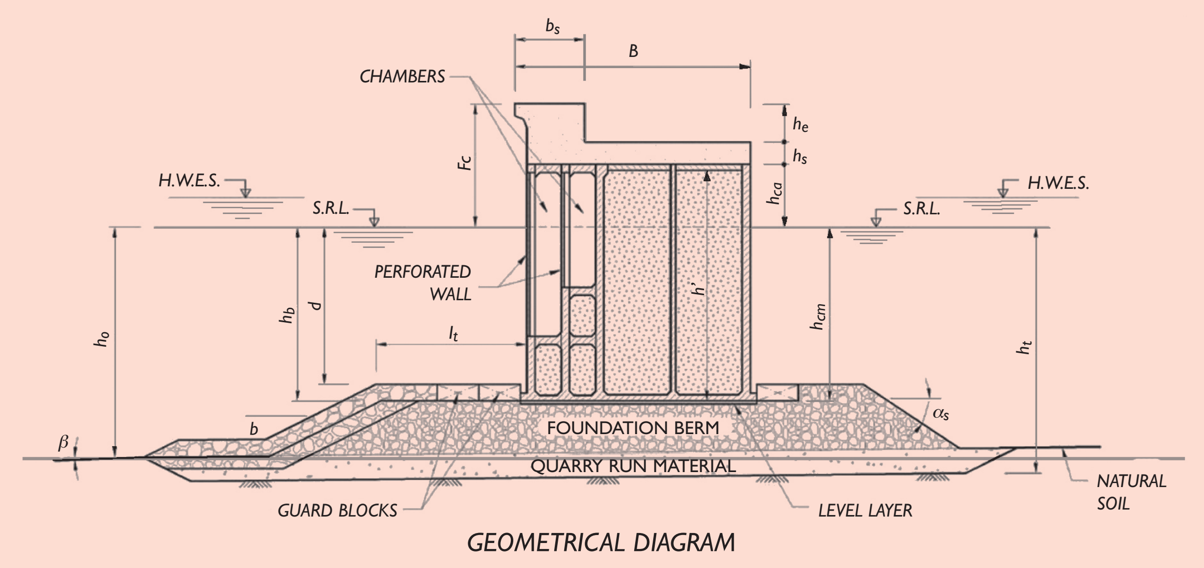
Figure 2.2.7. Vertical breakwater



Vertical breakwater with a special front face

In order to reduce wave reflection, recent studies have been made of vertical breakwaters with a sloping wall that may have perforations or slits, which either cover its entire surface or from a certain depth upwards (see figure 2.2.8). With these modifications of the seaward breakwater wall, incident and reflected wave trains have a phase lag, and the friction dissipation increases. This may reduce the wave height H_* at the toe of the breakwater and in its presence. Though the oscillation chamber helps to bring this about, special attention should be given to its width B_c and depth h_c since the reflection process depends on B_c/L_c where L_c is the wavelength inside the chamber. The oscillation of the water body inside the chamber can have a large amplitude, and when it is covered, the pressures on the chamber roof can be very great.

Figure 2.2.8. **Vertical breakwater with resonance and dissipation chambers**



Vertical breakwater with a protection layer/mantle

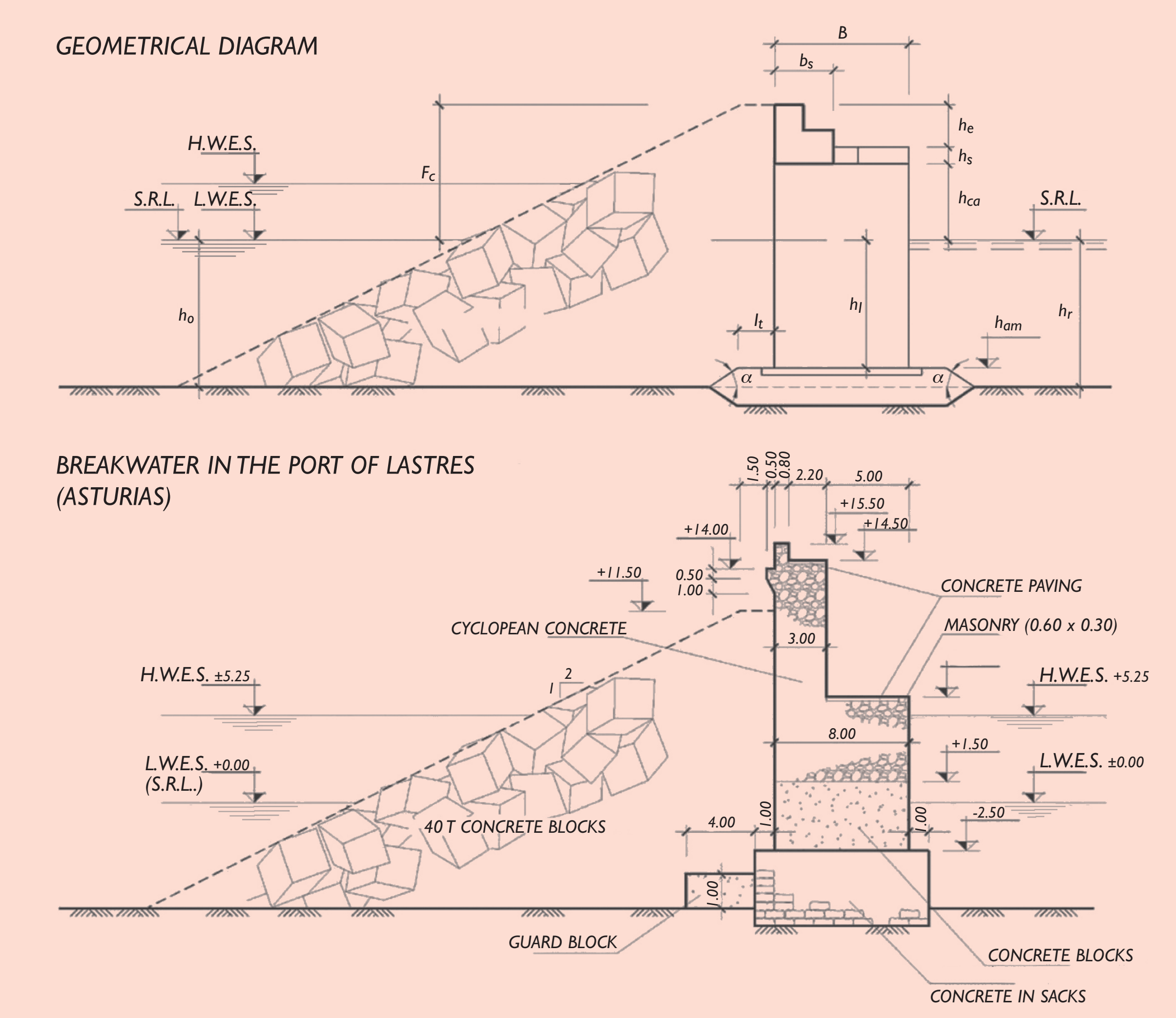
Figure 2.2.9 shows a vertical breakwater protected by a slope of armor units material. In this case the central portion or subset is formed by two structural elements: the slope made of concrete blocks and the vertical wall. Both structural elements generally extend above the water. In this case the vertical wall is also part of the superstructure. The presence of the slope transforms the reflecting breakwater into a partially reflecting and dissipative breakwater. The predominance of one action over the other depends on the characteristics of the incident wave, and the geometrical dimensions of the slope. Since there is an impermeable wall behind the granular slope, the level of reflection in this section and the stability conditions of the layer components are different from those of a rubble mound breakwater with a central portion made of quarry run material (see figure 2.2.11).

Note. In these recommendations, the term (toe) *berm refers to the formal structural element, composed of a series of quarry run filter layers as well as main and secondary layers, generally with a horizontally leveled crest. The transition of the berm to the foundation and to the central portion of the breakwater is usually carried out through the core or foundation material. These extend seawards and progressively level out. The same criteria can be applied to the breakwater head and to the construction of landward toe berms.*

2.2.3.2 Composite breakwater

When the foundation of the vertical breakwater occupies a large proportion of the water depth such that it significantly modifies the kinematics and the dynamics of sea oscillations, it is called a *composite breakwater* (see fig. 2.2.10). As can be observed, protection is afforded by the lower subset (which also provides a foundation) and central subset that extends above the water to become part of the superstructure. In the same way as the vertical breakwater, the wall of the superstructure can be entirely sloping or be partially covered by perforations, slits, chambers, etc. It may also have a wave screen. A composite breakwater has the same dimensioning requirements as a vertical breakwater. To guarantee the stability of the entire structure it may be necessary to build a quarry run toe berm, which, besides acting as a natural soil filter, also provides support for the protection layers. Depending on the water level and the characteristics of the incident wave in relation to the geometrical dimension of the breakwater, its performance can be predominantly reflecting, dissipative or a combination of the two. In other words, it can be partially reflecting and partially dissipative. The landward energy transmission of the breakwater is produced by overtopping at its crest or through its foundation, which can be significant if the filter layers are not properly constructed.

Figure 2.2.9. ***Vertical breakwater with a protection layer***



For the pre-dimensioning of the width of the central portion of a composite breakwater, the same criteria should be applied as for vertical breakwaters. It is thus necessary to consider the probability of wave breaking against the wall. Special attention should always be paid to the transformation of the wave by the foundation and berm, especially since in this particular breakwater type the relative width of the berm l_t / L_d and of the total seaward foundation $(l_t + [h-d] \cot \alpha) / L_d$ can have high values. This can increase wave steepness, eventually causing waves to break against the central portion of the breakwater.

If there is no wave breaking at the slope or against the central portion and the breakwater is designed so that wave overtopping cannot occur, the pre-dimensioning of the central portion should begin with a minimum width at depth d because of wave action such that:

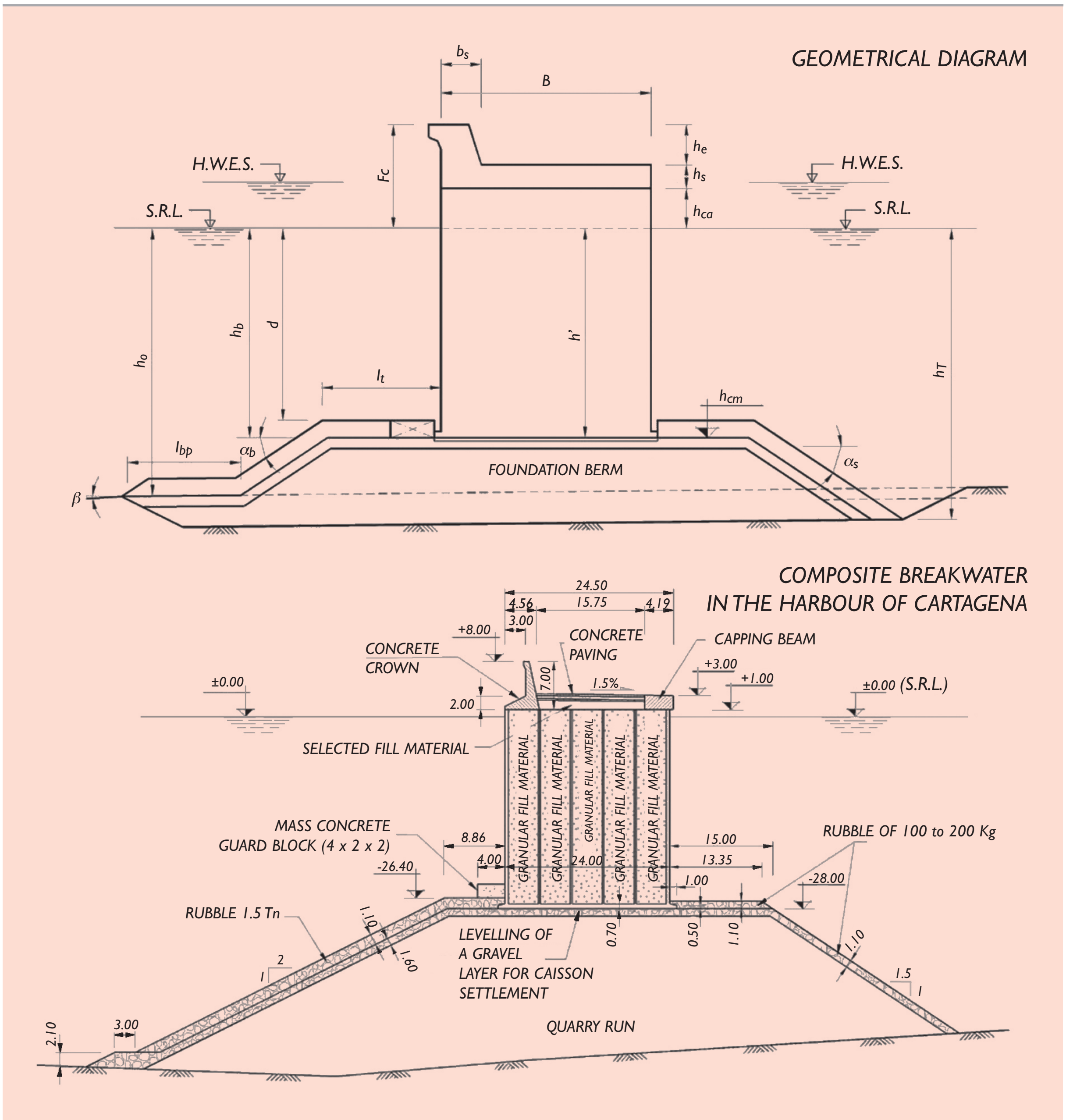
$$\frac{B}{H_*} > 1.0 \tag{2.22}$$

When the probability of wave breaking against the breakwater wall is not negligible, the minimum width to begin its pre-dimensioning should be:

$$\frac{B}{H_*} > 1.5 \tag{2.23}$$

Note. This structure is also referred to as a composite breakwater.

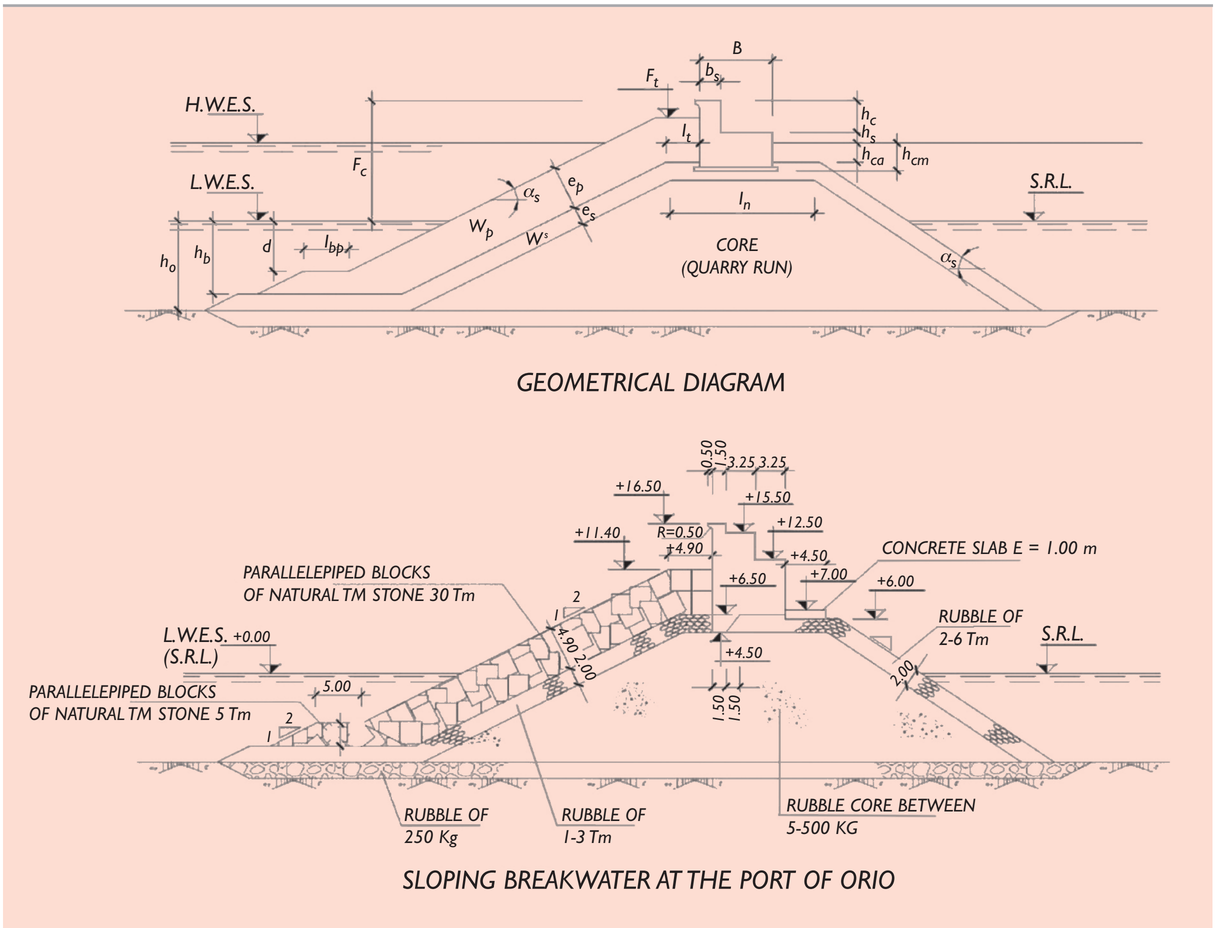
Figure 2.2.10. **Composite breakwater**



2.2.3.3 Sloping breakwater

Figure 2.2.11 shows a traditional (Iribarren) breakwater or rubble-mound breakwater with a crown. The central portion of the breakwater consists of a series of layers, which form a transition between the quarry run core and the main layer, which is made of natural or artificial unit pieces, and is the most resistant element to wave action. Unless the sea bottom is rocky, a toe berm is necessary to guarantee stability and slope form, since it protects the soil and foundation, besides giving support to the principal and secondary layers. A sloping breakwater may or may not have a superstructure. Depending on the characteristics of the incident wave action, particularly wave steepening and the slope of the main layer (whose quotient defines the Iribarren number), the action of the breakwater can be partially reflecting as well as partially dissipative. Energy transmission landwards from the breakwater can be produced by wave overtopping at the crown, and through the foundation and central portion of the breakwater. Such transmission can be significant when it is not prevented by protection layers that act as a filter for the energy flux.

Figure 2.2.11. ***Traditional Iribarren breakwater***



Generally speaking a sloping breakwater can be built to provide protection against any type of wave: non-breaking, breaking, or waves that have already broken. Insofar as possible, it is advisable to use natural stone as the principal component of the main layer and to adopt a slope angle α at the seaward wall of the breakwater within the interval $[1.5 \leq \cot\alpha \leq 3.0]$. If this is not feasible, there is also the possibility of using artificial blocks as the principal component of the main layer and of adopting a slope angle α on the seaward wall of the breakwater within the interval $[1.5 \leq \cot\alpha \leq 3.0]$. Still another option is to use cubic or slightly parallelepiped $(a * a * 1{,}3a)$ massive concrete blocks. In this case, the slope angles should fall within the interval $[1.5 \leq \cot\alpha \leq 2.0]$. For artificial cubic or parallelepiped concrete blocks, the pre-dimensioning calculation for the initiation of damage should be begun with a minimum weight per unit piece in the interval:

$$\frac{W}{\gamma_w R_s H_*^3} \gtrsim 0.020 \tag{2.24}$$

$$H_* \simeq 1.5 H_I \tag{2.25}$$

$$cot\alpha \simeq 1.5 \tag{2.26}$$

$$R_s = \frac{S_s}{\left(S_s - 1\right)^3}; \ S_s = \frac{\gamma_s}{\gamma_w} \tag{2.27}$$

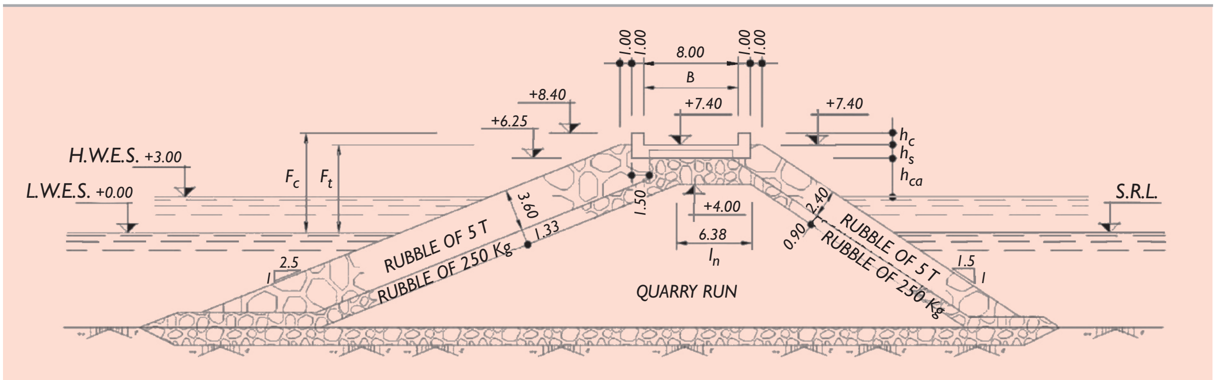
where H_* is a wave height at the toe of the breakwater and in its presence [14], representative of a climate state of extreme work and operating conditions; h is the water depth at the toe of the breakwater [15]; and d is the water height over the berm.

Note. For breakwaters impervious to overtopping and under collapsing or plunging breakers, the following interval holds: $1.3 \leq (\mu = H_* / H_I) \leq 1.7$. *In any case it is best to limit the width of the toe berm* l_t *such that* $l_t / L_d <$ $1/20$ *to prevent wave steepening. However, the berm should be of sufficient width to support the main layers with space left over for two to three blocks more.* L_d *is the wavelength over the berm, that is, at depth* d.

DIMENSIONS OF THE SUPERSTRUCTURE

The dimensions of the superstructure or crown can significantly influence the way that energy flux is controlled. For this reason, there are sloping breakwaters with a very small superstructure located at a level where wave action is negligible (see figure 2.2.12). In Spain it is customary to dimension sloping breakwaters with very large superstructures which control a substantial part of the incident wave energy. In order to make construction easier, they are usually supported above the low tide level, and thus, often have sawtooths. The crown of the breakwater generally has a parapet and a wave screen. If the crown built below the low tide level, the breakwater then resembles a composite breakwater with a protection layer. Until now, it has been rare to find crowns having an outer sloping wall with tiny holes, slits or chambers.

Figure 2.2.12. ***Sloping breakwater with an access path***



When the breakwater has a crown to prevent overtopping, it is advisable for the crown levels of the main layer and the actual crown to meet the following conditions when measured over the design sea level:

$$\frac{F_t}{H_*} \gtrsim 0.60 \tag{2.28}$$

$$\frac{F_c}{H_*} \gtrsim 1.0 \tag{2.29}$$

where the freeboards of the main layer and crown, F_t y F_c, respectively, are measured vertically in relation to the simultaneous sea level compatible with the sea state in which wave height H_* can occur (as an initial estimate, $H_* = 1.5 H_I$). The dimensions of the different parts of a sloping breakwater, particularly the extension of the main layer

(14) In other words, taking into account the modification of the sea oscillation by the harbor or coastal area, breakwaters, river and estuary mouths, changes of alignment, etc. (see Chapter 3).
(15) This depth should be simultaneous and compatible with the sea state in which wave height H_* can appear. When necessary, it is a good idea to analyze more than one depth.

and the height above the crown foundation can have a wide range of variation so as to optimize the availability of materials, construction resources, and economic costs while satisfying project design requirements.

SLOPING BREAKWATER WITHOUT A SUPERSTRUCTURE

When there is no superstructure, the resulting structure is a breakwater without a crown (see fig. 2.2.1.3). This type of breakwater is rare in Spain although it is often used in leeside breakwaters and groins. For no overtopping to occur at the breakwater, the crown level of the main layer must satisfy the following condition:

$$\frac{F_t}{H_*} > 0.9 \tag{2.30}$$

measured vertically in relation to the simultaneous sea level compatible with the sea state in which wave height and period H_*, T can occur $(H_* \approx 1.5H_I)$. When the breakwater cannot be overtopped, the wave energy transmitted landwards from the breakwater depends on the relative values of the freeboard F_t/H_*, wave steepness H_*/L, wave height H_*/h, and crest width B_c/L.

This type of breakwater is the one normally used in the construction of groins perpendicular to the coastline to control sediment transport in surf zones or at river mouths. In this case they also act as inlet groins, offshore breakwaters to protect the coastal area from wave action, etc. Generally, such groins do not have a superstructure though sometimes a concrete slab is placed on them to facilitate access.

2.2.3.4 Berm breakwaters

A berm breakwater whose central portion is a continuation of its foundation is known as a *berm breakwater*. It consists of granular material of non-uniform grain-size, commonly known as *rip-rap*. The geometry of the breakwater resembles a "reclining S" with the gentlest part of the slope near the average sea level in order to favor wave breaking. The following subsections describe three types of berm breakwaters.

BERM BREAKWATER WITH CORE AND LAYERS

The breakwater section, core and outer layers, are built with rock sizes distributed on the same granulometric curve. The slope of the S is not pronounced and is rather flat so as to minimize destabilization due to gravitational force. This type of breakwater is vulnerable to deformation because the unit pieces at the middle level of the structure can suffer extensive displacement in the same way as sand in a beach profile. This movement evidently affects the durability of the breakwater, and its construction requires great quantities of material.

Figure 2.2.15 shows a berm breakwater with a main rubble layer, constructed in such a way that the grain-size of the material is in consonance with a specific granulometric curve. The sizes of these rocks should satisfy the filter condition for all of the sizes of the material in adjacent layers.

The mobility of these rocks under wave action is much less than that of the rocks in a breakwater without layers, although when the sea state threshold is exceeded, these rocks can suffer displacement that substantially modifies the geometry of the breakwater and causes the appearance of rocks belonging to the core. Such movements make the rock less durable. In the same way as a breakwater without layers, the construction of this breakwater type also requires great quantities of material though in slightly lesser amounts.

This type of berm breakwater is seldom built in Spain because of the quantity of material needed and the space required. However, such breakwaters are often used as provisional/temporary breakwaters in the construction phase to afford protection for bays where the quarry is loaded or to create a safe shelter for the winter months.

Figure 2.2.13. Sloping breakwater without a superstructure

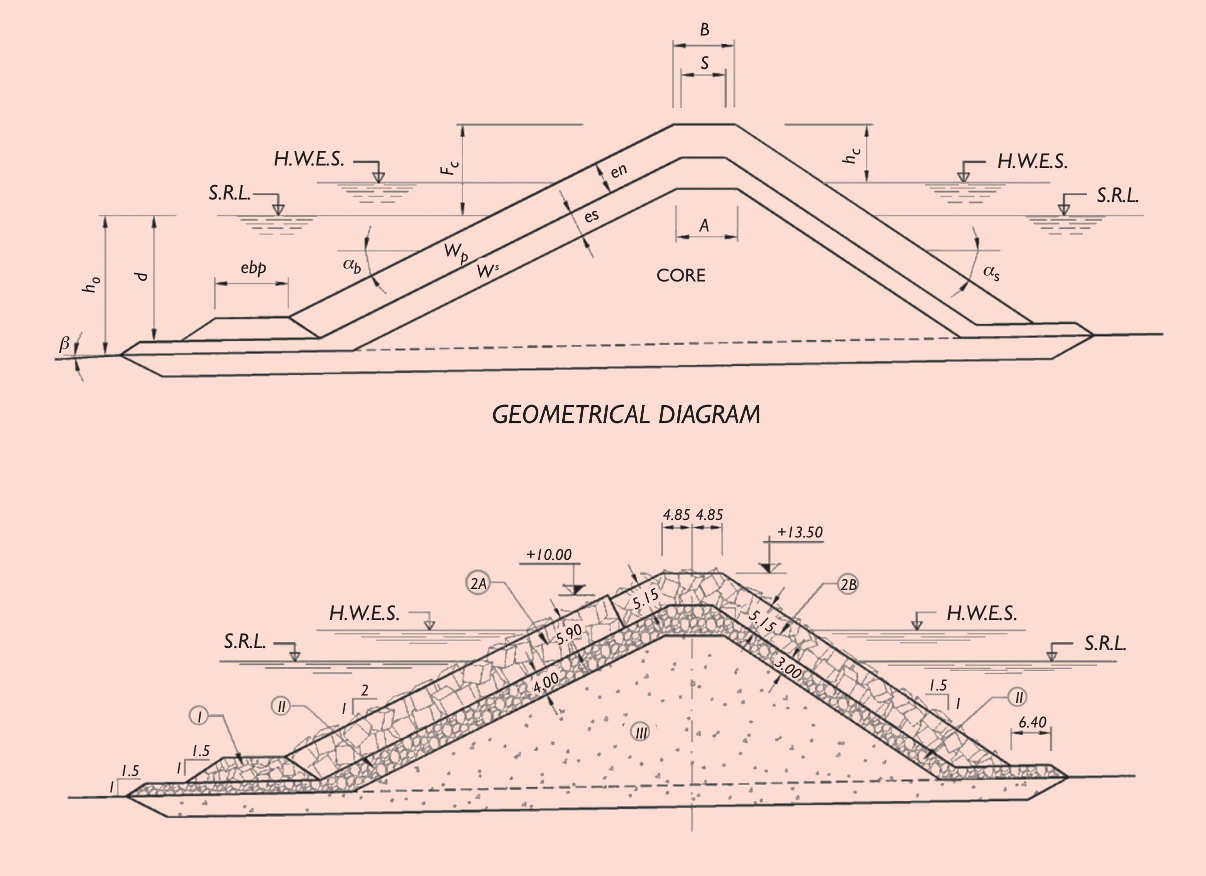


Figure 2.2.14. Constructed and reshaped berm breakwater

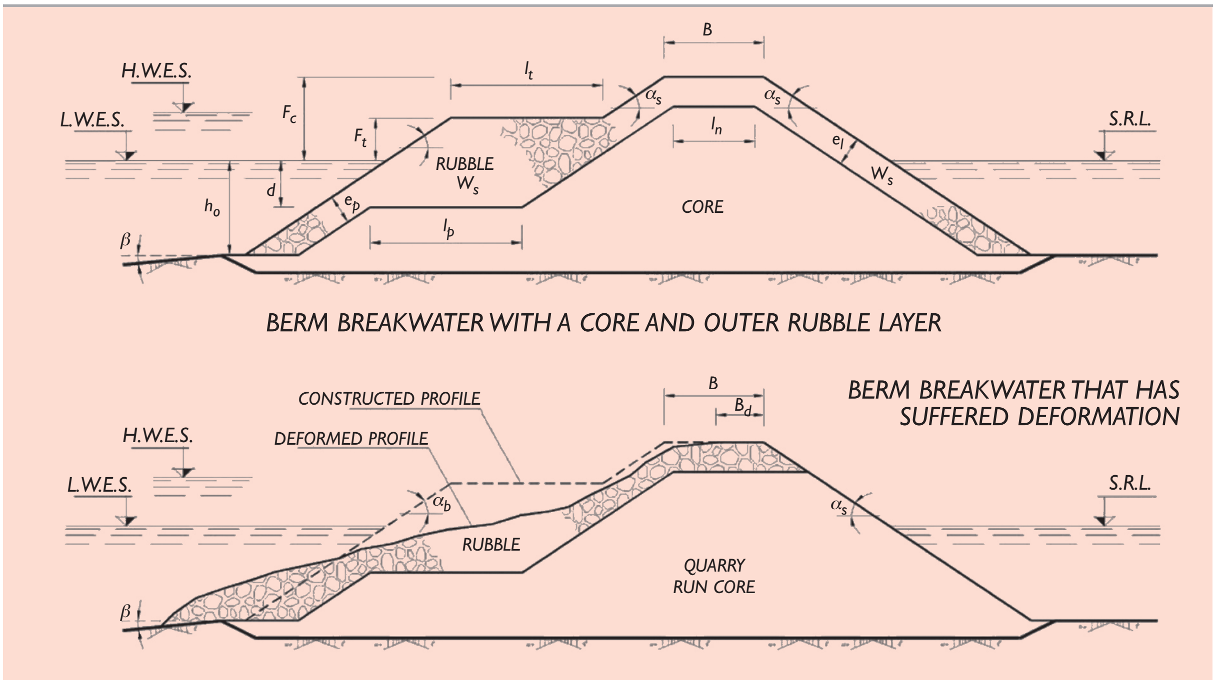


Figure 2.2.15. **Berm breakwater with rubble layers**

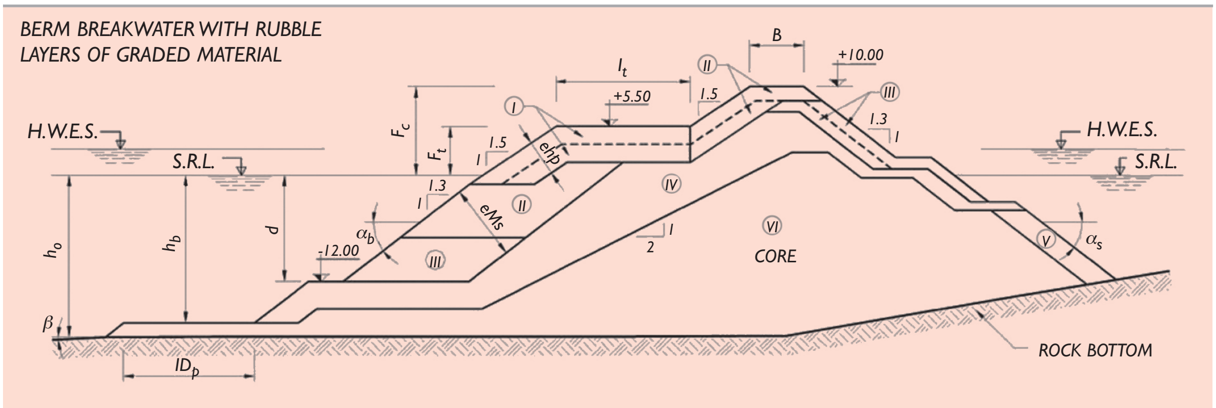
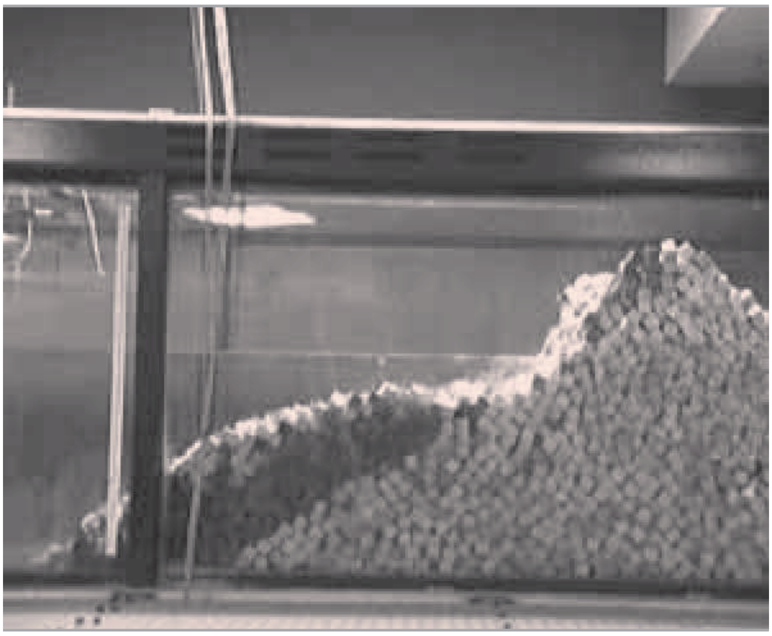


Figure 2.2.16. **S-shaped breakwater with an armored outer layer**



BERM BREAKWATER WITH A MAIN LAYER (ARMORED BREAKWATER)

Figure 2.2.16 shows a berm breakwater with a main layer of natural or artificial rocks of equal size, whose function is to armor the slope. The geometry of the profile is polygonal with three slopes. The length of the central polygon depends on the weight of the rock. In work conditions associated with an ultimate limit state, the breakwater is basically dissipative, and the reflected and transmitted flows are negligible. The central portion of the breakwater can be made of any type of material, but it should provide adequate support so that the rocks in the main armor layer can be arranged with the prescribed orientation. The upper section of the polygon can be replaced by a rigid, permeable, rough superstructure with the same slope. This breakwater has absolute stability for all wave heights lesser than or equal to the design wave height.

2.2.3.5 *Submerged breakwater*

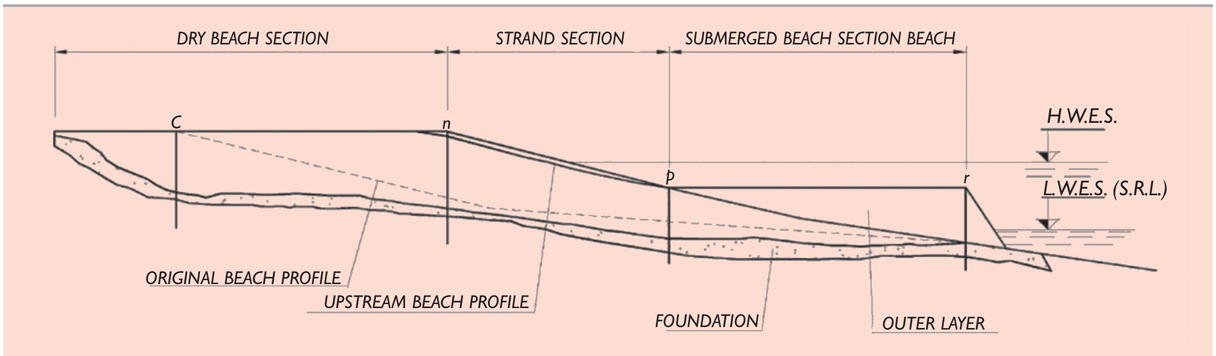
The crown height of the superstructure defines whether the breakwater is submerged. If it is above the average reference sea level, the breakwater is defined as *emerged*, and if it is below that level, then the breakwater is submerged. In the first case, the crown height is known as the *freeboard*, F_c. In the case of a submerged breakwater, the water depth above the crest of the breakwater is called the submersion or immersion depth, which is represented in this ROM by the letter d.

$$F_c > 0, \text{ emerged breakwater} \tag{2.31}$$

$$0 \leq \frac{d}{h} < 1, \text{ submerged breakwater} \tag{2.32}$$

Breakwaters whose crown height is near the average sea level can indeed be emerged or submerged, depending on the meteorological or astronomical tide of the moment. Groins, whose function is to prevent sand and sediment transport at beaches and river mouths or estuaries, generally have an emerged section without a superstructure, and another submerged section that extends towards the sea (see Fig, 2.2.17).

Figure 2.2.17. Rubble mound breakwater: groin



SUBMERGED SLOPING BREAKWATER

The geometry of submerged sloping breakwaters is very similar to that of emerged breakwaters without a crown since a series of layers are built on top of a quarry run core until the outer or main layer is reached. The main layer should extend along the crest, and depending on its width, may even continue along the landward layer.

Figure 2.2.18. Submerged sloping breakwater

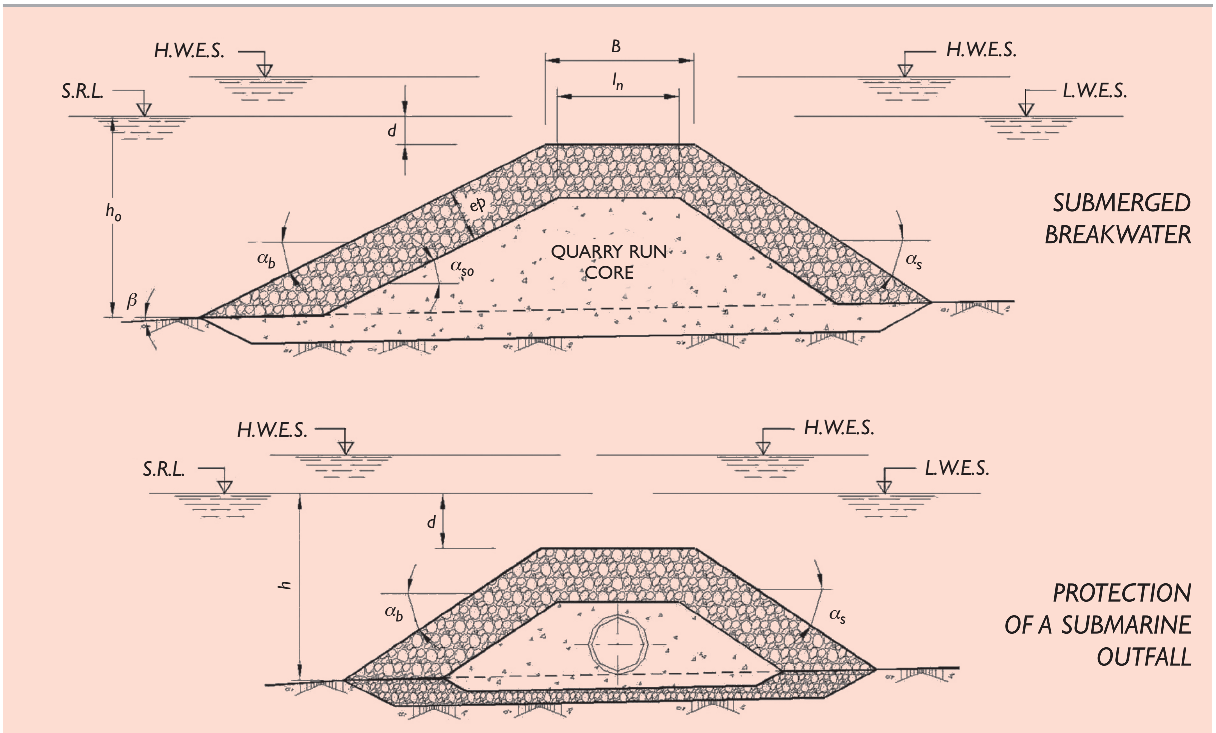
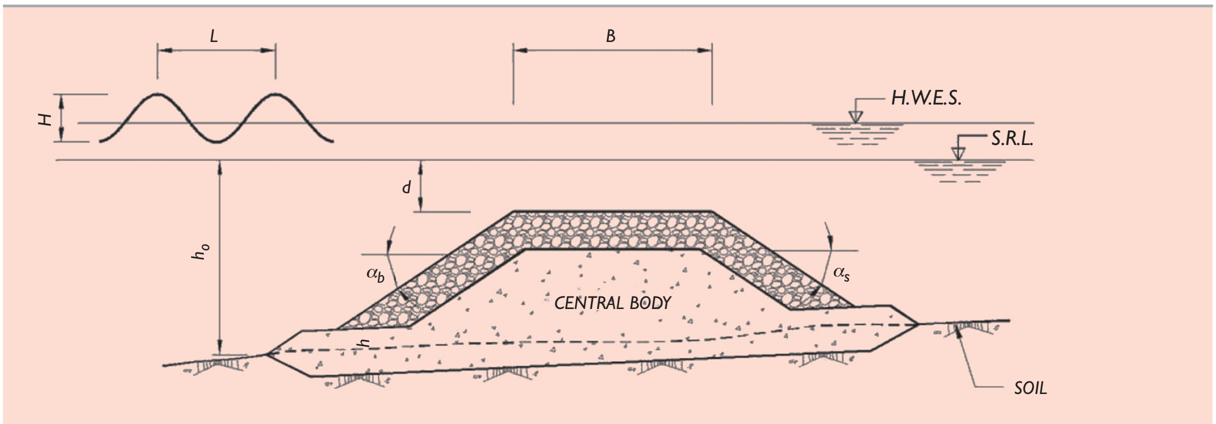


Figure 2.2.1.8 shows the cross-section of a submerged sloping breakwater. Sometimes the breakwater only has a central portion and main layer. It may even be built of only one type of material. Such a breakwater is generally constructed without a superstructure, and only consists of a foundation berm and a central portion. The characteristics of the foundation depend on the nature of the seabed. Unless the seabed happens to be rocky, the foundation berm and perimeter of the breakwater should be carefully constructed so as to guarantee its stability and position.

The energy transmission in submerged breakwaters occurs when waves go over the crest as well as through the granular central portion. The latter type of transmission will be very slight when the structure has been provided with the necessary number of filter layers. The quotient of the seaward and landward wave energy fluxes of the breakwater or the transmission coefficient (16) depends on the steepness of the wave, H_* /L or the relative wave height, H_* /h, depending on if it is in a Stokes or Boussinesq wave regime, the relative depth at the toe of the breakwater, h /L, as well as the relative depth and width of the crest d /h and B /L.

If necessary for construction purposes, the submerged breakwater can be built with prefabricated elements, e.g. concrete blocks with or without perforations, whose shape can be cubic, triangular, trapezoidal, etc. Other times, the breakwater can be built so that it resembles a reef. This causes the highest waves to break and thus dissipate energy and allows the smaller waves with a lower energy content to pass over the breakwater (see figure 2.2.19).

Figure 2.2.19. ***Rubble reef breakwater***



2.2.3.6 *Floating breakwater*

The type section of a floating breakwater (see fig. 2.2.20) is generally composed of a parallelepiped central portion of height h_f and width B_c. The flotation depth d and the freeboard F_c depend on the load state of the floating structure satisfying the equation:

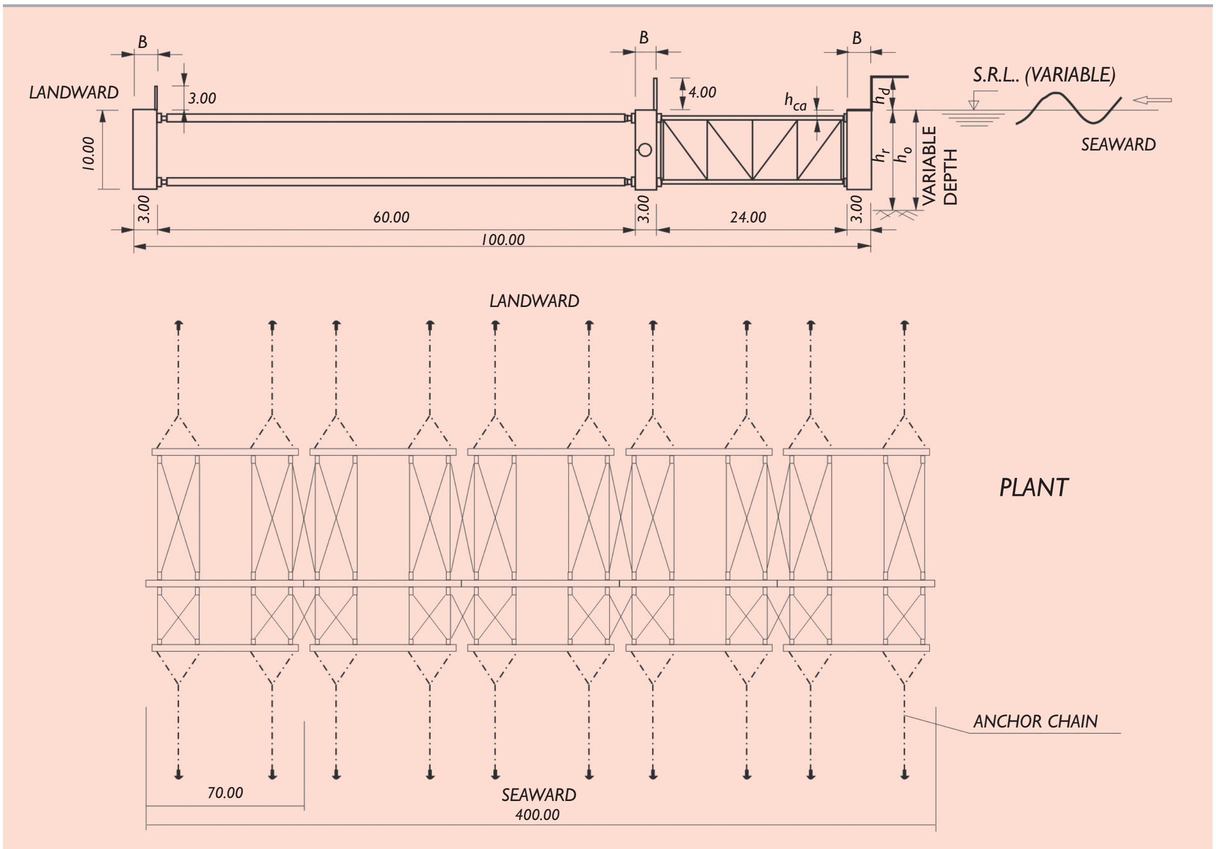
$$h_f = d + F_c \tag{2.33}$$

The central portion of the breakwater can be secured by using chains to anchor it to the seabed, to massive concrete dead weights or to other fixed structures. The chains can also be fastened to piles driven into the seabed with elements which facilitate vertical movement along the length of the pile, but prevent the horizontal movements and turning of the floating structure.

(16) This coefficient is defined for the most propagating wave modes or inthe case of wave action, for the incident and transmitted energy spectra, assuming that the peak period does not change during the propagation.

This type of breakwater does not normally have a superstructure although occasionally, it may have a wave screen on top of the central portion. The wave screen is generally made of impermeable, watertight fiber glass. Although a variety of materials can be used in the construction of floating breakwaters (e.g. pipes, tires, iron caissons, reinforced, pre-tensioned concrete, etc.), the choice of construction material generally depends on the importance of the sheltered area.

Figure 2.2.20. **Floating breakwater**

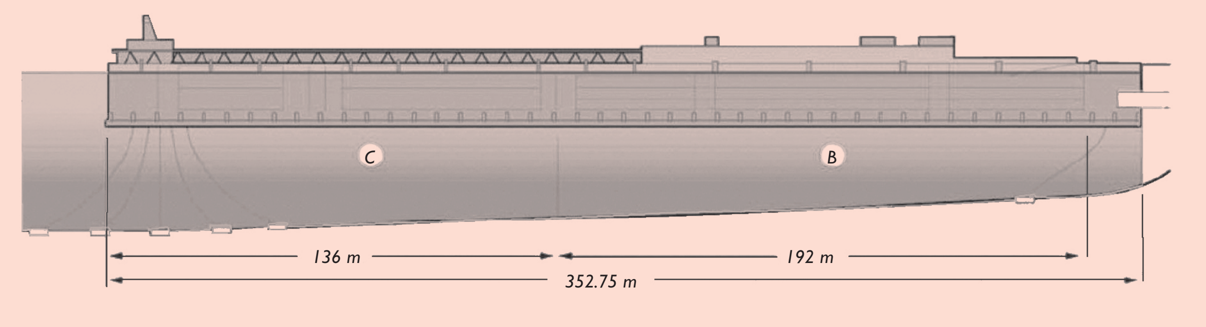


The main purpose of this type of breakwater is generally to control small, short-period waves that do not break. The capacity of the floating breakwater to control incident wave energy depends on the relative depth d/L and width B_c/L. When the floating structure is impermeable and its relative width is small $[B_c/L < 0.10]$, the control of the energy transmitted only depends on the relative flotation depth. In these circumstances, to totally control the incident energy flux it is necessary that $d/L > 0.5$. In this case the breakwater acts as a fully reflective structure. The flux of transmitted energy rapidly increases as the relative depth decreases. When $d/L \approx 0.25$, more than half of the incident energy is transmitted landwards from the breakwater. As a rule, floating breakwaters are not very efficient when it comes to controlling long-period wave action.

The wave height seaward from the breakwater H_* depends on the location of the breakwater, changes in alignment, etc. (see chapter 4). As an initial approximation, H_* can be regarded as twice the incident wave height H_I. For this reason, in order for the breakwater to be impervious to overtopping, the following should be true: $F_c > H_I$.

However, recently, there have been a few experiments that use floating breakwaters as protection against extreme sea states. In these experiments, steel caissons or reinforced, pre-tensioned concrete were used as a solution for sites with very demanding morphological or environmental conditions (see figure 2.2.21).

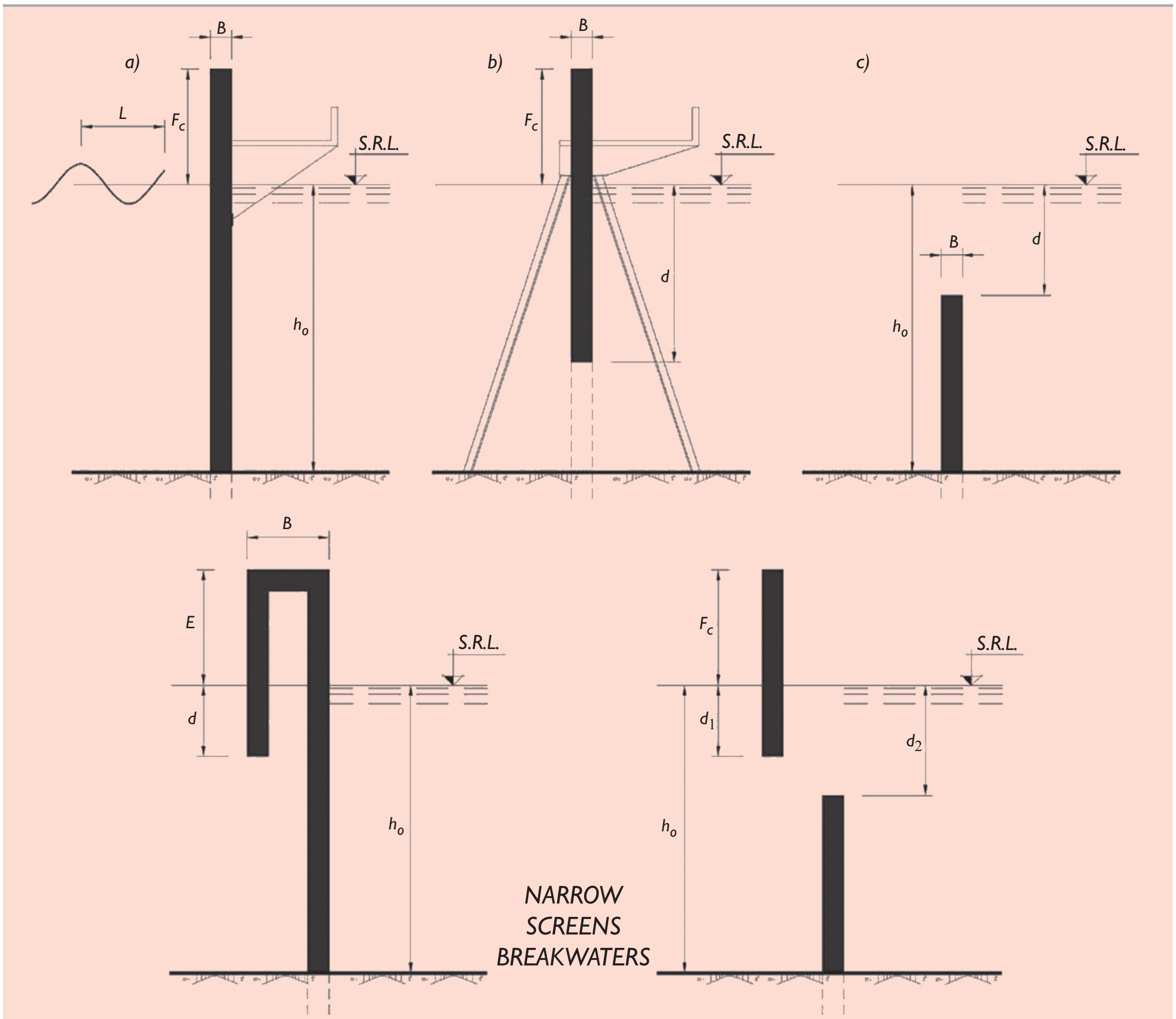
Figure 2.2.21. **Large floating breakwater. Monte Carlo, Monaco**



2.2.3.7 Narrow-section breakwaters and multiple systems

When wave action is not significant (of the order $H_I < 2\ m$ and $< T_z < 7\ s$) and there is no wave breaking (i.e. inside wharfs, tidal reaches, estuaries, and confined seas), a protected area can be created with narrow-section breakwaters, consisting of reinforced concrete screens or of prefabricated elements tied to piles, concrete blocks, etc. (see figure 2.2.22).

Figure 2.2.22. **Arrangement of narrow screens**

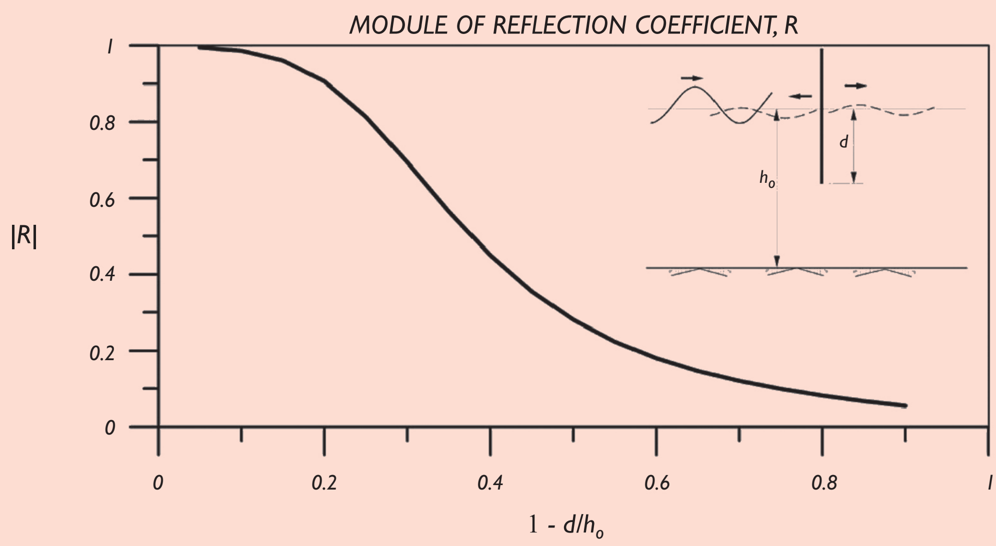


Depending on the characteristics of the wave action, the screen can be submerged to depth *d,* can have slits, or can be extended from the seabed to a specific depth d without emerging. Generally speaking, if it does not have slits, dissipation is negligible, and the energy conservation equation can be used to relate the reflection and transmission coefficients (see figure 2.2.23).

$$K_R^2 + K_T^2 = 1 \tag{2.34}$$

The pressures on the screen can be transmitted to the soil by means of piles which should be sloping if there is a strong horizontal force.

Figure 2.2.23. **Reflection coefficients of a narrow screen**



Note. These screens mainly act as reflectors. The minimum freeboard for the screen to totally prevent wave overtopping is $F_c/H_* \approx 0.5$*, where* $H_* = (1 + K_R)H_I$ *and* K_R *depends on* d/L.

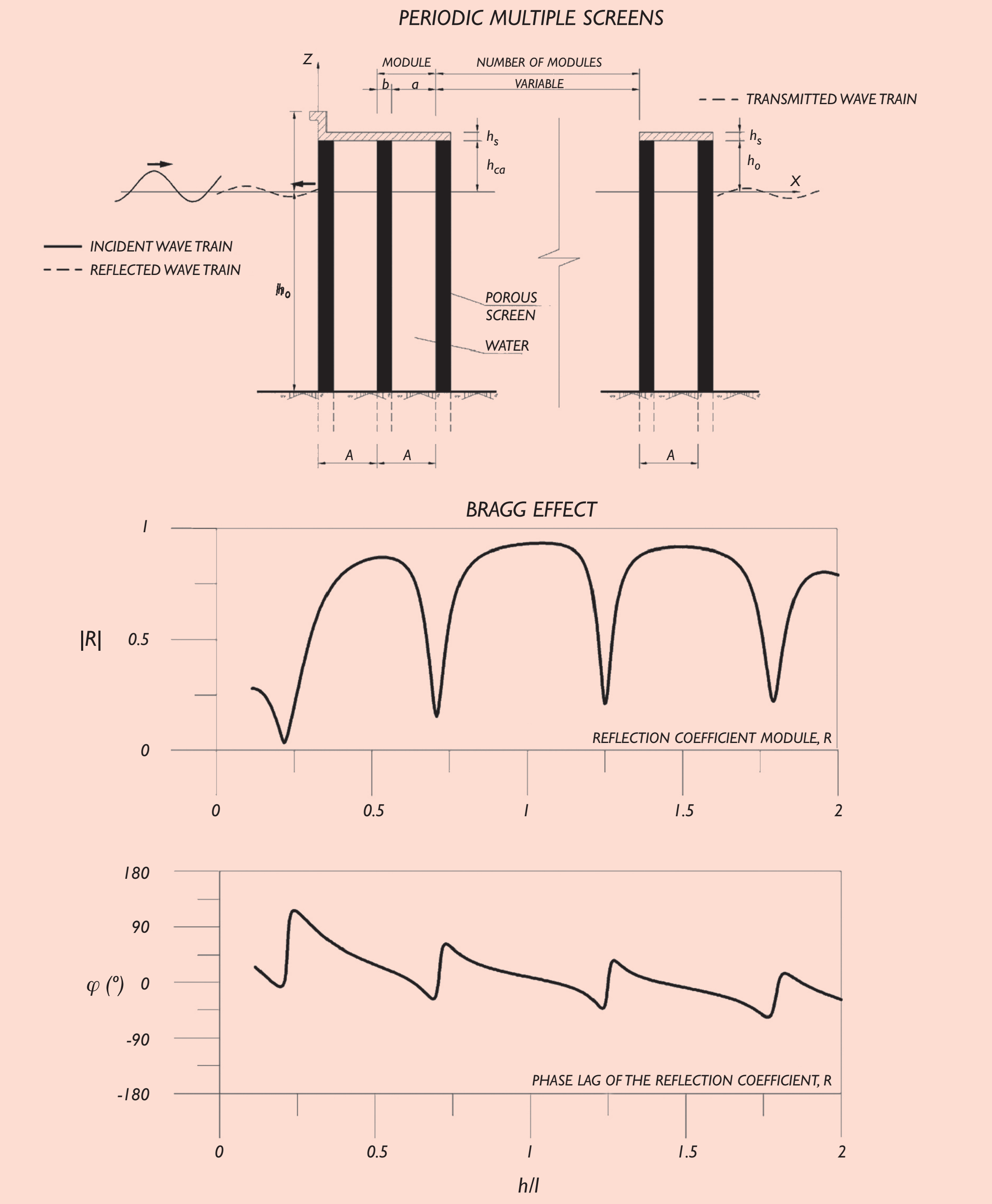
MULTIPLE SCREEN SYSTEMS

Sometimes there can be two screens with a confined water body between them (see figure 2.2.22). Depending on the distance between the screens and the depth of each, the resulting breakwater can act as a perfect resonator for certain wave periods. It thus reflects the maximum amount of energy, but transmits all of the energy for other wave periods. This effect can be reinforced by building periodic sequences of porous screens of a given width Λ. In this case the breakwater operates like a Bragg resonator, increasing the reflection for certain values of $k\Lambda$, and letting past the incident energy for other values (see figure 2.2.24). This type of breakwater is very useful to control wave reflection in experimental tanks, as well as the water agitation produced by the wakes of boats in harbor areas.

SCREENS MADE OF PILES AND SUBMERGED ELEMENTS

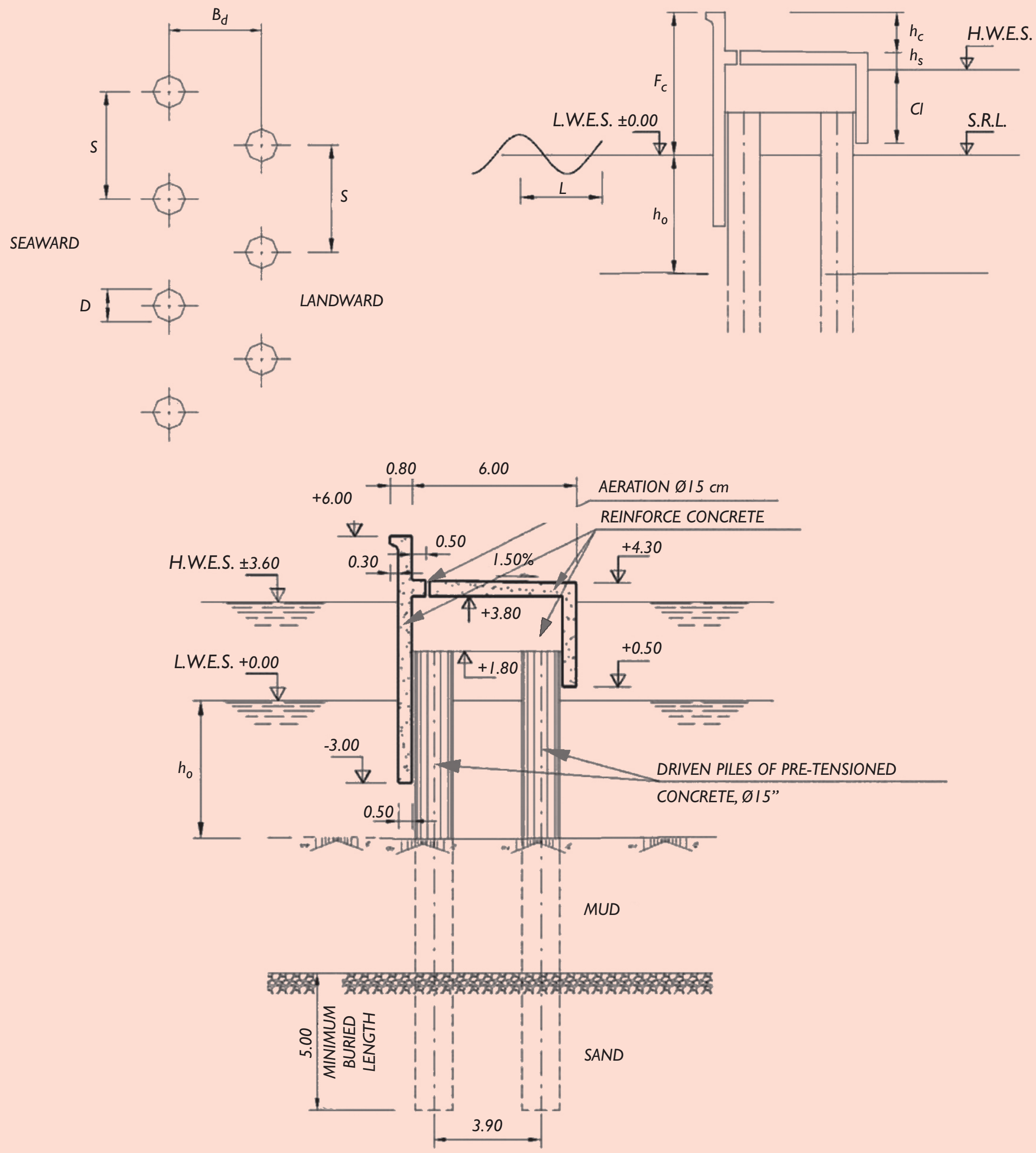
Similarly, it is possible to create porous structures consisting of rows of piles that dissipate frictional energy as well as wakes, eddies, etc. At the same time, such structures also dissipate part of the reflected energy and part of the transmitted energy. The efficiency of these screens depends on the number of rows and the separation between them, which specifies the relative width of the pile field B/L, their diameter D, and the separation s between piles in each row (see figure 2.2.25).

Figure 2.2.24. **Periodic system of porous screens**



In some cases it is not strictly necessary for the porous screens or the elements of the field of obstacles to be emerged (above the water). It is sufficient that they form a sequence of elements like the submerged bars in a beach profile. In such cases their energy reflecting efficiency can be significant. The principal drawback of these multiple types of breakwater is their selectivity in relation to the wave period. For some wave periods a virtually perfect reflector is obtained, while for other periods there is almost full and total transmission.

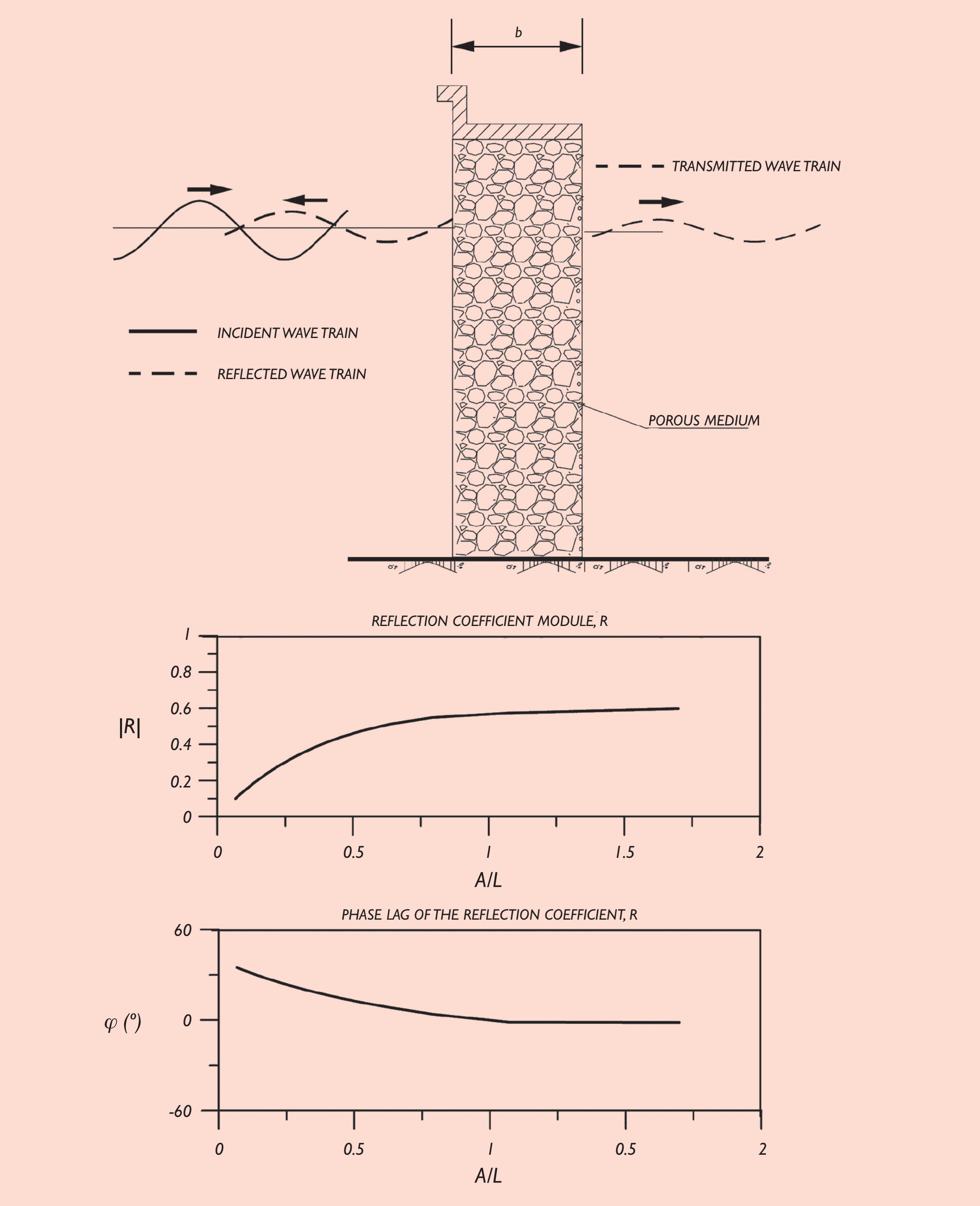
Figure 2.2.25. ***Double-row pile breakwater. Bonanza, Sanlúcar de Barrameda, Cádiz***



2.2.3.8 Other breakwater types

Previous sections of this ROM describe the most common breakwater types built for harbor and coastal areas. However, at the moment environmental restrictions regarding the use of granular material, the transient nature of many breakwaters, and the specificity of their function, not to mention recent technological advances, all point to the advisability of creating new types of breakwater that are better adapted to project requirements. For this purpose, it is best to follow the work method already mentioned, and define the parts and elements of the breakwater as well as their function in all aspects pertaining to energy control and structural stability (see figure 2.2.26).

Figure 2.2.26. **Porous vertical breakwater**



2.2.4 Criteria for breakwater type selection

When choosing the most suitable breakwater type for each subset, the following criteria should be taken into account regarding the adaptation of breakwater type to the requirements:

1. The agents of the physical environment, soil, use and exploitation, materials, and construction methods and procedures;
2. Use and exploitation, and the determination imposed by morphological, environmental, and construction constraints as well as those related to the local maintenance, repair, and dismantling materials;
3. Coastal morphodynamics, water quality, and the surrounding environment.

Generally speaking, if more than one breakwater satisfies the first two criteria and fulfills the environmental requirements of the third, it is best to choose the least expensive option. In the economic evaluation one should consider the initial investment and costs, which in this case would include maintenance and repair work as well as the effects of this work in harbor operations.

Due to severe environmental and climatic conditions that breakwaters are generally subject to, it is usually less costly to select robust, simple, and durable breakwater types, which will need very little maintenance during their useful life and which are easy to build and repair.

2.2.4.1 Breakwaters versus maritime agents

The performance of the breakwater subject to the action of wave climate agents depends on its geometry and the configuration of its components and elements related to wave action, particularly, the wave action at the toe of the breakwater and in its presence (i.e. the interaction of waves with the structure and the possible occurrence of wave breaking) and the water depth h. Evidently, breakwater performance cannot be determined until the climate agent regimes at the breakwater site are calculated and until the plan and elevation of the breakwater are dimensioned.

Selection criteria depending on climate agents

Table 2.2.2.1 lists the most suitable breakwater type according to the wave action and water depth characterized by H_*, L, and h.

Table 2.2.1. Breakwater type according to climate agents

Breakwater typology	Wave action at the breakwater	Water depth (m)
Sloping	All wave types	$0 \leq h_* < 35 - 45$
Vertical	No wave breaking	$15 \leq h_* < 40 - 50$
Composite	No wave breaking	$20 \leq h_* < 60 - 80$
Berm	All wave types	$0 \leq h_* < 35 - 40$
Submerged	All wave types	All depths
Floating and screens	Small, short period waves, no breakers	All depths

2.2.4.2 Soil behavior

A crucial factor in the selection of breakwater type is the adequacy of the sea bed to withstand the forces transmitted by the breakwater and the sea oscillations. More specifically, this refers to: (1) its compressibility or deformation capacity, which entails variations in volume when compression loads are applied to its surface; (2) its resistance to shear stress and the capacity of the seabed to resist the sliding that may occur between adjacent soil when it is subject to this stress; (3) the capacity of seabed particles to resist displacement when they are subject to sea dynamics.

Rock and granular soil. Regardless of possible deterioration, rocky seabeds are generally appropriate for any breakwater type. Soil and fill material of loose, non-cohesive materials, large-grain sand, and gravel are also apt for any type of breakwater because of their high permeability. This permeability allows them to drain interstitial fluid with relative facility when they are subject to cyclical loads. Even so, it is necessary to consider the time necessary for the soil to expel the interstitial water in the presence of the structure, and be aware that an excessive amount of interstitial pressure can arise inside the soil. This surplus pressure can produce a variation in the effective soil stress. In loose granular soils, special attention should be paid to the dynamic effects associated with wave movements because of their possible liquefaction.

Weak cohesive soils. Soils made up of fine and very fine-grain particles generally have low degree of permeability and high compressibility. As a result, their shear resistance is greatly influenced by drainage condition, the velocity of application of the action, and the stress history of the soil. In such cases, it is necessary to study the accumulation and excess of interstitial pressure inside the soil since it can cause a decrease in the rigidity module of the soils in the case of NC clay, or its increase in the case of SC clay. This type of soil is not the most appropriate to receive concentrated loads and to control settlements

Soil-breakwater interaction. In any case, it should be pointed out that the presence of the structure can modify soil resistance as well as the wave regimes outside and inside the soil. Vertical breakwaters located on cohesive or low-quality soils and infill should be avoided since wave action can produce a high concentration of differential loads and settlements. In this case a more resistant and less deformable foundation berm is necessary. Furthermore, because of their geometry and construction, vertical breakwaters are impermeable to flow, and thus, they can substantially modify the drainage patterns of this soil and infill. This effect becomes even more detrimental when the soil is less permeable.

Berms and fills. Generally speaking, breakwaters are built on a foundation composed of berms and coarse-grained, highly permeable fill material, which facilitates load distribution and the liberation of interstitial pressure. This type of soil/foundation is resistant to sheer stress and low deformability. In the case of rocky beds it would be possible to level them with submerged concrete. The more apt the soil is for the breakwater, the thinner the foundation should be. If the soil structure satisfies the geotechnical requirements (ROM 0.5-05), except in the case of great depths, the widths of the berms and fill materials are only large enough to homogenize and level the foundations.

Surface erosion. Surface erosion depends on the composition and granulometry of the soil, as well as its surface and bottom wave regime. Since the presence of the breakwater significantly modifies this regime, except in the case of rock, soil of all types should be protected against erosion.

SELECTION CRITERIA ACCORDING TO SOIL PROPERTIES

Based on the previous indications, the most optimal types of breakwaters in respect to soil characteristics are the following (table 2.2.2):

Table 2.2.2. ***Optimal breakwater type according to soil type***

Soil type	Optimal breakwater type
Rock	All types
Loose granular soil	Some types
Hard granular	All types
Soft, cohesive soils or low-quality fill material	All types except vertical breakwaters
Homogeneous, permeable infill	All types

2.2.4.3 Breakwater optimality according to morphological constraints

The combination of the availability of ground space and natural water depths at the breakwater site can condition the selection of a breakwater type. Generally speaking, breakwaters, except for floating breakwaters or vertical breakwaters built with screens or cofferdams occupy a large surface area (especially when there is significant water depth) either because of their own size or because they need large foundation berms. For this reason, such breakwaters are not viable in areas where there are limitations of space or where the sea bed is affected. Furthermore, breakwaters should not be built in places where there are steep slopes and the soil quality is such that extensive dredging is necessary. It is generally the case that vertical breakwaters require a smaller volume of borrow materials when the breakwater is constructed in deep water ($> 25\ m$).

2.2.4.4 Breakwater optimality according to constraints related to construction materials and processes

The availability of adequate amounts of high-quality material in all project phases as well as sea and land construction equipment significantly condition the choice of breakwater type.

Borrow material. A sloping or berm breakwater with granular layers is the best option when granular material is available in the proximity of the building site. However this type of breakwater should be ruled out when there is a lack of quarry run material for the rock core in the underlayer and armor layer. Other factors that also make the choice of this type of breakwater unwise are excessive water depth ($h > 40 - 50\ m$) and the lack of the necessary construction equipment.

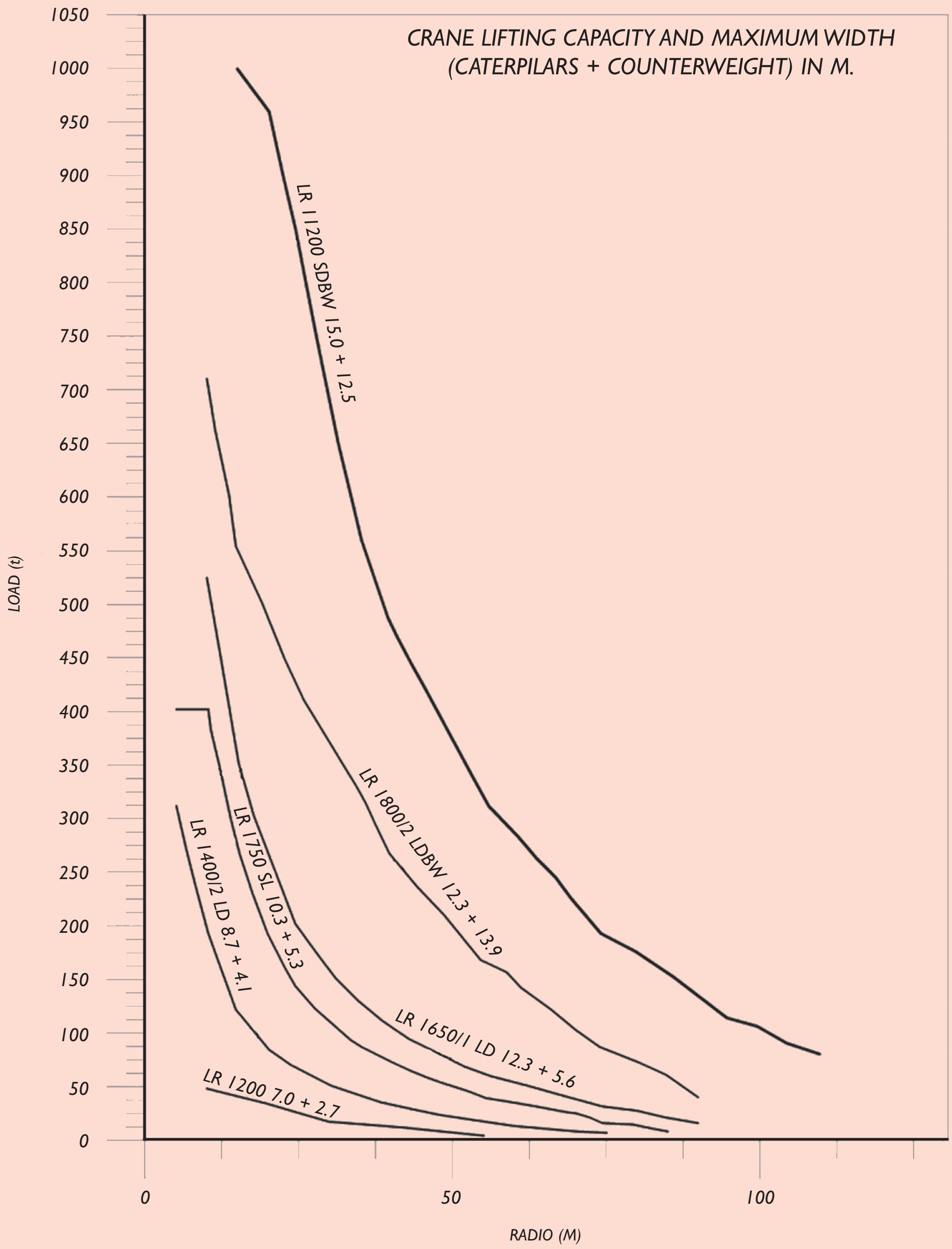
Prefabricated caissons: dimensions. When for construction or environmental reasons a sloping breakwater is not feasible, a possible solution can be a vertical breakwater built with prefabricated caissons, transported to the location and anchored to the site, or a composite breakwater with a caisson level layer as an anchorage mechanism. The dimensions of prefabricated caissons have increased considerably in recent years. Presently, maximum caisson dimensions are approximately $60\ m$ (length), $40\ m$ (width) and $35\ m$ (transport depth). To settle caissons, it is necessary for them not to exceed certain threshold values for the climate, maritime, and atmospheric agents, which depend on caisson dimensions, water retention, and above all, the means used for the control and development of the whole operation. This signifies that their use can affect construction schedule, which depends on the available climate windows during the year. Threshold values depend on the dimensions and displacement of the caisson. For caisson lengths on the order of $\sim 25\ m$ and normal settlement means, values indicative of their order of magnitude are and $H_s \approx 1\ m\ y\ T_p \lesssim 9s$.

Crane capacity and dimensions. The dimensions of the unit pieces and their configuration in the main layer, berm, and head also condition the choice of breakwater type since they define the dimensions of the crane, its lifting capacity, and arm. These dimensions, in turn, condition the advance width in the crown of the breakwater when it comes to the supply and positioning of the unit pieces without affecting the rest of the structural units. In Spain the normal crane size is up to $4000\ t \cdot m$. When a larger size is required (e.g. $7500\ t \cdot m$), it is often necessary to hire or construct special cranes. Presently, their lifting capacity exceeds $10000\ t \cdot m$ (see fig. 2.2.27).

Dumping from a hopper barge. Generally speaking, dumping material into the sea from hopper barges or other means does not affect the selection of breakwater type, except for maritime areas with rough seas. In those cases in which dumping large volumes is necessary, there should be a suitable location at the construction site that is designated as a loading, storage, and sheltered area. The volume of material normally loaded onto a hopper barge is $600 - 1200\ m^3$. The wave action conditions the operationality and precision of the pouring process by the hopper barge. The relation between the length of the barge and the wavelength is a good indicator of its oscillatory response. Experience has shown that for lateral dumping as well as dumping from the bottom of the barge, sea states must satisfy the condition $H_s < 2.5\ m$, depending on the positioning techniques, the precision required and type of material.

Forced stoppages and construction interruptions. Finally, in the construction of any maritime structure, it is necessary to make allowances for the necessity of technical interruptions during which the unfinished breakwater section may be subjected to climate agents which it is still not prepared to withstand. Such interruptions may be expected (e.g. winter stoppages) or unexpected (e.g. a storm event). Given the quantity of meteorological information available, unexpected interruptions almost always refer to when the building contractor does not have enough time to make adequate preparations for the extreme weather event. In both cases it is necessary to provisionally reinforce the structure so that it will be able to resist sea action without significant damage or with only limited damage. Certain breakwater types are easier to protect than others. Generally speaking, the temporary

Figure 2.2.27. **Graph of the lifting capacity of cranes and base width of cranes**



protection of a rubble mound breakwater is relatively simple, inexpensive, and rapidly recoverable. In contrast, vertical breakwaters, and especially composite breakwaters, are not so easily protected. This is the case for their central part as well as their foundation berm.

The usual protection is for all practical purposes a provisional head, whose hydrodynamic performance is similar to that of a permanent head. Consequently, even though it is provisional, its dimensioning and construction should follow the recommendations in the section on heads in the chapter on sloping breakwaters.

Before beginning construction, it is necessary to specify the threshold sea states whose exceedance makes it advisable to halt the building process. They should be calculated on the basis of the construction equipment, availability of materials, state of the structure and its elements as well as those parts of it that need protection.

Construction equipment. It is not a good idea to choose solutions that demand the use of extremely high-tech equipment or equipment of limited availability. Accordingly, the solutions incorporated into a breakwater project design should be simple, flexible, and applicable to a wide range of construction processes that can be adapted to the experience and available resources of constructors. When it is necessary to reduce construction time to a significant degree, project design will depend on local circumstances: availability of materials and construction equipment, as well as the experience and productivity associated with them.

Selection criteria according to the volume of materials and construction processes. In consonance with these general observations, the most advisable breakwater types in terms of constraints stemming from materials and construction processes are listed in table 2.2.3.

Table 2.2.3. ***Breakwater type in relation to volume of material and construction processes***

Breakwater type	Volume of borrow marerial	Construction processes and equipment	Adaptability
Sloping	Very large quantity	Dumping; heavy crane	Possible
Vertical	Small quantity	Caisson settlement; dumping	Difficult
Composite	Large quantity	Loading, pouring; crane; anchoring	Very difficult
Berm	Very larga quantity	Dumping; crane	Possible
Submerged	Depends on objetive	Dumping	Possible
Floating	None	Floating; driving	Possible
Screens	None	Floating; driving	Possible

2.2.4.5 Breakwater suitability according to climate during use and exploitation

This section analyzes the capacity of the breakwater to control incident energy flux and the implications of this during the use and exploitation of the harbor area.

The distribution of reflected, transmitted, and dissipated wave energy fluxes shows the influence that the selection of breakwater type can have in the use and exploitation of the sheltered area. Generally speaking, the ideal solution is when the incident flux is totally dissipated by the breakwater, but as things presently stand, there is no breakwater type capable of achieving this. When the quantity of wave energy dissipated is small and the fluxes of reflected and transmitted energy are great, the interferences in the use and exploitation of the harbor due to sea oscillations can also be significant. This may occur seawards in the access channels and harbor mouths, thus making navigation more difficult and increasing the agitation inside the harbor either by overtopping or by energy transmission through the breakwater.

Except in the case of wave overtopping, the vertical breakwater is the type of breakwater that transmits the least amount of energy through its central portion, and which reflects the most energy. Sloping breakwaters and berm breakwaters reflect the least energy and dissipate the most energy, generally transmitting very little energy except by wave overtopping. The composite breakwater has an intermediate performance between that of granular breakwaters and vertical breakwaters. The performance of the floating breakwater can vary, depending on its depth in the water in respect to the wavelength. If it is properly located, the greater part of the incident energy is reflected. In contrast, if its depth is insufficient, most of the energy is transmitted landwards.

Subsection of the construction. It is important always to bear in mind the principal function of each subsection, and also how the interaction of the breakwater with sea oscillations can affect the structure. In this regard, special consideration should be given to the influence of the energy radiated from alignment changes in the access to the sheltered area and the harbor mouth as well as reflected energy fluxes from different alignments of the structure.

Breakwater selection criteria based on climate requirements for use and exploitation. Based on these general observations, the most optimal types of breakwater in view of use and exploitation requirements are listed in table 2.2.4.

Table 2.2.4. **Optimal breakwater types according to climate requirements in use and exploitation**

Breakwater type	Energy distribution
Sloping	Dissipation and reflection
Vertical	Reflection
Composite	Dissipation and reflection
Berm	Dissipation
Submerged	Dissipation, reflection and transmission
Floating	Reflection and transmission
Screens	Reflection and transmission

2.2.4.6 Breakwater type optimality based on maintenance, repair and dismantling requirements

When selecting the most optimal type of breakwater type, maintenance costs should be analyzed in order to assure the durability of the structure during its useful life. Repair costs should also be included in the project and analyzed on the assumption that the structure will suffer a certain amount of repairable damage during its serviceability phase. Finally, it is also necessary to consider dismantling costs as well as costs related to the restoration of the coastal area.

Maintenance. The feasibility and cost of maintenance depends on to the number of elements that make up each part of the breakwater. Evidently, the number of possible failure modes and transitions between elements and parts (weak zones) is directly related to number of structural components. Vertical breakwaters have the fewest elements, whereas sloping breakwaters with a crown and composite breakwaters have the most elements and transitions.

Repair. The feasibility of repairs depends on the importance of the failure mode in the overall stability of the structure, on the possible correlation (concatenation, induction, and progressive collapse) with other failure modes, repair time, and the operationality conditions once the failure has occurred.

Generally speaking, granular, sloping, and berm breakwaters offer greater resistance to destruction although the repairs of the principal failure mode, detachment of unit pieces from the slope, is usually slow and requires the same equipment used in the actual construction of the breakwater. This naturally makes it more costly. However, the operationality of a harbor or coastal area is generally not severely affected. Furthermore, the failure modes, overturning and sliding of the vertical breakwater are very difficult to repair. In these cases, it is usually necessary to dismantle the breakwater in order to reconstruct the section. Needless to say, this means that during this repair phase the operationality of the area is greatly reduced.

Dismantling. Since the total dismantling of large breakwaters is a rare occurrence, there is little experience available regarding this type of event, something which logically makes it difficult to evaluate. The dismantling of granular breakwaters is particularly complicated because of the quantity of material used in their construction. In most cases, dismantling this type of structure is far from easy, given the precariousness of its construction. The work sequence in the dismantling of various parts of the breakwater plays a crucial role in the safety of the structure. One dismantling procedure that seems viable, at least in theory, is the refloating of the caisson, once it is freed from the upper slab, and then the emptying of the cells.

Selection criteria based on maintenance, repair, and dismantling. Based on the previous observations, the various breakwater types satisfy the following maintenance, repair, and dismantling conditions (see table 2.2.5).

Table 2.2.5. ***Breakwater types in relation to maintenance, repair and dismantling***

Breakwater type	Maintenance	Repair	Interaction	Dismantling
Sloping	Feasible	Slow, expensive	High	Complicated, difficult
Vertical	Complicated	Fast, expensive	Low	Simple
Composite	Complicated	Slow, expensive	Low/Medium	Complicated, difficult
Berm	Simple	Slow	High	Complicated
Submerged	Simple	Fast	Low	Simple
Floating	Simple	Fast	High	Simple
Screens	Simple	Fast	High	Simple

2.2.4.7 Choice of breakwater type based on environmental requirements

The construction of a breakwater can cause significant modifications in the land and sea areas near the construction site. These changes can be related to the opening and exploitation of quarries, the transport and dumping of construction material or the removal and pouring of dredging material. All of these activities can restrict the choice of breakwater types that require significant quantities of borrow materials or which must carry out large volumes of dredging until adequate foundation levels are reached. Vertical breakwaters generally have the least environmental impact except when significant amounts of dredging or soil material are required. Although the environmental impact of floating breakwaters is generally slight, it is advisable to analyze their effect on the morphodynamics of the coastline, which in certain cases can be important. Moreover, it is also true that the construction of any harbor or coastal area interacts with the coastline, modifying morphodynamic processes and the quality of coastal waters. The extent of such modifications mainly depends on the ground plan of the area and its degree of protection against wave action.

Modification of the circulation system. The ground plan of the area modifies the pattern of the tide and ocean currents as well as wave action characteristics, mainly, wave height and direction. Consequently, when their principal generating mechanism undergoes any sort of change, the coastal circulation systems are also substantially modified.

Modification of sediment transport and other substances. Although wave action is the principal mechanism behind seabed sediment movement, it is not necessarily the main transport mechanism for sediments and substances, which in the sea are conveyed by tides and currents. All sheltered areas create spaces where the effects of marine dynamics are intensified or mitigated. As a result, processes of erosion as well as sediment and substance transport and deposit can be significantly altered. All breakwaters tend to produce similar effects although the most reflective types intensify and extend these effects over a wider area than the more dissipative types.

On sandy beds, breakwaters can produce bars that run parallel to the line of crests that propagate radially to the head of the breakwater, forming sand lens. The relevance of this effect is in direct relation to the intensity of wave reflection at the breakwater.

Effect of the porosity of breakwater components. The porosity of certain breakwater components is also the source of important differences between breakwater types. More specifically, granular breakwaters can act in the same way as sand sinkholes, silting the layers and significantly modifying the water depth at the toe of the breakwater.Vertical breakwaters can behave in the same way as impermeable barriers, and, consequently, can retain sand.This naturally produces changes in depth, but dredging is easier in the case of vertical breakwaters than for rubble mound breakwaters.

Furthermore, the porosity of this type of structure can have beneficial effects since it increases the oxygenation of the water and provides niches for a great variety of marine species.

Breakwater type in relation to environmental requirements. Based on the previous observations, table 2.2.6 shows how different breakwaters types relate to the environment.

Table 2.2.6. ***Breakwater type and their relation to environmental requirements***

Breakwater type	Volume of materials	Interaction	Oxigenation of water and niches
Sloping	Large	Significant	High aoxygenation; many niches
Vertical	Small	Significant	low oxygenation; few niches
Composite	Medium	Significant	Medium oxygenation; some niches
Berm	Maximum	Significant	High oxygenation; many niches
Submerged	Depending on objective	Significant	High oxygenation
Floating	Minimum	Slight	Low oxygenation; some niches
Screens	Minimum	Significant	Low oxygenation

2.3 CALCULATION PROCEDURES FOR BREAKWATERS

The dimensioning of a breakwater is based on its performance and interaction in plant as well as its elevation with the agents envisaged in the project design. These agents can be related to gravity, the environment, soil characteristics, use and exploitation, materials, and construction procedures. Their actions are described in the specification of failure and stoppage modes. Through the analysis of breakwater performance, it is possible to describe and classify the mechanisms that lead to the failure or operational stoppage of the structure (failure or stoppage modes), based on their origin. This classification includes the principal agents responsible for the operational stoppage.

Calculations should be made to verify that for each failure mode, project requirements regarding reliability, functionality and operationality are fulfilled. If possible, these calculations should be in consonance with the general calculation procedure known as the "limit states method" (see the ROM 0.0) which entails verifying failure or stoppage modes solely for those project design states which have foreseeable limit situations.These limit situations can arise in the breakwater because of problems related to its resistance (ultimate limit states, ULS), structural and formal properties (serviceability limit states, SLS), and use and exploitation (operational limit states, OLS).

Note. Once the structural and formal response states of the breakwater have been determined, they can be verified in terms of the ultimate limit states or serviceability limit states. In the case of ultimate limit states, failure modes are definitive, and are associated with extreme states or extreme manifestations of certain agents. The verification equation provides information regarding whether the failure has happened, but it does not indicate its magnitude. Nevertheless, when such a failure mode occurs, repair work must inevitably be carried out in order to recover project design requirements.

In the case of serviceability limit states, the verification equation frequently gives information regarding the reduction in use and exploitation suffered by the breakwater or one of its elements. As a result, the breakwater also suffers a loss of structural and formal properties due to the appearance of limit states, whose descriptors have values that exceed a certain threshold. Consequently, this analysis offers the information necessary for maintenance and any necessary repair work to preserve or recover project requirements. Unfortunately, there is still relatively little data available to verify the performance of structures, soil, and the environment.

2.3.1 The performance of the structure and its subsets

This section analyzes the performance of the breakwater and each of its subsets in plan as well as elevation. The following types of performance are considered:

- Hydraulic performance as well as performance under the actions of other agents in the physical environment.
- Structural performance.
- Geotechnical performance.
- Performance derived from construction processes.
- Morphodynamic performance.
- Environmental performance: evaluation of the effects of the breakwater on environmental quality parameters, such as water quality in the harbor and coastal area according to the EU Water Framework Directive, and specifications in the ROM 5.1-05.

Note. *The analysis of the hydraulic, geotechnical, structural, morphodynamic and environmental performance of the sheltered area and the breakwater, soil, and coastal environment is an integral part of the project design process. It is advisable to use such a study to interpret the structural, formal, and operational response of the breakwater, and accordingly, to define stoppage and failure modes as well as their corresponding verification equations.The scope of the study depends on whether it is carried out for the predimensioning, draft, or actual project design of the structure.The annex at the end of Chapter 4 gives an overview of the theoretical premises of the performance of such structures under sea dynamics.*

2.3.1.1 Breakwater performance and agents of the physical environment

It is necessary to analyze the performance of the breakwater when it is subjected to the action of climate, atmospheric, and marine agents. Marine agents include sea oscillations, as well as seismic, biogeochemical and thermal agents.

Breakwater performance under sea oscillations

Characteristics to be evaluated in the breakwater are its capacity to reduce wave action and its energy efficiency when it is affected by sea oscillations. Other features that should be assessed are the structure's vulnerability to overtopping, its wave pressure and subpressure regimes, and the operationality of the sheltered harbor or coastal area.

This can be analyzed in the temporal domain or in the frequency domain. In the first case, the regimes of the set of sea oscillations (wave regimes) should be determined.This includes short, medium, and long-term description at the site. In the second case, the frequency spectra associated with the sea states and the average sea level are specified. Both descriptions should be representative of normal and extreme work and operating conditions.

Once the plan and elevation of the structure and each of its subsets has been pre-dimensioned, the wave regimes and frequency spectra are again obtained at the breakwater, as described in Chapter 4. Based on these results, the seaward and landward wave regime at the toe of the section should be determined for each subset, according to the recommendations in the corresponding sections.This is performed with a view to evaluating the energy efficiency, wave overtopping, and the wave actions, pressures and subpressures in the structure and the soil.

When the structural response of a part or element of the breakwater section is found to be oscillatory, the dynamic performance of the structure should be studied as explained in this ROM, as well as the ROM 0.3-91 and ROM 0.4-95 in those sections pertaining to the action produced by waves, other sea oscillations, and wind.

Note. *An energy efficiency analysis quantifies the breakwater's capacity to resist the energy of climate agents such as reflection, dissipation, and transmission.The overtopping probability estimate quantifies the capacity of the structure to control wave overtopping volumes and their frequency.The actions and responses provide the information necessary to verify the structural and formal stability of infrastructures during the design phase.*

PERFORMANCE DURING SEISMIC MOVEMENTS

When the probability of occurrence of earthquakes or tsunamis is not negligible, it is advisable to analyze the soil-breakwater response, based on the regulations currently in force. Depending on the specific context and site of the breakwater, the extreme regime of the seismic agent at the building site should be determined.

On the basis of this regime, it is necessary to define the frequency spectra and the corresponding time series of representative accelerations for exceptional, extreme, and normal work and operating conditions.

The ROM 0.0 recommends the specification of the simultaneity and compatibility of the seismic agent with climate, atmospheric and marine agents, particularly tsunami. This study should be in consonance with the sections in this ROM that describe each breakwater type and seismic movements in general.

2.3.1.2 Structural performance of the breakwater

Important factors to assess are the stability, carrying capacity, and deformability of the materials used to construct the elements of the breakwater section, which are vulnerable to the action of project design agents. This assessment should follow national and international standards and regulations (*i.e.* EHE, EHP, EAE, *Quarry quality,* UNE-EN 13383-1/AC, Eurocodes) for each type of material affected by project design agents as described in the ROM 0.1 and this ROM (section 3.5.1).

2.3.1.3 Breakwater performance and soil structure

It is also important to assess the load carrying capacity and deformability of the underlying soil and the breakwater section, taking into account the sea oscillations and other agents of the physical environment.

In the event that the soil structure is found to be significantly affected by sea oscillations, an analysis must be made of the shear stresses, deformation, and interstitial pressures in the ground, foundation, and fill material. The temporal and spatial scales of the drained and undrained soil should be considered as well as their dependence on the existing oscillatory conditions at the breakwater site as set out in the ROM 0.5-05 and ROM 1.1.

In such cases the meteorological state can be characterized by means of spectral or statistical descriptors, or in terms of wave heights and periods that are representative of normal and extreme work and operating conditions.

Whenever possible, appropriate techniques should be used to evaluate the geotechnical state of the soil structure, shear stresses, and deformation, considering temporal series of sea oscillations. When their geometry is excessively complex, the results should be contrasted with the results of experimental studies.

2.3.1.4 Breakwater performance and construction processes

Another important aspect to be evaluated is the incidence that construction processes and procedures have on geotechnical and structural performance. Their effect on the agents of the physical environment of the breakwater needs to be studied. This assessment should be made as described in the ROM 0.1 and those sections in this ROM that characterize different breakwater types.

2.3.1.5 Morphodynamic performance

An evaluation should also be made of coastal processes, seabed stability, the development of bedforms with different spatial scales, and the evolution of the coastline. The possible impact that the breakwater can have on the environment, and particularly the coastal environment, should be quantified, following the guidelines in this ROM as well as the ROM 5.0 and 5.2 (in press).

Also to be studied and assessed are the following ways in which the breakwater may interfere with the environment: (1) seabed morphodynamics and the development of small, medium, and large-scale bedforms; (2) coastal processes; (3) coastal evolution during the useful life of the breakwater (according to the ROM 5.2, to appear). When necessary, the possible evolution of the coastline should be predicted, and measures taken to avoid sudden local variations and significant temporary modifications of the coastline during the various project design phases. For this purpose, models can be applied that are suitable for the area to be studied, and for evaluating uncertainty.

SPATIAL EXTENSION OF THE EVALUATION

The evaluation of the interaction of the breakwater and each of its subsets with the coastal zone should cover all areas where the presence of the breakwater significantly modifies the agents of the physical environment.

Note. The presence of the breakwater modifies the wave regimes of marine dynamics. The structure produces changes in the magnitude and sometimes in the direction of the state descriptors and basic variables. In this analysis it is important to consider the transformation of sea oscillations by the breakwater, by reflection and radiation, particularly, wave height and the direction of propagation. Generally speaking, this modification affects the spatial and temporal gradients that are the driving forces behind coastal processes as well as the diffusion and advection of substances. The phrase "to significantly modify" should be applied to the value of the agent as well as its spatial and temporal gradients.

2.3.1.6 Environmental performance

The impact of the breakwater should be evaluated according to parameters of environmental quality. This includes the quality of the water in the harbor and coastal area according to the *DMA* (17) in Spain, the EU Water Framework Directive, and the specifications in the ROM 5.1-05.

At the very least, it is advisable to study water circulation, diffusion-advection processes of substances, and the spatial and temporal evolution of the water quality in the sheltered area and in the surrounding port and harbor area within the boundaries beyond which the presence of the breakwater no longer significantly modifies (18) the agents of the physical environment, following the recommendations in the ROM 5.1-05 and the DMA.

2.3.2 Failure and stoppage modes

A failure or stoppage mode can be defined as the way, form, or mechanism in which an operational failure or stoppage can happen. Such a failure or stoppage is generally described as part of a limit state. For each type of breakwater it is essential to consider the set of operational failure and stoppage modes described in the failure or stoppage diagrams included in this ROM. When feasible, this set should be as complete as possible and composed of mutually exclusive modes (19). When this is not an option and the complete set contains modes that are not mutually exclusive, it should be specified if they are statistically dependent or independent (20).

The elements described in the following paragraphs should be taken into account in the elaboration of failure and stoppage diagrams as well as in the description of modes.

(17) *Water Framework Directive* published in DO L 327 of 22/12/2000 and ratified by the government of Spain.
(18) The delimitation of the affected area should be carried out according to the criteria described in the previous section.
(19) A complete and mutually exclusive set of modes signifies that the occurrence of one mode excludes the occurrence of other mode(s). Consequently, through the operations of union, intersection, empty set, and complementary event, it is thus possible to describe any type of subset performance.
(20) Two modes that are not mutually exclusive are statistically independent if their joint probability is equal to the product of their marginal probabilities. Otherwise, they are regarded as dependent.

Spatial domain

The description of the area where the failure or stoppage occurs or the area which it affects should distinguish between the breakwater section, its components [21], elements and subelements, and the surrounding environment. This study should state whether the structure affects coastal morphodynamics, water quality or the coastal ecosystem.

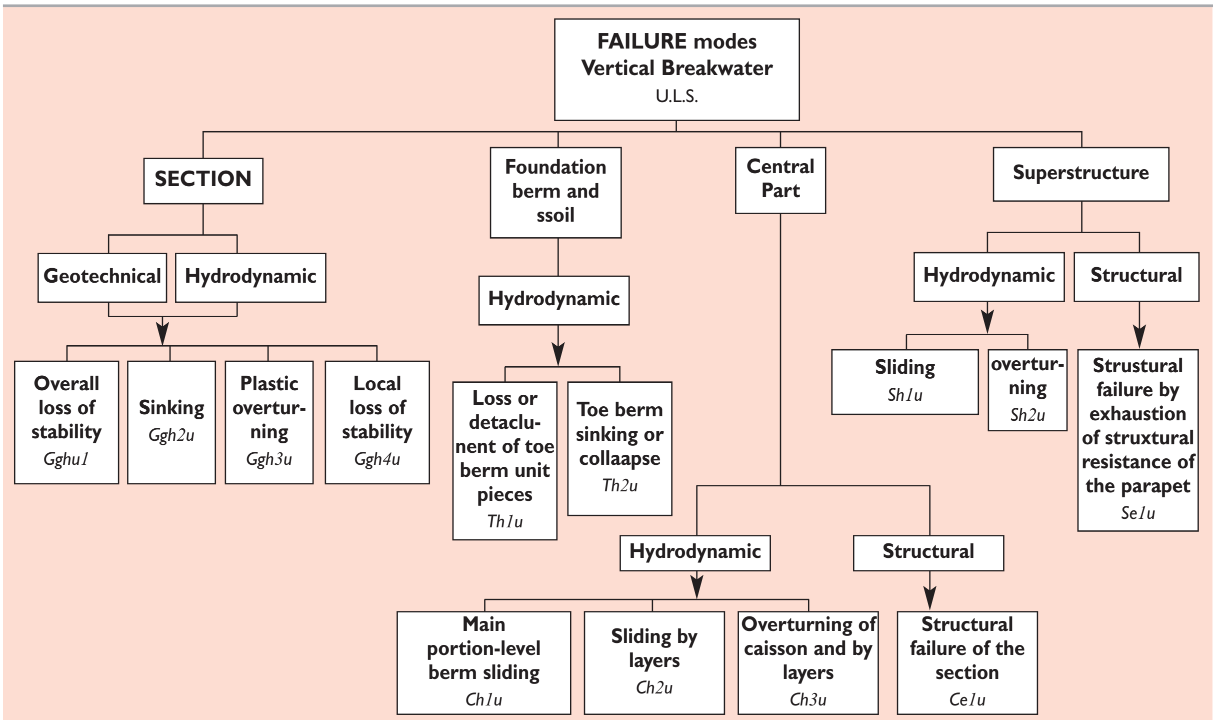
A failure or stoppage mode is said to affect a breakwater section when it affects two or more of its parts. The failure or stoppage mode affects a part or element of the breakwater when it affects two or more elements or two or more sub-elements, respectively.

Mechanism

The mechanism or means by which the failure or stoppage occurs is described by evaluating its importance and consequences for the safety, service, and use and exploitation.

Note. The following diagram shows the failure modes of a vertical breakwater, and also provides an outline of hydrodynamic failure modes.

Figure 2.2.28. Failure modes of a vertical breakwater



Predominant agents and other agents

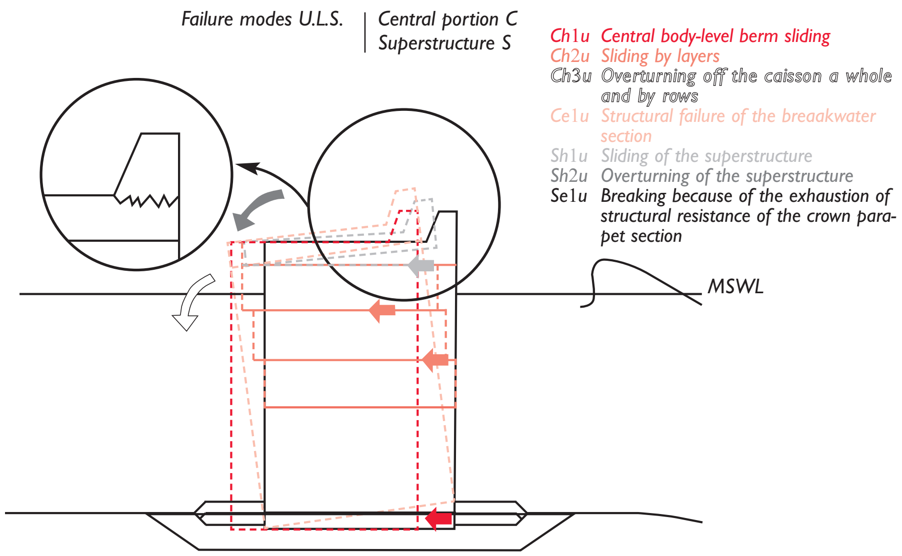
All of the prevailing agents and other agents that can participate in the triggering and evolution of the failure or stoppage mode should be specified. The agents are classified according to their origin, and an analysis should be made of their mutual interdependence.

(21) The parts of the breakwater section are described in section 2.2.1.

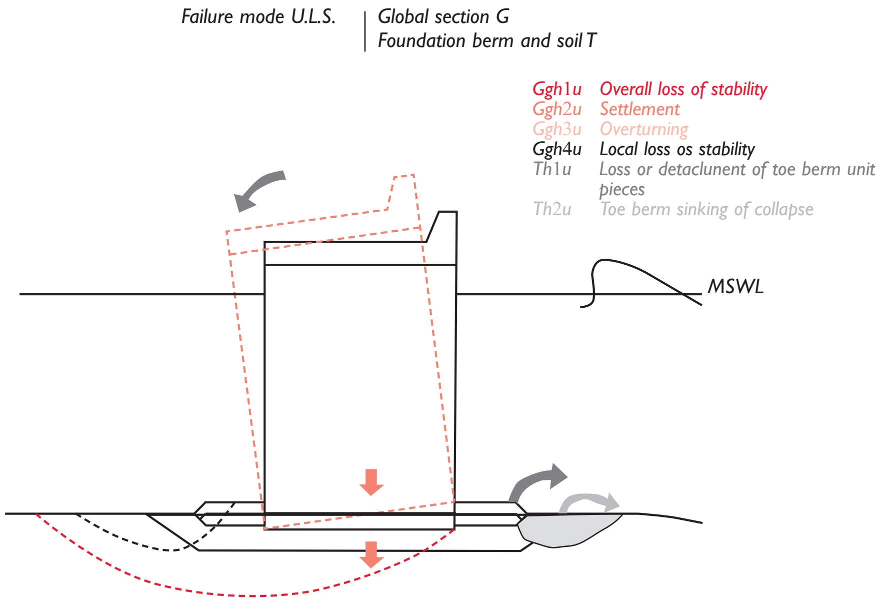
TEMPORAL DOMAIN

The time period in which the failure or stoppage mode can occur should also be characterized. As a rule, this is the (meteorological, geotechnical, etc) state which is regarded as a project design limit state. Within the state, it is crucial to define the descriptors of the agents, for example, significant wave height, average wave period, average direction and duration. In each case, the failure or stoppage mode should be assigned to an ultimate limit state of operational serviceability or stoppage. An analysis is made of the work and operating conditions in which the mode can appear, and also of the possible assignment of the mode to more than one limit state.

Figure 2.2.29. Diagram of the failure modes assigned to the ULS of a vertical breakwater



* In cofferdams these are regarded as particular failure modes.



Verification mode

The occurrence of the operational failure or stoppage mode during a given state can be verified by means of the corresponding verification equation. This equation establishes the functional relations between project factors that define the condition of the operational failure or stoppage. Generally speaking, this equation is a state equation, and more than one equation is normally used to verify a mode.

The analysis of the verification equations that can evaluate the failure or stoppage mode should specify the following: (1) format; (2) analysis of their theoretical, experimental, and numerical foundation, as well as their range of application; (3) time period, which is generally regarded as the state; (4) project factors, their basic variables and descriptors; (5) terms of the equation and a specification of whether they are deterministic or random, permanent or non-permanent, and favorable or unfavorable; (6) failure or stoppage criterion; (7) standards, regulations, and recommendations to be observed in the verification of the failure and stoppage mode.

When there is an applicable verification equation, numerical or contrasted experimental methods are used to verify the failure and stoppage mode. When these techniques are applied, the same verification procedure should be observed, particularly in the evaluation of the uncertainty of the input and output data and of the numerical or experimental technique in the laboratory or *in situ*.

Designation of failure or stoppage mode as the principal mode

A failure or stoppage mode should be classified as principal or non-principal. An analysis should also be made of the possible actions geared to reducing its contribution as principal mode to the probability of its presentation. Whenever possible, this analysis should be based on the economic optimization of the structure, evaluating the consequences of the mode for the construction, maintenance and repair costs. The mode is usually not regarded as a principal mode, when a negligible failure load can be reached with only slight changes in the geometry and mechanical properties of the element or component of the breakwater.

Observation and monitoring of the failure or stoppage mode

It is advisable to specify the possible observation and monitoring techniques of the mode and its occurrence, indicating the repair thresholds, and in the case of the stoppage modes, the stoppage thresholds.

Statistical dependence and independence

The analysis should show if the occurrence of the failure or stoppage mode excludes the occurrence of other modes. If this is the case, the modes can be regarded as mutually exclusive, and the intersection of both events is an empty set. When the opposite is true, the failure or stoppage mode is not mutually exclusive, and thus, can be regarded as statistically dependent or independent of other modes [22]. Consequently, in order for the set of modes to be complete, it should include all of the individual modes, the event comprising the intersection of modes, and the non-failure event.

Furthermore, for some breakwater types, it is prudent to analyze the possible evolution of the structure towards a progressive collapse. This analysis should take into account its correlation with simultaneous agents or the structural and formal response. It is also advisable to specify if the failure mode is induced or if it acts as an inducer.

Note. *The interdependence (exclusion and statistical dependence) of failure and stoppage modes is a fertile area of research which needs to be explored in greater depth so that maritime structures can be more reliable,*

(22) By definittion, if the modes are independent $\Pr[AB] = \Pr[A]\,\Pr[B]$, and if they are dependent, $Pr[AB] = \Pr[A \mid B]\,\Pr[B]$.

functional, and operational with an optimal total cost. At the moment, however, there is a general lack of verification equations that permit the analysis of the reciprocal dependence of response modes. Nevertheless, it is possible to make an initial analysis of common agents that intervene in each mode. Their respective individual verification equations can be used to study if each fails with the same agent values. In this case, both modes could presumably be said to occur simultaneously, and the effects of the failure probability distribution calculated as only one mode.

However, this may not be a good idea for two reasons. The first reason is associated with the capacity of the verification equation to represent the processes involved in the mode. The second is that the response of the structure to this failure mode may or may not lead to its failure (or vulnerability), once the necessary level of danger has been reached. In other words, the event may not occur in a sequence that entails first the appearance of agents (danger), and secondly, the induction of the failure mode (vulnerability). The various factors not considered in the formulation, the associated uncertainties (see the ROM 0.0, section 3.2), and the intrinsic randomness of many of the phenomena can result in the "non-failure" of the structure.

For example, let us consider the main layer of a sloping breakwater with a unit piece weight W_0 that fails at height H_0. The probability of exceeding the wave height in the time period is the dangerousness $Pr[H \geq H_0]$, and the probability of exceeding the required weight, assuming that the wave height has been exceeded, $Pr[W \geq W_0 \mid H \geq H_0]$, is the vulnerability. The probability of occurrence of the failure mode is the probability that both events will occur simultaneously, in other words, the exceedance of the wave height and the weight, or the joint probability of W and H.

$$\Pr[W\,H] = \Pr[W \geq W_0 \mid H \geq H_0]\,\Pr[H \geq H_0] \tag{2.35}$$

As can be observed, the failure probability is equal to the product of the vulnerability and the dangerousness. If the event $H \geq H_0$ has occurred, the value of $Pr[W\,H]$ depends on the value of the conditioned probability, $\Pr[W \geq W_0 \mid H \geq H_0]$. If this is assumed to be equal to their union, or that once the event $H \geq H_0$ occurs, the event $W \geq W_0$ is true, then

$$\Pr[W \geq W_0 \mid H \geq H_0] = 1 \tag{2.36}$$

$$\Pr[W\,H] = \Pr[H \geq H_0] \tag{2.37}$$

When this assumption holds, the failure probability is equal to the exceedance probability of the agent.

2.4 GENERAL PROJECT DESIGN CRITERIA

The general criteria that should be used to define the project design of a breakwater are the following: (1) the spatial organization of the structure in subsets and the temporal organization in phases: (2) the general and operational nature of each subset in each phase; (3) the project requirements pertaining to the structure's safety, serviceability, and use and exploitation. These criteria, which initially appeared in the ROM 0.0, are further described and specified in the sections that follow.

2.4.1 Spatial organization: breakwater subsets

According to the ROM 0.0 (section 3.3), the intrinsic or random variability of project factors for breakwater subset is generally regarded as spatially uniform and stationary in time. This facilitates its description by means of specific probability models.

A breakwater should be defined and organized in terms of subsets. As part of its project design, it is thus divided into homogeneous subsets of the same formal and structural type. These subsets are differentiated when there are significant variations in one of the project factors (*i.e.* geometry of the construction and the soil structure, characteristics of the topography, physical environment, and construction materials, and the values of agents and actions) throughout the building site, or when there are significant variations in the repercussions in the case of an operational failure or stoppage occurs.

Each breakwater can be said to have, among others, five subsets (see figure 2.2.30):

- Junction of the breakwater with the soil;
- Main alignment that controls and protects against the prevailing wave action;
- Secondary alignments that link the different subsets of the breakwater;
- Transition or subset between two alignments or types;
- Head or upper part of the breakwater;

An alignment can have more than one subset, and each subset can be designed and constructed with a typology that differs from the breakwater's (see section 2.2 for a description of breakwater types).

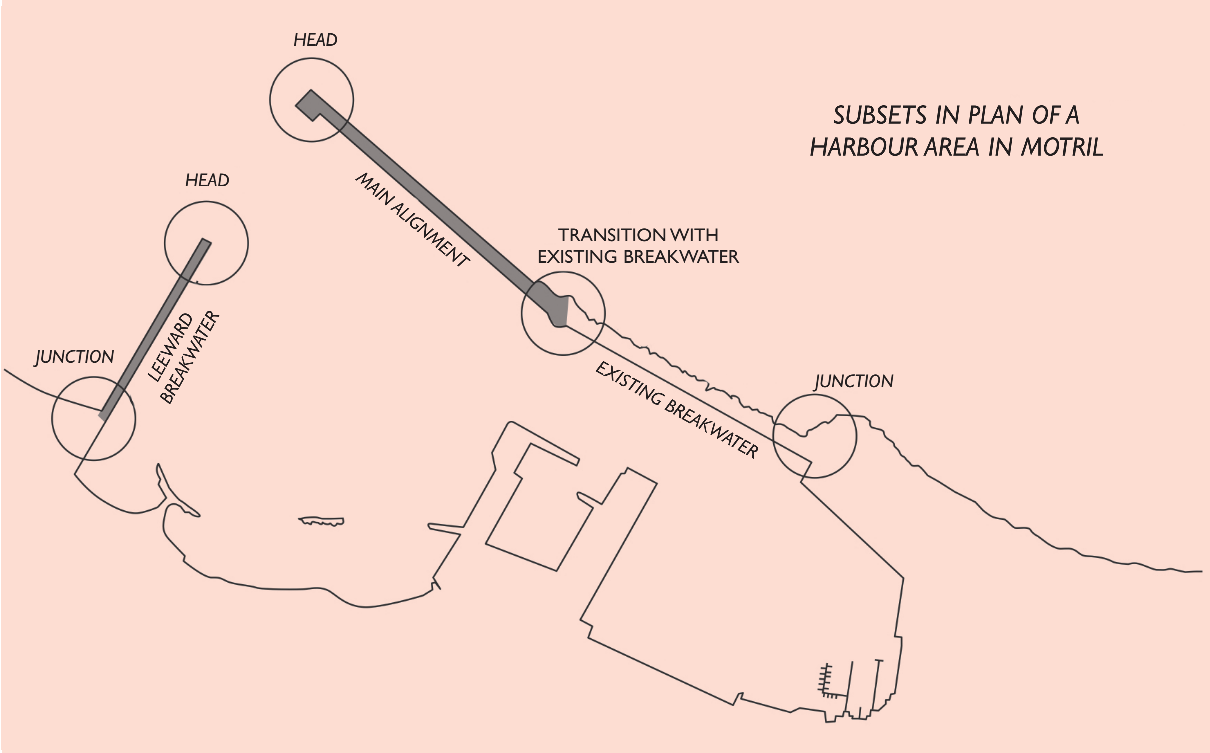
When the implementation of the structure is in phases, each phase is regarded as a different subset if the phase lag between the operational start-up of one phase and another is more than five years.

Note. *A subset is a continuous set of sections (or breakwater alignment) which fulfils a specific function that is relevant to the objectives and exploitation requirements of the structure. This set of sections is subject to the same action levels of all agents, particularly the predominant agents, and they all belong to the same formal and structural type.*
Apart from the uniformity of climate agents, a breakwater subset is defined in terms of the homogeneity of the soil structure (i.e. soil nature, load carrying capacity, and response).

2.4.2 Temporal organization: project phases

To facilitate the verification of a breakwater, it is necessary to define the different project phases, specify their duration, temporally organize the study, evaluate the probability of presentation of the agents, and define the state regimes, principally those related to meteorology, soil structure, and use and exploitation.

Figure 2.2.30. Subsets and alignments in the plan configuration of a harbour area



2.4.2.1 Project design phases

The following are regarded as project design phases:

- Construction.
- Serviceability or useful life.
- Repairs, maintenance, and dismantling.

These project phases are subdivided in subphases only when they affect the dimensioning of the construction or of its components.

Note. *For example, caisson settlement is a construction subphase that should be differentiated in vertical breakwaters with prefabricated caissons.*

2.4.2.2 Duration of the project design phase

The duration of each project phase should be decided by the project developer, considering issues pertaining to construction, materials, soil structure, maintenance, serviceability, finances, and administration.

When the duration of the project design phase of the subset has not been previously specified, or when it does not fulfill minimum values, the following reference durations will be regarded as valid:

DURATION OF THE SERVICEABILITY PHASE: USEFUL LIFE

The duration of the serviceability phase or useful life (V) of maritime structures is:

- Provisional structures and subsets: $V \leq 5$ years.
- Definitive structures and subsets: $V > 5$ years.

USEFUL LIFE OF PROVISIONAL STRUCTURES OR STRUCTURES BUILT IN PHASES

The construction of a breakwater is regarded as being carried out in phases when the phase lag between the operational start-up of the first phase and the final phase is greater than five years. In such cases, the useful life should be established for each phase of the structure or subset.

Accordingly, when the construction of a phase could significantly affect the value of project factors in a preceding phase, two subphases should be considered. The useful life of the first subphase is limited by the beginning of the subsequent subphase. It is thus essential to include in the project design the structural adaptations that would be necessary if subsequent phases were not implemented.

DURATION OF THE CONSTRUCTION PHASE

The construction phase of the subset is the time that passes from the moment construction begins until the moment when the breakwater is capable of satisfying project design requirements. The duration of this phase is determined on the basis of the technical and economic means available as well as the construction procedures used to implement the breakwater or subset.

In structures whose construction entails soil consolidation (foundation, fill, central portion of loose material, etc.) and permanent soil settlements, and in which these elements can be instrumental in the triggering of one or more of the failure modes and operational stoppages, the duration of the construction phase and subphases should be at least sufficient to reach the consolidation levels specified in the project.

Minimum useful life of definitive breakwater subsets

Generally speaking, as stated in the ROM 0.0, the duration of the serviceability or useful life project phase for permanent structures is at least the value designated in table 2.2.7, depending on the ERI, economic repercussion index of the maritime structure.

Table 2.2.7. Minimum useful life in the serviceability project design phase for the permanent structures

ERI	Useful life in years
≤ 5	15
6 – 20	25
> 20	50

However, section 2.7 specifies these values for breakwaters according to the ERI of the subset and breakwater type.

Duration of the repair and dismantling phases

The repair phase of the subset is the time that passes from the moment repairs begin until the moment when the structure is capable of satisfying project design requirements. It is the job of the person in charge of the repairs or dismantling to specify the duration of these phases, depending on the technical and economic resources available as well as the construction procedures and environmental requirements.

The repair phase should be as short as possible, and furthermore, it is important to avoid time periods in which extreme values of the agents are most probable.

The dismantling phase is the time that passes from the moment that dismantling begins until the moment when the area is restored to the condition defined in the corresponding construction project. The dismantling of the structure should not be of indefinite length. Lack of experience in this area makes it impossible to specify with any degree of exactness a minimum duration for this phase. However, roughly speaking, the dismantling of a structure can last about two or three times the duration of the construction phase.

2.4.2.3 Climate time sequence of the project phases

When the predominant agents of the breakwater are climate-related, it is advisable to temporally sequence each of the project phases, particularly the useful life of the structure and each subset by using the sequence of states or *state curve*, which defines the solicitation and operationality cycles, meteorological years, and hypercycles.

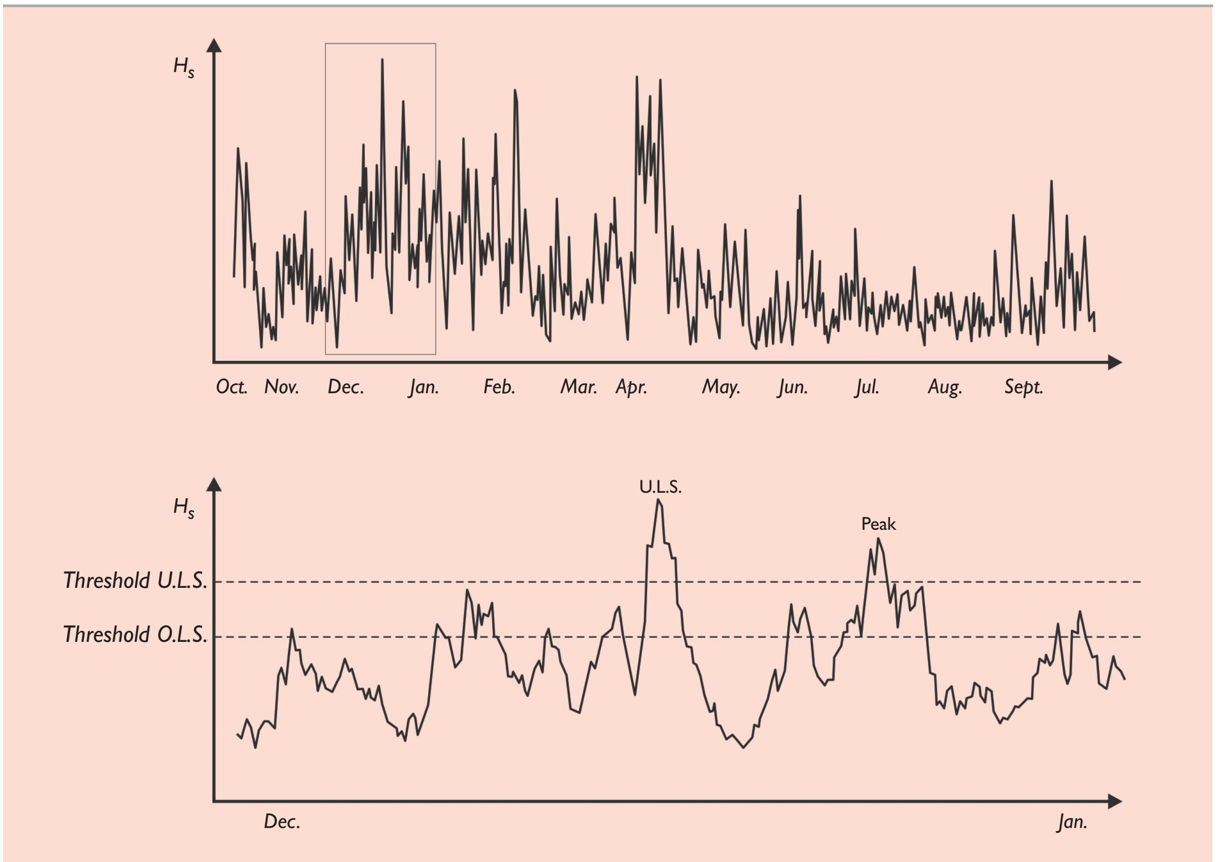
For this purpose, it is necessary to identify the safety and serviceability threshold values of the agents whose exceedance can significantly affect the reliability and functionality of the structure or subset. The solicitation cycle in respect to safety and serviceability is the time period between the two consecutive cut-off points of the state curve with the line that represents the threshold value of the safety agent or serviceability agent, respectively.

During this period, a maximum value is reached in the state curve. Similarly, it is possible to identify the threshold value of the use and exploitation agents, whose exceedance can significantly affect the operationality of the structure or subset. The operationality cycle pertaining to use and exploitation [23] is the time period between

(23) In navigation channels in seas with tides, a "window" is the time period in which the depth exceeds a certain threshold value without a continuity solution.

two consecutive cut-off points of the state curve with the line that represents the threshold value of the agent under study. During this period a minimum value is reached in the state curve. The identification of each cycle should assure their statistical independence (see figure 2.2.31).

Figure 2.2.31. ***Sea state curve of and threshold value of ultimate and operational limit states***



METEOROLOGICAL YEAR

In middle latitudes, whenever possible, the continuous sequence of solicitation and operationality cycles should be divided into meteorological years, and the duration of the serviceability or useful life project phases of the subset should be calculated in meteorological years. The useful life of the structure consists of a finite sequence of meteorological years. In other latitudes, other meteorological scales can be used. This pertains to the verification of certain project phases, for example, those phases that last less than a year, or when operationality is seasonal.

Although climate processes are known to take place with greater or lesser intensity in groups of meteorological years or annual hypercycles, if no other information is available, it can be assumed that the manifestations of agents that occur in each of them are statistically independent. Otherwise, this circumstance should be taken into consideration by organizing the climate time sequence in hypercycles

RANDOM VARIABLES OF STATE, CYCLE, METEOROLOGICAL YEAR, AND PROJECT PHASE

The manifestations of agents in each state, solicitation and operationality cycle, meteorological year, and project phase can be regarded as random variables described by a probability model.

Chapter 3 (section 3.1) summarizes the different temporal and spatial scales of the manifestations of (mainly climate) agents as well as their descriptions and characterization.

2.4.2.4 Regimes of random variables

Once the time period is selected for the description of the agent or agents, and once random variables are defined that characterize the process, the regime is the function of the joint, conditioned or marginal distribution of the representative variables of the agent or agents.

Depending on the time period, the probability models can be representative of the agent in the state, cycle, meteorological year or project phase.

Depending on the type of value, the distribution functions can belong to the extreme upper or lower values, or fall within the middle group of values that the variable can take during the time period. Generally speaking, extreme upper or lower values correspond to the occurrence of extreme events, included in extreme work and operating conditions (extreme regime), whereas the middle values refer to normal work and operating conditions (middle regime).

Note. The climate data base of Puertos del Estado [(24)] *is now sufficiently large to allow a joint analysis of the climate states. The regimes can belong to any of the joint manifestations of climate agents. For example, the analysis of the stability of the unit pieces of the main level of the breakwater should include the regime of maximum sea states (representative wave height, period, and direction) and maximum and minimum sea levels (astronomical and meteorological tide).*
This work method can also be applied to actions. For example, to study the sliding of a vertical breakwater, it is advisable to calculate the joint regimes of maximum wave pressures and subpressures/shear stresses and substresses and sea level, obtained from the meteorological regimes.

2.4.2.5 Regimes of other agents and actions

Any agent or its action on the breakwater can be organized according to the recommendations for climate agents in this ROM. Once there is a data base for an agent whose manifestations are random, it is possible to identify for the useful life of the structure the states, sequence of states, solicitation and operationality cycles, meteorological years as well as the corresponding extreme peaks over threshold regimes and middle regimes. As an aid for the use and exploitation of the harbor and coastal area, and for the characterization of the operational stoppage modes, it is a good idea to define the medium and extreme regimes of other agents, besides those agents mentioned in the previous sections, such as precipitation, fog, ice, etc.

Even if no database exists, functional relations and theoretical premises can be used to obtain medium and extreme regimes of other agents and their actions on the basis of the climate agent regimes (*i.e.* maximum pressures and subpressures on the vertical breakwater due to the presence of wave action). From a statistical perspective, those regimes are derived from the distribution function of variables. Although in many cases numerical techniques must be used instead of analytical methods to obtain such functions, they are a basic tool in the application of probabilistic methods.

Note. One of the project objectives is to verify that in all climate and geotechnical states, the subset of the structure is reliable, functional and operational according to project design requirements. The present lack of knowledge regarding the sequence of climate and soil states (i.e. the possible manifestations of environmental agents during the useful life of the breakwater) makes it necessary to define strategies or verification hypotheses

(24) The National Port Authority of Spain.

pertaining to the safety, serviceability, and use and exploitation of the structure. One of these strategies is to verify the structure and its subsets in reference to the worst possible climate and geotechnical states. These are the limit states that can occur during the useful life of the structure.

The meteorological state describes and characterizes the simultaneous manifestation of the atmospheric agents (wind velocity and direction, precipitation, fog, etc.) and marine agents (wave action, meteorological and astronomical tide, other long-period agents, and currents). Similarly, the geotechnical state describes and characterizes the stresses in the solid, interstitial pressures, and the deformation rates of the soil. The duration of a state depends on the temporal variability of the agent.

The time sequence of the states is known as a state curve, and reflects the evolution of a descriptor that is representative of the climate and soil agents over time. This curve shows the time periods of great activity known as solicitation or storm cycles, and other periods of low or zero activity, or operational cycles or calms.

The threshold safety states generally define the beginning and end of a storm or solicitation cycle, and are usually related to extreme work and operating conditions, in other words, to severe meteorological states. The threshold states of use and exploitation define the beginning and end of a period of calms or an operational cycle in which the structure and its installations can function. Consequently, during this period use and exploitation is feasible, and is generally associated with normal work and operating conditions.

Solicitation cycles generally occur in Spain when there are extratropical storms. Randomness is an intrinsic quality of this sequence. A threshold climate state can be defined in terms of any of the manifestations of the agent. For example, the threshold value of the use and exploitation of a breakwater can be defined by the magnitude of the wave height inside a harbor (wave agitation inside harbors) or by the magnitude of the harbor access in terms of water depth, etc.

2.4.3 Nature of the subset

In each of the project design phases, the subset has a general nature and an operational nature. These are determined by external studies, and in their absence, by indexes of economic, social, and environmental repercussion as described in the ROM 0.0 (section 2.1.4).

2.4.3.1 General nature

The general nature of a subset is an indicator of its importance. It is measured by the economic, social, and environmental repercussions when it is irreparably damaged or when it suffers an irreversible loss of functionality. Accordingly, a subset's general nature indicates the magnitude of the consequences stemming from the eventual failure of the breakwater once it has begun to operate.

The general nature of a subset should be specified by the project developer on the basis of independent studies. At the very least, the results should be the same as those obtained with economic repercussion indexes (ERI), and the social and environmental repercussion indexes (SERI) as defined in the ROM 0.0. This ROM also describes the procedures for the determination of the principal failure mode, generally ascribed to ultimate limit states.

Figures 2.2.32 and 2.2.33 (section 2.7) show the recommendations for the economic repercussion index (ERI), and the social and environmental repercussion index (SERI) for breakwaters. Based on these recommendations, they also specify the useful life and the maximum joint probability for the failure modes ascribed to ultimate limit states and serviceability limit states.

2.4.3.2 Operational nature

The operational nature of a structure is an indicator of the economic, social, and environmental repercussions that can occur when the operating conditions of the breakwater are not reached or are reduced in the sheltered area or its accesses. As a result, the operational nature reflects the magnitude of the consequences produced by operational stoppages in the serviceability phase.

The operational nature should be specified by the developer of the breakwater, and should be at least the same as that obtained with the operational indexes of economic repercussion (OIER) and social and environmental repercussion (OISER) as defined in the ROM 0.0. This ROM also describes the procedures for determining the principal operational stoppage mode.

Figures 2.2.34 and 2.2.35 (see section 2.7) show the recommended indexes of economic repercussion (OIER) and the indexes of social and environmental repercussion (OISER) for breakwaters. Based on these indexes, it also shows the recommended minimum operationality and the annual average number of operational stoppages, respectively.

Figure 2.2.36 specifies the probable maximum duration of the operational stoppage according to the OIER and OISER.

2.4.3.3 Nature of the breakwater in other project design phases

The general and operational nature of a breakwater should also be defined for the construction, repair, and dismantling phases and subphases, and is based on the economic, social, and environmental repercussions generated in case of the destruction or the operational stoppages of the structure. The nature of the structure should also be specified by the developer of the breakwater based on independent studies. It should be at least the same as that obtained with the operational indexes, ERI, SERI, OIER, and OISER, as defined in the ROM 0.0. Generally speaking, the SERI of the construction phase and subphases can be regarded as insignificant. In the absence of specific precautions, the repair and dismantling phase can have a SERI equal to the SERI of the serviceability phase.

2.5 PROJECT DESIGN REQUIREMENTS

In each project design phase, each subset of the breakwater should satisfy minimum safety, serviceability, and use and exploitation requirements. These requirements are specified by delimiting the probability of exceedance regarding safety and serviceability during the phase, and the probability of no-exceedance of the operationality during the year.

In each phase and subphase of the project, the structure or subset and their respective components should fulfill the requirements stipulated in current regulations as well as by the project developer in matters of safety, serviceability and exploitation for all project states in the phase. The objective is to specify the probabilities of a failure or operational stoppage of the breakwater within acceptable limits as defined in terms of the possible consequences of the failure or operational stoppage. Accordingly, the safety, service and exploitation requirements for a given structure or subset should be defined by means of the following parameters (see ROM 0.0):

- Reliability: Complementary value of the joint probability of failure in the phase or subphase of the project within the context of the failure modes ascribed to ultimate limit states.
- Capacity for service or functionality: Complementary value of the joint probability of failure in the project phase or subphase within the context of the failure modes ascribed to the serviceability limit states.
- Operationality: Complementary value of the probability of stoppage in the phase or subphase of the project within the context of the stoppage modes ascribed to the operational stoppage limit states.

The reliability index β is also used as a measurement for each of these concepts. Along with the corresponding probability of failure or operational stoppage p, this index has the following bi-univocal relation: $\beta = \Phi^{-1}(p)$, where Φ is the normalized, standard accumulated probability function.

Note. If a verification equation is linear, and if the terms X_1 and X_2 are independent random Gaussian variables, the safety margin $S = X_1 - X_2$ is also a Gaussian variable. The failure domain is defined by $S \leq 0$. If S is a normal variable with an average of μ_S and a standard deviation σ_S, the reduced variable $\beta = (S - \mu_S) / \sigma_S$ is known as the reliability index and represents the number of typical deviations that separate the average value of the function S of the origin. The probability of failure or stoppage, $p = Pr[S \leq 0]$, can be obtained from the Gauss *distribution function, $\Phi(\beta)$, $p = 1 - \Phi(\beta) = \Phi(-\beta)$, and the reliability, functionality, and operationality of the structure or subsection within the context of the mode is $r = \Phi(\beta) = 1 - \Phi(-\beta)$.*

2.5.1 Requirements during the useful life of the structure

In the serviceability phase or useful life, each subset of the breakwater should satisfy all of the following project design requirements related to safety, serviceability, and use and exploitation.

2.5.1.1 Safety requirements

The minimum safety requirements for a breakwater (or any of its subsets) within the context of the set of limit states that can arise in the serviceability phase is a function of the consequences of the failure or the destruction of the structure. These consequences can be globally evaluated by the general nature of the breakwater, whose value cannot be less than that obtained with the economic repercussion indexes (ERI) and social and environmental repercussion indexes (SERI) (see section 2.4.3.1). In this sense, safety should be greater when the social or environmental consequences of the breakage are more serious.

It is essential that the maximum admissible failure probability of a breakwater within the context of all the possible failure modes ascribed to limit states $p_{f,ULS}$ be less than the maximum values included in the ROM 0.0 for the social and environmental repercussion index (SERI). According to the ROM 0.0, table 2.2.8 shows the values applicable to breakwaters based on these criteria:

Table 2.2.8. Maximum joint probaability in the serviceability phase or the useful life of the breakwater for ultimate limit states (ULS)

SERI	$P_{f,ULS}$	β_{ULS}
< 5	0.20	0.84
5 – 19	0.10	1.28
20 – 29	0.01	2.32
≥ 30	0.0001	3.71

Section 2.7 specifies these values according to the SERI and the type of sheltered area.

Note. According to table 2.2.8, the social and environmental repercussion for breakwaters is usually insignificant or low. The assignment of a reliability value should be based on economic optimization calculations. However, in the case of breakwaters with adjacent storage or handling areas for hazardous cargo, or coastal defense structures for flood protection, the repercussion should be regarded as high or very high. The reliability value should never be less than the value in table 2.2.15.

In the case of breakwaters with storage or handling areas for hazardous cargo or coastal defense structures for flood protection, necessary precautions should be taken to avoid any possible damage. The failure probabilities in table 2.2.8 (10^{-2} and 10^{-3} respectively) are only a formal maximum reference. In such cases it is best to adopt the standard reliability values in civil engineering for each failure mode, on the basis of which accurate and precise project design rules have been developed.

2.5.1.2 Serviceability requirements

The minimum functionality for a breakwater (or each of its subsets) within the context of the set of serviceability limit states that can arise during the structure's useful life is a function of the consequences of a serviceability failure. For the serviceability phase or useful life of a structure, these consequences can be globally evaluated by the general nature of the structure. This general nature can be specified in the same way as the reliability, given that some of the failure modes ascribed to serviceability limit states may also entail repairing the structure in order to recover project requirements. In the same way as for reliability, the functionality or service capacity of the structure should be greater when the social and environmental consequences of failure are more important.

As stated in the ROM 0.0, the maximum admissible failure probability of a breakwater for all the failure modes ascribed to serviceability limit states, $p_{f,SLS}$, should be less than the maximum values shown in table 2.2.9, based on the social and environmental repercussion index (SERI) of the structure.

Table 2.2.9. Maximum joint probability in the serviceability phase or useful life for serviceability limit states (SLS)

SERI	$P_{f,SLS}$	β_{SLS}
< 5	0.20	0.84
5 – 19	0.10	1.28
20 – 29	0.07	1.50
≥ 30	0.07	1.50

Section 2.7 specifies these values according to the SERI and the type of sheltered area.

Note. *It is perhaps surprising that for low SERI values, the joint failure probabilities in ultimate and serviceability limit states are the same, whereas for the two highest SERI levels, they are different.*
However, in the case of the sliding of a vertical breakwater, its verification as a failure mode ascribed to an ultimate limit state is performed by selecting an extreme wave action state, for example, from the extreme wave regime. This calculation shows that the structure fails when the sliding begins. Furthermore, the verification of this failure mode ascribed to a serviceability limit state is also carried out by defining a threshold sea state, which, if exceeded, means there is a significant probability that the caisson will slide. Part of the verification process is also the evaluation of the failure probability and the distance of structural displacement at all of the states beyond the threshold. In this case the criterion used to define the failure is that the total accumulated displacement should not exceed the pre-established value.
From this perspective, the calculation in terms of ultimate limit states can be understood as a particular instance in which the threshold is the maximum sea state. In any case, it is clear that both calculations should agree and give the same caisson dimensions. Therefore, the joint probabilities of failure should be similar for those modes in which the progressive failure does not cause loss of human life or serious environmental damage (i.e. SERI < 20). In the absence of a better criterion and bearing in mind that maritime structures generally have a low SERI, the proposed values seem like a reasonable first choice.
Moreover, the lack of experience in the calculation for serviceability limit states for high SERIs, and in the absence of more reliable data, the same probability of failure was adopted for the two highest SERI levels.

2.5.1.3 Requirements for operational stoppage modes

The minimum operationality required for a breakwater (or each of its subsets) in view of the set of limit states of operational stoppage that can arise during the serviceability phase, as well as the average number of stoppages and maximum duration of a stoppage, depends on the consequences stemming from the operational stoppage. Regarding the serviceability phase, these consequences can be globally evaluated by means of the operational nature of the structure, whose value cannot be less than the value obtained by means of the corresponding operational indexes of economic repercussion (OIER) and of social and environmental repercussion (OISER) (see section 2.4.3.2). In this sense, the operationality should be greater when the economic consequences of operational stoppage are more important.

Section 2.7 gives the minimum operational values, the average number of stoppages, and the maximum admissible duration of a stoppage, depending on the OIER and OISER as well as the type of breakwater.

Minimum operationality

In a specified time interval (generally an average year of useful life), it is advisable that within the context of the set of all possible operational stoppage modes $r_{f,OLS}$, the minimum admissible operationality of a sheltered area or an area protected by a breakwater (and its accesses, when they exist), be greater than the minimum values given in the ROM 0.0 according to the operational index of economic repercussion (OIER) (see table 2.2.10).

Table 2.2.10. Minimum operationality during the serviceability phase

OIER	Operationality $r_{f,OLS}$	β_{OLS}
≤ 5	0.85	1.04
6 – 20	0.95	1.65
> 20	0.99	2.32

Regarding the lower limits on the table, the most suitable operationality level for each case should be calculated on the basis of economic optimization studies.

Average number of stoppages

Within the specified time period (generally an average year of useful life) the sum of the average number of operational stoppages N_m, regarded as admissible for the sheltered area or the area protected by a breakwater (and its accesses, when they exist), should be less than the maximum values given in the ROM 0.0 according to the operational index of social and environmental repercussion (OISER), as shown in table 2.2.11):

Table 2.2.11. Average number of operational stoppages in the time period

OISER	Number
< 5	10
5 – 19	5
20 – 29	2
≥ 20	0

In the same way as the minimum operationality, regarding the upper limits shown on table 2.2.11, the foreseeable number of stoppages should be calculated on the basis of economic optimization studies.

Maximum duration of a stoppage

Within the specified time period (generally an average year of useful life), the most probable maximum duration of an operational stoppage (t_{max}) cannot exceed the value in hours in table 2.12 based on the OIER and OISER, according to the ROM 0.0.

Note. *The values shown in table 2.2.12 should be applied when an operational stoppage has occurred. Generally speaking, when two of the operationality requirements are satisfied, the third is automatically fulfilled.*

Table 2.2.12. ***Most probable value of the maximum duration of an operational stoppage (hours)***

OIER \ OISER	< 5	5 – 19	20 – 29	≥ 30
≤ 5	24	12	6	0
6 – 20	12	6	3	0
≥ 20	6	3	1	0

2.5.1.4 Requirements when there are changes in use and exploitation

When structure (regardless of whether or not it has expanded in size) is subject to changes in use and exploitation during its useful life, the requirements of serviceability, functionality and operationality during this phase should be revised in accordance with the new exploitation conditions. The general and operational nature of the structure associated with these conditions should thus be redefined. Similarly, new project requirements should be established in the manner described in previous sections.

2.5.1.5 Requirements for a partial start-up of the structure

When breakwater or one of its subsets provisionally begins operation during the construction phase, the admissible probability of failure during this transitional phase should be as specified in the project design. This specification should take into account the social and environmental consequences of the failure in this situation and includes considerations pertaining to the construction. In any case, it should be equal to or less than the SERI value in table 2.2.8 and the limit states considered.

2.5.2 Requirements for other project phases

During the construction, repair, and dismantling phases, each subset of the structure should also satisfy project requirements pertaining to safety, serviceability, and in some cases, to use and exploitation. In the same way as during the useful life of the structure, the minimum levels of safety, functionality, and operationality required in these phases should be in accordance with the consequences of the failure, and when relevant, of the possible operational stoppages during these phases. These consequences can be assessed by following criteria similar to those for the useful life of the structure, based on the evaluation of the partial ERI, SERI, OIER, and OISER.

For this purpose the structure should be divided into discreet time segments and the corresponding construction, repair, and dismantling subphases. For each of these it is necessary to evaluate the nature of the structure or the relevant subset. The highest admissible probabilities of failure in these phases should be less than the maximum values in the ROM 0.0, depending on the structure's partial social and environmental repercussion index (SERI). Accordingly, the operationality should be greater than the minimum values in the ROM, depending on its OIER.

Once the construction or repairs of the subset has finished, the probability of failure and stoppage should be the same as that of the project design. In this way it is possible to sequence the "subset satisfaction" of the project requirements.

2.5.2.1 Reliability and functionality in other project phases

As a general rule, for the construction, repair, and dismantling phases, the social and environmental repercussions of a failure are not significant. For this reason, the structure's reliability and functionality can be determined by economic optimization criteria. Nevertheless, in certain cases, particularly in structures located near large population centers, sensitive environmental settings, etc., the social and environmental factors can be significant, and thus should be taken into account, following the general procedure described in section 2.6.

2.5.2.2 *Operationality in other project phases*

Regarding the structure's operationality during the repair phase, the developer should be the one to decide whether it is partially or totally limited. For this reason, the minimum operationality values for the serviceability phase do not apply in such cases, and the general procedure in section 2.6 can be used as a guide. During the construction phases, there is generally no use and exploitation of the installations except in the case of a partial start-up and in the dismantling phase,

2.5.3 Calculation of the joint probability of failure or stoppage in a phase

The set of failure or stoppage modes, regardless of whether they are mutually exclusive, satisfies the project requirement pertaining to safety, serviceability, and use and exploitation if the linear superposition of the probabilities of occurrence in the project phase is less than or equal to the joint probability of failure or stoppage in the project. In other words,

$$\sum_{j=1}^{N} p_{f,LS,j} \leq p_{f,LS} \tag{2.38}$$

where $p_{f,LS,j}$ is the probability of failure mode j in the project phase, and $p_{f,LSL}$ is the probability of failure or stoppage of the set of N failure or stoppage modes ascribed to the ultimate limit states or the serviceability or operational stoppage limit states, respectively (25).

Only in those cases in which two or more modes occur with the same agent values, can they be considered to have the same individual failure or stoppage probability, and their contribution to the joint probability is thus added once.

Note. If N principal modes are mutually exclusive, the probability that at least one of them occurs once in its useful life is

$$p_f^* = \sum_{i=1}^{n_p} p_{f,EL,i} \leq p_{f,EL} \tag{2.39}$$

If the n_p modes are not mutually exclusive and are independent, the failure probability of the set of modes is:

$$p_f^* = 1 - \prod_{i=1}^{n_p}\left(1 - p_{f,EL,i}\right) \approx \sum_{i=1}^{n_p} p_{f,EL,i} \tag{2.40}$$

If the n_p modes are not mutually exclusive and statistically dependent with a positive correlation, the joint probability of failure has the following upper limit, $p_{f,Sup}$:

$$p_{f;Sup} = 1 - \prod_{i=1}^{n_p}\left(1 - p_{f,EL,i}\right) \lesssim \sum_{i=1}^{n_p} p_{f,EL,i} \tag{2.41}$$

In other words, its upper limit is the complementary value of the product of the non-occurrence of each individual mode. This limit can be further specified. For example, when only three principal failure modes ($N = 3$), are considered, this gives:

$$p_{f;Sup} = 1 - \left\{1 - \left(p_{f,1} + p_{f,2} + p_{f,2}\right) + \left(p_{f,1}p_{f,2} + p_{f,1}p_{f,3} + p_{f,2}p_{f,3} - p_{f,1}p_{f,2}p_{f,3}\right)\right\} \tag{2.42}$$

and bearing in mind that the probability values are small, their products are of a lower order of magnitude than the order of magnitude of each one, separately. Consequently,

$$p_{f;Sup} \lesssim p_{f,EL,1} + p_{f,EL,2} + p_{f,EL,3} = \sum_{i=1}^{n_p} p_{f,EL,i} \tag{2.43}$$

(25) In what follows, the subscript indicating the limit state LS is omitted.

Furthermore, the failure probability of the set of modes has the following lower limit, $p_{f,Inf}$

$$p_{f,\mathrm{Im}\,f} = \max_{i=1}^{n_p}\left(p_{f,EL,i}\right) \tag{2.44}$$

The joint failure probability of the subset of the structure in the project phase for the failure modes ascribed to the ultimate limit states or serviceability limit states satisfies the following inequality:

$$\max_{i=1}^{n_p}\left(p_{f,EL,i}\right) \lesssim p^*_{f,EL} \lesssim \sum_{i=1}^{n_p} p_{f,EL,i} \tag{2.45}$$

Consequently, the project requirement is satisfied as long as the value of the joint failure probability, $p^*_{f,ELU}$ *is lower than the value given in Table 2.2.8,* $p^*_{f,ELU}$ *in other words,*

$$p^*_{f,ELU} \lesssim \sum_{i=1}^{n_p} p_{f,EL,i} \leq p_{f,ELU} \tag{2.46}$$

Similarly, for serviceability limit states, see table 2.2.9.

For operational limit states, the complementary value of the sum of the probabilities of occurrence of the principal stoppage modes during the year is greater than or equal to the recommended operationality value in table 2.2.10.

The following three types of mode make up the complete set: (1) *mutually exclusive modes;* (2) *dependent modes that are not mutually exclusive;* (3) *independent modes that are not mutually exclusive. For all three types, the sum of the probabilities of occurrence of the individual modes is an upper limit of the joint probability.*

On the other hand, the fact that the probability assigned to each failure mode depends on the number of modes does not signify an overdimensioning of the sections, but rather the recognition that, all project requirements being equal, the section has more weak elements (all of which thus contribute to the possible failure) than another section with fewer failure modes. In this sense, when economically feasible, it is best to have the fewest possible failure or stoppage modes categorized as principal modes.

This means that in order to be able to verify the requirement that the set of failure or stoppage modes satisfy in the project phase where their probability is specified, it is necessary to distribute the joint probability by ascribing a probability of occurrence to each mode. For this reason, complete sets of failure or stoppage modes should be specified.

2.5.3.1 Joint probability, complete set and diagram of modes

Whatever the method applied to obtain the joint probability distribution, the first step should always be the description and characterization of the set of modes and their organization in diagrams, according to the recommendations in section 2.3.2. For this purpose, the modes for each breakwater type are described in specific sections of this ROM. The section at the end of this chapter shows the failure modes of vertical breakwaters ascribed to ultimate limit states.

This first step is to simplify and reduce the complexity of the calculation process, with a view to selecting the minimum number of modes that contribute to the joint probability of failure or stoppage of the structure during its useful life. Precisely because this selection can greatly simplify the work, it should be done carefully and in such a way that the initial hypotheses do not excessively condition the final result.

The procedure to be carried out is the following: analysis of mutually exclusive occurrence of the modes; discussion of the statistical dependence of modes; elaboration of the complete set of modes; finally, the organization of the modes in sequential, parallel (redundant) or composite/mixed diagrams.

Exclusive occurrence and statistical dependence. From the complete set of modes, it is essential to identify the mutually exclusive modes, whose occurrence is incompatible with the occurrence of other modes, as well as those modes that are not mutually exclusive. In the latter case, it is necessary to specify if they are statistically independent, in other words, if the probability of two or more modes is equal to the product of their marginal probabilities. When this is not the case, the modes are dependent, and it is advisable to specify if they are positively or negatively correlated [26]. Positive correlation signifies that the failure or stoppage of a mode coincides with the failure of stoppage of another. Negative correlation means that the failure or stoppage of a mode corresponds to the non-failure or non-stoppage of another.

Unless otherwise stipulated, the N failure or stoppage modes should be specified for allocating the joint probability distribution in the project phase. The N modes must make up a complete set [27] of modes, and in this case, they are regarded as mutually exclusive.

This set should be organized in sequential, parallel or composite diagrams [28] as described in the ROM 0.0 (see section 2.9.4.1). A sequential diagram includes the modes whose individual occurrence determines the failure or stoppage of the subset. A parallel (or redundant) diagram includes those modes that cause the breakwater section to suffer failure or stoppage when there is the occurrence of two or more redundant failure modes. In a composite diagram a breakwater section is considered to have failed when there is the occurrence of at least one of the sequential failure modes as well as one of the subset of redundant failure modes.

Note. The diagrams help to visualize the performance of the subset of the structure and contribute to its dimensioning. For example, in the case of a sloping breakwater, a sequential diagram of failure modes includes, among others, the sliding and overturning of the superstructure, detachment of the armorstone units of the main layer of the central part of the breakwater, and the detachment of units of the toe berm. It is customary for the main layer to have two layers of armorstone units. When upper layer units become detached from the structure, the armorstone units of the underlayer are then left exposed. These will inevitably fail since their support and friction are provided by another secondary layer made up of smaller-size blocks. Nevertheless, it is possible to build a sloping breakwater in which the failure mode detachment of the main layer *is redundant or parallel. This is done by building the main layer with three layers of armorstones. This way a main layer failure (appearance of the third layer of armorstones) is only produced when the two upper parts have failed.*
Accordingly, the performance of the subset is depicted in the complete sets of all possible failure modes, whose occurrence significantly affects the safety, serviceability, and use and exploitation of the structure. Whenever possible, these modes should be mutually exclusive. When this is not feasible, the complete set should contain the events that contain the occurrence of two or more modes.
The evident limitations of this procedure make it more suitable for the analysis of failure modes ascribed to ultimate limit states, which by themselves cause the failure of the structure, and can only be resolved by the urgent and immediate repair of the subset to avoid its destruction.
Nevertheless, given the present state of knowledge, the assessment of the evolution of the performance of the structure during its useful life is extreme difficult. Consequently, this methodology can also be applied to failure modes ascribed to serviceability limit states.

CLASSIFICATION OF FAILURE MODES FOR VERIFICATION PURPOSES

For verification purposes, failure or stoppage modes are classified as follows:

1. Modes that can be verified by other regulations, standards, and recommendations;
2. Non-principal modes and principal modes, which may become non-principal;
3. Principal modes.

Failure modes verified by other regulations, standards, and recommendations. A selection should be made of those failure modes in the complete set that obligatorily require the application of regulations, standards and recommendations, and which can thus be verified by deterministic methods. The other modes within the set

(26) If the failure and stoppage modes for maritime and harbor structures are not mutually exclusive and if they are statistically dependent, the correlation will generally be positive.
(27) The set is regarded as complete when all possible failures for a breakwater type are described and characterized by this set of modes.
(28) As a rule, failure modes for maritime and harbor structures are organized in sequential diagrams. In other words, the occurrence of one failure mode is sufficient for the subset of the structure to have suffered failure or stoppage.

are classified as principal and non-principal modes. The probability of occurrence of failure modes verified by mandatory regulations leads to the assumption that their contribution to the joint probability of failure during a structure's useful life is negligible.

Note. *Regarding breakwaters, this section can affect failure modes that are (a) related to the structural performance of a part of the breakwater or any of its elements, and verified by the EHE and other regulations; (b) related to the geotechnical performance of the structure, and verified by the ROM 0.5. Generally speaking, it can be assumed that the order of magnitude of the failure probability of a mode calculated by mandatory regulations (applying deterministic methods) is less than* 10^{-4}

However, when sea dynamics plays a major role in the occurrence of the structural or geotechnical failure mode, it is advisable carry out a more specific analysis since such failure modes may not be considered to be non-principal modes for economic reasons. Accordingly, the method and verification criteria should be justified according to the mechanisms set out in the regulations and standards to be applied in such cases. This ROM specifies when this is relevant, as well as the procedures to be used.

Certain regulations state that the probability of occurrence of a failure or stoppage mode is associated with the probability of exceedance of the predominant agent in the occurrence of the mode. In the case of maritime structures, this hypothesis may not be suitable. It is then advisable to evaluate the probability of occurrence of the failure mode by considering the uncertainty of the set of project factors that intervene in the verification equation.

Non-principal modes. A failure mode is regarded as non-principal when slight increases in the total cost of the structure can significantly improve the reliability, functionality or operationality of the subset.

The subset is dimensioned within the context of non-principal failure modes in such a way that its probability of occurrence in the structure's useful life is very small, and thus, it does not contribute to the joint probability of the subset during its useful life.

Note. *To carry out this verification, a level II method can be applied (see ROM 0.0, sections 6.3 and 6.6) which, apart from evaluating the probability of the failure mode in the state, provides information regarding the importance of each factor in the result. This method quantifies the sensitivity of the failure probability to slight changes in the values of the agents and parameters that intervene in the verification equation.*

Given these assumptions, the joint probability distribution only affects the set of major failure and stoppage modes such that the sum of their individual probability of occurrence should be less than or equal to the values in tables 2.2.8, 2.2.9, 2.2.10, 2.2.11, and 2.2.12.

The threshold value of the probability of exceedance fixed at 10^{-4} *should be understood as the order of magnitude below which this probability is negligible. Consequently, the result of the sum of the probabilities of occurrence of non-principal failure modes is of the same order of magnitude as the probability of each mode. In other words, the probability of occurrence in a structure's useful life of a non-principal mode can be regarded as an infinitesimal quantity, and the sum of such infinitesimal quantities is logically another infinitesimal quantity.*

Principal modes. Once identified the failure modes that must be verified with other sets of regulations, standards, and recommendations and the non-principal modes, the rest of the modes can be regarded as principal modes. Principal modes are those failure and stoppage modes for which it is extremely difficult to improve the reliability, functionality or operationality of the subset of the structure or for which such improvement is only viable if there is a significant increase in the cost of the structure.

For the sake of practicality, principal modes are regarded as mutually exclusive, and are the only modes that determine the calculation of joint probability. Specific sections of this ROM include the diagrams of failure and stoppage modes for each breakwater type. They include principal modes, non-principal modes, and principal modes which at a given time may become non-principal.

Note. *In a complete set the three types of mode are:* (1) *mutually exclusive modes;* (2) *non-mutually exclusive modes that are dependent;* (3) *non-mutually exclusive modes that are independent. The sum of the probabilities of individual modes is an upper limit of the joint probability.*

2.5.4 Methods for allocating the joint probability distribution

The joint probability distribution is the task of the project designer, who should dimension the subset of the structure, according to its structural and formal nature and function as well as to its performance under the action of different agents. This performance materializes in the failure or stoppage modes, whose occurrence can produce economic, social, and environmental consequences. Given that the joint probability distribution entails decisions that can produce effects at all levels, such a distribution should be an integral part of the analysis and evaluation of the Harbor Investment Project.

As a general rule, this analysis should be carried out within the framework of Decision Theory. This naturally signifies using optimization techniques for an objective function, subject to restrictions. Examples of such restrictions are the satisfaction of (1) the joint probability values in tables 2.2.1, 2.2.9 and 2.2.10; (2) the economic and financial profitability of the Investment Project and its social consequences; (3) environmental requirements specified in the corresponding environmental impact analysis, and in the ROM 5.0 and ROM 5.1-05 (29).

2.5.4.1 Investment project analysis

It is the job of the project developer, whether in the public or private sector, to provide the necessary information to define and optimize the objective function that evaluates the economic, social, and environmental importance of the subset of the structure and the consequences of its failure.

This evaluation of the Investment Project must tackle and effectively deal with the optimization of the economic and financial benefits during the useful life of the structure, and the economic effects of the different operations and agents involved. This means quantifying the investment in the construction, maintenance, and exploitation of the structure, which is naturally subject to the restrictions of technical parameters and the arcs of the transportation chains. Among other things, the function VAN_{eco} or VAE_{eco} can be optimized. However, these methods generally do not assess the benefits of environmental questions, but only consider the costs of the investment and of the maintenance and exploitation necessary to guarantee compliance with environmental requirements.

Currently, evaluating environmental consequences in the same way as economic (cost-benefit) and social consequences is still infrequent, but it is an issue that is far from trivial. Hopefully, this type of assessment will soon (30) be performed as a matter of course. Until a better method is available to evaluate environmental consequences, this ROM suggests using the SERI for this purpose.

Note. The economic (or financial) profitability of the subset and of the set of subsets of the structure can be measured by estimating the net economic value of the project. Using constant monetary values, it is possible to calculate the variations in the economic surpluses of all the agents affected by the project during each year of the structure's useful life and at market prices with the following equation:

$$VN_{eco,t} = (\nabla EO - \nabla INV)_t + \nabla(CO - PM)_t + \nabla ECL_t \quad (2.47)$$

where $VN_{eco,t}$ is the net economic value in the year t and the variations are expressed by:
∇EO : surplus of the operators (Port Authority).
∇INV : investment cost of the project.
$\nabla(CO - PM)$: differences between opportunity costs and market prices.
∇ECL : urplus of the clients.
The net economic value VAN_{eco} and the annual equivalent economic value VAE_{eco} are obtained by the following equation:

(29) ROM 5.0 *Estudio de Impacto Ambiental en las Obras Marítimas y Portuarias* is currently in press. ROM 5.1-05 *Calidad del Agua en Áreas Portuarias y Litorales* was published in December 2005.
(30) The *Directiva Marco del Agua* (DMA) [The Spanish Water Framework Directive] and other similar legislation that is presently being drawn up (e.g. *Directiva frente a inundaciones*) proposes the establishment of methods for the cost-benefit assessment of the environment.

$$V\ AN_{eco} = \sum_{t=x}^{X} \frac{V\ N_{eco,t}}{(1+r)^{t-x}} \qquad (2.48)$$

$$V\ AE_{eco} = \frac{V\ AN_{eco}}{\sum_{t=x}^{X}(1+r)^{t-x}} \qquad (2.49)$$

where X is the horizon year, which at most will be the useful life of a subset $(X < V)$, and x is the year in which the initial costs are produced that can be attributed to the investment project. Finally, *r* is the social discount rate for the updating of flows.

Since these methods are obviously not free of uncertainty, it is customary to include an analysis of the risk and uncertainty pertaining to the investment project. Among other things, this includes: (1) a direct sensitivity analysis based on three case scenarios: a favorable outcome, an unfavorable outcome, a neutral outcome; (2) an inverse sensitivity analysis which calculates the value that a specific variable should have so that the *VAN* or the *VAE* will take a specific value; (3) a statistical risk analysis by means of the probabilistic consideration of uncertainty and the distribution functions for the evaluation indicators *VAN* or *VAE* with a Monte Carlo simulation.

This ROM proposes the statistical calculation of expected damage at various levels of complexity. These calculations should include the influence of economic cycles in investment parameters, particularly in the social discount rate, and, when applicable, in the surplus of the operators and clients caused by the variation in the business volume.

2.5.4.2 Recommended methods

The probability distribution should be carried out within the context of the evaluation of the economic, social, and environmental repercussions of the Investment Project during the useful life of the structure. For this purpose, the following methods of different levels of complexity are recommended:

- Optimization of the total cost-benefit during the useful life of the structure;
- Optimization of the total cost of the breakwater during its useful life;
- Analysis of the total annual cost of the breakwater for three allocations of the joint probability distribution.

The best decision should be taken for the Investment Project on the basis of the information obtained with any of these three methods, along with the data pertaining to the rest of the subsets, the construction processes, and the achievement of the projected levels of economic and financial profitability.

In the first two methods, the probability distribution is obtained as one of the results of the optimization process. The third method, which is less complex, is generally most frequently recommended. It begins with the establishment *a priori* of the allocation of the distribution assumptions that satisfy the project requirements. The subset is then dimensioned, and the total costs are calculated.

In line with the ROM 0.0, it is advisable to optimize (maximize) the economic, social, and environmental objective function: total benefit minus the total cost $[B_T - C_T]$, using advanced programming techniques , especially designed for this type of problem. This analysis gives the values of the objective function and the probability associated with each mode. These methods should be applied when the nature of the structure has a high or very high SERI and a high ERI.

When the social and environmental consequences are insignificant, it is usually sufficient to optimize (i.e. minimize) the objective function: the total cost of the structure during its useful life $[C_{T,V}]$.

The costs of the subset should include the investment costs of the construction of the project, conservation and exploitation costs, and finally the expected costs for damages during the structure's useful life. The first group should include (along with their schedule) all of the fixed costs necessary for (1) the construction and start-up of the structure; (2) the restorations, adaptations, and enlargements which the structure will presumably undergo during its useful life; (3) environmental restorations and rectifications.

The conservation and exploitation costs (along with their respective schedules) include all the expenditures so that the subset has the necessary and sufficient conditions for the use and exploitation of the harbor area and installations (when they exist).

However, generally speaking, in the case of breakwaters the joint probability of failure should be distributed among the mutually exclusive principal modes.

This is done by evaluating (1) the total annual cost of the subset $[C_{T,annual}]$, including the cost of the expected damage per year; (2) the set of subsets belonging to the structure; (3) the construction processes. By means of different probability distributions, a selection can be made of those solutions that are closest to the minimum total costs.

Note. *Decision Theory is normally used to optimize the objective risk function. Probability is thus employed to weigh the consequences of a failure or stoppage, and determine the average Bayesian risk value (probability according to consequences). Each probability distribution is a decision that entails a risk value. This process can be represented by a decision tree that begins with a node indicating the shared probability to be satisfied. The branches that sprout from this node are the different probability distributions. Each decision involves an estimated value of joint risk.*
Furthermore, the joint probability distribution by means of optimization techniques is advisable for breakwaters that are located at great depths or where the wave height is not limited by the water-depth wave breaking.
The number of combinations of wave climates and sea level that can cause failures in different parts and elements of the breakwater, and, therefore, significantly contribute to the joint failure probability can be very high. In any case, a low failure probability does not mean that the failure will definitely not occur; rather it indicates that the possibility of occurrence is more or less remote.

2.5.4.3 Distribution a priori of the probability and quantification of the total annual cost of the subset

As a general rule, the joint probability can be distributed by using the following calculation method that evaluates the total annual cost for the set of subsets in the structure and the construction processes.

The first step, and prior to anything else, is to establish certain assumptions or decisions regarding the probability distribution that satisfy the joint probability requirement. The section is then dimensioned according to this probability distribution. Finally, the total cost of the investment is calculated such as the sum of the expected cost of yearly damage produced by each of the principal modes and the equivalent annual cost of the investment.

This method can be summarized as follows:

1. Description and characterization of the set of failure and stoppage modes;
2. Classification of the modes and selection of the principal modes regarded as mutually exclusive;
3. Allocation of joint probability distribution among principal failure modes;
4. When necessary, weighting of the previous result by means of the evaluation of the uncertainty pertaining to breakwater performance and to analytical or experimental verification processes, as well as their underlying theoretical premises. Other factors to be assessed are the relevant data as well as the knowledge and experience of the project designer and building contractor.
5. Dimensioning of the section and calculation of the total annual cost of the subset, including the annual investment cost and the projected cost of damages per year by using the distribution functions of the predominant agents.
6. Finally, one solution is chosen for each subset, once the total cost of the structure has been evaluated. Factors to be taken into account are the subsets of the structure and its transitions, construction processes, and the materials available.

Note. *In order to evaluate the total annual costs of a structure, it must first be dimensioned. Dimensioning a structure requires choosing a value for the predominant agents and the other project factors which, if their occurrence and magnitude are random, should be performed by selecting a probability of exceedance. If from previous*

studies, the percentage of economic repercussion on the total annual costs of the structure is already known for each principal failure mode, then it is possible to begin the process by distributing the joint probability among the failure modes, based on that repercussion. If this evaluation is based on probable annual damage costs, which are adimensionalized by the total annual cost of the structure, the distribution is more biased.
When data from previous experience is not available or there are doubts about its accuracy, the probability distribution should be obtained by optimizing the total annual minimum cost with optimization theory techniques. Alternatively, one can also proceed by trial and error on the basis of distribution assumptions. One way of doing this is to assign a very small probability (e.g. $0.1p_{f,V}$) to the worst-case scenario failure mode, a part of the residual probability that is greater than the previous value (e.g. $0.25p_{f,V}$) to the next worse failure mode, etc. Still another way is to begin the process in just the opposite way by assigning the highest probability value to the worst-case scenario failure mode and going downwards from there. These two extreme distributions together with the isoprobable distribution should provide the information necessary to establish the distribution with the minimum total annual cost.

EVALUATION OF UNCERTAINTY

In all cases, the joint probability distribution should be weighted [(31)] by means of the evaluation of the uncertainty pertaining to the performance of the breakwater and analytical or experimental verification processes, as well as their underlying theoretical premises. Other factors to be assessed are the relevant data as well as the knowledge and experience of the project designer and building contractor. When the predominant agent (e.g. soil, climate, atmosphere, sea, etc.) is the same in the induction of principal failure modes (i.e. subject to the same uncertainty as the input data), the safety coefficient adopted for a level I method can be indicative of the uncertainty that is currently attributed to the verification equation. Consequently, the same as the other factors, the joint probability of failure of the principal modes with the same predominant agent should be weighted by means of the quotient of the recommended safety coefficients of their respective verification equations.

Similarly, the probability distribution among principal modes with different predominant agents (e.g. agents pertaining to physical environment, soil or materials) should be weighted by (subjectively) evaluating the quality and quantity of the initial data, and the current knowledge and experience regarding the application of this method.

Note. The network of sea oscillation measurements available in Spain, their constant comparison and verification by Puertos del Estado*, and the accuracy and precision of the methods of analysis make this oceanographic data extremely accurate. This is in vivid contrast to geotechnical data, which because of its very nature as well as its unavailability, is characterized by a high level of uncertainty.*
Moreover, verification equations of failure modes whose predominant agent is wave action are derived from a significant number of experimental and theoretical studies, and have been subject to a wide range of studies and analyses of the processes included in these equations. This situation is usually not the same for geotechnical failure modes in which sea oscillations are the predominant agent in the triggering of the failure. For this reason, the failure mode, soft soil in-depth sliding, *whose parameters are uncertain when this type of soil constitutes the foundation of a vertical breakwater, has a smaller probability of failure than the failure mode* rigid sliding of the breakwater over the level berm.

CALCULATION OF THE EXPECTED COST OF DAMAGE PER YEAR

Since the useful life of the structure is generally measured in statistically independent years the total annual cost of the structure should be calculated as the expected cost of the damage per year, and the equivalent annual cost of the investment. Whenever possible, the structural damage costs should include the repair costs as well as

(31) The most widespread conception of the probability of an event is the *a priori* estimate obtained by purely deductive reasoning. Another conception of probability, which is the most widely used in maritime engineering (regimes) is *a posteriori* probability or relative frequency. Moreover, there is also subjective probability, where objective methods, theory, and observations are insufficient, incomplete or of doubtful validity. This probability is based on experience and personal judgment, and is performed by weighting values and results.

those costs produced by the cessation and influence of the economic activities directly related to the structure. These activities can be services offered after the structure has begun to operate or services demanded beforehand as described in the ROM 0.0 (section 2.11.1.2). If part of the joint probability attributed to each mode is $p_{f,LS,i}$, the annual probability of occurrence $p_{LS,i}$ of each of the principal failure modes $i = 1, \ldots, n_p$, can be evaluated on the basis of the following equation:

$$p_{EL,i} = 1 - \left(1 - p_{fEL,i}\right)^{\frac{1}{V}} \tag{2.50}$$

where V is the useful life of the subset. The expected cost for yearly damages for each failure mode can be calculated by means of:

$$C_D\left(D_{j,i}\right) \approx d\left(D_{j,i}\right) * f_{\vec{X}}\left(D_{j,i}\right) \tag{2.51}$$

where $d(D_{j,i})$ is the damage function of failure mode *i* assuming that the probability distribution $j, f_{\vec{X}}(D_{j,i}) = dF(D_{j,i}) / dD_j$ is the density function of mode $D_{j,i}$ and $p_{LS,i} = F(D_{j,i})$is its distribution function.

CALCULATION OF THE ANNUAL EQUIVALENT COSTS: INVESTMENT AND TOTAL

The equivalent total annual cost for each of the *j* probability distributions $(j = 1, 2, 3)$ is:

$$C_T\left(D_j\right) \approx C_{ta}\left(D_j\right) + \sum_{i=1}^{n_p} C_D\left(D_{j,i}\right) \tag{2.52}$$

where the summatory evaluates the expected costs of yearly damages produced by each of the failure modes and the equivalent annual cost of the calculated investment, for example, by the method described in this section.

Based on this information, it is possible to compare which of the three probability distributions produces the lowest annual total cost, and if necessary, to carry out a new possible distribution, closer to the minimum cost.

Note. The initial costs or construction costs of the subset increase as the probability of occurrence of the failure and stoppage modes in the structure's useful life decrease. However, the probable damages decrease with this probability since the subset is better prepared to withstand agents. Some of the difficulties in the application of these methods are (1) *the simulation of storm events whose number, intensity, and duration are random;* (2) *the management of a large number of random and deterministic variables;* (3) *and the fact that each failure mode is in itself an optimization problem.*

In many cases and more concretely in previous studies, pre-dimensioning and analysis of alternatives, the method of economic minimization admits simplifications that are viable when the number of failure modes is small and when the dimensions of the structure are constrained by one of these modes. In this case, the annual investment cost can be calculated for representative dimensions of the structure. The expected costs for yearly damages can be added on. When the differences between dimensions are significant, the annual investment cost must be re-evaluated.

To simplify things, a failure mode occurs because of the exceedance of a certain threshold value of the predominant agent $x \geq X_d$ and is verified by means of a state equation $g(x) \leq 0$. When $f_X(x)$ is the annual probability function of the agent (e.g. extreme wave action regime) and $d(x)$, the damage function, the expected cost of yearly damages is the following:

$$C_D(x) = \int_{g(\vec{x}) \leq 0} f_X(x)\, d(x)\, dx \tag{2.53}$$

When there are N mutually exclusive failure modes, it is possible to delimit the value of the probable damages by means of the linear superposition of the yearly damage produced by one of the failure modes, in other words:

$$C_{D,total}\left(\vec{x}\right) \lesssim \sum_{i=1}^{N} C_{D,i}\left(\vec{x}\right) = \sum_{i=1}^{N} \int_{g\left(\vec{X}\right) \leq 0} f_{\vec{X},i}\left(\vec{x}\right) d_i\left(\vec{x}\right) d\vec{x} \tag{2.54}$$

where $\vec{X}$ indicates the set of agents that intervene in each of the failure modes and $f_{\vec{X},i}(\vec{x})$ is the density function of the joint probability of the agents.

For each failure mode, the integral that determines the probable damages can be approached by the following difference scheme:

$$\begin{aligned} C_D^V(x) &= \sum_{j=1}^{\infty} \frac{d(x_{j-1})+d(x_j)}{2}\left\{F(x_j)-F(x_{j-1})\right\} \\ &= \sum_{j=1}^{\infty} d^*(x_{j-1},x_j)\,f^*(x_j,x_{j-1}) \end{aligned} \tag{2.55}$$

where $x_{j-1} = x_0$ is the initial value of the calculation of the damage function, $d^*(x_{j-1}, x_j)$ is the annual increase of probable damage, and $f^*(x_j, x_{j-1})$ is the density value of the probability of the interval. The superscript V represents the number of years accumulated in the calculation of probable damage in the structure at the end of its useful life.
The equivalent annual investment cost $C_{ta}(x)$ can be calculated, for example, by the method described in the Analysis of the Investment Project *(see section 2.5.4.1). The equivalent total annual cost is thus:*

$$C_T(x) = C_{ta}(x) + C_{D,total}(x) \tag{2.56}$$

Minimizing the total cost function $C_T(x)$ with respect to the value of the agent (i.e. giving its annual probability of exceedance), it is possible to obtain the values of the predominant agent and the probability of exceedance during the year $Pr[x < X_d] = F(X_d)$, which is the information necessary to calculate its probability of occurrence during the structure's useful life.
To facilitate calculation, the following tables should be elaborated (the numbers are indicative):

Table 2.2.13. Calculation of annual cost of damage

Year	$Pr[x < X_d]$ $F(X_d)$	$f^*(x_j,x_{j-1})$	Damage costs (euros*10^6)	$d^*(x_{j-1},x_j)$ (euros*10^6/year)
1	0.000	–	0.00	–
2	0.500	0.500	30	15
3	0.625	0.125	40	35
4	–	–	–	–

Table 2.2.14. Calculation of total annual cost

Year	$C_D(x)$ (euros*10^6/year)	Investment cost (euros*10^6/year)	Total cost (euros*10^6/year)
1	$C_D^V(x)$	0	$C_D^V(x)$
2	$C_D^{V-1}(x)$ + 7.50	15	$C_D^{V-1}(x)$ + 22.50
3	$C_D^{V-2}(x)$ + 4.38	25	$C_D^{V-2}(x)$ + 29.38
4	–	–	–

Finally, the representation of the total cost in relation to the probability of no exceedance shows the minimum total cost and the failure probability ascribed to the failure mode.
This simplified method reduces the complexity of the analysis of the optimization of the structure, as constrained by the joint probability. In order to do this, one has no choice but to make hypotheses, which at least in certain circumstances may not be totally correct. In any case, the calculation of the probability of failure, risk, and probable damage is an upper limit of their values.
Furthermore, the probability of failure ascribed to each failure mode evaluates the uncertainty of the occurrence of the mode, and not of the agents that cause it.
Consequently, the evaluation of the density function entails the knowledge of its distribution function for which a level II or III method should be used. In its absence, and if the failure mode is triggered by the exceedance of a certain value of the predominant agent (as is usual for breakwaters), the distribution function of the occurrence of the failure mode can be substituted by the distribution function of the predominant agent.

Finally, if all of the breakwaters of the harbor system in Spain were calculated with the simplified method, the expected cost of yearly damages for all of them would be an order of magnitude of the costs that the system should allocate for breakwater conservation throughout Spain.

2.5.4.4 Allocation of the probability distribution in the construction, repair, and dismantling phases

For the joint probability distribution of the failure or stoppage in another project phase, it is best to follow one of the methods recommended for the serviceability phase although often the abbreviated method is sufficient for most purposes (see section 2.5.4.3 (32). In this case, the method consists of the following steps:

1. The failure modes and the relevant stoppage modes of the phase (or subphase) should first be specified for each subset of the structure. Principal modes, non-principal modes, and principal modes that can become non-principal modes are then identified. These modes then form a complete set of mutually exclusive modes. Generally speaking, it is only necessary to distribute the joint failure probability corresponding to the failure modes ascribed to the ultimate limit states. The principal modes in these phases are not necessarily the same as those in the serviceability phase.
2. The ERI and SERI of the phase or subphase are determined by considering the worst-case scenario failure mode of the subset, which is then compared to the ERI and SERI of the subset in the serviceability phase. The joint failure probability $p_{f,LS,c}$ for the principal failure modes must be adjusted according to the joint probability of the subset in the serviceability phase $p_{f,LS}$. Unless otherwise stated, the minimum duration of the phase or subphase V_c in all cases will correspond to complete years.
3. The maximum joint probability is determined in the construction phase.
4. The probability calculated *a priori* is then distributed, for example, in accordance with the cost of one construction part or element as compared to the total cost of the section or part.
5. When necessary, the distribution is weighted by evaluating uncertainty.
6. The expected annual cost of damages and the total annual coast are evaluated.
7. The most suitable distribution is selected based on the construction processes, materials, and financing modes of the structure.

With the exception of the steps described in what follows, the rest of the steps are carried out as specified in section 2.5.4.3.

Maximum joint probability in the construction phase

The joint probability of failure in the construction phase is delimited by the SERI function during the serviceability phase (or during the construction phase if it is greater than the structure's useful life). For this purpose, unless otherwise specified, the maximum probability of failure during the phase or subphase should not exceed the value in table 2.2.15.

Table 2.2.15. Numerical value of the SERI of the subset and the maximum probability of failure admissible during the construction phase

SERI of the subset during the serviceability or construction phases	Maximum $p_{f,c}$ during the construction subphase
Insignificant, < 5	≤ 0.20
Low, $5 - 19$	≤ 0.10
High, $20 - 29$	≤ 0.05
Very high > 30	≤ 0.01

(32) In what follows, explicit reference is only made to the construction phase, but it should be understood that this is also valid for the repair and dismantling phases.

ALLOCATION OF THE JOINT PROBABILITY DISTRIBUTION OF FAILURE DURING THE CONSTRUCTION PHASE

The joint probability can be distributed, beginning with the economic minimization of the total annual cost (see section 2.5.4.3). This process should take into account the simultaneity and compatibility of failure modes in the subset and in the subphase or part of the subset.

Consequently, once the joint failure probability $p_{f,c}$ during the construction phase of subphase with duration V_c has been determined, the annual probability of failure of each mode can be obtained with the following equation:

$$p_{c,i} \approx 1-\left(1-p_{f,c,i}\right)^{\frac{1}{V_c}} \tag{2.57}$$

where $p_{f,c,i}$ is the probability of the mode in the construction phase. Next, the methods proposed for the useful life of the structure are used to calculate the expected cost of yearly damage to the structure. Finally, the expected costs for each failure mode and the annual investment cost are all added up to obtain the total annual cost of the subset during construction. In all cases, it is necessary to make sure that the probabilities fulfill the project requirement (see table 2.2.15)

$$\sum_{i=1}^{n_p} p_{f,c,i}, \leq p_{f,c} \tag{2.58}$$

and generally assure that total annual cost of the structure is kept at a minimum.

Note. *It may seem strange to delimit the joint probability in a construction subphase based on the SERI during the serviceability phase. However, it is necessary to guarantee that no breakdowns occur during construction that can leave "scars" on the finished structure or be the source of undesirable consequences. Accordingly, constructions with a low, high and very high SERI value should be implemented with construction methods, materials and machinery that are in consonance with the importance of the structure and the consequences of a possible failure.*

2.6 VERIFICATION PROCEDURE

Once the diagrams of failure and stoppage modes are specified, and the joint probability distribution carried out, it is necessary to verify that the dimensioned construction satisfies all requirements pertaining to safety, serviceability, and use exploitation in each of the project phases.

Accordingly, the verification equation should be formulated and resolved for each mode, and the probability of the occurrence of a failure or stoppage during the project phase should be calculated.

2.6.1 Verification of a failure mode and its probability of occurrence

The verification of a failure or stoppage mode in the structure and each of its subset, and the calculation of the probability of occurrence during its useful life (or another project phase) signifies formulating and resolving a state equation. Although formulation and resolution are generally regarded as only one step, it is a good idea to explicitly separate them into two since the formulation of the equation determines its format as well as the assignment of values to the terms in the equation. In contrast, its resolution provides information regarding the compliance or non-compliance with a failure or stoppage criterion and, when relevant, its probability of occurrence.

For this purpose it is advisable to follow the recommendations of the ROM 0.0 and previously define the following:

1. Project parameters and agents;
2. Project actions.

2.6.1.1 Formulation of the state equation

As stated in the ROM 0.0, the state equation can be written in the format of the overall safety or safety margin coefficient. Depending on the variability of the project factors one of the following three formulations is recommended: deterministic, deterministic-probabilistic, or probabilistic.

DETERMINISTIC FORMULATION

The values of the predominant and non-predominant agents and the values of the parameters are nominal or deterministic values. They are used to calculate the terms in the state equation. The combination of agents, the simultaneousness of their appearance, and the compatibility of their values are determined according to the recommendations in this ROM, the ROM 0.5-05 pertaining to geotechnical failures, and the ROM 0.0 (Chapter 5). However, the value of project parameters regulated by other regulations and standards are calculated according to the specifications given in them.

Since this formulation does not quantify the variability or randomness of agent values and physical environment parameters, its application should be restricted to the verification of those failure and stoppage modes in which they are not significant for the safety, serviceability, and use and exploitation of the structure. In these cases, the probability of occurrence of the failure mode can be very low. It can be a non-principal failure mode and, consequently, not participate in the calculation of the joint failure probability.

In this ROM the sections pertaining to different breakwater types provide both the partial and global coefficients that intervene in the verification equation linked to nominal or deterministic values of the most common project factors. For the cases not mentioned in this ROM, the values used should be properly verified or justified either on the basis of other experiences or other recommendations, regulations and standards.

*Note. These methods are mainly applied to structural failure modes according to the EHE (*Spanish Structural Concrete Standards*). When relevant, they can also be applied to geotechnical failure modes, following the recommendations in the ROM 0.5-05 for failure modes whose predominant agent is not related to climate, sea state or seismic movement.*
However, in the EHE Standards the assignment of certain parameters is performed on the basis of probabilistic criteria, following a deterministic-probabilistic formulation. For example, it is obligatory to assign the compression resistance values of the concrete on the basis of a probability function of this previously estimated resistance, assuming characteristic upper and lower values of 95% and 5%. The verification of the probability model or distribution function of the resistance is carried out afterwards, once the concrete has been manufactured, by taking a sufficient number of samples and subjecting them to the corresponding breaking test.

DETERMINISTIC-PROBABILISTIC FORMULATION

Representative values should be established for parameters and agents, particularly those pertaining to the physical environment, climate, and seismic movement, as well as to use and exploitation. When relevant, soil agents should also be specified. All of these values should be determined on the basis of their respective probability models, with which the values of the state equation terms can be calculated. Generally speaking, these representative values are regarded as characteristic values.

The sections of this ROM on each breakwater typology or the sections in the ROM 0.5-05 on geotechnical failure modes give the combination of agents, the simultaneity of their occurrence, and the compatibility of their values, as well as the minimum or partial global coefficients that intervene in the verification equation of each of the failure modes linked to the representative values of the project factors

More concretely, they expressly state the probability of occurrence of the mode that can be regarded as formally linked to the criteria adopted for the definition of the representative values of the project factors, taking into account the partial coefficients and the serviceability coefficients.

For the cases not mentioned in this ROM, the values used should be properly verified or justified either on the basis of other experiences or other recommendations, regulations and standards.

When the verification equation term evaluates the action of agents pertaining to the physical environment, the value of the partial coefficient of the term should also incorporate a quantification of the uncertainty of the equation and of the bias of the term. Recommendations regarding this issue can be found in this ROM.

Note. *Such methods are generally used for the verification of structural failure modes according to EHE Standards and EuroCodes, as well as the geotechnical failure modes according to the general procedure established in the ROM 0.5-05 for level I calculations.*

Probabilistic formulation

The values of the equation terms are determined by using their respective probability models in the phase analyzed, which are calculated on the basis of probability models of parameters and agents, and are a result of the same equation resolution process. The global safety coefficients and the partial coefficients of the verification equation for each mode are set equal to one.

2.6.1.2 State equation resolution methods and probability

The subsections that follow describe state equation resolution methods.

Level I methods

Level I methods can be used for equations formulated with deterministic and deterministic-probabilistic criteria. Reliability, functionality or operationality are introduced, depending on the type of combination, affecting nominal or representative values of the project factors that intervene in the verification equation of global or partial coefficients, as well as weighting, simultaneity, compatibility, and reducing coefficients (see the ROM 0.0, Chapters 4 and 5).

The analytical or numerical resolution of the equation only says whether the failure or stoppage mode has occurred with the assigned values in the state considered.

If the equation has been formulated with deterministic criteria, neither the probability of failure nor the operationality can be calculated. Consequently, it should not be applied to failure modes whose predominant agent is random. Unless justified or required by regulations currently in force, failure modes whose predominant agent pertains to climate, atmosphere, wave climate or seismic movement should not have a deterministic formulation.

If the equation has been formulated on the basis of a deterministic-probabilistic criterion, the probability of exceedance of the value of the predominant agent causing that value can be adopted as the probability of occurrence of the failure mode.

Note. *Evidently, in the majority of cases, this is an oversimplification, above all for those maritime structures in which the magnitude of the agent is not delimited, for example, the highest wave heights in the sea state, given that other states with descriptor values close to those of the chosen state can significantly contribute to the failure probability during the structure's useful life. As a result, these formulations should not be applied to subsets of the structure with a medium or high general nature.*
In seas with significant tide movements (e.g. Cantabrian Sea), and at medium and shallow depths, wave regimes and extreme storm regimes usually vary considerably throughout the tide cycle. Furthermore, the most severe conditions for berm stability generally occur at low tide, and for armor layer and crown stability, at high tide.
For this reason, the calculation of failure probability should be conditioned by the sea level as explained in Chapter 3 of this ROM.

LEVEL II AND III METHODS

Level II and III methods can be applied to deterministic-probabilistic formulations as well as strictly probabilistic formulations. Generally speaking, the solution can be obtained by means of numerical techniques. The result is the probability of occurrence of the mode in the state considered.

2.6.1.3 Criteria for the application of verification methods

Depending on the nature of the subset in the project phase analyzed, the resolution of the state equation corresponding to a principal failure or stoppage mode should be performed by one of the methods in table 2.2.16. The verification of non-principal failure modes can be carried out by level 1 methods.

Table 2.2.16. Verification methods according to ERI and SERI values

ERI	Insignificant	Low	High	Very high
Low	[1]	[2]	[2] and [3] or [4]	[2] and [3] or [4]
Medium	[2]	[2]	[2] and [3] or [4]	[2] and [3] or [4]
High	[2] and [3] or [4]	[2] and [3] or [4]	[2] and [3] or [4]	[2] and [3] or [4]

Any of these methods can be applied to a state equation of a failure or stoppage mode. The only differences reside in the establishment of project factors that intervene in the verification equation and in the acceptance criterion of the result. In projects in which a multiple verification procedure (a level I method along with a level II or III method) should be carried out of the principal failure modes, the calculation is acceptable when the two verification procedures show that the reliability, functionality or operationality required are fulfilled. Since level I methods must be included in these cases, and also since they are easy to use, it is best to begin the pre-dimensioning with this type of method (see Chapter 4), and use this result as a reference during the rest of the process.

In this regard, in order to clarify the goodness of each conflicting result, it may be useful to carry out sensitivity studies of the solution to small changes in agent and parameter values. This information is automatically obtained when a level II method is used.

Note. The number between brackets indicates: [1] *Overall Safety Coefficient Method (level I);* [2] *Partial Coefficients Method (level I);* [3] *Level II Methods;* [4] *Level III Methods. The subsets of the construction whose general nature is* $[r = r_3, s \geq s_1]$ *and* $[r \geq r_1, s \geq s_3]$ *should be verified by at least two methods. One of the methods should be the partial coefficients method and the other should be a higher level method.*

2.6.1.4 Definition of project states

To analyze a limit state of the structure or one of its subsets, it is necessary to establish its geometry, certain properties of the materials, physical environment and soil as well as certain agents and actions. This usually entails a simplification of reality, which is valid for a specific time period during which the project factors and functional or operational structural response of the construction are assumed to be statistically stationary. Such simplifications are known as *project states*.

The selection of project states to be considered in verification processes should be carried out after analyzing all of the possible conditions which the breakwater can be subject to during the project phases. At the same time it is necessary to consider when statistically stationary values can be adopted for the various project factors (i.e. geometry, properties, agents, and actions. The project states can be classified according to work and operating conditions based on the simultaneity and compatibility of predominant agents.

If the predominant agents that can affect a breakwater are principally those agents pertaining to climate, soil and, when relevant, seismic movement, the selection of project states in breakwater design should be in accordance with the following criteria related to the method adopted for the formulation and resolution of the verification equation.

2.6.1.4.1 Level I Methods

For each project phase, limit states should be selected for the project that represent the different solicitation and operational cycles. These limit states are principally climatic, and correspond to the climate actions that the breakwater can be subject to during this phase.

Extreme and exceptional work and operating conditions. For each of the principal failure modes a climatic limit state should be selected from the set of extreme regimes of wave action, sea level, and possibly other environmental disturbances. In the absence of joint regimes, it is possible to use extreme regimes of the predominant agent and dependent agents. This selection is based on the probability of exceedance, which should be less than or equal to the value of the probability of occurrence designated as a project requirement for the failure mode under consideration.

In each of the limit states, the selection of simultaneous project factors, the compatibility of values and combination types should be in consonance with the ROM 0.0 (Chapters 4 and 5), and in the sections of this ROM pertaining to different breakwater types. Another aspect also to be taken into account is how the failure mode is produced.

For the consideration of exceptional work and operating conditions, the recommendations in the ROM 0.0 should be applied.

Normal work and operating conditions. For each of the principal failure modes it is necessary to select the climatic limit state from the joint middle regimes of wave action, sea level, and other environmental disturbances that can cause operational stoppage modes. In the absence of joint regimes, extreme regimes of the predominant agent and dependent agents can be used.

The selection of safety and serviceability limit states is performed on the basis of the probability of exceedance, which should be less than or equal to the value of the probability of occurrence designated as a project requirement for the failure mode under consideration.

The selection of operational stoppage limit states is performed on the basis of the probability of non-exceedance, which should be greater than or equal to the value of the operationality, number of stoppages or duration of the stoppage designated as a project requirement for the stoppage mode under consideration.

2.6.1.4.2 Level II and III methods

The selection of the limit states (safety and serviceability as well as operational stoppage limit states) is carried out according to the method used. However, to avoid an excessive number of verifications, the verification of the occurrence of the mode in extreme work and operating conditions should be separated from verifications in normal work and operating conditions. It is thus necessary to "filter" those states in which the probability of occurrence of the failure or stoppage mode is negligible (i.e. less than 10^{-5}).

Note. *The probability of occurrence of the failure or stoppage mode in the time period is the product of the dangerousness of the mode multiplied by the vulnerability of the structure. The calculation of the dangerousness is based on the probability models of the climate agents (e.g. sea state or wave climate in a specific time interval, such as the year, useful life, etc.). The calculation of the vulnerability must quantify the uncertainty of each of the terms in the verification equation for the project state under consideration.*

For this reason it is essential to know the joint distributions of all the project factors that intervene in each of the terms. In certain cases when this calculation is very complicated or even impossible, there is no choice but to make simplifications.

One such simplification is to assume that the randomness of the term or action exclusively resides in the predominant agent, and that all of the other project factors can take previously specified nominal or statistical values. The presence of the project agents with their corresponding values inevitably produces the failure. The vulnerability is equal to one, and the probability of failure in the time interval is equal to the dangerousness.

However, given that the verification equation has an intrinsic uncertainty, each term should be multiplied by a coefficient that quantifies (corrects) its bias. This is the proposal of the Technical Committee of the PIANC. Observe that this correction coefficient is different from and in addition to the partial coefficient that evaluates the simultaneity and compatibility of terms that are listed in the ROM 0.0 (Chapter 5), previously described in the ROM 0.2-90. This correction coefficient should be duly weighted. Its main limitation is that its value is local or specific to each displacement.

When the expression of verification equation terms is based on state descriptors (e.g. significant wave height), the vulnerability of the structure is assumed to be equal to one and its possible deviation should be reflected in the equation by means of the correction coefficient and confidence levels. When the expression of equation terms is based on the basic variable (e.g. the wave height and wave period), and this is a random state variable, the vulnerability then has a value lesser than or equal to one, and should be calculated accordingly.

One way to do this is to determine the probability model of the term, bearing in mind the joint probability models of the intervening project factors. This calculation can be performed analytically, and the resulting functions are known in statistics as "derived" probability functions. They can also be obtained by using numerical simulation techniques (e.g. Monte Carlo Method).

Nevertheless, even with this technique, it is best to only consider those project factors that significantly contribute to the uncertainty of the term, and to use nominal values or state descriptors for the rest of the project factors. In this event, the resolution of the state equation should consider the probability model of each term, and evaluate the vulnerability or probability of occurrence of the failure mode in the state, which does not necessarily have to be equal to one. The probability of occurrence of the failure mode during the time period is equal to the product of the dangerousness (presence of the state) multiplied by the vulnerability (probability that the failure mode will occur in the state). This formulation of the state equation is probabilistic, and the method used to resolve it should be in consonance with this formulation (i.e. level II or level III method).

The main difference between a deterministic-probabilistic formulation and a probabilistic formulation is that in the first type of formulation the probability of occurrence of the failure mode in a time interval is evaluated by means of the uncertainty of the predominant agent and those agents of the same origin. In contrast, in the second type, it is necessary to evaluate the uncertainty of the term (or action) and the probability of occurrence of each of the terms in the time interval.

2.6.2 Verification of the joint probability in relation to the principal failure and stoppage modes

One of the objectives of the project is to verify that the dimensioned structure meets the safety, serviceability, and use and exploitation requirements in each of the project phases. This calculation can only be accurate if the verification equation of the principal failure and stoppage modes has been formulated and resolved by using level II or III methods.

In the case of verification equations formulated with determinist-probabilistic methods, the joint probability in relation to the principal modes in the project phase should be evaluated, following the criteria in the section *Probability Distribution*, in which the probability of each mode is the probability of exceedance of the respective predominant agent.

In the case of verification equations that are formulated and resolved with level II or III methods, the following schema should be used to calculate the probability of occurrence of each principal failure or stoppage mode in the project phase.

2.6.2.1 Generation of annual sequences of agent and action states

Based on the probability models of the agents (33), annual sequences of the solicitation or operationality cycles are numerically generated in each case for a number of years equal to the useful life of the structure. For each state in each cycle, a verification equation should be resolved using level II or III methods. It is necessary to see whether it complies with the failure criterion, and the safety margin value or safety coefficient must also be calculated.

2.6.2.2 The useful life as an experiment and inference of the failure probability of each mode

The duration of a project phase and particularly the useful life can be understood as an experiment whose final result ascertains if the subset of the structure has failed or not due to one or more of the failure modes ascribed to the ultimate limit states and the serviceability limit states. In the case of breakwaters, this result depends on the specific sequence of meteorological states grouped in solicitation cycles and use and exploitation cycles that have arisen during the structure's useful life in each meteorological year. If the experiment were repeated, the result would be different each time, given the random nature of the occurrence of the agents and their manifestations. And this would happen in each repetition.

The final result of the experiment can only be predicted probabilistically. Precisely for this reason, it is necessary to know the probability models of the occurrence of the agents and their manifestations as well as the models of structural response. In real life it is evidently impossible to repeat the experiment so many times, and on the basis of the results obtained, elaborate a statistically significant sample, which would make it possible to infer the failure probability of each of the principal modes.

If the joint probability distributions of the agents are known, and if we have the models or verification equations for each of the failure modes, techniques of numerical simulation (e.g. the Monte Carlo Method) can be used to perform a great number of experiments, randomly generating the presence and magnitude of the agents in each of the solicitation cycles and use and exploitation cycles during the structure's useful life. Each experiment gives a value for the frequency of occurrence of each of the failure modes, calculated as the quotient of the number of states with a failure and the number of states in the useful life of the structure.

Once all of the repetitions of the useful life experiment are performed to assure the required degree of statistical significance, a sample of the frequency values of all of the failure modes can be obtained. These repetitions give an estimate of the joint probability of all the principal failure modes and an estimate of the marginal probability of each one. The average value of each sample is the best estimate of the probability of occurrence of each failure mode and of the set of modes during the structure's useful life.

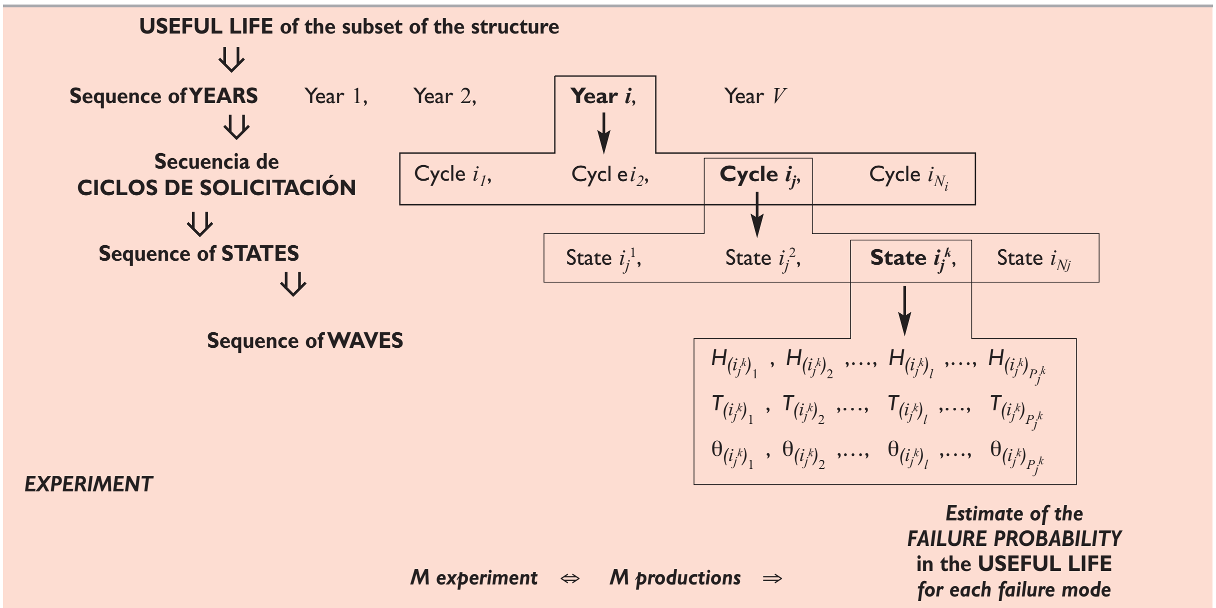
This work scheme is summarized in figure 2.37.

- The useful life of the structure is an experiment.
- Each experiment consists of a known number of meteorological years or tests.
- Each test consists of a random number of solicitation cycles and use and exploitation runs of random duration and intensity.
- Each run consists of a specific number of meteorological states, tests of known intensity and duration.
- The result of each of the states or tests is the occurrence or non-occurrence of each of the failure modes. In other words, each test is a Bernouilli event.

Similarly, the result of each run is the number of times that each failure mode occurs. The result of the test is the number of times that each mode occurs per year. Finally, the result of the experiment is the number of times that each mode occurs during the structure's useful life.

(33) These are principally the peak values in the solicitation cycle, duration of the threshold exceedance, and the associated sea level.

Figure 2.2.32. ***Experiments of the useful life of a subset of the structure***



2.6.2.3 Statistical inference of the minimum safety margin (34)

The previously described process results in a sample of the safety margin (or safety coefficient) values. On the basis of this sample, its probability model can be inferred. The samples of the minimum safety margin values are especially relevant. One value per year is taken, considering the year as a test, or one value in each of the *M* useful lives or experiments.

For this purpose, the minimum safety margin values for each solicitation or operationality cycle are selected, values which comprise a sample of minimum values in the experiment or useful life. The simulation process of "useful lives" should be repeated several times (35) (e.g. M, until there is a set of M independent samples or results of each of the *M* experiments). Once this sample is obtained with the result of the *M* experiments, it is possible to estimate the "relative frequency" of exceedance of the different safety margin values during the structure's useful life A probability model can be adjusted and on the basis of this model, one can calculate the value of the probability that the safety margin will be less than a certain value (e.g. zero, in the useful life). This value is an estimate of the probability of occurrence of the failure or stoppage mode in the useful life of the subset.

This procedure can also be used for the failure modes assigned to serviceability limit states. Accordingly, it is necessary to define the limit state threshold after which the deterioration of the element or part of the breakwater begins, and an accumulative failure law that usually follows the Palmgren-Miller Model.

2.7 CALCULATION AND DISTRIBUTION OF THE JOINT PROBABILITY

This annex includes calculations of the joint failure probability and its distribution among the principal failure modes of a vertical breakwater. The set of failure modes does not strictly comply with the calculation hypotheses. Nevertheless, in the majority of cases, the results err in favor of safety.

(34) In Engineering, the application of statistics to a data sample gives relevant information about the population of the sample (which includes all possible observations regarding the process or phenomenon), in this case of the minimum safety margin. This process is known as statistical inference.
(35) When Monte Carlo techniques are applied, the minimum number of simulations should be in consonance with the ROM 0.0 (section 6.4.8.2).

2.7.1 Joint failure probability

If we consider a vertical breakwater with three principal failure modes or sample elements: (1) sliding, s_1; (2) overturning s_2; (3) berm erosion s_3. These elements along with the null event, ϑ, make up a complete set, and thus comprise the sample space of the failure modes (or events):

$$\Omega_s \equiv \{s_1, s_2, s_3, \vartheta\} \tag{2.59}$$

2.7.1.1 *Set of events*

These sample elements, their complementary events and combinations are the components of different collections or sets of possible failure mode occurrences. The most elementary events are those that contain each failure mode and the corresponding complementary events: $S_1(s_1)$, S_1^c, $S_2(s_2)$, S_2^c, $S_3(s_3)$ y S_3^c. If we assume that the probability of each principal failure mode in a year is $\Pr[S_1] = 0.050$, $\Pr[S_2] = 0{,}001$ and $\Pr[S_3] = 0{,}010$, the fact that each event and its complementary event are mutually exclusive signifies that $\Pr[S_1^c] = 0.950$, $\Pr[S_2^c] = 0.999$ and $\Pr[S_3^c] = 0.990$.

The formulation of other events is based on the union and intersection of events. More concretely, the event that is the occurrence in a one-year period of overturning, sliding, and berm erosion is the union event $S_1 \cup S_2 \cup S_3$. In contrast, the complementary event of the union is the non-occurrence of the three failure modes, $\vartheta = \{S_1 \cup S_2 \cup S_3\}^c$, which is equal to the intersection of the complementary events $\vartheta = \{S_1^c\, S_2^c\, S_3^c\}^c$.

2.7.1.2 *Mutually exclusive failure modes*

When the failure modes, S_1, S_2 and S_3, are mutually exclusive, their intersections are the empty set, and they cannot occur simultaneously. The possible states which the vertical breakwater is subject to in a given time interval (i.e. a year) are sliding, overturning, berm erosion, or non-failure. Consequently, a complete set of mutually exclusive modes, Ψ, consists of the following combinations:

$$\Psi \equiv \{S_1, S_2, S_3, S_1^c\, S_2^c\, S_3^c\} \tag{2.60}$$

The probability that in a year the failure event will not occur is:

$$\Pr[S_1^c\, S_2^c\, S_3^c] = \Pr[\vartheta] = 1 - 0.061 = 0.939 \tag{2.61}$$

It is a good idea to verify that the complementary events of the failure modes are not statistically independent, given that:

$$\Pr[S_1^c]\Pr[S_2^c]\Pr[S_3^c] \neq \Pr[\vartheta] = \Pr[\{S_1^c\, S_2^c\, S_3^c\}] \tag{2.62}$$

On the basis of this information it is possible to calculate:

1. The probability that the breakwater will fail because of sliding, overturning or berm erosion in a one-year period:

$$p_{f,1} = \Pr[S_1 \cup S_2 \cup S_3] = \Sigma_{i=1}^{3}\Pr[S_i] = 0.050 + 0.001 + 0.010 = 0.061 \tag{2.63}$$

   Evidently, this probability can be calculated on the basis of the probability of the complementary event of the union or intersection of complementary events:

$$p_{f,1} = 1 - \Pr[\{S_1 \cup S_2 \cup S_3\}^c] = 1 - \Pr[\{S_1^c\, S_2^c\, S_3^c\}] = 0.061 \tag{2.64}$$

2. The reliability or probability of non-failure in the one-year period.

$$1 - p_{f,1} = 0.939 \tag{2.65}$$

3. The probability of breakwater failure due to any of the three failure modes in 25 years is:

$$p_{f,25} = 1 - \{\Pr[S_1 \cup S_2 \cup S_3]\}_{25\ \text{years}} = 1 - (1 - p_{f,1})^{25} = \\ = 1 - \{1 - \Pr[S_1 \cup S_2 \cup S_3]\}^{25} = 0.7927 \qquad (2.66)$$

2.7.1.3 Breakwater overturning and berm erosion are not mutually exclusive and are statistically dependent

It has frequently been observed that a vertical breakwater can overturn after berm erosion occurs. In the case of such erosion, the probability that the breakwater will overturn is not null.

Let us assume that $\Pr[S_2 \mid S_3] = 0.5$ (or that the two failure modes are not mutually exclusive, and are also dependent) and that the (marginal) probabilities of sliding, overturning and berm erosion, regarded as individual failure modes, are $\Pr[S_1] = 0.050$, $\Pr[S_2] = 0{,}001$ aand $\Pr[S_3] = 0.010$. In these circumstances, a complete set of non-mutually exclusive failure modes, Ψ, consists of the following combinations:

$$\Psi = \{S_1, S_2, S_3,\ S_2S_3,\ S_1^c\ S_2^c\ S_3^c\} \qquad (2.67)$$

The probability of the intersection event, breakwater overturning and berm erosion, is obtained according to the conditioned probability:

$$\Pr[S_2S_3] = \Pr[S_2 \mid S_3]\Pr[S_3] = 0.50 \cdot 0.010 = 0.0050 \qquad (2.68)$$

With this data it is possible to calculate that:

1. The probability that the breakwater will fail in a year due to sliding, overturning or berm erosion is:

$$p_{f,1} = \Pr[S_1 \cup S_2 \cup S_3] = \\ = \Sigma_{i=1}^{3}\Pr[S_i] - \Pr[S_2S_3] = 0.0610 - 0.0050 = 0.0560 \qquad (2.69)$$

2. The probability that the breakwater will not fail in a year is 0.944 in the same way as the previous case:

$$p_{f,1} = 1 - \Pr[\{S_1 \cup S_2 \cup S_3\}^c] = 1 - \Pr[\{S_1^c\ S_2^c\ S_3^c\}] = 0.0560 \qquad (2.70)$$

3. The probability that the breakwater will fail in 25 years due to one of the three failure modes is:

$$p_{f,25} = 1 - \{\Pr[S_1 \cup S_2 \cup S_3]\}_{25\ \text{years}} = \\ = 1 - \{1 - \Pr[\{S_1 \cup S_2 \cup S_3]\}^{V} = 1 - (1 - p_{f,1})^{25} = \\ = 1 - (1 - p_{f,1})^{25} = 0.7632 \qquad (2.71)$$

2.7.1.4 Breakwater overturning and berm erosion are not mutually exclusive and are statistically independent

Under this assumption, the set of events is the same as in the previous example. However in this case these non-mutually exclusive failure modes are regarded as statistically independent. For this reason, the probability of the occurrence of breakwater overturning after berm erosion is:

$$\Pr[S_2 \mid S_3] = \Pr[S_2] \qquad (2.72)$$

and the probability of the intersection event is:

$$\Pr[S_2S_3] = \Pr[S_2]\Pr[S_3] = 10^{-5} \qquad (2.73)$$

This data can be used to calculate that:

1. The probability that the breakwater will fail in a one-year period due to sliding, overturning or berm erosion is:

$$p_{f,1} = 0.06099 \quad (2.74)$$

2. The probability that the breakwater will not fail in a year is:

$$1 - p_{f,1} = 0.9390 \quad (2.75)$$

3. The probability that the breakwater will fail in 25 years due to one of the three failure modes is:

$$p_{f,25} = 0.7926 \quad (2.76)$$

2.7.1.5 The three failure modes are mutually exclusive and statistically dependent

Next we consider the case in which breakwater sliding and overturning are failure modes that are dependent on berm erosion, in other words:

$$\Pr[S_2 \mid S_3] = 0{,}50; \ \Pr[S_1 \mid S_3] = 0.25 \quad (2.77)$$

and the possibility also exists that the breakwater can overturn after sliding and berm erosion have occurred:

$$\Pr[S_2 \mid S_1 S_3] = 0.80 \quad (2.78)$$

Furthermore, the joint probability of overturning and berm erosion is small:

$$\Pr[S_1 S_2] = 0.0001 \quad (2.79)$$

The definition of conditioned probability and the individual probability values of S_1, S_2 and S_3 can be used to obtain ($\Pr[S_1] = 0.050$; $\Pr[S_2] = 0.001$ and $\Pr[S_3] = 0.010$),

$$\begin{array}{c} \Pr[S_2 S_3] = 0.0050 \qquad \Pr[S_1 S_3] = 0.0025 \\ \Pr[S_2 S_1 S_3] = \Pr[S_2 \mid S_1 S_3] \ \Pr[S_1 \mid S_3] \ \Pr[S_3] = 0.0020 \end{array} \quad (2.80)$$

In this way, a complete set of events, Ψ, which are not mutually exclusive and are statistically independent, is made up of the following combinations of failure modes:

$$\Psi = \{S_1, S_2, S_3, \ S_1 S_2, \ S_1 S_3, \ S_2 S_3, \ S_1 S_2 S_3, \ S_1^c \ S_2^c \ S_3^c\} \quad (2.81)$$

Based on this information, it is possible to calculate that:

1. The probability of breakwater failure in a year due to overturning, sliding or berm erosion is:

$$p_{f,1} = \Pr[S_1 \cup S_2 \cup S_3] = \Sigma_{i=1}^{3} \Pr[S_i] - \Pr[S_1 S_2] - \Pr[S_1 S_3] - \Pr[S_2 S_3] + \Pr[S_1 S_2 S_3] = 0.0554 \quad (2.82)$$

2. The probability that the breakwater will not fail due to these failure modes in a year is:

$$p_{f,1} = 0.9445$$

3. The probability that the breakwater will fail at least once due to one of the failure modes is

$$p_{f,25} = 0.7601$$

2.7.2 Distribution of the joint failure probability

The SERI of the subset is assumed to be low, given that the breakwater has pipelines attached to the breakwater that transport pollutants and which can break. Since sliding, can produce the same effects as the worst-case scenario failure mode, overturning, its SERI is also high. Furthermore, if the breakwater is subject to continuous inspection, berm erosion can be controlled, and any possible failure in the breakwater for this reason is thus mitigated since it can be immediately repaired. The SERI for berm erosion is thus insignificant because its occurrence does not produce movement of the breakwater. The joint failure probability should not exceed 0.1 (see table 2.2.8). The failure modes are organized in two groups. Overturning and sliding belong to one group, and berm erosion belongs to the other. The next step is to make three distribution assumptions:

D_1 : The probability of the worst possible failure mode and of the modes with similar consequences is much less than the joint probability, for example, $0{,}1p_{f,\mathrm{LS}}$

$$p_{f,1} = p_{f,2} \approx 0.005$$

The residual probability is assigned to berm erosion,

$$p_{f,3} \lesssim 1 - \sum_{i=1}^{2} 0.005 \approx 0.090$$

D_2 : The probability of all the failure modes that produce less serious consequences than the worst possible failure mode is much less than the joint probability, for example, $0{,}1p_{f,\mathrm{LS}}$.

$$p_{f,3} \approx 0.010$$

The residual probability is distributed equally between overturning and sliding.

$$p_{f,1} = p_{f,2} \approx 0.0445$$

D_3 : The probability of each principal mode is similar.

$$p_{f,1} = p_{f,2} = p_{f,3} \approx 0.0330$$

The uncertainty of the verification equations for overturning and sliding are similar, but the uncertainty associated with berm erosion is greater than those of the other failure modes, given the lack of research data and information regarding its real-life performance. The choice of the most suitable probability for the investment project can be made by calculating the total annual cost of the subset for the three assumptions, selecting the least expensive option, and taking into account the possible solutions for the other subsets of the breakwater, especially the transitions and changes of alignment.

Note. The distribution assumptions incorporate certain requirements that affect breakwater performance and inspection. The first assumption relaxes the berm failure, which has the most uncertain verification equation. If this solution is adopted, it must be studied in greater depth, for example, by carrying out more laboratory studies. In the event that the first distribution is adopted, the berm is over-sized, and its failure is thus very unlikely. In this case, given the uncertainly of the equation, it is advisable to increase the weight of the berm, approximate this mode to a non-principal mode, and establish a series of inspections to monitor breakwater performance.

2.7.3 Joint failure probability of the combinations of failure modes

The last case study described greatly resembles the real performance of a vertical breakwater under the action of climate agents with three statistically related principal failure modes. The marginal and conditional probability values are the same as those in the previous section. The objective here is to analyze the failure probability of the breakwater in a one-year as well as a twenty-five year period in order to better define conservation strategies and repair work.

The calculation of the total risk is performed in much the same way as the calculation of the joint probability since this evaluates the consequences weighted by the respective probabilities of occurrence.

- Questions:

1. What is the probability that the breakwater will fail due to the occurrence of the three failure modes?
2. What is the probability that the breakwater will fail due to the occurrence of two failure modes: sliding and berm erosion; overturning and berm erosion; sliding and overturning?
3. What is the probability that the breakwater will fail due to the simultaneous occurrence of different combinations of two of the three failure modes?
4. What is the probability that once sliding has occurred, the breakwater will subsequently fail due to overturning and berm erosion?

- Answers:

1. Occurrence of three failure modes.

$$p_{f(3),1} = \Pr[S_1 S_2 S_3] = \Pr[S_2 \mid S_1 S_3]\Pr[S_1 S_3] = 0.80 \cdot 0.25 \cdot 0.010 = 0.0020 \quad (2.83)$$

$$p_{f(3),25} = 1 - (1 - p_{f(3),1})^{25} = 0.0048 \quad (2.84)$$

2. Occurrence of two failure modes:
Sliding and berm erosion:

$$\begin{aligned} p_{f(2),1} &= \{\Pr[S_1 S_3]\}_1 = 0.0025 \\ p_{f(2),25} &= \{\Pr[S_1 S_3]\}_{25} = 0.0607 \end{aligned} \quad (2.85)$$

overturning and berm erosion:

$$\begin{aligned} p_{f(2),1} &= \{\Pr[S_2 S_3]\}_1 = 0.0050 \\ p_{f(2),25} &= \{\Pr[S_2 S_3]\}_{25} = 0.1178 \end{aligned} \quad (2.86)$$

sliding and overturning:

$$\begin{aligned} p_{f(2),1} &= \{\Pr[S_1 S_2]\}_1 = 0.0001 \\ p_{f(2),25} &= \{\Pr[S_1 S_2]\}_{25} = 0.0025 \end{aligned} \quad (2.87)$$

3. Occurrence of at least one of the possible combinations of two failure modes.

$$\begin{aligned} p_{f(2),1} &= \Pr[(S_1 S_2 S_3^c) \cup (S_1 S_2^c S_3) \cup (S_1^c S_2 S_3)] = \Pr[S_1 S_2 S_3^c] + \Pr[S_1 S_2^c S_3] + \Pr[S_1^c S_2 S_3] = \\ &= \Pr[(S_1 S_2 S_3^c) \cup (S_1 S_2^c S_3) \cup (S_1^c S_2 S_3)] = 0.0075 \end{aligned} \quad (2.88)$$

4. Probability that some of the combinations of two possible failure modes will occur simultaneously, at least once in 25 years is:

$$p_{f(2),25} = 1 - \{1 - \Pr[(S_1 S_2 S_3^c) \cup (S_1 S_2^c S_3) \cup (S_1^c S_2 S_3)]\}^{25} = 0.1716 \quad (2.89)$$

5. Probability of the occurrence of berm erosion and overturning failure after the occurrence of sliding:

$$\Pr\left[S_2 S_3 \middle| S_1\right] = \frac{\Pr\left[S_1 S_2 S_3\right]}{\Pr\left[S_1\right]} = 0.040 \quad (2.90)$$

Complete set of mutually exclusive failure events. Let us assume that in extreme work conditions, the only possible failure modes for a vertical breakwater are sliding (S_1), overturning (S_2), and berm erosion (S_3). Therefore, these sample elements form a complete set.

The failure mode space should be comprised of the combination of these sample elements, S_1, S_2, and S_3. As described in previous sections, the following failure probability values can be obtained for a one-year time period:

$$\Pr[S_1] = 0.045; \quad \Pr[S_2] = 0.020; \quad \Pr[S_3] = 0.050;$$

If these failure modes are assumed to be mutually exclusive, the joint probability of failure of the subset during a useful life of 25 years for one of the three failure modes is $p_{f,ELU} = 0.9528$. This very high probability value can be reduced in various ways. Generally speaking, the cost of protecting the level berm is not significantly raised by increasing the size of the protection elements. When this happens, $\Pr[S_3]_1 \approx 0$, which means that $p_{f,ELU} = 0.8137$. Given these conditions, it is necessary to reduce the magnitude of the probability of occurrence of failure modes S_1 and S_2, ($\Pr[S_1] = 0.0045$; $\Pr[S_2] = 0.0020$) in order to obtain $p_{f,ELU} = 0.1504$.

Set of non-mutually exclusive events. If the individual events are not mutually exclusive, the calculation is more complicated because certain combinations of events must be considered, which were previously regarded as negligible. Overturning and sliding are assumed to be mutually exclusive. In other words, if the breakwater overturns, then sliding cannot occur. As a result, the intersection $S_1 S_2 = \phi$, and $\Pr[S_1 S_2] = 0$. Similarly, in the case of sliding and berm erosion, $S_1 S_3 = \phi$. However, it is assumed that the simultaneous occurrence of berm erosion and overturning is possible, $S_2 S_3 \neq \phi$, and that the probability of the breakwater overturning after berm erosion is $\Pr[S_2 \mid S_3] = 0.20$. Consequently,

$$\Pr[S_2 S_3] = \Pr[S_2 \mid S_3]\,\Pr[S_3] = 0.20 * 0.050 = 0.010$$

In this case, the space of mutually exclusive events contains the following combinations of failure modes, $S_2 S_3$, $S_2 S_3^c$, $S_2^c S_3$, since now

$$\Pr\left[S_2 S_3^c\right] = \Pr\left[S_3^c | S_2\right]\Pr\left[S_2\right] = \left\{1 - \Pr\left[S_3 | S_2\right]\right\}\Pr\left[S_2\right] = \left\{1 - \frac{\Pr\left[S_3 S_2\right]}{\Pr\left[S_2\right]}\right\}\Pr\left[S_2\right] = 0.010 \neq \Pr\left[S_2\right]$$

The space of mutually exclusive events is formed by

$$\Psi_S = \{S_2 S_3,\ S_2 S_3^c,\ S_2^c S_3,\ S_1 S_2^c S_3^c,\ S_1^c\ S_2^c\ S_3^c\} \quad (2.91)$$

The probability that at least one of the failure modes (S_1 or S_2 or S_3) will occur is the probability of the union of mutually exclusive modes that include all of the failure events. In other words,

$$\Pr[(S_1 S_2^c S_3^c) \cup (S_2 S_3) \cup (S_2 S_3^c) \cup (S_2^c S_3)] = \Pr[S_1 S_2^c S_3^c] + \Pr[S_2 S_3^c] + \Pr[S_3 S_2^c] + \Pr[S_2 S_3] \quad (2.92)$$

such that the probability that a failure mode will not occur (i.e. the occurrence of $S_1^c\ S_2^c\ S_3^c$) can now be written as:

$$\Pr[S_1^c S_2^c S_3^c] = 1 - \Pr[(S_1 S_2^c S_3^c) \cup (S_2 S_3) \cup (S_2 S_3^c) \cup (S_2^c S_3)] \quad (2.93)$$

A Venn diagram can be used to show that the probability of occurrence of at least one failure mode (S_1 or S_2 or S_3) is:

$$\Pr[S_1 \cup S_2 \cup S_3] = \Pr[S_1] + \Pr[S_2] + \Pr[S_3] - \Pr[S_2 S_3] = 0.1050 \quad (2.93)$$

2.8 RECOMMENDED VALUES FOR BREAKWATER CONSTRUCTION AND DEFENSE

This section presents a series of tables that show ERI and SERI values, and thus, the nature of the maritime and harbor structures that are generally designed and built in Spain. Furthermore, in the same way as in the ROM 0.0 (Section 2.10) as well as in this ROM (section 2.5.1), values are proposed for their useful life and joint probability of failure and stoppage.

Figure 2.2.33. ERI, SERI and minimum useful life for different types of sheltered area

TYPE OF SHELTERED OR PROTECTED AREA			ERI [7]		MINIMUM USEFUL LIFE (L_m) [7] (years)
HARBOR AREAS	COMMERCIAL PORT	All vessel types	r_3	High	50
HARBOR AREAS	COMMERCIAL PORT	Specific vessel types	$r_2(r_3)$ [1]	Medium (high) [1]	25 (50) [1]
HARBOR AREAS	FISHING PORT		r_2	Medium	25
HARBOR AREAS	MARINA		r_2	Medium	25
HARBOR AREAS	INDUSTRIAL PORT		$r_2(r_3)$ [1]	Medium (High) [1]	25 (50) [1]
HARBOR AREAS	NAVAL PORT		$r_2(r_3)$ [2]	Medium (High) [2]	25 (50) [2]
HARBOR AREAS	PROTECTION OF FILL MATERIAL OR SHORELINE		r_2 (r_3) [3]	Medium (High) [3]	25 (50) [3]
COASTAL AREAS	DEFENSE AGAINST EXTREME FLOOD EVENTS [4]		r_3	High	50
COASTAL AREAS	PROTECTION OF WATER INTAKE OR DISCHARGE STRUCTURE		$r_2(r_3)$ [5]	Medium (High) [5]	25 (50) [5]
COASTAL AREAS	SHORELINE PROTECTION AND DEFENSE		$r_1(r_3)$ [6]	Low (High) [5]	15 (50) [7]
COASTAL AREAS	BEACH DEFENSE AND NOURISHMENT		r_1	Low	15

1 The ERI increases to r_3 when shipping is related to energy supply or to raw minerals with strategic value, and when there are no alternative installations available for their handling and storage.
2 The ERI increases to r_3 when the naval port is crucial for national defense.
3 For structures that protect fill material or the shoreline, the ERI is the same as that of the harbor area where they are located.
4 Breakwaters that offer protection against extreme flood events are those which in the event of a failure can cause important flood damage in the surrounding area.
5 The ERI increases to r_3 when the point where water is drawn off or drained is associated with the urban water supply or with energy production.
6 The ERI increases to r_2 when there are factory buildings or industrial installations in the same area.
7 The indexes lower than r_3 on the table increase in one degree for each 30 M€ of the initial investment cost of the construction

Figure 2.2.34. **SERI and joint probability of failure for ULS and SLS**

TYPE OF SHELTERED OR PROTECTED AREA				SERI		$P_{f,ULS}$	$P_{f,SLS}$
HARBOR AREAS	COMMERCIAL PORT	Storage areas or areas for passengers and/or cargo handling adjacent to the breakwater [1]	Hazardous cargo [2]	s_3	High	0.01	0.07
			Passengers and non-hazardous cargo [1]	s_2	Low	0.10	0.10
		No storage areas or areas for passengers and/or cargo handling adjacent to the breakwater		s_1	Insignificant	0.20	0.20
	FISHING PORT	Storage or operational areas adjacent to the breakwater		s_2	Low	0.10	0.10
		No storage or operational areas adjacent to the breakwater		s_1	Insignificant	0.20	0.20
	MARINA	Storage or operational areas adjacent to the breakwater		s_2	Low	0.10	0.10
		No storage or operational areas adjacent to the breakwater		s_1	Insignificant	0.20	0.20
	INDUSTRIAL PORT	Storage or cargo handling areas adjacent to the breakwater [1]	Hazardous cargo [2]	s_3	High	0.01	0.07
			Non-hazardous cargo	s_2	Low	0.10	0.10
		No storage or cargo handling areas adjacent to the breakwater		s_1	Insignificant	0.20	0.20
	NAVAL PORT	Storage or operational areas adjacent to the breakwater [1]		s_3	High	0.01	0.07
		No storage or operational areas adjacent to the breakwater		s_1	Insignificant	0.20	0.20
	PROTECTION *	Storage area adjacent to the breakwater [1]	Hazardous cargo [2]	s_3	High	0.01	0.07
			Non-hazardous cargo	s_2	Low	0.10	0.10
COASTAL AREAS	DEFENSE AGAINST EXTREME FLOOD EVENTS [3]			s_4	Muy alto	0.0001	0.07
	PROTECTION OF WATER INTAKE OR DISCHARGE STRUCTURE			s_2 (s_3) [4]	Low (High) [4]	0.10 / 0.0001	0.10 / 0.07
	SHORELINE PROTECTION AND DEFENSE			s_2 (s_4) [5]	Low (Very high) [5]	0.10 / 0.0001	0.10 / 0.07
	BEACH DEFENSE AND NOURISHMENT			s_1	Insignificant	0.20	0.20

* PROTECTION OF FILL MATERIAL OR SHORELINE.

1 When the surface adjacent to the breakwater has buildings (maritime installations, fish markets, etc.), warehouses or silos that would be negatively affected if the breakwater were damaged, the SERI is then very high (s_4) ($p_{f,ULS}$ = 0.0001; $p_{f,SLS}$ = 0.007).

2 Hazardous cargos are those substances included in Annex X of the *Water Framework Directive* (2455/2001/EEC), in the European inventory of contaminating emissions (EPER: 2000/479/EEC), and in the *Reglamento Nacional de Admisión, Manipulación y Almacenamiento de Mercancias Peligrosas* (RD 145/1989) (see ROM 5.1-05).

3 Breakwaters that offer protection against extreme flood events are those which in the event of a failure can cause important flood damage in the surrounding area

4 The SERI increases to s_3 when the point where water is drawn off or drained is associated with the urban or industrial water supply or with energy production

5 The SERI increases to s_4 when there are factory buildings or industrial installations in the same area which could be negatively affected by the failure of the breakwater.

Figure 2.2.35. ***OIER and minimum operationality***

TYPE OF SHELTERED OR PROTECTED AREA				OIER		$r_{f,OLS}$
HARBOR AREAS	COMMERCIAL PORT	Storage areas or passengers and/or cargo handling areas adjacent to the breakwater, which would be affected by overtopping		r_{o3}	High	0.99
HARBOR AREAS	COMMERCIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater or adjacent to areas not affected by overtopping	Shipping of bulk cargo	r_{o2} [2]	Medium	0.95 [1]
HARBOR AREAS	COMMERCIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater or adjacent to areas not affected by overtopping	Vessels with passengers and general cargos	r_{o3} [2]	High	0.99 [1]
HARBOR AREAS	COMMERCIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater or adjacent to areas not affected by overtopping	Tramp steamers with general cargos	r_{o2} [2]	Medium	0.95 [1]
HARBOR AREAS	FISHING PORT			r_{o3}	High	0.99 [1]
HARBOR AREAS	MARINA			r_{o3}	High	0.99 [1]
HARBOR AREAS	INDUSTRIAL PORT	Storage areas or passengers and/or cargo handling areas adjacent to the breakwater, which would be affected by overtopping		r_{o3}	High	0.99
HARBOR AREAS	INDUSTRIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater or adjacent to areas not affected by overtopping		r_{o2}	Medium	0.95 [1]
HARBOR AREAS	NAVAL PORT			r_{o3}	High	0.99
HARBOR AREAS	PROTECTION OF FILL MATERIAL OR SHORELINE			r_{o3}	High	0.99
COASTAL AREAS	DEFENSE AGAINST EXTREME FLOOD EVENTS			r_{o3}	High	0.99
COASTAL AREAS	PROTECTION OF WATER INTAKE OR DISCHARGE STRUCTURE			r_{o3} (r_{o2}) [3]	High (Medium) [3]	0.99 (0.95) [3]
COASTAL AREAS	SHORELINE PROTECTION AND DEFENSE			r_{o1} (r_{o3}) [4]	Low (High) [4]	0.85 (0.99) [4]
COASTAL AREAS	BEACH DEFENSE AND REGENERATION			r_{o1}	Low	0.85

1 When shipping is seasonal, the minimum operationality refers to that time period.
2 When the intensity of the demand is low (exploitation of the sheltered area < 40%), the indexes can be reduced by one degree.
3 The OIER can be reduced to r_{02} when the demand can be adapted to the operational stoppage
4 The OIER should be increased to r_{03} when an urban or industrial area can be affected.

Figure 2.2.36. ***OISER and maximum number of annual stoppages***

	TYPE OF SHELTERED OR PROTECTED AREA			OISER		N_m
HARBOR AREAS	COMMERCIAL PORT	Storage areas or areas for passengers and/or cargo handling adjacent to the breakwater, which would be affected by overtopping	Hazardous cargos [1]	s_{o3}	High	2
HARBOR AREAS	COMMERCIAL PORT	Storage areas or areas for passengers and/or cargo handling adjacent to the breakwater, which would be affected by overtopping	Passengers and non-hazardous cargos	s_{o2}	Low	5
HARBOR AREAS	COMMERCIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater or adjacent to areas not affected by overtopping		s_{o1}	Insignificant	10
HARBOR AREAS	FISHING PORT			s_{o2}	Low	5
HARBOR AREAS	MARINA			s_{o2}	Low	5
HARBOR AREAS	INDUSTRIAL PORT	Storage areas or passenger and/or cargo handling areas adjacent to the breakwater, which would be affected by overtopping	Hazardous cargos [1]	s_{o3}	High	2
HARBOR AREAS	INDUSTRIAL PORT	Storage areas or passenger and/or cargo handling areas adjacent to the breakwater, which would be affected by overtopping	Non-hazardous cargos	s_{o2}	Low	5
HARBOR AREAS	INDUSTRIAL PORT	No storage areas or cargo handling areas adjacent to the breakwater, which would be affected by overtopping		s_{o1}	Insignificant	10
HARBOR AREAS	NAVAL PORT	Storage or operational areas adjacent to the breakwater, which would be affected by overtopping		s_{o3}	High	2
HARBOR AREAS	NAVAL PORT	No storage or operational areas adjacent to the breakwater		s_{o1}	Insignificant	10
HARBOR AREAS	PROTECTION*	Storage area adjacent to the breakwater which would be affected by overtopping	Hazardous cargos [1]	r_{o3}	High	2
HARBOR AREAS	PROTECTION*	Storage area adjacent to the breakwater which would be affected by overtopping	Non-hazardous cargos	s_{o2}	Low	5
COASTAL AREAS	DEFENSE AGAINST EXTREME FLOOD EVENTS			s_{o4}	Very high	0
COASTAL AREAS	PROTECTION OF WATER INTAKE OR DISCHARGE STRUCTURE			s_{o2} (s_{o3}) [2]	Low (High) [3]	5 (2)
COASTAL AREAS	SHORELINE PROTECTION AND DEFENSE			s_{o1} (s_{o3}) [3]	Insignificant (high) [3]	10 (2) [3]
COASTAL AREAS	BEACH DEFENSE AND REGENERATION			s_{o1}	Insignificant	10

* PROTECTION OF FILL MATERIAL AND SHORELINE.

1 Hazardous cargos are those substances listed in Annex X of the *Water Framework Directive* (2455/2001/EEC), in the European inventory of contaminating emissions (EPER: 2000/479/EEC), and in the *Reglamento Nacional de Admisión, Manipulación y Almacenamiento de Mercancias Peligrosas* (RD 145/1989) (see ROM 5.1-05).

2 The OISER will be increased to s_{o3} when the operational stoppage can have important environmental consequences.

3 The OISER should be increased to s_{o3} when an urban or industrial area can be affected.

Figure 2.2.37. ***Probable maximum duration of an operational stoppage***

OIER	OISER			
	Insignificant	**Low**	**High**	**Very high**
Low	24 hours	12 hours	6 hours	0
Medium	12 hours	6 hours	3 hours	0
High	6 hours	3 hours	1 hours	0

Chapter III
Climate agents at the project site

Table of contents

CHAPTER III. CLIMATE AGENTS AT THE PROJECT SITE

3.1 INTRODUCTION

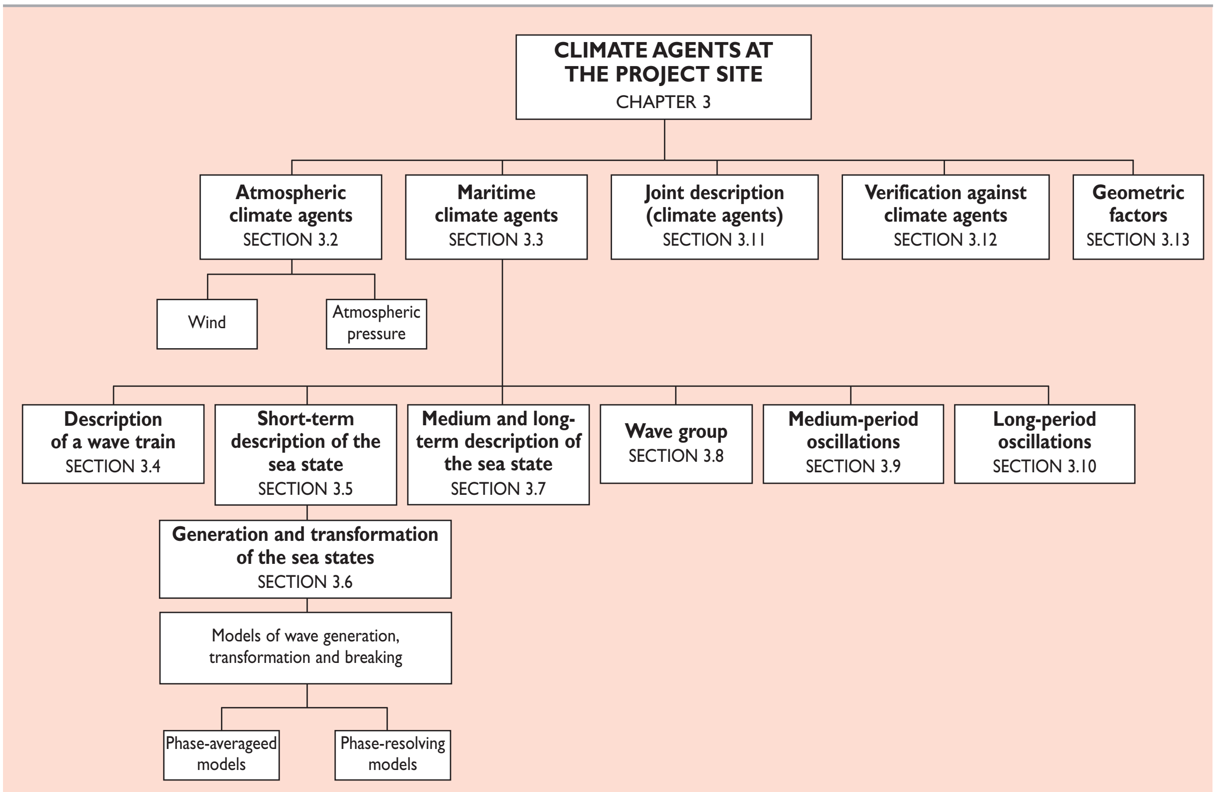
The ground plan of the harbor area as well as the typology and geometric dimensions of breakwaters are determined by the predominant project factors. In most cases, these factors are marine climate agents (e.g. wave action and sea level) as well as soil. At each point of the project site and over time, marine agents appear in the form of sea oscillations that can cause vessel movement and/or lead to actions that affect constructions. These include, for example, horizontal thrusts on vertical breakwaters or interstitial pressures in the soil. The temporal variability of these agents is related to spatial oscillations by means of the corresponding dispersion equation, which generally relates the period of oscillation with its speed or wavelength and environmental conditions (e.g. water depth).

To develop and implement any project in a harbor area, whatever the scope of the project, and the design and construction of maritime structures at the site, project factors must be calculated in order to perform the following tasks: Predimension the project; carry out preliminary studies of the construction materials to be used as well as the marine and geotechnical aspects of the project; design the construction project and do the work; put the structure into immediate operation for its use and exploitation; repair the structure during its useful life; finally, dismantle the structure and restore the construction site to its original state. The scope of these tasks depends on the importance of the work, which is classified, according to its nature. The scope of the tasks and work is directly linked to Level I, II, or III verification methods, as well as to the project phase.

3.1.1 Objectives and organization of the chapter

The objective of this chapter is to provide the methods as well as the necessary analytical and numerical tools to describe and characterize the set of climate agents and other project factors at the site without the presence of the structure. The final result of these calculations should be the joint regimes of mean and maximum meteorological states at the site as well as those of other agents in the physical environment. Figure 3.3.1.shows the organization of this chapter.

Figure 3.3.1. ***Organization and sections of Chapter 3***



3.1.1.1 Basic concepts and justification

The Annex provides a description of key concepts, justifies the recommendations given in this chapter, and also explains how to apply them. These Recommendations include the concepts, criteria, and methods that are at the basis of Maritime Engineering. At all times, an effort has been made to avoid particular case studies in order to facilitate the incorporation of recent advances into the general body of knowledge pertaining to this field.

Note. Unless otherwise stated, all magnitudes are expressed in SI (International System of Units): for example, length, height, depth and distance in meters; period and time in seconds; velocity in m/s and acceleration in m/s^2.

3.1.1.2 Other recommendations to be considered

Since various atmospheric and maritime climate agents have the same origin, it is best to describe them together as a set (see 3.1.3.1.4). For this reason, these Recommendations summarize part of the information included in the ROM 0.4-95. In Spain, whenever possible, the description of climate agents is based on specific measurements performed at the project site. When this is not feasible, the description is based on data supplied by the Spanish government agency, *Puertos del Estado*, and the Spanish National Meteorological Institute, *Instituto Nacional de Meteorología* (www.inm.es). These data are then applied to the site by means of numerical models that have been previously verified and validated.

Moreover, the seismic agent can be regarded as independent of other agents in the physical environment, and as such, it is regulated by the Seismic Resistance Construction Norm (NCSR-02), currently in force. A tsunami produced by a seaquake, whose epicenter is in the seabed can be described as a statistical function derived from that of the seaquake, and is regarded as independent of other climate agents. The aspects related to the description and characterization of the soil will be in accordance with the ROM 0.5-05.

3.1.2 Hypothesis and basic premises

Climate agents, particularly sea oscillations, are considered to be random processes, whose statistical descriptors are more or less constant during a time interval, known as a state. This state has a certain temporal duration. In this state, both the instantaneous and basic variables are random variables that follow probability models, whose parameters can be used, among other things, as state descriptors.

3.1.2.1 Temporal description

Climate agents can be described and characterized according to the following schema:

1. State variables: instantaneous, basic, and state descriptors;
2. Statistical and frequency description of state variables or short-term description;
3. Temporal evolution of the states and medium-term description: state curves, loading cycles, calms, and operational cycles;
4. Long-term description of states grouped in longer time intervals: seasonal, annual, pluri-annual, and useful life;
5. Regimes or probability functions (seasonal, annual, pluri-annual, and useful life) of the state variables, regardless of whether they belong to loading cycles or periods of calm.

3.1.2.1.1 LONG-TERM AND SHORT-TERM TEMPORAL VARIABLES

Wave action is generally the main climate agent affecting breakwaters. The wave period, which can be measured in seconds, is a measurement of temporal variability. Although at a certain point in the sea, the wave action can vary very quickly, the temporal variability of astronomical and meteorological tides can be measured in hours. Long-term manifestations (as compared to wave action) are defined as those whose oscillation period is several times greater than the wave period. Examples of such manifestations include wave groups, medium oscillations, and meteorological and astronomical tides.

3.1.2.2 Spatial work domains

Depending on the importance of the generation, evolution, and transformation processes of climate agents, five spatial work domains can be defined: (I) ocean and open sea: (II) outer continental shelf; (III) inner continental shelf and coastal areas, including surf zones; (IV) port areas; (V) relatively shallow confined seas. The importance of these processes in each domain is determined by the quotient of the domain dimensions and the time length necessary for a significant change to occur in the field of marine (and atmospheric) climate oscillations.

3.1.2.3 Other project factors

In the project design of maritime structures, and breakwaters in particular, safety, serviceability, and operationality depend on seismic and climate agents of the physical environment, which act directly on the structure or force the appearance of other agents.

Generally speaking, in maritime engineering, all project factors (i.e. parameters, agents, and their actions) can be described in terms of the information given in the two previous sections.

Note. The general description of climate agents as well as other agents permits a uniform treatment of data and its application to the verification of safety, serviceability, and operationality. Accordingly, agents as different as wave action, interstitial pressure in the soil, pier corrosion, and water temperature as well as its spatial and temporal variability, can be described and analyzed in much the same way. This method makes it possible to coherently, homogeneously, and accurately organize the analysis of work and operating conditions; the simultaneousness and compatibility of the values of agents and actions; the calculation of joint probability failure; and the use and exploitation during the useful life of the structure or during any other time interval.

3.1.3 Description of climate agents

According to section 3.1.2.1, climate agents (and other project factors such as soil and seismic agents) can be described in terms of three time scales: short-term, medium-term, and long-term scales.

3.1.3.1 Short-term description: states

For all practical purposes and considering relevant constraints, in any given state, there is a set of manifestations of an agent or agents belonging to a stationary and homogeneous random process, for which temporal and spatial statistic descriptors remain constant. This is known as a short-duration or short-term description.

*Note. In Spain, sea state [*estado del mar*] is also known as the wave state [*estado del oleaje*]. However, these Recommendations have opted for extending the definitions of these terms so that the scope of each corresponds to its name: wave action, sea level, atmospheric state, and meteorological state. Consequently, the sea level state includes the long-term manifestations of the free water surface. The meteorological state includes the set of manifestations of climate agents forced by atmospheric activity (e.g. wind, atmospheric pressure, wave action and meteorological tide, and when relevant, meteorological tsunamis).*

3.1.3.1.1 Time and frequency domains

The description of a random process in the state can be performed in one or two ways: (1) a statistical analysis of a time series of basic variables can be used to obtain the probability models and the statistical moments pertaining to the process; (2) a frequency analysis of a series can be performed to obtain the frequency and directional spectrum of the process as well as its spectral moments. Generally speaking, the statistical state descriptor is a statistical value of a sample (e.g. the mean value, the mean square value, etc.). The state frequency descriptor is chosen from among the spectral moment values or from a combination of values (e.g. the lower part of the spectrum or the zero-order moment, which represents the total energy of the process).

For cases in which the local evolution of instantaneous variables (near their mean value) follows a Gaussian probability model, it is usually possible (with additional hypotheses) to establish theoretical relations between statistical and spectral descriptions, and between their respective state descriptors (see section 4.3.1).

Note. The short-term description of sea oscillations is based on the ergodicity hypothesis, which marks the statistical stationarity and the spatial statistical homogeneity of random processes.

3.1.3.1.2 Spatial dimension in the state description

The description of sea state oscillations is generally of an instantaneous nature. However, depending on the wavelength of the oscillation, it can affect adjacent areas. Generally speaking, the description of the state at a certain point is generally assumed to be valid for the square (or circumference) of the center of a point and the side (radius) on the order of an eighth of the representative wavelength of the oscillation. If the bathymetry is regular, and there are no obstacles, the area of validity can be expanded.

3.1.3.1.3 Description of cinematic and dynamic variables

In certain contexts, it may be necessary to describe other instantaneous variables related to kinematics and atmospheric or maritime dynamics, such as the velocity and acceleration of water particles, pressures, and tangential stresses on the soil, whose spatiotemporal variability is related to the oscillations that cause them. Within the framework of linear theory, these variables can be described in terms of their vertical displacement on the free surface by means of the corresponding transfer functions.

3.1.3.1.4 Duration of the state and joint description of instantaneous and slow-changing variables

The duration of the state corresponds to the time that should pass for there to be a significant change in the manifestation of the process, and consequently, the time during which the underlying hypotheses can be fulfilled. Generally, the duration of the state is imposed by the rate or velocity of the change in the wave action energy. In such cases, a time period of one hour is adopted.

The agents whose rate of change (or evolution in the state) is significantly greater than the duration of the state can be assumed to be constant in the state. As a result, slow-changing variables normally associated with other simultaneously acting agents can be represented by their mean value in the state. This frequently occurs in the case of wave state descriptors and sea level variations caused by meteorological and astronomical tides. In all cases, the joint description of the agents and their actions on the structure should be based on the concomitance of their time scales.

3.1.3.2 Medium-term description: cycles

The temporal evolution of states depends on the evolution of forcing and restoring agents, as well as on the geometric characteristics and the roughness of the water surface where they occur. Their graphical representation is known as state curves.

Generally speaking, wave action is the state that evolves most rapidly. In only a few hours, the sea can go from being perfectly calm to being altered by storm conditions as it almost immediately responds to changes in wind speed and atmospheric pressure fields. At the other extreme, the semi-diurnal astronomical tide requires a little over twelve hours to go from low water to high water. Its rate of change is more or less constant in each of the twelve hours of the wave period. The state curve is thus best organized in cycles.

3.1.3.2.1 *Loading cycles*

The loading cycle is the time in which the values of the variables defining the state remain consistently over the threshold value. The loading cycles of the sea state are also known as storms. The loading cycle of meteorological states include the joint evolution of the sea state, wave groups, and the meteorological tide. The loading cycle of the astronomical tide includes the oscillations of spring tides. The occurrence of the loading cycles of tsunamis and the astronomical tide (spring and neap tides) are not related to the loading cycles of meteorological tides.

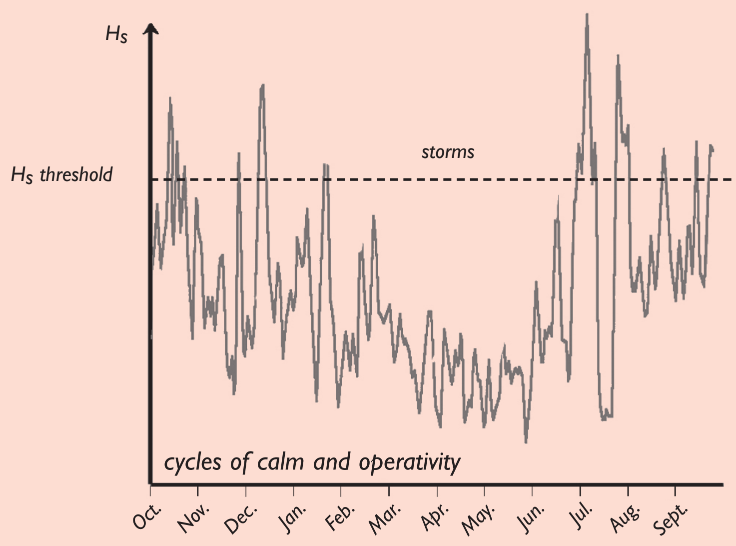
The peak value of the loading cycle and the duration of the cycle are two of the random state variables. The peak values can be used to create a sample to fit the peak over threshold regime. The duration or persistence of the cycle is the exceedance time of the threshold value that defines the cycle. The duration of random processes should be divided into three phases: (i) generation and growth; (ii) maximum point or peak; (iii) decline or dissipation. Duration is a random value, and the mean value of the loading cycles is on the order of days.

Note. *During the exceedance time of the threshold value of the state descriptor* operational stoppage *(which occurs between two continuous operational cycles), there are no conditions for the use and exploitation of the structure or any of its subsets, and consequently, it is a time when no operations take place.*

3.1.3.2.2 *Calm cycles and operational cycles*

The calm cycle is defined as the time during which the values of the variables defining the state remain continuously below a threshold value of calmness or operationality. The occurrence of calms is associated with good weather conditions. Their duration is a random variable, and the mean value of calm cycles is generally on the order of weeks (see Figure 3.3.2).

Figure 3.3.2. *Loading cycles, calm cycles and operational cycles in a meteorological year*



3.1.3.3 Long-term temporal description: years and useful life

Long-term descriptions reflect the sequence of states in terms of meteorological years. This type of description identifies the safety and serviceability threshold values of the agents, whose exceedance can significantly affect the reliability and functionality of the structure. Furthermore, it is possible to identify the threshold values of use and exploitation agents, whose exceedance can significantly affect the operationality of the structure or one of its subsets.

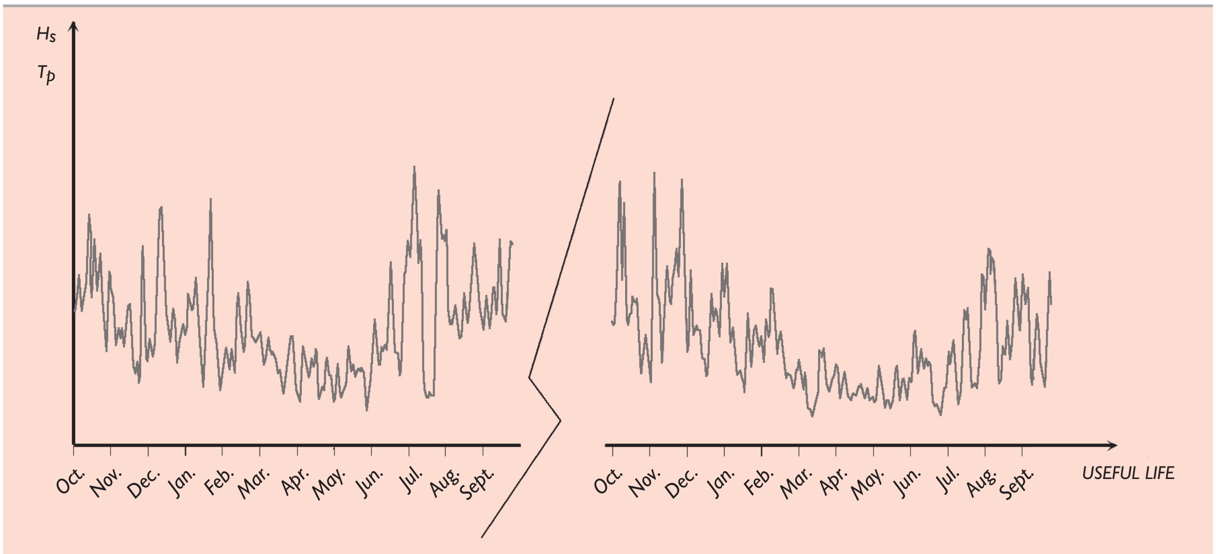
The meteorological year is usually the time unit for the long-term description and characterization of climate agents. Nevertheless, depending on the local climate, another time unit may be used (e.g., winter, monsoon, etc.).

Note. If the data are available, the description of climate agents can be divided into pluri-annual cycles of 11-13 years. However, when there is no such information, it can be assumed that each meteorological year is the result of independent statistical tests (see section 3.7.1.7).

3.1.3.3.1 Description of the useful life

Generally speaking, the duration of a project phase, and particularly the useful life of the structure or duration of the serviceability phase, is measured in meteorological years. For this reason, the description of the agents and actions in a project phase can be based on the values of the descriptors during the year and the verification of the structure during its useful life with the year as the statistical test (see Figure 3.3.3).

Figure 3.3.3. Wave action state curves ordered according to meteorological years



3.1.3.4 Regimes and probability functions

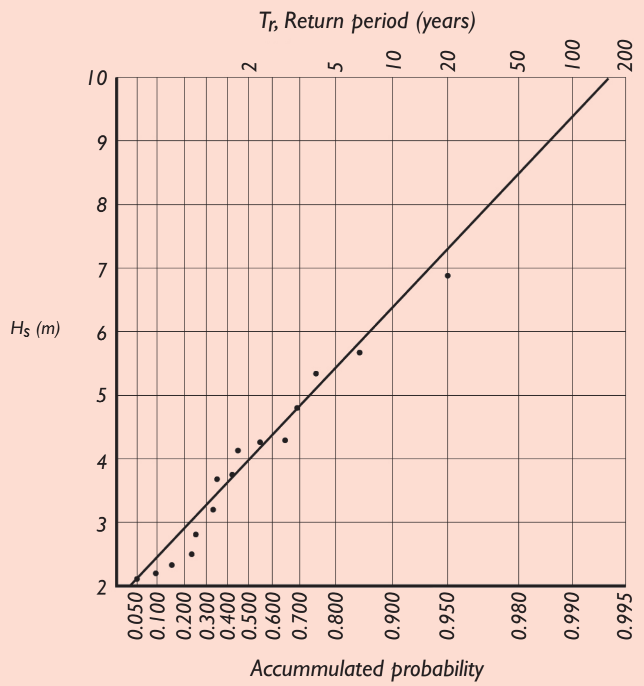
A regime is the distribution function of one or various state variables in a specific time interval. If the values of the variables belong to the upper or lower tails, the regime is of maximums or minimums, generally, of extremes (see Figure 3.3.4). If the values are centered, the regime is of mean values. The regimes are representative of the statistical variability of the agents during the specified time interval.

In middle latitudes and for climate agents, the hypothesis of the meteorological year can be regarded as the pulse of the planet. Accordingly, the year is considered to be the basic independent statistical test or Bernoulli event.

The results of the test are a sample from which extreme and medium regimes can be constructed, according to the meteorological year and useful life of the structure. At specific locations the same bases can be used to elaborate the regimes for other time intervals, such as seasons, dry and rainy periods, etc.

Note. *In Spain, the distribution function of sea states in the mean year is known as the sea regime. Various probability models have been applied, such as the Gauss function, log normal function and the Weibull distribution.*
The distribution function of the maximum sea state of the year is known as the storm regime. The significant wave height is used as a state descriptor of the maximum wave action of the year. Various probability models have been applied, such as the double exponential (or Gumbel) function and the Weibull function.

Figure 3.3.4. Extreme regime of Hs. Probabilistic role of the Gumbel Maximum



3.1.4 Joint description of climate agents

Many of the atmospheric and maritime agents (1) have the same forcing agents. For this reason their joint description should be based on spatial and temporal scales as shown in Table 3.3.1.

Table 3.3.1. Atmospheric and maritime agents and theeir spatial and temporal scales

Denomination	Dimension	Spatial scale	Ocean/atmosphere
Planetary	20.000 km	Macroscale	Global circulation
Synoptic/plaque	2.000 km	Macroscale	Cyclone/anticycle
Local-regional	2-200 km	Mesoscale	Fronts
Limit c. surface	2-200 m	Macroscale	Surface/bottom
Turbulence/dissipation	2-200 mm	Micro/viscose	Fluid

(1) These agents aare described in Sections 3.2 and 3.3.

Table 3.3.1. Atmospheric and maritime agents and their spatial and temporal scales (continuation)

Denomination	Phenomena/processes	Temporal scale
Planetary	currents/astronomical tide	Weeks
Synoptic/plaque	Wave/meteorological tide/tsunami	Days
Local-regional	Metro-tsunami	Hours/min.
Limit c. surface	Wave breaking / surface drag	Min./seconds
Turbulence/dissipation	Erosión/sedimentation	miliseconds

Consequently, climate agents can be individually described in terms of their respective atmospheric or maritime states, or jointly, in terms of a meteorological state, represented by their instantaneous basic variables or state descriptors (see Table 3.3.2).

Table 3.3.2. Instantaneous variables, basic variables and state descriptors of climate agents

Climate agent	Instantaneous/basic v.	Descriptor	Time passed
Atmospheric pressure	$p_a(t)$	$\overline{p_a}$	1 – 2 h.
Wind	$u(t),\ \theta_u(t)$	$\overline{u_{10}},\ \overline{\theta_u}$	10 – 20 min.
Barometric pulse	$p_b(t)$	$\overline{p_b(t)}$	10 – 20 min.
Waves	$\zeta_w(t),\ H,\ T_z,\ \theta$	$\overline{\eta_w},\ \overline{H_s},\ \overline{T_z},\ \overline{\theta}$	30 – 60 min.
Wave groups	$\zeta_{env}(t)$	$\widetilde{\eta}_{BWL},\ H_{max},\ \overline{T_g}$	30 – 60 min.
Tsunamis	$\zeta_m(t),\ U_m,\ V_m$	$\eta_{m,max}$	10 – 20 min.
Meteorological tide	$\zeta_{MM}(t),\ U_{MM},\ V_{MM}$	$\eta_{MM,max}$	1 – 2 h.
Astronomical tide	$\zeta_{MA}(t),\ U_{MA},\ V_{MA}$	$\eta_{MA}(t),\ A_{MA}$	1 – 2 h.

Meteorological states are generally assumed to be unrelated to tsunami states of tsunamis or to astronomical tide states. When the amplitude of the astronomical tide is a significant fraction of the water depth (e.g. 15-20%), the sea level should be taken into account in the description of wave action.

Figure 3.3.5 shows a flow chart for calculating the joint regimes of climate agents at the project site, regardless of the existence of a structure.

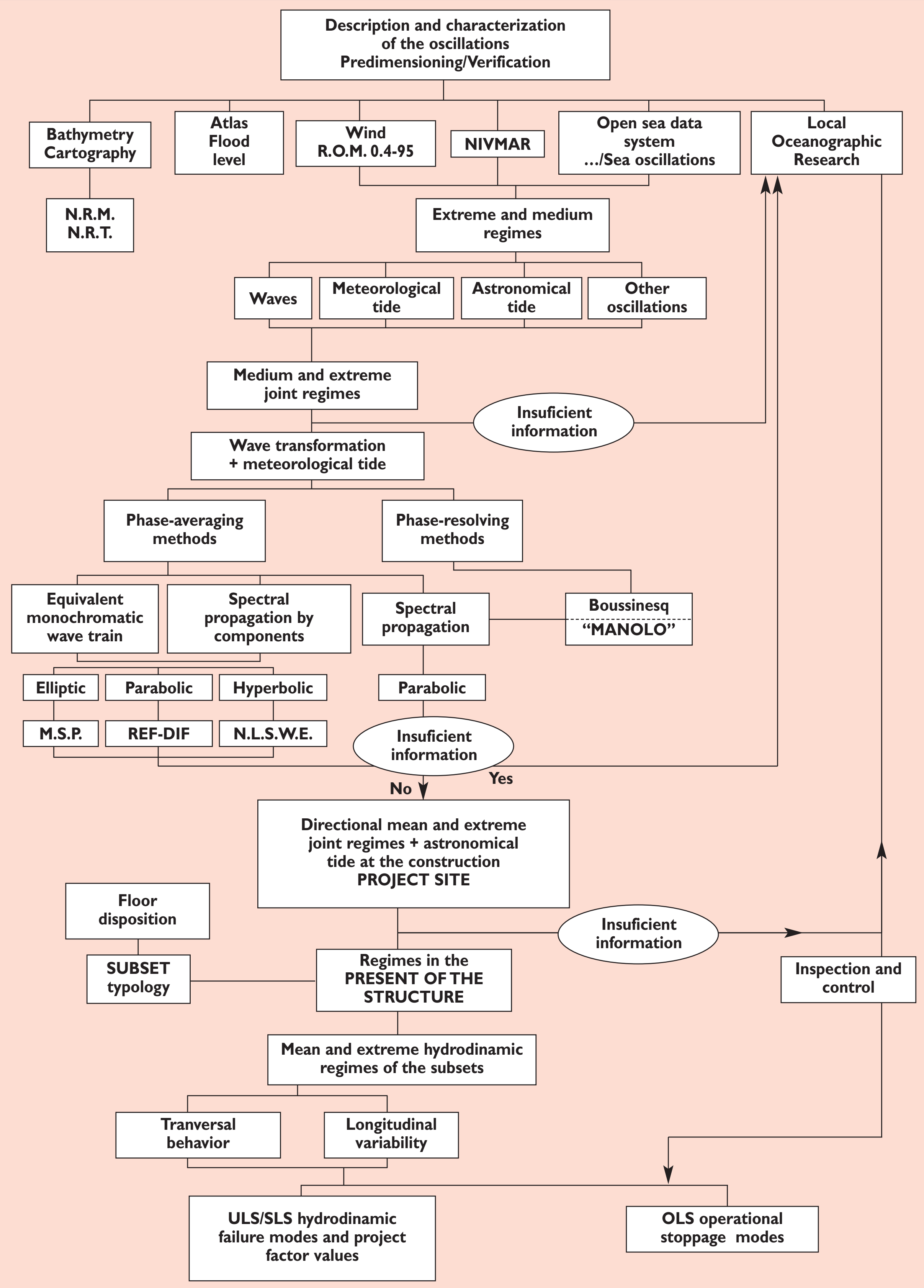
3.1.5 Data sources and databases

In order to describe the climate agents at the site according to the flow chart in Figure 3.3.5, sufficient data must be available. If there is not enough local information, it must be taken from databases pertaining to other sea locations, and must be transferred to the site by applying generation and transformation models methods and models. For this purpose, one of the best data sources is the oceanographic and meteorological database of *Puertos de Estado* (Spanish Port Network), which can be consulted interactively at the following website: www.puertos.es.

3.1.5.1 Visual data

Visual data are one of the most ancient sources of information regarding wind speed and direction as well as visual wave height and period, as collected by vessels during sea voyages. From a statistical perspective, such data series have certain limitations related to the observer and his perceptions and representation of extreme conditions. Visual data are visual state descriptors, and before they can be effectively applied, they must be converted into statistical state descriptors.

Figure 3.3.5. **Flow chart for the calculation of meteorological state regimes (wind, wave actions and sea level) at the project site, regardless of the existence of a structure. See section 3.13**



3.1.5.2 Retroanalysis of meteorological states and pressure fields

The oceanic database has been created by applying numerical models to the information collected in synoptic maps since 1952. This database can be consulted interactively on the web page of *Puertos del Estado*: (www.puertos.es/es/oceanografia_y_meteorologia/index.html).

3.1.5.2.1 WANA NETWORK

Puertos del Estado in cooperation with the Spanish National Meteorological Institute (1996) has applied wind field generation models (HIRLAM) and wave generation models to obtain time series (state curves) of the significant wave height (mean and peak spectral periods and mean direction of origin) and of the wind (mean velocity and mean direction of origin). The WANA data are generally valid for the entire Spanish coastline. However, this information should be applied cautiously to the areas of the Strait of Gibraltar, the northern Catalonian Sea, and the southern part of the Canary Islands.

3.1.5.2.2 SIMAR-44 DATA SET (THE HIPOCAS PROJECT)

The SIMAR-44 data set is composed of time series of atmospheric and oceanographic parameters, resulting from the high-resolution modeling of the atmosphere, sea level, and wave action of the coastal environment in Spain. The simulation of atmospheric phenomena and sea level in the entire work domain as well as the simulation of the wave action in the Mediterranean basin was performed by *Puertos del Estado* within the framework of the European Project HIPOCAS (Hindcast of Dynamic Processes of the Ocean and Coastal Areas of Europe). The simulation of the wave action in the Atlantic Ocean was also performed by *Puertos del Estado*, though independently. The time series in the database refer to 1958-2001 with one datum obtained every three hours.

Wind data for this data set was obtained by using the regional climate model REMO with NCEP, forced by NCEP reanalysis data. Sea level variation due to the action of atmospheric pressure and wind was simulated by the ocean circulation model HAMSOM with atmospheric data from the REMO model. Wave action fields were generated by using the WAM wave prediction model.

3.1.5.3 Simulation

With the WANA and Simar-44 databases as well as the data from measurement networks, it is possible to fit the joint, conditional, and marginal probability functions of climate agents. Numerical simulation techniques (e.g. Monte Carlo methods) can then use these functions to generate discrete time series for agents whose descriptors satisfy the original probability models.

Continuous time series can also be generated, which are based on measured time series. Their behavior and tendencies are analyzed, and the data is assimilated. These techniques involve the development of Bayesian prediction techniques to forecast the behavior of the agents in a specified time interval, based on available information. The time intervals of prediction and uncertainty depend on previous information and the temporal variability of the agents.

Note. To ensure reliable results, all simulation models should be calibrated before their use in any type of application.

3.1.5.4 Measurement data

The instrument network of *Puertos del Estado* is composed of a deep sea network and coastal network. It also includes a network of tide gauges in port areas. More detailed information concerning the access to this instrument measurement data can be found at the *Puertos del Estado* website:
(www.puertos.es/en/oceanografia_y_meteorologia/index.html).

3.1.5.4.1 Deep sea network

The REDEXT data set is made up of measurements from the network of deep sea buoy stations (outer network). This network unifies, expands, and updates the former networks, RAYO and EMOD, The buoys in this network are located in deep water far away from the coastline (at depths of 200-800 *m*). The measurements obtained are generally not affected by the seabed. For this reason, each buoy station provides observations that are representative of large coastal areas.

3.1.5.4.2 Coastal Network

The REDCOS data set is made up of instrument measurements from the *Puertos del Estado* network of coastal buoys. This network expands and updates the former network of scalar buoys, REMRO. The buoys in this network are located near port installations, and are anchored at depths of less than 100 *m*. In most cases, the measurements are affected by the coastal profile as well as by the effects of the seabed on the waves. The observations of the Coastal Network are only representative of local conditions.

3.1.5.4.3 Tide gauge network

The REDMAR data set is made up of measurements from the tide gauge network of *Puertos del Estado*. The main objective of this network is the real-time monitoring of sea level and the generation of historical data series for their further study and exploitation. The tide gauges in this network are located in port installations on a dock or breakwater. The oldest tide gauges have been providing data since July 1992.

3.1.5.5 Laboratory

A wave tank that directionally generates waves can help to provide time series of basic and instantaneous wave action variables in port and coastal areas. The results depend on the time series that activates the paddles and their response.

A boundary layer wind tunnel can provide time series for basic and instantaneous wind variables. The results depend on the accurate modeling in the tunnel of the turbulent boundary layer.

3.2 ATMOSPHERIC CLIMATE AGENTS

Generally speaking, in the project design of breakwaters, predominant atmospheric agents are atmospheric kinematics and dynamics, as evaluated by the instantaneous variables, wind speed and direction at a certain altitude above the sea surface and the atmospheric pressure at the sea surface. Furthermore, in certain contexts, precipitation in the form of rain, snow, ice, and fog should also be considered. The generation, transformation, and complete description of these atmospheric agents can be found in the ROM 0.4-95.

Basic variables are the relative amplitude of the wind speed or maximum displacement in reference to the mean wind speed, and the relative amplitude of the atmospheric pressure in reference to the mean pressure. The interval between these amplitudes is usually on the order of seconds though this depends on turbulence fluctuations that are on the order of milliseconds and microscales.

The atmospheric state descriptors are the mean wind speed and the mean atmospheric pressure. Usually, the average duration of the atmospheric state is assumed to be ten minutes though under storm conditions this duration can be an hour. The temporal evolution of a meteorological state has at least three scales: (i) mesoscale (barometric pulses and tornados); (ii) synoptic (cyclones and anticyclones); (iii) annual, associated with the variation of solar radiation throughout the year due to changing positions of the Earth and Sun. Pluri-annual variations (11-13 years) seem to be related to the activity of sunspots.

3.2.1 Wind speed and direction

On the Spanish coastline during the passage of a storm or a loading cycle, the atmospheric boundary layer is assumed to be neutral. Consequently, the vertical structure of the wind fits a logarithmic profile. In stable atmospheric conditions (i.e. good or anticyclonic weather night and day) and unstable atmospheric conditions, windspeed profiles can vary considerably.

3.2.1.1 *Basic and instantaneous variables*

Instantaneous wind variables are the three wind speed components $U[u(z,t),v(z,t),w(z,t)]$ and the wind direction $\theta_u(z,t)$ at a given altitude z in reference to the sea surface. The direction indicates where the wind is blowing from in reference to the geographic north. The wind speed components fluctuate around an average value $[\overline{u}(z), \overline{v}(z), \overline{w}(z)]$, depending on turbulence intensity and heat flux.

$$u(z,t) = \overline{u}(z) + u'(z,t) \tag{3.1}$$

The basic variable of the wind speed is its amplitude in reference to the mean wind speed $\overline{u}(z)$. In other words, they are the peak values of the fluctuation speed. The windspeed units in SI are *m/s*. The description of the peak values (amplitudes) establishes the limits for the gusts of wind and their duration. If $u'(z,t)$ is a zero-mean Gaussian random process, the amplitudes follow a Rayleigh distribution, whose statistical descriptor is related to the lower area of the frequency spectrum of the time series.

3.2.1.2 *Wind state descriptors*

The mean velocity $\overline{u}(z)$ in a time interval (of about ten minutes) is a state descriptor value. It is customary to calculate the mean value in a ten-minute time interval and at an altitude of $z = 10\ (m)$. For this reason, the state descriptor is identified by $\overline{u_{10}}$ and the mean wind direction at that altitude by $\overline{\theta_{10}}$.

On the open sea, far from the coastline, where there are no geographic landforms or manmade structures, and for a neutral atmosphere, the variation of the mean wind speed in reference to the distance from the sea surface more or less follows a logarithmic or power function,

$$\overline{u}(z) \approx a\left(\frac{z}{z_0}\right)^{\frac{1}{7}} \tag{3.2}$$

$$\overline{u}(z) = \frac{u_*}{K}\ln\left(\frac{z}{z_0}\right) \tag{3.3}$$

where a is a constant; u_* is the friction velocity (m/s); $K \approx 0.4$ is the von Karman constant; and z_0 is the altitude that evaluates the mechanical roughness that the water surface exerts on the air flow. On the sea surface, the value of z_0 depends on the wave conditions. If no other information is available, the following range of variation can be considered:

$$0.0002 \leq z_0 < 0.01$$

where the lower values of this range correspond to the sea surface without appreciable wave action or a swell of extremely reduced wave height. Similarly, the shear speed u_* can be calculated with the following expression:

$$\begin{gathered} u_*^2 = C_D\left(\overline{U(z)}\right)^2 \\ 0.03 \leq C_D \leq 0.02 \end{gathered} \tag{3.4}$$

where the range of values for the drag coefficient corresponds to the same conditions described for the range of values of z_0.

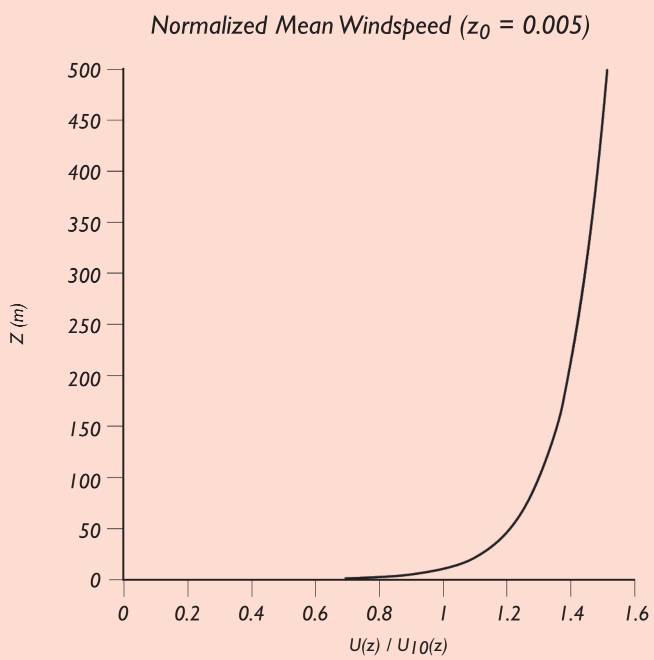
When atmospheric conditions are unstable, the wind speed profile of the surface layer is exponential, and when atmospheric conditions are stable, the logarithmic profile is completed with a linear term. In this way, the wind speed near the ground is less than the wind speed in the neutral atmosphere. However, at an altitude of approximately $100\ m$, it is greater (for more information, see ROM 0.4-95),

$$\overline{u}(z) = \frac{u_*}{K}\left[\ln\left(\frac{z}{z_0}\right) + 6\frac{z}{L}\right] \qquad (3.5)$$

where L is the Obukhov length (meter units) that quantifies the mechanical turbulent energy in contrast to the sensible heat flux on the surface.

Near the sea surface, the horizontal wind component is the principal component. For this reason, the statistical descriptors of the wind state are the two horizontal components of mean velocity and mean wind direction $[\overline{u}(z), \overline{v}(z), \overline{\theta}(z)]$. It is generally sufficient to work with the mean horizontal wind speed $\overline{u_{10}}$ and mean direction $\overline{\theta_{u10}}$, obtained as a ten-minute time average of altitude measurement $z = 10\ (m)$ of the sea (see Figure 3.3.6).

Figure 3.3.6. **Wind speed profile**



3.2.1.3 Probability function of the wind speed

The variability of the wind speed and direction in the state near the mean values $(\overline{u_{10}}, \overline{\theta_{u10}})$ is determined from specific measurements at the site. If no information is available for the site or when the frequency spectrum approaches the narrow-band model, it can be assumed that the marginal distribution of wind-speed peaks or amplitudes in respect to the mean velocity follows the mean square velocity parameter of the Rayleigh model, $\overline{u}_{rms}$, as described in the ROM 0.4-95.

3.2.1.3.1 Variance and standard deviation of the wind speed

The variance of each wind speed component is defined by:

$$\sigma_u^2 = \frac{1}{N}\sum_{n=1}^{N}\left(u_n - \overline{u(z)}\right)^2 = \frac{1}{N}\sum_{n=1}^{N}(u_n)^2 = \left(\overline{u_n'}\right)^2 \quad (3.6)$$

where N is the size of the sample, which depends on the time interval of the sampling. Similarly, the variance of the components can be defined by windspeed components u, w. The standard deviation is simply the square root of the variance, σ_u.

For a neutral atmosphere (bad weather conditions), the vertical variation of the standard deviation of each windspeed component can be determined by ascertaining the thickness *ha* of the atmospheric boundary layer:

$$\sigma_u = 2u_*\left[1 - \frac{z}{h_a}\right]^{\frac{3}{4}} \quad (3.7)$$

$$\sigma_v = 2.2u_*\left[1 - \frac{z}{h_a}\right]^{\frac{3}{4}} \quad (3.8)$$

$$\sigma_w = 1.73u_*\left[1 - \frac{z}{h_a}\right]^{\frac{3}{4}} \quad (3.9)$$

h_a can range from 200 to 2000 meters. Moreover, the mean thickness usually varies throughout the day.

3.2.1.4 Spectrum of wind frequency and direction

For each windspeed component, a frequency and direction spectrum can be defined $S_v(f, \theta)$, which represents the directional and frequency distribution of the energy density in the wind speed component, and is determined from specific measurements at the site. Generally speaking, this spectrum can be expressed as a product of a frequency part and another directional and frequency part:

$$S_v(f, \theta) = S_v(f)\, D_v(\theta, f) \quad (3.10)$$

If no information is available for the site, a theoretical-experimental frequency spectrum [2] can be adopted $S_v(f)$ which incorporates roughness parameters of the sea surface and atmospheric stability [3]. The directional spectrum $D_v(\theta, f)$ can be represented by a cosine function of the half-angle formed by the direction considered to be θ and the principal wind direction, θ_0. The spectrum frequency units of the wind in SI are (m^2/s).

3.2.2 Atmospheric pressure on the sea surface

Storms that arrive at the Spanish coastline bring bad weather. Extreme work conditions generally have a well-defined circular or elliptic isobaric representation on the sea surface, a low-pressure center, and a spatial pressure gradient, which remains more or less constant at the radius or larger axis of the ellipsis.

3.2.2.1 Instantaneous and basic variables

The instantaneous variable is the atmospheric pressure at a point of the sea surface, $p_a(\vec{r},t)$, where $\vec{r}$ is the vector radius of the point in reference to the center of the storm. This fluctuates on various scales. The smallest ones are on the order of the second, and are related to the turbulence intensity of the wind. The basic pressure

(2) These spectra can be expressed in spectral parameters or in terms of state descriptors.
(3) See Dawwson and Hino.

variable is the relative amplitude in respect to the mean reference pressure. According to the SI, atmospheric pressure is measured in pascals.

Note. In synoptic maps, contour lines of equal atmospheric pressure on the Earth's surface (isobars) are represented in millibars (More specifically, 1013.25 *millibars is equal to* 101325 N/m^2 = 101325 *Pa).*

3.2.2.2 State descriptor of atmospheric pressure

In maritime engineering, it is generally sufficient to work with a mean atmospheric pressure $\overline{p_a}$ in a time interval, which is generally on the order of 10-20 minutes, in other words, by the state descriptor.

$$\overline{p_a(\vec{r})} = \frac{1}{d_t}\int_{t_i}^{t_i+d_t} p_a(\vec{r},t)\,dt \qquad (3.11)$$

where t_i is the instant of time in which the meteorological state begins; d_t represents its duration; and $\vec{r}$ is the vector radius of the position of the point, which is being considered in reference to the center of the storm.

This information can be obtained from measurements at the site. If these data are not available, synoptic maps are a good data source, above all for the mean pressure and its spatial distribution during the passage of storms or of a high pressure area, as explained in the ROM 0.4-95. This information makes it possible to obtain the mean atmospheric pressure gradient, $\overline{p_a(\vec{r}=0)}$, and to identify the mean pressure at the center by $\partial\overline{p_a(r)}/\partial r$. Both are state descriptors, and both are necessary for the calculation of wave action states and the meteorological tide with generation models.

For the analysis of wind generation processes or oscillation processes involving vessels or port infrastructures, it is advisable to use atmospheric pressure measurements and wind speed measurements on a turbulent scale.

3.2.2.3 Probability function of the atmospheric pressure

In a meteorological state, the instantaneous atmospheric pressure on the free sea surface, in reference to a mean pressure value, follows a zero-mean Gaussian distribution. Its typical deviation is related to the area under the fluctuating pressure spectrum. When it is a narrow-band process, the relative amplitudes follow a Rayleigh model. It is customary to take the mean atmospheric pressure as a distribution parameter.

In the same way as other Gaussian processes and with the necessary hypotheses, it is possible to obtain the distribution function of the maximum and minimum atmospheric pressures in the meteorological state.

3.2.2.4 Spectral density function of the atmospheric pressure

This density function represents the frequency distribution or the wave number of the fluctuations of the atmospheric pressure. In maritime engineering, this spectrum is rarely used since in the open sea and far from obstacles, it can be obtained from the fluctuation speed spectrum. Nevertheless, knowledge of it is crucial in studies of structural stability related to phenomena involving the coupling of structural movements and vortex emission, which occurs in the case of a docked vessel, port installations, or a crane.

3.2.3 Other atmospheric climate agents

Other atmospheric climate agents include precipitation, snow, or ice (on land or sea), which can affect breakwaters, particularly when they have installations attached, and fog reduces visibility. Such agents can be characterized by state descriptors. More specifically, precipitation in the form of rain should be described in terms of the mean precipitation in the state (one hour) in mm and the maximum precipitation during a smaller time interval (e.g. ten minutes). Snow precipitation should be described in terms of the depth of the accumulated snow in meters on a horizontal plane in the state. Ice precipitation should be described in terms of the thickness of the layer of accumulated ice in meters in the state.

3.2.3.1 Visibility and fog

Visibility must be treated separately since it especially affects navigational safety. One of the causes of loss of visibility is fog. The capacity of the human eye to identify the form and color of an obstacle depends on the luminosity and radiance of the object. Its luminosity and radiance depend in turn on wavelengths in a visible spectrum. These two properties define the contrast and their attenuation because of environmental conditions, mainly, due to the water-drop content, water vapor, and other particles in the air. The contrast threshold is the distance at which a dark object on the horizon can no longer be distinguished. The loss of contrast can be transformed into a physical distance, measured in meters, which can be used as a measure of the loss of visibility, and therefore, of the intensity of the fog. In navigation it is customary to consider a distance of 1000 meters.

The origin of the fog is important since in many cases, it determines the dissipation process, and consequently, the duration of the fog. Apart from fog formed by the adiabatic cooling of the air as it moves upslope, fog can also be the result of cooling, hydration, or a mixture of both. Radiation and advection fog are both formed when warm, moist air comes into contact with a relatively cooler surface. Frontal or precipitation fog is formed when relatively warm rain falls through cool, almost saturated air, and evaporation from the precipitation saturates the cool air.

The dissipation of the advection fog is controlled by mesoscale synoptic processes. Radiation and advection fog can also be dissipated when the ground is heated by solar radiation.

3.2.3.2 Duration of the state

Generally speaking, it can be assumed that the mean duration of the state of these atmospheric agents is on the order of an hour. The state curve of these agents represents the temporal evolution of state descriptors.

Note. Although icebergs are not found off the Spanish coastline, icebergs of different dimensions are a maritime-atmospheric agent, whose displacement can cause damage to structures, installations, and sailing vessels both at docks and at sea.

3.2.4 Models of atmospheric circulation: global scale and mesoscale

In recent years, there have been significant advances in our knowledge of and ability to predict atmospheric circulation thanks to the development of meteorological models that provide the temporal and spatial evolution of wind, atmospheric pressure, and other atmospheric agents.

3.2.4.1 Movement scales

Horizontal movements of the atmosphere cover a wide range of scales that go from the smallest turbulent eddies to the planetary scale of the jet stream to cyclones and tornados (see Table 3.3.1). The temporal scale of these horizontal movements is proportional to their horizontal scale λ *(m)*,

$$\tau \approx a\lambda \tag{3.12}$$

where $a \approx 1 s/m$.

Note. The movements associated with microscale turbulence last from 0.1 to 1 second, whereas local phenomena related to thermal processes with an extension of 0.5-1 kilometer last approximately 10-15 minutes. A cyclone with a diameter of 1000 km lasts approximately one week.

3.2.4.2 Global air circulation

Solar radiation causes a net heating of the tropics and net cooling of the poles. Global air circulation is a compensatory mechanism for this temperature difference. Because of the vertical distribution of temperature, atmospheric circulation basically occurs at the level in the troposphere, the lowest major layer of the atmosphere, which extends from the Earth's surface to a height of 6-10 mi (10-16 km). The air transports latent heat as well as sensible heat. There are three wind cells or circulation belts between the equator and each pole. In the Northern Hemisphere between the Equator and 30º latitude (tropical cell), air circulation from the East (direct circulation) is observed in the same way as in the polar cell at 60º-90º latitude.

However, in the mid-latitude cell at 30º-60º, air circulation is in the opposite direction (indirect circulation). These three atmospheric circulation cells are governed by the equilibrium between buoyancy (floatability) associated with the differential heating of the surface, which generates a vertical movement that also transports momentum.

This air mass compensation is not complete. In middle latitudes, it produces the jet stream, an equilibrium between the force of Coriolis and the thermal winds. This jet stream is unstable, forming large-scale meanders or planetary (Rossby) waves, which transport sufficient heat and movement to invert the direction of the mid-latitude cell, generating atmospheric "ridges and troughs". This circulation pattern produces high pressure systems (air masses) and low pressure systems (cyclones). The upwards movement of air in troughs and low pressure systems causes the dry and moist adiabatic cooling or the air, thus generating clouds and precipitation. The downwards movement of air in ridges and high pressure system causes a stable surface boundary layer.

3.2.4.2.1 Air masses

Air masses form when air remains on a surface sufficient time for it to acquire its properties. This is frequently the case in systems of high pressure and light winds. These fronts define the boundary between air masses, and thus, between wind, temperature, humidity, visibility, etc. Throughout these systems, one can find low-lying clouds, low pressure areas, and precipitation. Air masses are drained by means of low pressure areas, which strengthen baroclinic behavior and cyclonic circulation. Cyclones or storms are areas of low pressure, cyclonic vorticity, and upward movements, characterized by clouds, precipitation, and strong winds. Extratropical cyclones, as opposed to tropical cyclones, are cold-core systems.

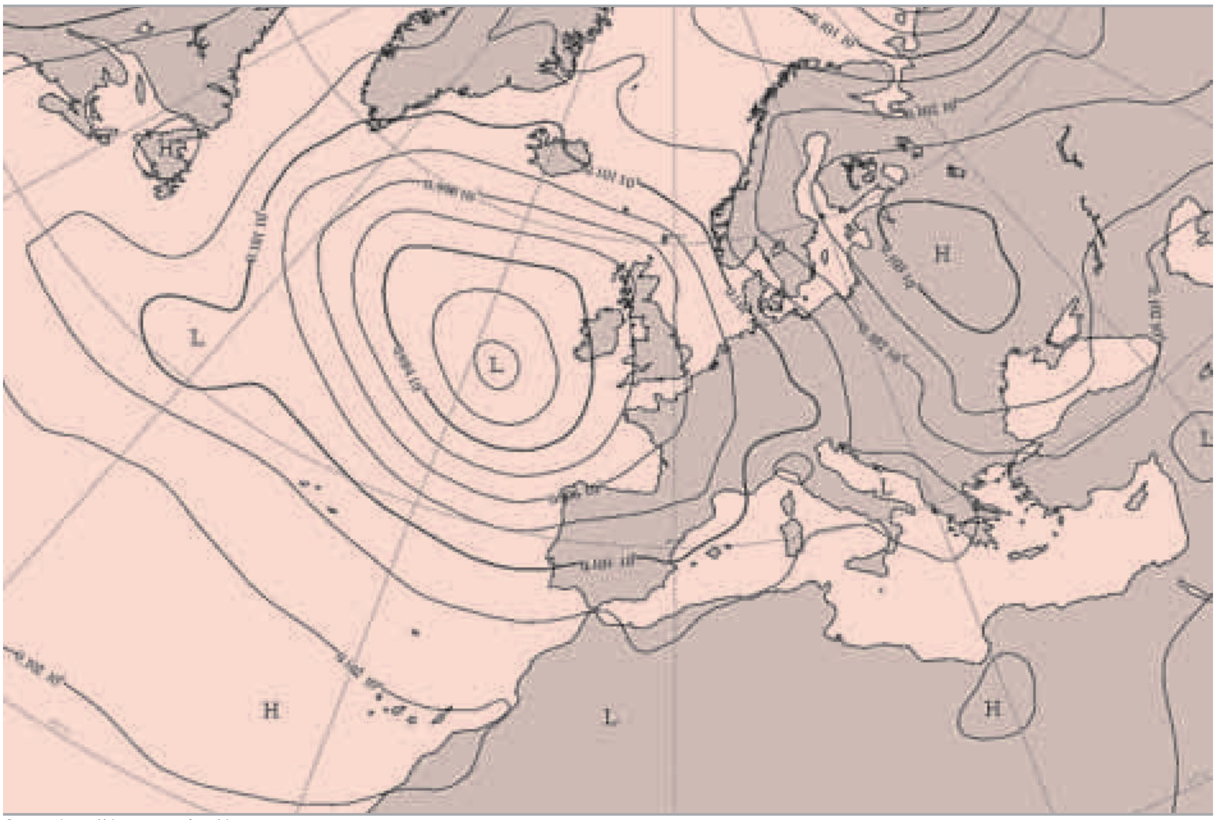
High and low pressure areas, air masses, and fronts are the main components of the synoptic pattern of meteorological weather. This can be reflected in a sequence of maps. Their spatiotemporal evolution can be predicted by resolving primitive equations.

These meteorological conditions can vary on a regional and local scale because of the effects of topography, valleys, and mountains on the winds and insolation. In the first case, the modification of wind and pressure fields is primarily because of mechanical reasons. In the second case, which is associated with good weather conditions, the reason lies in the local temperature gradient.

3.2.4.3 Parametric models of atmospheric circulation

A synoptic chart is the representation of an isobaric field on the surface at one instant for a large area of the Earth (see Figure 3.3.7). This information is representative of an atmospheric state. For several decades now, synoptic charts for each three-hour period have been available. Consequently, when there is a lack of instrumental measurements or visual observations, the interpolation of synoptic charts can be used to determine the spatiotemporal evolution of the mean atmospheric pressure field of the sea surface and the geostrophic wind field. From the geostrophic wind field, it is possible to obtain the baric wind fields in the surface boundary layer. The mean wind speed in the surface boundary layer can be used to determine the tangential stress on the water surface. These data can be used to ascertain the joint regimes of atmospheric agents. When there are storms, the atmosphere is assumed to be neutral and the mean wind speed profile (state descriptor) follows a logarithmic or power function, at least at an initial altitude of 100-200 meters.

Figure 3.3.7. Synoptic chart of the North Atlantic Ocean, isobaric patterns network on the Earth's surface



Source: http://data.ecmwf.int/data

3.2.4.4 Numerical models of atmospheric circulation

According to the spatial scale of the integration domain and its relation to models of wave generation and evolution, there are three types of numerical models: oceanic, regional, and local models. On a local scale, it is also possible to evaluate atmospheric circulation by means of wind tunnel experiments.

3.2.4.4.1 HIRLAM

In today's world, the data provided by measuring instruments permits the application of numerical techniques. A case in point is the HIRLAM model (or HIRHAM in its latest version) that is capable of making predictions on various temporal scales for a specific area of the Earth. The model's output data are the temporal evolution of the temperature, humidity, wind speed, and direction in a 3-D grid, and the atmospheric pressure and surface temperature in a 2-D grid.

The model is able to make predictions as well as perform the retroanalysis of loading and operational cycles, since it provides the input necessary to obtain the fields of sea oscillations, wave action, and meteorological tide, which are principally caused by atmospheric action.

The HIRLAM model integrates mass and momentum conservation equations for air and atmospheric components (e.g. water) and the thermodynamic equation. The initial conditions, obtained from recent observations should be fed into the model. The results obtained can be used to calculate the most relevant phenomena of atmospheric and marine climate agents.

Note. AEMET routinely uses the HIRLAM model for meteorological forecasts. It is also used by Puertos del Estado for wave action retroanalysis.

3.2.4.5 Regional transformation of the field of wind speed and atmospheric pressure

The presence of the coast, mountains, and valleys can significantly modify meteorological states. In high pressure conditions, local temperature gradients generate anabatic and katabatic thermal winds. In low pressure conditions and with strong winds, the topography can modify wind and pressure fields, producing phenomena such as bores, Foehn effects, hydraulic jumps, and mountain waves. Bores, Foehn wind events, and hydraulic jumps are related to sudden temperatures changes between two layers of air, a warm upper layer and a colder bottom layer, circulating above an obstacle (e.g. a mountain). Mountain waves occur when in potential temperature conditions that gradually intensify with height, the air is forced to circulate above the geographic landform. The oscillation frequency of the pressure and wind speed fields are related to the horizontal scale of the process. For a statically stable atmosphere, its wavelength λ is obtained from the Brunt-Vaisala frequency, N_{BV},

$$\lambda = 2\pi \frac{\overline{u_{10}}}{N_{BV}} \quad (3.13)$$

And the amplitude of the oscillation depends on the Froude number, defined by

$$F_r = \frac{\lambda}{2W_a} \quad (3.14)$$

where W_a is the width of the mountain. The amplitude of the oscillation can reach the base of the mountain when $Fr \approx 1$. If the Froude number is much greater than the unit, the airflow to the lee side of the mountain can cause a backward movement in the flow. It thus expands, forming a wake. Depending on the magnitude of the obstacle, regional transformations of the state can be analyzed by regional, local, or physical numerical models in a wind tunnel.

3.2.4.6 Regional and local circulation

The PSU/NCAR mesoscale model (known as MM5 (4)) is designed to simulate or predict mesoscale atmospheric circulation, and is supported by several pre- and post-processing programs. The initial input consists of isobaric meteorological data, sufficiently enhanced to work on a regional scale, as well as the vertical variation of the atmospheric pressure. Since it is a regional model, their interpolation requires a series of initial conditions, boundary conditions, and lateral conditions that cover the entire time domain under study.

The model can be adapted to any area on the Earth, and includes three map projections: Polar stereographic, Lambert conformal, and Mercator. The resolution varies, depending on the terrain elevation, land use, soil type, deep soil temperature, and vegetable fraction. The MM5 can be coupled with global models and other regional models in different ways as an initial approximation or boundary condition.

From the perspective of maritime structures, this model provides the wind and pressure fields on the sea surface with resolutions of up to 500 m. For this reason, it can be regarded as a useful tool for the analysis of meteorological conditions of a loading cycle in a port area.

3.2.4.6.1 Local circulation models

The presence of port infrastructures, ships docked along side of each other, buildings, etc. can significantly modify the mean wind speed and direction, as well as small-scale fluctuations of atmospheric pressure associated with the resulting turbulent fluctuations (see Figure 3.3.8). In these cases, the coupling between the frequency of vortex emission and the same oscillation frequency of all or certain components of the infrastructure can produce oscillations that are unacceptable from the perspective of safety or operationality. These oscillations can also affect the durability of the structure. The vortex structure on the leeside of the obstacle and the

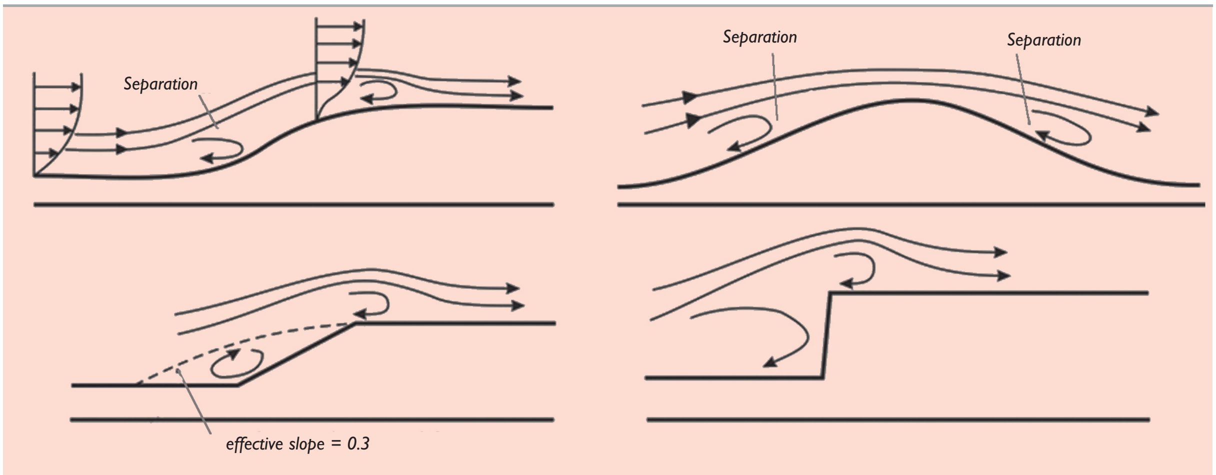
(4) http://www.mmm.ucar.edu/mm5

frequency of vortex emissions depends on the obstacle's dimensions and the wind speed. When the structure is a Von Karman Street vortex, the Strouhal number, approximately $S_t \approx 0.2$, provides a good estimate of the frequency f of vortex emission:

$$S_t = \frac{fD}{V} \tag{3.15}$$

where D is the dimension of the obstacle to the flow.

Figure 3.3.8. ***Modification of the wind by obstacles***



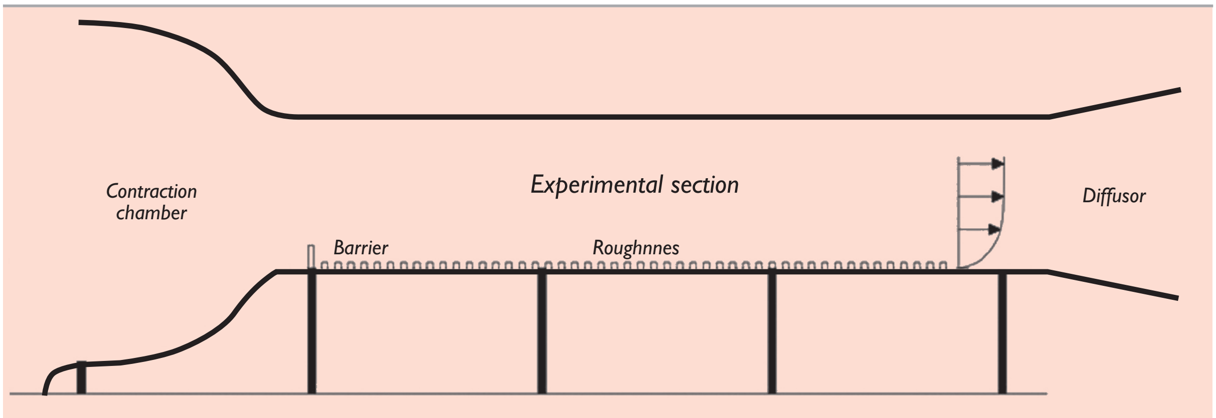
3.2.4.6.2 *WIND TUNNEL EXPERIMENTS*

Generally speaking, when there are structural stability problems as well as significant oscillations related to atmospheric dynamics affecting vessels and port infrastructures, which are locally caused by turbulent mechanics, the boundary layer should be analyzed in a wind tunnel since the main vertical wind speed gradients of mechanical origin are produced in this layer.

Once the geometric scale of the model (conservation of the Froude number) is selected, in order to accurately model the atmospheric boundary level, it is necessary to make a scale model of the profiles of the mean wind speed $\overline{u}(z)$ and of the turbulence as well as the spectral shape $\sigma_u^2(f)$. As long as the dimensions of the area of the experiment are relatively small and the turbulent processes are due to mechanical rather than thermal causes, the similarity of the flow can be achieved by keeping the Reynolds number above 10^5. In any case, if the turbulence similarity requirement is not satisfied, additional roughness should be added. Although it is not possible to conserve the Rossby number in the wind tunnel, for small sections of the study area, this is not a limiting factor.

The generation of the air flow is achieved with a turbine, and the dimensions of the tunnel should be such as to be able to readapt this flow to experimental conditions by means of the different sections: (1) conditioning of the flow to isotropic turbulence; (2) formation and generation of the turbulence, spectral shape, and profile of the windspeed required in the experiment; (3) area of the experiment; (4) output. The models of the installations and structures, which can be non-rigid or deformable, are where the transformation of wind speed and pressure fields is measured as a consequence of the structure-flow interaction. They can also be aeroelastic so as to reproduce the deformability of the structure, and thus, the non-linear coupling between the air flow and the structure. Figure 3.9 is a diagram of an open-circuit wind tunnel.

Figure 3.3.9. ***Diagram of an open-circuit wind tunnel***



3.2.5 Short, medium, and long-term descriptions

The previous sections explained the short-term description or state of the atmospheric agents, wind and atmospheric pressure. This section explains their joint description in the state, atmospheric loading cycle, meteorological year, and long-term description. It is advisable to follow the work method outlined in the description of marine climate agents (see section 3.3).

3.2.5.1 Description in the atmospheric state

The set of manifestations of atmospheric dynamics, wind and atmospheric pressure, are described in an atmospheric state in which they are statistically stationary. In Spain, during the passing of a storm or a low pressure system, it is customary for extreme work conditions to simultaneously occur in atmospheric and marine climate agents. For this reason, whenever possible, both agents and actions should be described jointly.

3.2.5.2 Description of an atmospheric loading cycle and a calm cycle

During the passage of a storm over a certain location, each of the meteorological states that comprise the meteorological evolution of the phenomenon should be described in terms of the following simultaneous and compatible state variables: mean wind speed and direction at an altitude of $z = 10\ m$ and mean atmospheric pressure at sea level, in other words,

$$\left[\overline{u_{10}}, \overline{\theta_{10}}, \overline{p(z=0)}\right].$$

Moreover, it is advisable to analyze the increase and decrease of the state curve, obtain the maximum values of the module of wind speed and mean atmospheric pressure as well as the duration of these maximums. These maximum values do not have to be simultaneous.

Once the loading and calm threshold values have been selected (see section 3.1.3.2), the loading cycles and calm cycles and their most representative state values (e.g. cycle, duration or persistence, etc.) are defined. This information can be relevant, among other things, for the determination of the sea state and meteorological tide in the area under analysis.

3.2.5.3 Long-term joint description and regimes

From each of the loading cycles, it is possible to select the values for wind speed and air pressure (i.e. minimum sines) as well as the duration of their exceedance. These values make up a sample, which can be used to fit a probability model or joint regime of peaks over-threshold of the meteorological agents. Similarly, it is possible to work with the maximum values of the peaks by taking one per meteorological year. This gives the extreme regime of atmospheric states. This description can be applied to time intervals related to project phases (e.g. useful life) by following the same methods as for sea states and sea level.

Note. *The ROM 0.4-95 gives an in-depth description of the statistical and frequency description of wind fields. This is based on the same theoretical foundations as the description of sea oscillations, particularly wave action. Accordingly, the description of atmospheric climate agents should be performed by following the same work schemas as for maritime climate agents.*

3.3 MARITIME CLIMATE AGENTS

This section classifies the maritime climate agents in the physical environment in order to facilitate their organization and description. This is a necessary step before characterizing their interaction with the project structure and quantifying their actions on it.

3.3.1 Classification and organization

Marine climate agents cause oscillations on the free sea surface in different frequency bands. Their instantaneous variable is the vertical displacement of the free surface $\zeta(\vec{r};t)$ in regards to a fixed reference level. Based on this variable, it is possible to define the basic variables of oscillatory motion: amplitude (or height) and period.

Maritime climate agents are organized according to their representative period T:

1. ζ_{SP}: Short Period $(0,5 < T(s) < 300)$,
2. ζ_{IP}: Medium Period $(5 \leq T(min) < 180)$,
3. $\zeta\zeta_{LP}$: Long Period $(T(h) \geq 3)$

$\zeta(\vec{x};t)$ can be obtained by the linear superposition of three components associated with the short-period bands, SP, medium-period bands, IP, and long-period bands, LP.

$$\zeta(\vec{x};t) = \zeta_{SP}(\vec{x};t) + \zeta_{IP}(\vec{x};t) + \zeta_{LP}(\vec{x};t) \qquad (3.16)$$

3.3.1.1 Short-period oscillations

Short oscillations have a period in the interval $[0.5 < T(s) < 300]$, and are very irregular. It is possible to identify at least three different temporal (period) and spatial (wavelength) scales: capillary waves, waves, and wave groups. Capillary waves have periods in the interval $T < 3$s. Wave steepness and celerity are limited by gravity as well as by surface tension. In maritime engineering it is usually not necessary to consider capillary waves, except in the laboratory.

3.3.1.1.1 Wave action $\zeta_{wave}(\vec{x};t)$

The wave period, which is important for the safety, serviceability, use, and exploitation of maritime structures, lies within the time interval $[3 < T(s) < 30]$. In this interval, it is possible to admit that the surface tension does not determine its kinematics and dynamics. In maritime engineering, the wave height [and thus, the height of the water wave] is the basic variable of the ondulatory cycle, and represents the maximum vertical variation of the free surface in a time interval, known as a wave cycle period. The most basic description of a progressive wave train (or long-crested waves) is composed of three parameters of description: (i) wave height; (ii) wave period; (iii) direction of propagation.

3.3.1.1.2 Wave groups, $\zeta_{BLW}(\vec{x};t)$

The sea at a certain point is usually in the form of time sequences of larger waves followed by smaller ones. These are known as wave groups. This phenomenon becomes even more pronounced the greater the age of the waves are and the farther they are from their fetch. The number of waves N in each of the sequences also depends on the age of the wave group and the speed of the generating wind. However, generally speaking, the number is in the interval $[3 < N < 6]$. For this reason, a complete sequence of large and small waves has a wave number of $[6 < N_t < 12]$. Its duration or mean period $\overline{T_g}$ lies in the interval $[6\overline{T_z} < \overline{T_g} < 12\overline{T_z}]$, which is in the range $[30 < \overline{T_g}(s) < 300]$.

Linked to this time sequence of waves is a variation of the sea level η_{BLW} with a mean oscillation period related to the temporal structure of the wave groups, in other words, with $\overline{T_g}$. This bound wave travels with the same celerity as group C_g. The amplitude of this wave depends on the relative sea depth, and in shallow water and surf zones, it can reach values of 10-20% of the amplitude of the largest wave in the group. The bound long wave is in the opposite phase of the envelope of the group. In other words, the peak of the envelope coincides with the sine of the bound long wave. When the wave group is transformed, and the height of the individual waves in the group is modified, the amplitude of the bound long wave is also modified. This also generates free oscillation modes η_{FLW}, which travel with celerity $\sqrt{gh}$.

Note. Something worth highlighting is the adscription of wave groups to short-band oscillations, in other words, less than five minutes.

3.3.1.1.3 Decomposition of short-period oscillations

Short-period oscillations can be decomposed into the following instantaneous variables:

$$\zeta_{SP}(\vec{x};t) = \zeta_{wave}(\vec{x};t) + \zeta_{BLW}(\vec{x};t) + \zeta_{BHW}(\vec{x};t) + [\zeta_{area}(\vec{x};t)] \tag{3.17}$$

where ζ_{BLW}, ζ_{BHW} represent subharmonic and superharmonic oscillations, generated by the interaction of wave components. This gives the oscillations a group structure as well as asymmetry (for subharmonic oscillations) and wave steepening (for superharmonic oscillations). Generally speaking, wave steepening is included in $\zeta_{wave}(\vec{x};t)$. The wave action contributes to the instantaneous variation of the sea level, and is the principal action of port and maritime structures. Wave groups and superharmonic wave groups produce time variations in the amplitudes of the waves. $\zeta_{area}(\vec{x};t)$ represents the oscillations of the wharf or semi-closed water body, when they are forced by wave groups. Otherwise, they should be included in the medium-period oscillations, and for this reason, they appear in brackets.

3.3.1.2 Wave action state

Short-period oscillations can be decomposed into components: (i) a component that is constant over time and equal to its mean value in the state; (ii) an oscillatory or deflecting component, according to the following definition:

$$\overline{\eta_W}(\vec{x}) = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{d_{i+1}}} \zeta_{wave}(\vec{x},t)\,dt \tag{3.18}$$

$$\overline{\eta_{wave}}(\vec{x},t) = \zeta_{wave}(\vec{x},t) + \overline{\eta_w}(\vec{x}) \tag{3.19}$$

In shallow waters and as long as the waves do not break, it can be assumed that $\overline{\eta_W}(\vec{x}) \approx 0$. The spectral and statistical treatment of the waves should be carried out based on the time series $\zeta_{wave}(\vec{x};t)$, in such a way that its mean value is zero. This is known as a sea state. When the waves appear in groups, the mean sea level also varies over time $\overline{\eta_W}(\vec{x};t) \neq 0$ with a mean period equal to that of the wave group. In other words, it slowly changes in the state. In this case, the mean sea level due to the wave action can be decomposed into two components: (i) a component that remains constant; (ii) a component that varies over time:

$$\overline{\eta_W}(\vec{x};t) = \tilde{\eta}_{BLW}(\vec{x}) + \tilde{\eta}_{BLW}(\vec{x};t) \quad (3.20)$$

Consequently,

$$\tilde{\eta}_{SP}(\vec{x}) = \tilde{\eta}_{BLW}(\vec{x}) + [\tilde{\eta}_{area}(\vec{x})] \quad (3.21)$$

3.3.1.3 Medium-period oscillations

The medium-period oscillation band ($5\ min \leq T < 3\ hours$), in its most general form, contains information pertaining to tsunamis (either due to seaquakes or landslides) $\zeta_{ms}(\vec{x},t)$; meteo-tsunamis and barometric pulses $\zeta_{ma}(\vec{x},t)$; and, when relevant, oscillations in confined water bodies $\zeta_{area}(\vec{x},t)$, such as port and shoreline areas. Such oscillations are known as seiches [5].

3.3.1.3.1 Tsunamis, $\zeta_{ms}(\vec{x};t)$

Tsunamis produce oscillations of the free sea surface with a period on the order of 5-10 minutes over several hours. The basic variable is the maximum amplitude of each of the water waves of the oscillation. The number of oscillations depends on the origin and magnitude of the seaquake as well as the propagation dynamics related to the bathymetry of the shelf. Generally speaking, because of their seismic origin, tsunamis in Spain occur independently of other climate agents. For this reason, it is not strictly necessary to include them in a joint description of climate agents and their actions. Other agents that can generate tsunamis are surface landslides of coastal landforms, as well as landslides occurring at greater depths.

3.3.1.3.2 Barometric pulses, $\zeta_{ma}(\vec{x};t)$

Spatial and temporal atmospheric pressure gradients produce temporal and spatial variations in the vertical displacement on the free sea surface in regards to a certain reference level. These pulses can be due to a wide range of factors. Some are related to the fast pulses of mean pressure and wind associated with storm dynamics, whose period is within a range of 10-30 minutes, whereas others are the result of the interaction between atmospheric dynamics and the local topography. Still others are of thermal origin. In these Recommendations, such oscillations are also called meteorological tsunamis or meteo-tsunamis, and usually last various hours.

3.3.1.3.3 Forced oscillations in semi-confined water bodies and wharfs in port areas, $\zeta_{area}(\vec{x};t)$

A semi-confined body of water with its lateral boundaries and depth has a water volume with its own oscillation period. For wharfs with dimensions on the order of hundreds of meters and a depth on the order of tens of meters, this period is generally in the interval of 0.5-30 minutes. Because of the wide range of dimensions characteristic of wharfs and the possible types of docking (e.g. longitudinal, transversal), the range of oscillation frequencies is very wide. If the mass is forced by oscillations, whose period lies in this interval, the forcing is said to be resonant, and the amplitude of the oscillation can increase significantly. Apart from gravity, which is the main restoring force, its damping depends on possible losses because of bottom friction and turbulence, at the lateral boundaries and at the mouth or opening. It also depends on the radiation of the oscillation, which is caused by this.

Forced oscillations can be of short, medium, or long duration, depending on the period of the forcing agent. However, it is necessary to take into account that the oscillatory response of a water body to a certain forcing depends on the forcing mechanism. If this mechanism is linear, the period remains the same, but if the mechanism is non-linear, the oscillation of the water body may have harmonic and subharmonic oscillations of the forcing period.

(5) Generally speaking, medium and long-period oscillations are called long waves.

When wharf oscillations are due to a wave group, it is advisable to include $\zeta_{area}(\vec{x};t)$ in the decomposition of short-period oscillations.

Note. To approximately specify the limits of the range of expressions, analytical expressions can be used, which are obtained for wharfs with a simple geometry and a constant depth,

$$T_n = \frac{2l}{n\sqrt{gh}} \quad n = 1,2,\ldots \tag{3.22}$$

where l is a length that is representative of the dimensions of the area of generation; h is the water depth in the epicenter zone; and n is the number of nodes. Strictly speaking, forced oscillations in wharfs should not be regarded as an agent of the physical environment. However, they are agents that generate certain vessel movements.

3.3.1.3.4 DECOMPOSITION OF MEDIUM-PERIOD OSCILLATIONS

The medium-period signal can be regarded as the sum of three oscillations:

$$\zeta_{IP}(\vec{x},t) = \zeta_{ms}(\vec{x},t) + \zeta_{ma}(\vec{x},t) + [\zeta_{area}(\vec{x};t)] \tag{3.23}$$

Medium-period oscillations mainly contribute to variations in water depth, in other words, in the position of the mean sea level and velocity field on the order of tens of minutes. All of these oscillations should be considered when determining the local sea level along with the astronomical tide.

3.3.1.4 State of medium-period oscillations

In the state, the medium-period signal can be decomposed into two components, such that:

$$\zeta_{IP}(\vec{x},t) = \tilde{\eta}_{IP}(\vec{x}) + \eta_{ms}(\vec{x},t) + \eta_{ma}(\vec{x},t) + \eta_{area}(\vec{x};t) \tag{3.24}$$

$$\tilde{\eta}_{IP}(\vec{x}) \approx \tilde{\eta}_{ms}(\vec{x}) + \tilde{\eta}_{ma}(\vec{x}) + \tilde{\eta}_{area}(\vec{x}) \tag{3.25}$$

where,

$$\tilde{\eta}_{ms}(\vec{x}) = \overline{\zeta_{ms}(\vec{x})} = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{di+1}} \zeta_{ms}(\vec{x},t)dt \tag{3.26}$$

$$\tilde{\eta}_{ma}(\vec{x}) = \overline{\zeta_{ma}(\vec{x})} = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{di+1}} \zeta_{ma}(\vec{x},t)dt \tag{3.27}$$

$$\tilde{\eta}_{area}(\vec{x}) = \overline{\zeta_{area}(\vec{x})} = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{di+1}} \zeta_{area}(\vec{x},t)dt \tag{3.28}$$

When the oscillation period is on the order of twenty minutes, the duration of the state should be increased to one hour and a half or even two hours in order to determine the mean value of the signal.

3.3.1.5 Long-period oscillations

Long-period oscillations lie in the period band $[3 < T(h) < 24]$, and mainly include meteorological tides related to the occurrence of storms, astronomical tides, and other long ocean waves that generally do not significantly contribute to variations in sea level, which usually have periods lasting days or weeks. Long-period oscillations mainly contribute to hourly variations in water depth (i.e. the position of mean sea level).

3.3.1.5.1 METEOROLOGICAL TIDE, $\zeta_{MM}(\vec{x};t)$

The vertical displacement of the free sea surface, forced by variations in atmospheric pressure and the tangential action of the wind on the sea surface is called the meteorological tide, and can be relevant during the occurrence of storms and cyclones. The basic variable is vertical displacement or amplitude in respect to a reference level. It is a slow-changing variable since, generally speaking, its most significant temporal variations occur in a time interval on the order of one hour. The duration of the meteorological tide at a point on the coastline depends on the duration of the storm that generates it, and which can last 1-7 days.

3.3.1.5.2 ASTRONOMICAL TIDE, $\zeta_{MA}(\vec{x};t)$

The principal agent of the astronomical physical environment, including climate agents, is the astronomical tide. This type of tide is the sea oscillation forced by celestial bodies and their movement. It manifests itself in the temporal displacement of the free sea surface in regards to a fixed reference level. The main period of the astronomical tide on the Spanish coastline is around 12 hours.

The basic variable of the astronomical tide is the vertical displacement of the free sea surface in regards to a fixed reference level. Consequently, it can be described and thus predicted by a time series, reconstructed on the basis of the amplitude and phase of its harmonic (or frequency) constants at the project site. Since it is possible to predict its value at a certain point over time, this is a deterministic variable. Furthermore, it is a slow-changing basic variable because, generally speaking, the significant temporal variations occur in a time interval of one hour.

3.3.1.5.3 DECOMPOSITION OF LONG-PERIOD WAVES

Long-period oscillations ($T \geq 3$ *hours*) contain the signal of the astronomical tide, ζ_{MA}, generated by astronomical dynamics and the meteorological tide, ζ_{MM}. This tide is generated during the formation and evolution of atmospheric storms, whose occurrence affects the project site:

$$\zeta_{LP}(\vec{x},t) = \zeta_{MA}(\vec{x},t) + \zeta_{MM}(\vec{x},t) \tag{3.29}$$

3.3.1.6 State of long-period oscillations

The state of long-period oscillations describes the manifestations of the free sea surface, forced by the astronomical and meteorological tides, which in the state can be regarded as constant, $\eta_{MA}(\vec{x})$, $\eta_{MM}(\vec{x})$, and equal to the mean value of the time interval $\Delta t_d = t_{di+1} - t_{d_i}$:

$$\eta_{MA,i}\left(\vec{x}\right) = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{di+1}} \zeta_{MA}\left(\vec{x},t\right) dt \tag{3.30}$$

$$\eta_{MM,i}\left(\vec{x}\right) = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{di+1}} \zeta_{MM}\left(\vec{x},t\right) dt \tag{3.31}$$

Δt_d represents the duration of the state (approximately one hour).

In the same way as other oscillations, long-period oscillations can be decomposed into two components: (i) a component that is constant over time and equal to its mean value in the state; (ii) an oscillatory component that deflects oscillations:

$$\zeta_{LP}(\vec{x},t) = \tilde{\eta}_{LP}(\vec{x}) + \eta_{LP}(\vec{x},t) \tag{3.32}$$

where,

$$\tilde{\eta}_{PL}(\vec{x}) = \eta_{MA}(\vec{x},t) + \eta_{MM}(\vec{x},t) \tag{3.33}$$

Generally speaking, for the definition adopted, the following equation should hold:

$$\eta_{LP}(\vec{x},t) = 0 \tag{3.34}$$

3.3.1.7 State of (mean) sea level

This state includes the slow-changing manifestations of the free sea surface, and defines the mean sea level (MSL or MW) in movement. Slow manifestations (in regards to wave action) are those whose oscillation period is several times greater than the wave action period (i.e. wave groups, medium-period oscillations, and meteorological and astronomical tides).

$$\tilde{\eta}_{NM,i}(\vec{x}) = \tilde{\eta}_{SP,i}(\vec{x}) + \tilde{\eta}_{IP,i}(\vec{x}) + \tilde{\eta}_{LP,i}(\vec{x}) \tag{3.35}$$

$$\tilde{\eta}_{NMm,i}(\vec{x}) = \tilde{\eta}_{BLW,i}(\vec{x}) + \tilde{\eta}_{ms,i}(\vec{x}) + \tilde{\eta}_{ma,i}(\vec{x}) + [\tilde{\eta}_{Area,i}(\vec{x})] + \eta_{MA,i}(\vec{x}) + \eta_{MM,i}(\vec{x}) \tag{3.36}$$

Where i is the state duration $\Delta t_d = t_{di+1} - t_{d_i}$ (approximately one hour). The value of the mean sea level can be regarded as constant in the state. When the probability of tsunamis and meteo-tsunamis is negligible, then the mean sea level in the state outside of the port area is:

$$\tilde{\eta}_{NMm,i}(\vec{x}) = \tilde{\eta}_{BLW,i}(\vec{x}) + \eta_{MA,i}(\vec{x}) + \eta_{MM,i}(\vec{x}) \tag{3.37}$$

Outside of the surf zone, it can be assumed that:

$$\tilde{\eta}_{NMm,i}(\vec{x}) \approx \eta_{MA,i}(\vec{x}) + \eta_{MM,i}(\vec{x}) \tag{3.38}$$

The mean sea level evolves in time at the state scale, forced by the astronomical tide and atmospheric variability, wind speed, and atmospheric pressure.

Note. The mean sea level (MSWL or MSL) at rest defines a reference level in the absence of any oscillation or current. Conventionally, the low water equinoctial spring level (LWES) is the level used although alternately, any other water level is viable, which refers to the local topographic zero. In this sense, the definition of topographic zero is ambiguous since it is measured in reference to the mean sea level in Alicante, which is not constant, but rather changes over time. Nevertheless, on a scale of tens of years, its variation is practically negligible.

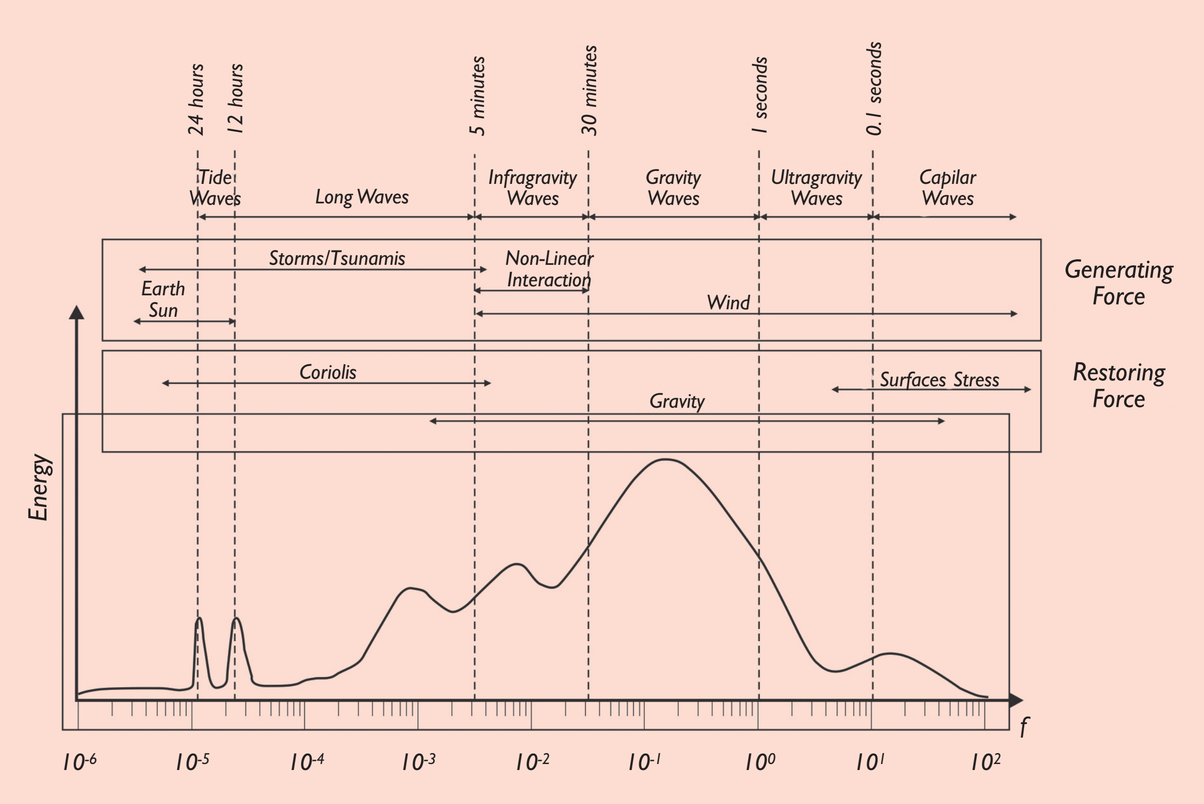
3.3.1.8 Energy content of sea oscillations

Figure 3.3.10 shows the energy content of the various sea oscillations in accordance with their representative periods and of the generative and restoring forces. The greatest energy content can be found in the wave action band. Sometimes the interaction between the oscillations, boundaries, and the structures can cause resonance situations in which energy accumulations can be a determining factor in the behavior of the harbor area.

3.3.1.9 Continental boundaries and coastal zones

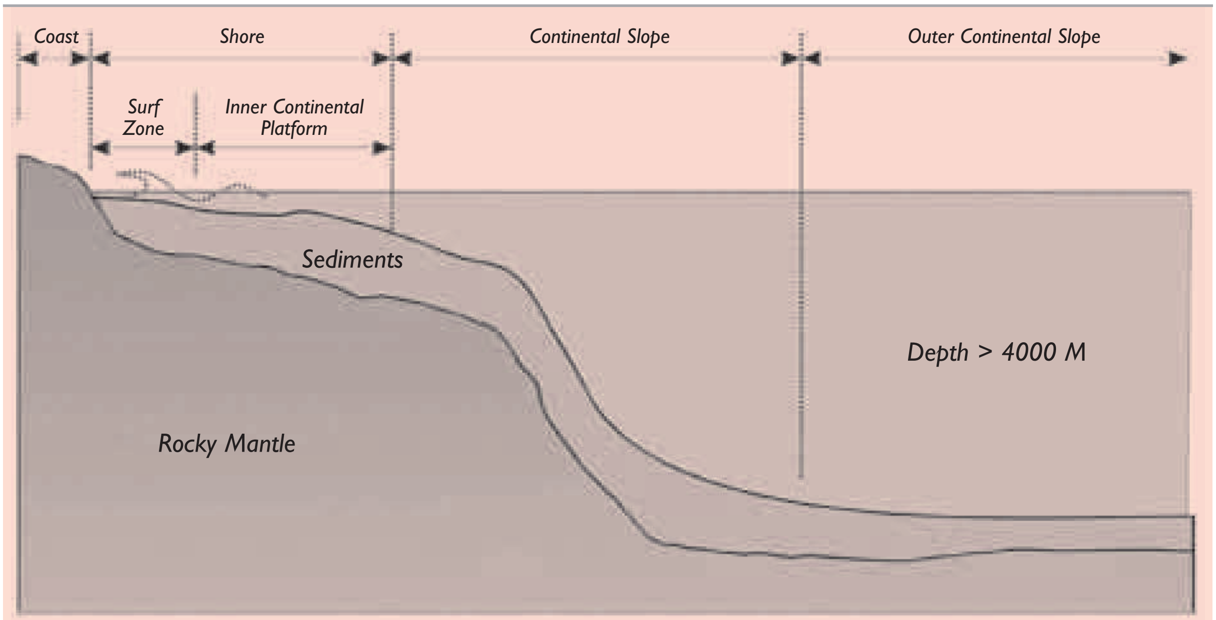
The world's oceans have a mean depth of 4000 m and an area that exceeds 5000 km. The continental borders are formed by the continental slope, the outer continental shelf and the coastline, which includes the inner continental shelf, the surf zone, and the inland waters. The continental slope is the transition area between the abyssal zone with a depth of 1000-2000 m and the outer continental shelf with a depth of 200-400 m. Generally speaking, the gradient of the continental slope is on the order of 10º. For this reason, its horizontal extension l_t is small in comparison to the tidal wavelength. In contrast, the continental shelf has a slope of approximately 1°, an order of magnitude that is smaller than that of the continental slope, which has a horizontal extension on the order of tens of kilometers.

Figure 3.3.10. Energy content of sea oscillations and their frecuency distribution



The shoreline includes two major zones: (i) the inner continental shelf; (ii) the surf zone. This is the area where port and shoreline areas are usually located. As a result, it is where most maritime and port installations are constructed (see Figure 3.3.11). The inner continental shelf generally extends from a depth of 20 meters to the coastline. It is where shoaling and wave refraction are most significant.

Figure 3.3.11. Continental shelf, border and slope



Moreover, it is the zone where sediments most actively participate in coastal landforms. Its width is on the order of hundreds of meters. The surf zone is characterized by wave breaking and associated processes. Its width can vary from tens to hundreds of meters. Finally, inland waters are formed by inlets and estuaries which long waves can only access by a channel or opening. During their propagation from the open sea towards the coast, sea oscillations become transformed in order to adapt themselves to the geometric characteristics of the area, particularly, to the depth h. The capacity to feel the seabed depends on the relative depth h/L, where L is the wavelength. Three propagation regions can be defined: (i) deep water ($h/L > 1/2$) in which the oscillatory motion is not affected by the seabed; (ii) medium depths ($1/20 < h/L < 1/2$) at which the oscillatory motion reaches the seabed; (iii) shallow water ($h/L < 1/20$) at which all of the water column is subjected to the approximately the same horizontal field of velocities. Short-period oscillations propagate throughout the three regions. Medium-period propagate through medium and shallow depths, whereas long-period waves always propagate through shallow waters.

3.4 DESCRIPTION OF THE WAVE TRAIN

The most basic description of a progressive wave train (long-crested waves) is performed in the wave cycle, and is made up of wave height, period, and direction of propagation. The wave train is an indefinite repetition of the wave cycle. Some of the basic premises of wave theory are described in the Annex.

3.4.1 Mathematical description

The monochromatic wave is an oscillation of the free sea surface that is identically repeated in time and space. The instantaneous variable of the wave train is the vertical displacement of the free sea surface $\eta(t, \vec{x})$ over time, in regards to a fixed reference level.

3.4.1.1 Basic variables

The basic variables of the wave train are the period T, length L, wave height H, and the direction of propagation θ. The profile of the progressive wave propagates with celerity $C = L/T$ whose dependence on the propagation depth h is determined by the dispersion equation. The stationary wave is not propagated, but its basic variables, L, T also satisfy it.

3.4.1.2 Dispersion equation

When the wave is propagated through depths where the effect of the sea bed is not negligible, the functional relation establishes a dependence among the three previous parameters and the depth h, $f(C,L,T,h) = 0$, in such a way that if three parameters are known, the value of the fourth is already determined. If the celerity of the wave depends only on period, the wave is both frequency and period-dispersive. When wave height A is not negligible and is not a scale factor, the functional relation establishes a dependence between the four previously mentioned parameters and height, $f(C,L,T,A) = 0$. It is thus known as amplitude-dispersive wave. Generally speaking, the celerity of the wave depends on its depth, amplitude, and period:

$$f(A, h, L, C, T) = 0 \qquad (3.39)$$

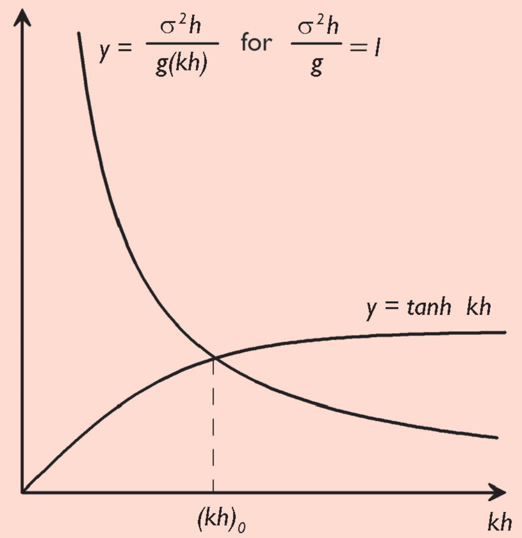
The wave is both frequency and amplitude-dispersive. This equation is known as the dispersion equation. Once the values of the four parameters are known, the value of the fifth is the root of the dispersion equation. In linear wave theory, waves are frequency-dispersive, but not amplitude-dispersive. When there is no current, the dispersion equation is the following:

$$f(h, L, T) = \sigma^2 - gk \tanh kh = 0 \qquad (3.40)$$

$$k = \frac{2\pi}{L} \quad \sigma = \frac{2\pi}{T} \quad C = \frac{L}{T} \qquad (3.41)$$

In each case, depending on objectives and necessities, it is necessary to determine the wavelength of the monochromatic wave train, the wavelength of the spectral components of the wave action or the wavelength of their state descriptors with the corresponding dispersion equation. Generally speaking, in *t I, II, III* and *IV* spatial domains, it is sufficient to apply the linear dispersion equation (see Figure 3.12). In domain V, in shallow waters and in breaking conditions, it may be necessary to resolve the dispersion equation by taking into account the influence of the wave amplitude (third-order Stokes waves, Boussinesq, and non-linear long waves).

Figure 3.3.12. ***Real square of the wave number of the linear equation of the dispersion***



3.4.1.2.1 WAVE-CURRENT INTERACTION

It is advisable to analyze the interaction between the current, whatever its origin, and its wave action. This interaction is reflected in the wave dispersion equation, varying the functional relation between the wave period and wave length. The linear dispersion equation is:

$$\sigma^2 \left(1 \mp \frac{U}{\sigma / k}\right)^2 = gk \tanh(kh) \tag{3.42}$$

where U *(m/s)* is the velocity of the current; $\sigma = 2\pi / \overline{T}_z$ *(cycles/s)*, $k = 2\pi / \overline{L}_z (1/m)$ are respectively the angular frequency and representative wave number of the wave train; and *h* is the depth in meters. The negative sign in the equation corresponds to the direction [6] of the current U, which is the opposite to that of the wavefronts. The positive sign corresponds to when wave and current advance in the same direction. When the current propagates obliquely to the wavefront, U is taken as the velocity component in the propagation of the wavefront.

The change of wavelength, and thus, of wave celerity because of the current can produce shoaling, refraction, reflection, and wave breaking. Accordingly, the presence of wave action can vary the module and the propagation angle of the current. These effects are usually important in the surf zone as well as at the mouths of rivers and estuaries. In such cases, it is possible to use some of the models described in section 3.4.4.

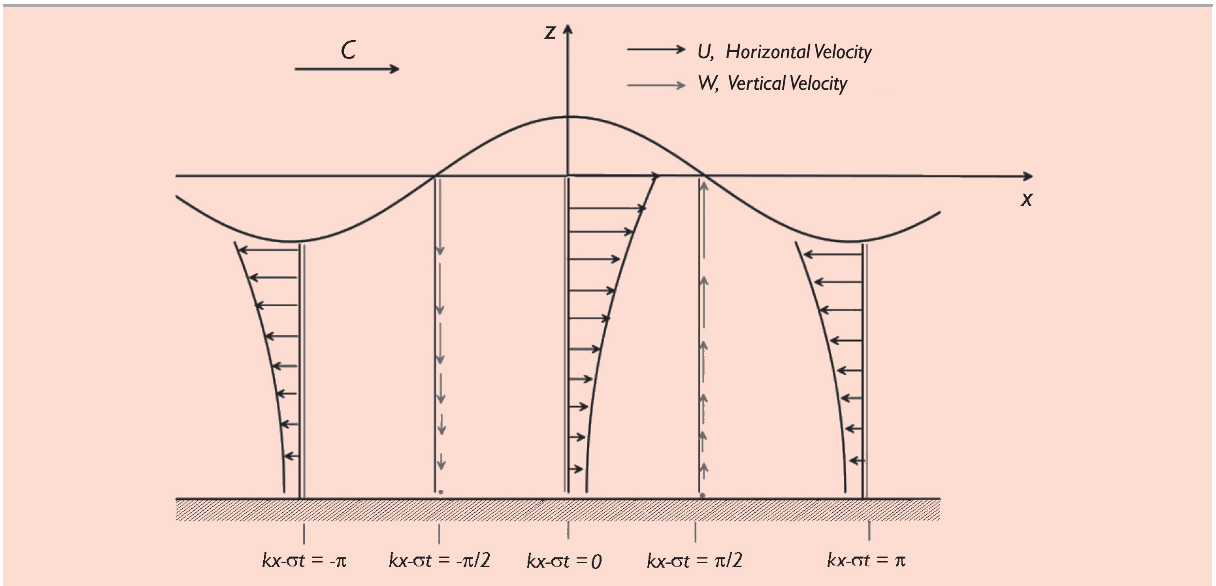
(6) The propagattion angle of the current with respect to N, for example, shows the direction of propagation. In contrast, the propagation angle of wind and wave action indicates the direction from which they are coming.

3.4.1.3 Types of oscillatory motion

The mathematical representation of oscillatory motion should take into account one of the three following types of movement:

- Progressive movement: the profile progresses in only one direction with celerity C and without modifying its shape (see 3.3.13);
- Stationary movement: the profile does not progress, but only oscillates without changing its shape around a series of fixed points (nodes), between which are other points of maximum oscillation (antinodes);
- Partially stationary movement: the profile slowly progresses, slightly oscillating around almost fixed points (quasi-nodes), between which are other points of maximum oscillation (quasi-antinodes).

Figure 3.3.13. Variation of the free sea surface and the horizontal and vertical field of velocities for different phase values of a progressive wave train



In any case, it should be remembered that in linear theory, stationary and partially stationary movements can be modeled by the linear superposition of two or more progressive wave trains, and that regardless of the type of movement that they participate in, the cinematic properties of the waves satisfy the (linear) dispersion equation.

Note. Stationary and partially stationary waves can be formed when two waves, traveling in opposite directions, meet. In the first case, stationary waves have the same height, the same period, and the same phase, as occurs at the front of a vertical breakwater. Partially stationary waves can be formed by modifications in height, period, and phase, as occurs at the front of sloping breakwaters or on reflecting beaches.

3.4.1.3.1 Wave height variation along the crest

Wave trains can be long-crested in infinite theories, or short-crested or finite. By extension, long-crested waves are applied to swells. Their wave height is constant throughout the train, or varies only slightly. The term, *short-crested wave train*, refers to the meeting of waves that come from different directions, and to the type of wave action that is representative of generation states. Wave height varies significantly along the crest though over shorter distances than the wavelength.

Note. In these circumstances, the diffraction o lateral transfer of energy throughout the crest is not negligible.

3.4.1.4 Mathematical characterization of the wave

Generally speaking, the mathematical description of a constant wave train should be represented as the product of three adimensional monomials: (i) relative depth h/L (ii) wave steepness H/L; (iii) relative amplitude H/h:

$$F = f_1\left(\frac{H}{L}\right) f_2(kh) f_3(k,\sigma) \tag{3.43}$$

where F generically represents the wave profile, the kinematics or the dynamics of oscillatory motion; f_1 defines the magnitude of oscillatory motion, which is generally the function of the steepness H/L or H/h; f_2, the function of the depth, indicates how and the extent to which the oscillatory motion spreads with depth. It thus depends on the relative depth h/L; and f_3, known as phase, defines the shape of the wave profile and its movement type (progressive, stationary, or partially stationary) as well as the variation of the wave height along the crest.

Note. The phase function is expressed in terms of wave celerity and wave period. The phase function of the profile of a linear progressive wave is the cosine function. The argument or phase can be expressed as follows:

$$\vec{k}(\vec{x}-ct) = \vec{k}\cdot\vec{x} - \sigma t$$

This wave with period $T = 2\pi/\sigma$ and with wavelength $L = 2\pi/|k|$ propagates in the sense of the x axis with celerity $c = L/T = \sigma/k$. If the wave is stationary, the phase function is formed by the product of two terms, one related to space $\cos(|k|x)$, and the other related to time, $\sin(\sigma t)$. This wave is not propagated, and oscillates around mean sea level, maintaining fixed points (nodes) and other points (anti-nodes), which have a maximum amplitude of oscillation.

In linear theory, the depth function is the hyperbolic cosine $f_2 \sim (|k|(h+z))$ and the magnitude function depends on the variable being defined. For example, for velocity $f_1 \sim (g/\sigma)Ak$.

3.4.1.5 Wave regimes and wave theory selection

The characteristics of oscillatory motion on a horizontal bottom can be described in terms of three adimensional monomials: (i) relative depth h/L; (ii) wave steepness H/L; and relative amplitude H/h. However, in most cases, two monomials are sufficient. The progressive wave theory should be selected, always taking into account calculation objectives and adimensional monomials as described in the following sections (see Figures 3.3.14 and 3.3.15).

Note. The public domain software program cnrt.for permits the selection of the most suitable wave theory, based on the values of the adimensional monomials.

3.4.1.5.1 Regimes according h/L

Stokes waves are those whose length is fairly small in comparison to their depth. Generally speaking, the application field of the following two wave regimes are differentiated by their parameter, relative depth (see Table 3.3.3).

Table 3.3.3. Wave regimes according to relative depth

Regime	h/L
Stokes	≥ (1/10) (7)
Boussinesq	< (1/20)

(7) Although the lower limit to consider intermediate depths is $(h/L) > (1/20)$, the Stokes regime, that's obtained as a mathematical solution of a boundary problem, shouldn't be applied to lower depths of $(h/L) < (1/10)$. Through the Ursell number we can fix the range of validity for Stokes regime with more accuracy.

Figure 3.3.14. **Diagram of the application of progressive wave theories**

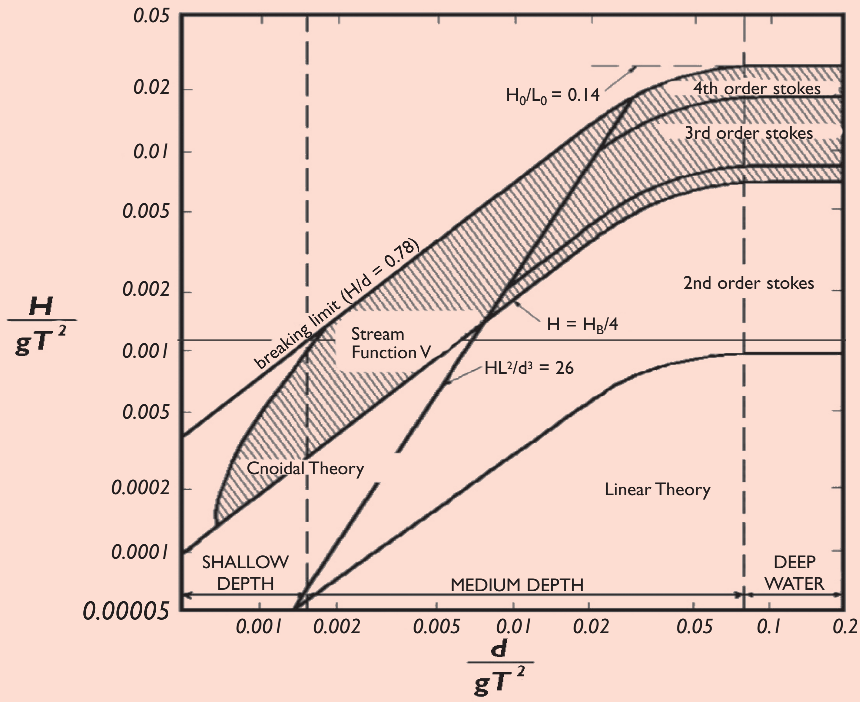
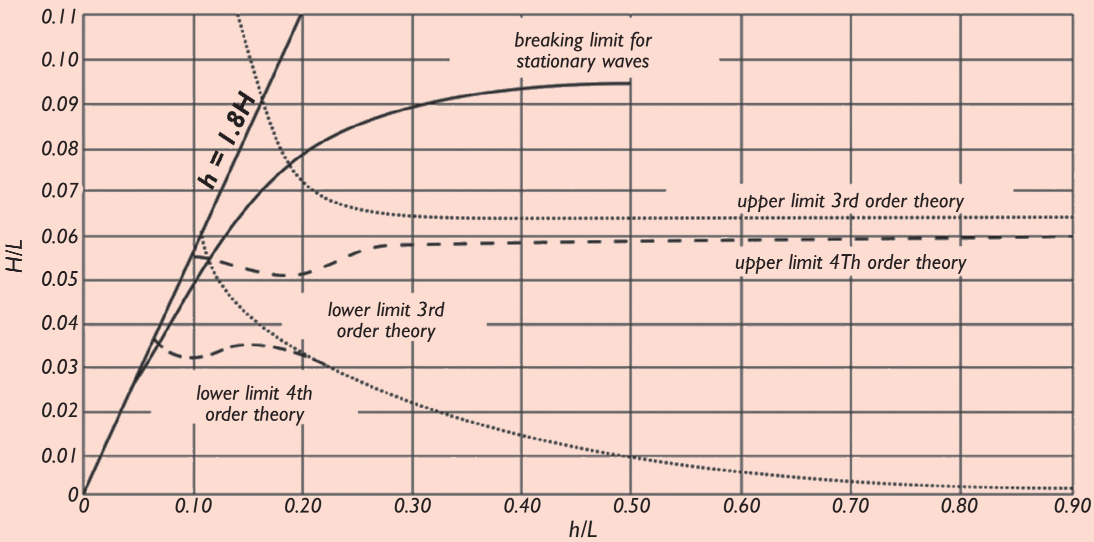


Figure 3.3.15. **Application range of stationary wave theories**



Stokes waves adequately represent oscillatory motion in deep water ($h/L > 1/2$) and at medium depths ($1/20 < h/L < 1/2$). In regards to long waves (or a long-wave or Boussinesq regime), the wavelength is large in comparison to its depth ($h/L < 1/20$). For each regime and for each of the relative depths, the waves can have low steepness. Therefore, they can be described by linear theory. However in the case of a high steepness, the description of the behavior of the wave train requires the use of higher-order theories, i.e. non-linear theories. The Ursell number, which combines both parameters, allows the field of validity to be delimited for each regime.

Stokes Regime. In the Stokes regime, higher-order waves are formed by a series expansion in which each term is formed by a magnitude, proportional to an n power of the wave steepness $(Ak)^n$ and a phase term, which is a harmonic of the linear term $m(kx - \sigma t)$, $m \geq 1$. The number of terms that appear in the expansion define the Stokes wave. Accordingly, the linear wave only has the fundamental mode ($m = 1$), and is called a first-order Stokes wave. In contrast, a wave with a fundamental mode ($m = 1$) and a *harmonic* ($m = 2$) is known as a second-order Stokes wave. These two waves are frequency-dispersive, and have the same dispersion equation. Third-order Stokes waves are both frequency and amplitude-dispersive, and thus, the dispersion equation is modified to include wave steepness.

Note. *The steepening of a sinusoidal wave can be described by adding an additional term to the surface profile expression (e.g. a half-period sine function). In the Stokes regime, which includes the sinusoidal wave in deep water, the vertical displacement of the free surface is described by a series expansion, in which each term in the series has a phase with half the value of the phase of the previous term. Consequently, as the wave steepens, more (non-linear) terms need to be added to the mathematical description.*
Linear wave theory is obtained as the first solution of the Stokes regime. It is also known as a first-order Stokes wave or small-amplitude wave, since, mathematically speaking, it is only valid when wave steepness is very low. However, except for certain phenomena and specific processes, the description of the kinematics and dynamics of the movement of this linear wave is adequate for maritime engineering. Stokes-regime descriptions that include non-linear terms to represent oscillatory motion properties associated with wave steepening are called second-order Stokes wave, third-order Stokes wave, etc. However, it should be remembered that the range of relative depths for which the Stokes regime is valid is independent of the non-linear order considered.

Long waves (Boussinesq regime). The selection of a wave theory for the long-wave regime $h/L < 1/20$ is carried out, based on the relation between the orders of magnitude of the parameters, h/L and H/h, in accordance with one of the three possible situations in Table 3.3.4.

Table 3.3.4. Long-wave regimes

	Theory
$H/h << (h/L)^2$	Onda larga lineal
$H/h \sim (h/L)^2$	Onda de Boussinesq
$H/h >> (h/L)^2$	Onda larga no lineal

In the first case, $[H/h \ll (h/L)^2]$, $U_r \ll 1$ the theory obtained corresponds to waves with an extremely small amplitude. Consequently, it is known as long-wave linear theory. This theory is extensively applied in the chapter on long waves in the study of astronomical and meteorological tides, tsunamis, and other long waves.

The second case $[H/h \sim (h/L)^2]$, $U_r \sim 0(1)$ is Boussinesq's theory. The solution waves of this theory are regarded as weakly non-linear since the celerity or propagation velocity of the wave depends only very slightly on wave height. Furthermore, Boussinesq equations can be used to derive the Korteweg–de Vries equation (KdV equation), which provides analytical wave solutions that propagate, while maintaining their shape. These solutions are the solitary wave and the cnoidal wave. Their main characteristic is that in order to conserve their shape, there must be an equilibrium between their frequency dispersion and amplitude dispersion.

The third case $[H/h \gg (h/L)^2]$, $U_r \gg 1$ is the range of non-linear theory for long waves, also known as Airy waves or shallow-water waves. It is used to locally describe meteorological and astronomical tides as well as waves breaking in oscillation on a gentle slope. The linear long waves defined as the first case are obtained as an asymptotic solution, assuming that $H/h \rightarrow 0$.

Note. *Long-wave linear theory does not change the shape of its profile during propagation, whereas non-linear theory provides a surface profile that becomes progressively steeper, thus making the front of the wave more vertical. This is evidence of its capacity to analyze situations that are very close to wave breaking.*

In general, the Boussinesq theory is applied to shallow waters though in recent years, progress has been made in the extension of the theory to deep water. In contrast, the Stokes wave theory has conventionally been applied to medium-depth and deep water. The zone of medium depths, $[1/20 < h/L < 1/10]$*, is ambiguous because Stokes theory should not be applied to it. Moreover, in order to apply the Boussinesq theory, the wave celerity has to be suitably modified for it to be representative. The following sections discuss wave breaking in deep, medium, and shallow water depths. It is obvious that in the deep and medium depths, the Stokes wave is the theoretical model to apply, whereas in shallow water depths, the model to apply is the solitary wave, which is the solution of the Boussinesq equations. Wave breaking in the region of relative depths* $[1/20 < h/L < 1/10]$ *can be defined by the interpolation of the results obtained for the other two depths.*

3.4.1.6 Frequency domain

The basic variables of the wave train in the frequency domain are the following: (i) the mean energy per unit of horizontal surface $E(f, \vec{x})$; (ii) the frequency $f = 1/T$ (or the angular frequency $w = 2\pi/T$); and (iii) the velocity of energy propagation $C_g(f)$ where,

$$E(f,\vec{x}) = \frac{1}{8}\rho_w g H^2 \tag{3.44}$$

$$C_g = \frac{1}{2}C\left(1+\frac{2kh}{\sinh 2kh}\right) \tag{3.45}$$

$$k = \frac{2\pi}{L} \tag{3.46}$$

where h is the water depth; g is the acceleration of gravity; and ρ_w is the sea water density.

Note. *Phase-integrated wave generation and transformation models are formulated in terms of the mean energy per unit of horizontal wave surface or of each one of the spectral components in each point of the propagation domain expressed by the following:*

$$S(f,\vec{x}) = \frac{E(f,\vec{x})}{\rho_w g} \tag{3.47}$$

3.4.1.6.1 Mean oscillatory energy flux

The vertical distribution of the mean energy flux per unit of crest width f_E, which defines the quantity of energy passing through a section from the seabed to the mean sea level, depends on the relative depth of the wave. In linear wave theory, this mean flux at elevation z in the direction of wave propagation in a period or oscillatory motion cycle can be calculated with the following expression:

$$\overline{f_E(z)} = \frac{1}{T}\int_t^{t+T} p_d u dt = \frac{1}{T}\int_t^{t+T} \rho g \sigma \eta^2 \frac{\cosh^2 k(h+z)}{\cosh kh \sinh kh} dt \tag{3.48}$$

where h represents the water depth; z is the vertical coordinate measured in respect to the mean sea level or still water level; p_d is the dynamic pressure, u is the velocity of water particles; and η is the vertical displacement of the free surface due to oscillatory motion. This equation represents the work of by the pressure forces on one side of the section. The total energy flux that passes through the section from the seabed to the mean sea level (8) or still water level $(z = 0)$.

(8) As an approximation, it is sufficient to integrate to the mean sea level. This is coherent with the calculation of the energy flow, where only the work due to pressure forces is considered.

$$\int_{-h}^{0} \overline{f_E(z)}dz = \left(\frac{1}{8}\rho g H^2\right) C_g = EC_g \tag{3.49}$$

E is the mean energy per unit of horizontal surface associated with the oscillatory motion. In the same way as any other conservative mechanical system, it is equally distributed in potential energy and cinematic energy. C_g is the energy propagation velocity, and is related to the celerity or propagation of the profile or phase velocity by the following expression:

$$C_g = \frac{C}{2}\left[1 + \frac{2kh}{\sinh(2kh)}\right] \tag{3.50}$$

whose asymptotic approximations with respect to relative depth are the following:

$$kh \geq \pi, \quad C_g \approx \frac{C}{2} \tag{3.51}$$

$$kh \leq \frac{\pi}{10}, \quad C_g \approx C \tag{3.52}$$

Note. The calculation of the vertical distribution of the energy flux requires the application of a wave theory that provides information concerning the depth function. In linear theory, this function is the hyperbolic cosine, and the argument is the relative depth, kh. In deep water, a flux lower than $h > L/2$ is negligible, whereas in shallow waters, the energy flux is practically constant throughout the water column. Floating breakwaters only control the energy flux above their draught. To control the energy flux of shallow-water wave action, it is necessary to protect it all the way to the seabed.

The lower order to which the energy flux can be evaluated is on the order of $P_d u \sim O((Ak)^2)$ since in linear theory, the dynamic pressure is $P_d \sim O(Ak)$, and the horizontal velocity due to oscillatory motion is also $u \sim O(Ak)$. The energy transmited by water particles is $u^2 u \sim O((Ak)^3)$. Simpler models of wave train shoaling can be formulated by conserving the energy in a control volume,

$$\frac{d}{dx}\left(EC_g\right) = 0 \tag{3.53}$$

3.4.2 Wave train transformation

Changes in depth, currents, obstacles, and boundaries transform the cinematic and dynamic characteristics of the wave train. The principal transformation processes of the wave train are: (i) shoaling and refraction; (ii) diffraction and radiation; and (iii) reflection and breaking.

Wave shoaling, refraction, and breaking occur during the propagation of the wave train from the open sea to medium and shallow depths. These processes happen because of the reduction in phase velocities and the propagation of energy as well as because of the shortening of the wavelength, which causes an increase in wave height (and thus in steepening), an increase in dynamic pressure on the seabed, a convergence of surface water particles with wave celerity, and an increase in the vertical acceleration of the particles up to values on the order of $g/2$. Finally the wave can break.

3.4.2.1 Shoaling

When a wave train propagates to shallow waters, besides decreasing its wave celerity, and consequently its wavelength, its amplitude varies because the energy propagation velocity decreases. These two modifications are translated into a change in steepening H/L, which is known as shoaling. Similarly, when a wave train propagates to greater depths, its steepening varies in the opposite direction. This is known as inverse shoaling.

The modification in the wavelength and amplitude of a wave train because of shoaling can be simply represented in the context of the Rayleigh Hypothesis, by assuming that the wave immediately adapts to the depth in such a way that at each point, its cinematic and dynamic properties (i.e. celerity and wavelength) are those of a wave moving along the horizontal bed at that depth.

Generally speaking, a shoaling problem can be resolved by assuming that the shoaling process is bidimensional; the wave period is constant; and the energy flux in the propagation direction of the wave train is conserved since there is no wave dissipation because of breaking or bottom friction, or wind energy contribution. These assumptions are applied in those situations in which the Rayleigh Hypothesis is acceptable.

3.4.2.2 Wave shoaling and refraction with gradual slope changes

When a wave train is obliquely propagated because of a gradually changing bathymetry, the points of the line defining the wavefront can be found at different depths. Since the velocity of propagation is greater as the water increases in depth, those segments of the wavefront where the draught is greater travel more quickly than the segments in more shallow depths. As a result, the wavefronts tend to be located parallel to bathymetric lines.

Assuming that the Rayleigh Hypothesis is valid, and that each point of the wavefront travels in a direction perpendicular to the front at a celerity corresponding to its depth, it is possible to draw lines tangent to the wave propagation direction at each point. These lines are called orthogonals, and when there are no currents, they coincide with lines running parallel to the energy propagation direction (i.e. rays). This hypothesis signifies that each ray behaves independently, and furthermore, that the wave instantaneously adapts itself to its depth.

The transformation models of linear wave trains because of shoaling and refraction should satisfy wave conservation equations, the irrotationality of the wavenumber, and the energy conservation equation, regardless of the inclusion of energy dissipation because of bottom friction and wave breaking.

3.4.2.2.1 Limitations of ray theory

Although wave theory allows an intuitive explanation of refraction, it has certain limitations, which must be taken into account when adopting a method to calculate wavetrain shoaling and refraction processes induced by changes in depth or by a current. Firstly, each point of the wavefront does not travel by itself with the celerity corresponding to its depth, but rather is affected by the local bathymetry around the point. Generally speaking, for a change in depth to affect wave propagation, it should have a characteristic dimension $D > 0.2L$, where L is the wavelength. Based on this property, Iribarren (1938) proposed the elaboration of "Wave Plans". This meant calculating for each point a celerity corresponding to the depth averaged with the depths of the corners of a square of side $L/4$ oriented in the local propagation direction of the wave.

Furthermore, depending on the bathymetry, the ray theory can predict very large wave heights, and even generate infinite values at those points where two rays converge. Since in these conditions, it is generally not verified that the wave has a small amplitude, the linear theory is no longer valid, and the oscillation should be described with a finite-amplitude wave theory that can account for the non-linear character of the wave. Furthermore, extreme wave height gradients can occur in the direction perpendicular to that of the wave propagation. There are two natural mechanisms that compensate these longitudinal wave height gradients. The first is wave breaking, and the second is diffraction or the lateral transfer of energy in the direction perpendicular to that of the wave propagation. When this transfer is not negligible, it invalidates the hypothesis of energy conservation between orthogonals.

Note. The parallelepiped method was proposed by Iribarren at the beginning of the 1930s. It can be regarded as a ray-type model, though with a fundamental difference, which is fruit of the intellectual and observational capacity of its creator. According to Iribarren, the propagation of a wave train does not depend on the celerity of each point, but rather on the average behavior of a set of points (i.e. the corners and central point of parallelepiped. Once the dimension of the parallelepiped side was selected (on the order of $L/4$), the method sidestepped the *problem of caustics or the crossing of rays, and the instantaneous response of the wave train to sudden changes in depth. The energy E that appears in the conservation equation is the average total cinematic and potential energy in the period per unit of horizontal surface of the wave (see the Annex on the linear theory of gravity waves). Consequently, it is an average magnitude in the phase, and the methods that resolve the propagation of the wave train in the phase are called phase-averaged methods. The methods that account for the "instantaneous" behavior of the free surface are known as phase-resolving methods (see section 3.4.4).*

3.4.2.3 Reflection produced by a breakwater or a cliff

When the wave nears the coastline, the change in depth or the presence of an obstacle can cause the reflection of part of the wave energy. A wave train propagating over an impermeable horizontal seabed can strike an infinitely wide, impermeable vertical wall with a crest height (or freeboard), sufficiently high to eliminate the possibility of overtopping. The incident energy is then totally reflected against the vertical face, and a stationary wave train is generated, due to the interference between the incident wave train and the reflected wave train. In these conditions, the reflection of the incident energy is total. In other cases, the reflection is partial, and the wave height of the reflected wave train is less than that of the incident wave train. Moreover, if the wall is not totally impermeable, oscillatory motion is transmitted, and consequently, the velocity of the wall is not equal to zero.

$$\frac{\partial \Phi}{\partial x}_{|x=0} = \frac{\partial \Phi_I}{\partial x}_{|x=0} + \frac{\partial \Phi_R}{\partial x}_{|x=0} \neq 0 \tag{3.54}$$

In this case, the reflection coefficient is:

$$R = |R| e^{i\varphi} = \frac{A_R^{'}}{A_I^{'}} = \left|\frac{A_R}{A_I}\right| e^{i(\varphi_R - \varphi_I)} \tag{3.55}$$

where $|R|$ represents its module ($|R| < 1$); $\varphi_R - \varphi_I$ is its phase lag between the incident wave and the reflected wave; and Φ is the velocity potential function. In linear theory, the reflection coefficient only depends on the wave period. For relatively long wave periods, whose steepening is not sufficient to produce wave breaking, an impermeable and sloping wall reflects practically all of the incident energy.

3.4.2.4 Diffraction

When variations in wave height in the direction of the crest are not small, and in all cases, smaller than the variations in the direction of propagation, it is necessary to consider the diffraction process. In a system of local coordinates (s, n) with s oriented in the direction tangent to the crest, and n oriented in the direction of propagation, the hypothesis is the following:

$$\frac{\partial H}{\partial s} \ll \frac{\partial H}{\partial n} \tag{3.56}$$

The calculation of diffraction requires equations that consider energy transfer all along the crest. In the same way as the refraction and the reflection, in linear theory, the diffraction only depends on the period as well as the shape and dimensions of the obstacle. The diffraction coefficient is defined as the quotient of the diffracted wave height and the incident wave height.

3.4.2.4.1 Diffraction produced by a semi-infinite breakwater

When the wave train reaches a semi-infinite breakwater, the turbulence behind the obstacle is produced by the diffraction of the incident wave train energy. The head of the semi-infinite breakwater acts like a wave emission source that generates waves in all directions. In the case of a reflecting breakwater, the reflected energy is also diffracted around the head, thus contributing to the turbulence downdrift from the breakwater. The area around the breakwater can be divided into three zones: (i) feeding zone; (ii) turbulent zone; (iii) non-turbulent zone.

Note. Diffraction can be analyzed by means of the Huygens Principle for the study of the propagation of light through a slit. One part of the wavefront that meets the obstacle is "revealed", whereas in the adjacent regions, the boundary condition must be verified. Downdrift from the obstacle, this creates a wave height gradient along the crest that compensates the lateral transfer of energy in a direction perpendicular to the direction of propagation.

3.4.2.4.2 ARBITRARY RADIATION FROM BREAKWATER HEADS AND OBSTACLES

The breakwater head, alignment changes, the mouth or any aperture in the breakwater or coastline can emit energy that radially propagates from the emission source. The set of all the radiations interact, thus causing longitudinal variations in the wave height. In some cases, this can considerably increase (or reduce) the wave height of the wave train, compared to what the situation would be if there was no aperture.

3.4.2.4.3 DIFFRACTION PRODUCED DURING REFRACTION

The refraction process can produce the concentration and divergence of energy in specific areas. When the waveheight gradient between contiguous regions is very large, part of the energy contained between orthogonal lines is laterally transferred through them. This phenomenon, whose origin is a waveheight gradient in the direction perpendicular to the crest, is also a diffraction process. This situation can also be the result of an abrupt change in the seabed, the navigation channel, or a submerged obstacle. It can also be the result of a diffraction process when the energy dissipation due to, for example, non-uniform breaking along the wavefront causes spatial differences in wave height.

3.4.2.5 Numerical models of wave train transformation

Generally speaking, wave transformation processes begin to be important when waves penetrate the continental shelf. Because of rapid changes in depths and boundaries that usually occur in domains *III-V* (i. e. inner continental shelf, port areas, and the coastal zone, including the surf zone), the application of generation and transformation models for medium and shallow depths make it necessary to expand the spatial resolution grid beyond reasonable limits. This significantly increases computation time and the necessary instruments.

Furthermore, the dimensions of this domain are usually small in comparison to those of the generation area. This means that most specific cases, it is possible to ignore the energy increase in the domain because of wind action and the quadruple resonance of the components. In these circumstances, especially in extreme conditions, it is sufficient to study wave propagation and transformation in domains *III-V* by applying one of the models described in the following sections. A wavetrain transformation model should be selected, according to the recommendations in section 3.4.5.

3.4.2.6 Classification of models

Wave transformation models can be classified in two groups:

1. Wave theory models.
2. Models for resolving Navier-Stokes and Reynolds equations.

3.4.2.6.1 WAVE THEORY MODELS

Wave theory models are the most frequently used. Generally speaking, their degree of approximation is usually sufficient in maritime engineering. Accordingly, these models can also be subclassified into the following groups:

1. Phase-averaged models (time-independent models)
2. Phase-resolving models (non-stationary models)

3.4.2.6.2 *Phase-averaged and phase-resolving descriptions*

A model is regarded as time-independent if the mean energy flux is expressed only in respect to the waveheight and propagation velocity of the energy, and if the phase does not explicitly appear since all of the energy has been integrated into a wave period. The description of the moment is thus said to be phase-averaged, and characterizes the overall or integrated manifestation of the oscillatory motion. Models that analyze the wave transformation by means of the conservation equation of this mean energy belong to the group of phase-averaged models. In order to ascertain the nature of the water profile or its kinematics and dynamics due to oscillatory motion, theoretical wave should be adopted, which has parameters of waveheight and energy propagation velocity that satisfy the mean energy flux equation.

The models that analyze the instantaneous behavior of the free surface by resolving mass and momentum conservation equations, expressed in terms of instantaneous variables, vertical displacement of the free surface, η, and horizontal velocity, u, are known as phase resolving methods since they analyze the behavior and evolution of oscillatory motion over time.

3.4.2.7 *Public domain software of wave transformation models*

Currently, the following public domain software packages of wave propagation models are available: Ref-Dif, MSP, Funwave, etc.

Note. *The analysis of refraction and shoaling when waves propagate over a mildly sloping seabed can be performed by using the energy conservation equation in definite control volume between two lines running perpendicular to the wave crest, known as orthogonals or rays. The energy flux in the volume control boundaries is specified by means of oscillation parameters, height H, wave group celerity C_g, and wavelength L, in other words, by means of an integrated or averaged representation of oscillatory motion. To calculate the temporal evolution of the horizontal velocity field near the seabed at a given point, the movement is described by taking the phase into account (e.g. by the sinusoidal wave, as shown in Figure 3.3.14). The wave plan method proposed by Iribarren (1938) uses the trocoidal wave theory and the parallelepiped as the control volume.*
In contrast, the phase-resolving description is a solution of wave propagation along the slope, in which each instant of time is taken into account. Generally, numerical methods should be used for this type of description.

3.4.3 Phase-averaged models in the Stokes regime

Phase-averaged models can be formulated in two irrotational wave regimes (i.e. Stokes or Boussinesq). However, most of them are formulated in the Stokes regime, and particularly, in linear theory or first-order Stokes waves, which is the basis for the expansion of the mild slope equation (MSPE). They simultaneously resolve the wave energy conservation equation, the wavenumber irrotationality equation, and a dispersion equation (usually, the linear equation). It should be remembered that the presence of a current alters the irrotationality condition of oscillatory motion, the dispersion equation, and the energy conservation equation.

Processes that can be regarded as phase-averaged models in the Stokes regime are listed in Table 3.3.5:

Table 3.3.5. Physical processes in phase-averaged wave transformation models

Process	MSP	Parabolic	Hyperbolic	Snell-Refrac
Bottom refraction	Yes	Yes	Yes	Yes
Current refraction	No/Yes	Yes	Yes	No/Yes
Reflection	Yes	No	Yes	No
Diffraction	Yes	< 40º	No	No
Bottom-induced breaking	No	Yes	Yes	Yes
Bottom friction	No/Yes	Yes	Yes	Yes

These models can be used to study the transformation of the following:

1. A regular wave train represented by basic variables, H_0, T, θ_0, (the subscript 0 stands for the initial reference point), whose most simplified version is ray theory (Snell's Law and the hypothesis of energy flux conservation between orthogonals, which is valid when there is no current).
2. An irregular wave train represented by state descriptors, $H_{rms,0}$, $\overline{T_z}$, $\overline{\theta_0}$
3. The linear version of the spectral density function represented by each of the spectral components and formulated in the frequency domain $S(f, \theta)$ or its equivalent in the wavenumber domain $S(\vec{k}, \theta)$.

Note. The numerical model, Refrac, resolves the shoaling and refraction problem for regular bathymetries with energy conservation equations, regardless of the inclusion of bottom friction dissipation terms.

3.4.3.1 Numerical models formulated with the mild slope equation (MSPE)

When there is a mild variation in depth $a \ll kh$, and when the waves are linear or almost linear, the governing equations and boundary conditions of the waves can be expanded in a Stokes series. This results in the mild slope equation (MSPE). In order for the shallow-water application of this equation not to diverge significantly, it should satisfy the condition $H/h \ll (kh)^2$. This formulation also permits a non-stationary resolution (see section 3.4.4.1).

Its stationary form assumes that the waves are harmonic. Generally speaking, the equation to be resolved is elliptic. Depending on the dissipation and reflection conditions imposed on the oscillatory motion at the boundaries, the MSPE equation can have three approximations (9),

1. Elliptic (generically, MSPE): reflection and dissipation, depending on the boundary type;
2. Hyperbolic: reflection and dissipation according to characteristic lines and boundary type;
3. Parabolic (Ref-dif): only dissipation at the boundary and diffraction for small incidence angles.

The solution obtained with the mild slope equation can be applied to a regular wave train as well as to the state descriptors and to the spectral density function components.

3.4.3.1.1 Elliptic model

The mild slope equation (MSPE), which includes the first-order contributions to seabed variation, describes shoaling, refraction, and radiation. This signifies the description of reflection (because of boundaries as well as bathymetric variation) and diffraction, conserving the energy in the absence of currents. It can include terms that model wave energy dissipation by whitecapping and bottom friction with empirical models as well as the non-linear interaction of waves with a stationary current that is constant in time. When the bathymetry is very irregular and the slope is not mild, second-order terms should be included in the seabed variation. This gives what is generically known as a modified mild slope equation.

In its complete form, the equation is elliptic. Thus, before it can be resolved, it is necessary to specify boundary conditions in the entire domain, the curve enclosing it, and the simultaneous solution for all of its inner points. This gives the temporal evolution of the free surface at any point in the inner domain (see the section in the Annex on numerical models based on the first-order Stokes regime).

Generally speaking, elliptical models are applied to small areas on the order of 2-4 kms (i.e. 10-20 wavelengths). In its elliptic version without energy dissipation terms or current interaction, calculations only take a matter of minutes. In recent years, wave breaking, wind action, and bottom friction are being incorporated in these models. In this case, since the non-linear terms should be resolved iteratively, the numerical resolution time is greater unless the friction is formulated in equivalent linear terms.

(9) Hyperbolic, elliptic, and parabolic are the three basic types of second-order differential equations.

Note. The public domain software program, MSPE-UGR, resolves the elliptic approximation of the mild slope equation, including second-order terms in seabed variation. It also models the effects of diffraction, reflection, shoaling and refraction. The MSPE equation is resolved with finite elements in a closed domain, once specified the reflection and dissipation conditions at the boundaries that define and enclose the domain. The model also allows the inclusion of dissipation effects because of whitecapping and bottom friction by means of different empirical models.

3.4.3.1.2 PARABOLIC APPROXIMATION

The waveheight variation along the rays may be negligible in regards to its variation along the crests. This occurs, for example, when the wave reflection at the boundaries is negligible. In such cases, it is sufficient to take the lateral diffraction (or lateral energy transfer) into account. The equations can thus be resolved by progressing from deep to shallow water without imposing any boundary condition for the dryland area. When diffraction is also negligible, the equation is of the conventional type used for shoaling and refraction without bottom-induced energy dissipation. If, in addition, the bathymetry is not very irregular, the parabolic model is greatly simplified, and the solution obtained is a Snell's equation.

Parabolic models work somewhat less well in the case of obstacles that cause the reflection and diffraction of the incident wave train. Consequently, parabolic models should not be used when the wave train meets a reflecting obstacle or one that generates an energy "shadow zone", or when there is significant reflection at the boundaries of the integration domain. Evidently, parabolic models should not be used for breakwaters, since these structures significantly contribute to the radiation and reflection of wave energy. In such cases, it is best to apply the elliptic MSPE model.

Note. The Ref-Dif free software program includes dissipation by bottom friction, slow current variation, non-linearity in the dispersion equation, and corrections of the lateral transfer effect so that its results can be considered valid for wavefront rotations of up to 60º. Terms not usually included in these models are wind action source, wave breaking, and quadruple interaction. Accordingly, it should be applied in nearshore areas where these processes are negligible. There are two versions, one for monochromatic wave trains, and another that resolves the propagation of spectral components by providing a spectrum for any point of the sea.
The Sistema de Modelado Costero (SMC) *[Coastal Modeling System] applies a parabolic propagation model coupled with another vertically integrated model that calculates the equations of the current and free surface oscillations in the surf zone.*

3.4.3.1.3 HYPERBOLIC APPROXIMATION

The hyperbolic approximation to elliptic MSPE equations differs from the hyperbolic approximation for mild slopes and non-stationary movement. Strictly speaking, the hyperbolic approximation can only be applied in the case of wave trains, in other words, in the case of a periodic solution, as is the case of a sinusoidal wave train.

For the hyperbolic resolution of the time-dependent mild-slope equation, a pseudo-flux unknown is included, and it is rewritten as a first-order system of two equations. The problem thus maintains its spatial and temporal derivatives. It is resolved with a finite-difference scheme that requires a relatively large spatial and temporal discretization in terms of time periods and wavelength nodes. Other approximations reinterpret temporal dependence as a kind of pseudo-time that is used to perform iterations until arriving at a stationary solution. The hyperbolic form takes into account the total or partial reflection at the boundary of the resolution domain as well as the energy dissipation because of whitecapping or bottom friction (by introducing a source function).

Note. The hyperbolic approximation changes the MSP equation to a transitory equation, and is resolved in order to obtain results for the stationary state. In comparison with the elliptic model, this approximation offers the advantage of reducing computation time, particularly in 2D domains, and allows the inclusion of arbitrary reflection boundaries as well as diffraction and diffraction mechanisms.

3.4.3.2 Propagation and transformation coefficients

Whenever phase-averaged linear models are applied, the period is conserved and the waveheight is only a scale factor. Moreover, certain processes can produce a lag between incident and transformed wave trains. Results of the model are the wave train transformation coefficients at a certain given point in the sea, more specifically, the module of shoaling coefficient K_S, refraction coefficient K_R, reflection coefficient K_r, and diffraction coefficient K_d, which once multiplied by the wave height in the open sea H_0 provide the local wave height (at that point) $H(x, y)$ and its corresponding phase $\varphi_{S,R,r,d}$.

The four coefficients are defined in terms of the energy flux, in other words, their effect on the square of the wave amplitude or wave height. Generally speaking, it should be expressed as a complex magnitude with module and phase φ as shown in the following expression:

$$K_m(f, \theta, x, y) = |K_m(f, \theta, x, y)| \exp[i\varphi(f, \theta)]; \quad m = S,R,r,d \tag{3.57}$$

where

$$\left|K_m(f,\theta,x,y)\right| = \frac{H_m(x,y)}{H_0} \tag{3.58}$$

and $m = S,R,r,d$ generically expresses the shoaling processes $m = S$, refraction $m = R$, reflection $m = r$, and diffraction $m = d$. The frequency $f = 1/T$ is the inverse period.

3.4.3.2.1 Shoaling and refraction

Generally speaking, it can be assumed that shoaling and refraction processes do not produce a lag, and the vertical displacement of the free surface is expressed by $\varphi = 0$,

$$\eta_m(x,y,t) = \frac{\left|K_m(f,\theta,x,y)\right| H_0}{2} \cos\left(kx\cos\theta + ky\sin\theta - 2\pi ft + \varphi\right) \tag{3.59}$$

3.4.3.2.2 Reflection, diffraction, and radiation

In reflection, diffraction, and radiation processes, the oscillation at the point is formed by the linear superposition of incident or diffracted and reflected wave trains. It can be expressed by:

$$\eta(x,y,t) = \eta_I + \eta_m = \frac{H_0}{2}\left[1 + K_m(f,\theta,x,y)\right] \tag{3.60}$$

The difference in respect to shoaling and refraction is important, especially in the calculation of the spectrum by components since the energy of an irregular wave train is proportional to η^2.

$$\eta^2 = \left(\eta_I^2 + \eta_m^2 + 2\eta_I\eta_m\right) = \left(\frac{H_0}{2}\right)^2 \left[1 + \left|K_m(f,\theta,x,y)\right|^2 + 2\left|K_m(f,\theta,x,y)\right| \cos\left(\vec{k}\vec{x} - \sigma t + \varphi(f,\theta)\right)\right] \tag{3.61}$$

The term $\eta_I\eta_m$ modulates the (total height) of the oscillatory wave train at the point, and allows the evaluation of the correlation between the incident and transformed wave train. At a sufficient distance from the transformation (reflection and diffraction) focus point, the contribution of this term is negligible and the local wave height is no longer modulated.

Note. This result is relevant for the computation of maritime structural stability and the circulatory system on beaches. It shows that the transformation coefficient module should only be applied to shoaling and non-reflective refraction processes. Elliptical numerical models (e.g. MSP-UGR) and parabolic models (e.g. Ref-Dif) resolve wave *propagation in the complex field. For this reason, the solution contains the amplitude and phase of the wave train at each point.*

3.4.4 Phase-resolving numerical models and advanced models

Phase-resolving numerical models and advanced models are advisable for shallow water and the surf zone. They can be used to quantify medium-period oscillations generated by spatial and temporal variations of the waves and wave groups that can force resonant or non-resonant oscillations in wharfs and the surf zone.

These models are formulated by adopting approximations inherent to the derivation of the Stokes or Boussinesq regime. They resolve the conservation equations formulated in the instantaneous variable of free-surface vertical displacement and of the (horizontal) velocity field in the time domain, $\eta(\vec{x},t)$ and $u(\vec{x},t)$ or $u(\vec{x},t)$ and $p(\vec{x},t)$. Their main advantage lies in the fact that they can be initiated with an irregular signal or a regular wave train.

They are applied in a grid nested in the transformation model grid at the inner continental shelf or from the open sea. Generally, a wave celerity function is considered, which optimally adapts itself to the relative depth. Among the models that should be used for this are numerical models based on the time-dependent mild slope equation (Stokes regime) and on the Boussinesq equations as shown in Table 3.3.6.

Table 3.3.6. ***Physical processes in phase-resolving wave transformation models***

Process	MSP-t	Boussinesq
Bottom refraction	Yes	Yes
Current refraction	No/Yes	Yes
Reflection	Yes	Yes
Diffraction	Yes	No/Yes
Bottom-induced breaking	No/Yes	Yes
Bottom friction	No/Yes	Yes

3.4.4.1 The non-stationary mild slope equation MSPE-t

This equation is appropriate for modeling non-stationary linear processes related to the following: (i) the propagation of wave groups and irregular waves in a wide range of relative depths (Stokes regime); (ii) the propagation of small-amplitude waves generated by structures and moving bodies (e.g. vessels moored to docks or in movement), oscillations of wharfs, and entry channels, etc.

Unlike the stationary mild-slope equation (MSPE) that assumes that the wave time component is harmonic, the non-stationary MSPE equation is able to model the non-harmonic temporal evolution of the free surface, but still using the Stokes linear approximation. It also incorporates the effects of wave breaking, wind action, and bottom friction.

The time-dependent mild slope equation is hyperbolic. For this reason, boundary conditions in the entire domain should be open, and totally or partially reflecting, and the initial condition of the free surface or velocity potential should be at all points of the domain (see section 3.4.5.1).

Open boundary conditions can be modeled with an extended version of the parabolic approximation of the radiation conditions (which takes into account the non-harmonic nature of incident and radiated wave action) or by using a technique known as "sponge layers". MSPE-t models can be applied to small areas on the order of 2-4 km (10-20 wavelengths). When non-linear processes are important, MSPE-t models should not be applied, and Boussinesq models are preferable.

3.4.4.2 Boussinesq models

Boussinesq-type models are best for the study of wave and wave-group transformation in shallow waters and surf zones. The numerical models based on this type of equation are phase resolvers. They can be used to obtain the free surface and horizontal velocity field over time at any point in space, considering radiation processes (i.e. diffraction and reflection, shoaling and refraction, and wave-current interaction), particularly the non-linear nature of some of these processes. They adopt Stokes or Boussinesq-type approximations, and resolve conservation equations formulated in the instantaneous variable of free-surface vertical displacement and of the (horizontal) velocity field in the time domain.

Their application is carried out in a grid nested in the transformation model grid at the inner continental shelf or from the open sea. Generally, a wave celerity function is considered, which optimally adapts itself to the relative depth. However, these models should be applied to small areas, regardless of the incorporation of deep-water whitecapping, wind action, and bottom friction. Certain models incorporate the bottom-induced breaking term by modeling the roller or water volume that simulates the behavior, propagation, and evolution of the tidal bore In the same way as the MSPE-t, Boussinesq models are hyperbolic. Therefore, the boundary conditions should be open and totally or partially reflection. The initial condition can be a record of the free surface or velocity potential.

From the perspective of the area of study, Boussinesq models can be applied to areas on the order of 2-3 *kms* (10-20 wavelengths). In the case of larger areas, phase-averaged models should be used.

The type of Boussinesq model obtained depends on the representative horizontal velocity, the celerity expression, and the numerical method used,

Note. The public domain software program, Funwave, is a good example of a Boussinesq-type model. Created at the University of Delaware (USA), this model resolves equation for normal wave incidence as well for oblique wave incidence. Coulwave, created at the University of Cornell (USA) is another example.

3.4.4.3 Boussinesq models in the frequency domain

By substituting the vertical displacement of the free surface and the horizontal velocity $\eta(\vec{x},t)$, $u(\vec{x},t)$ for its Fourier series expansion (for waves propagating in one direction, though possibly in both directions), Boussinesq equations can be used to obtain evolution equations for one of the two variables, depending on their complex amplitudes (spectral components). In this way, a Boussinesq model is obtained in the frequency domain, which analyzes the evolution of spectral components, even in the surf zone.

3.4.4.4 Numerical models based on exact Navier-Stokes equations

If none of the monomials and the slope are sufficiently small to determine wave transformation, what is required are models that provide an exact resolution of the equations without approximations, and at the same time, resolve the vertical structure as well as the horizontal variations of the wave train.

In recent years, thanks to research advances, the increase in computational power, and the implementation of numerical schemes in finite volumes interalia, significant progress is being made in the direct integration of Navier-Stokes equations. It is thus no necessary to assume a certain type of fluid movement (e.g. harmonic). This has led to models of wave breaking and the subsequent rise of wave on the slope, the impact of wave breaking against a vertical wall, and the arbitrary overtopping of a breakwater, among other phenomena related to oscillatory motion.

3.4.4.4.1 N-S and RANS Models

The numerical resolution of Navier-Stokes equations, 2DV (two-dimensional problems), and RANS (Navier-Stokes equations averaged in the scale of turbulent flow) by means of finite-volume techniques are quite unlike numerical models based on Boussinesq equations or shallow-water non-linear equations. For one thing, they do not need to assume any initial wave theory, and the quantification of wave breaking is based on a turbulence model. Currently, the main disadvantage of these models is their 2DV formulation, computational time, and the fact that these models are not public domain software.

3.4.5 Selection of a wave transformation model

As long as wave generation processes are not relevant, the selection of a wave transformation model for domains *III-V* should be in consonance with the following criteria and recommendations (see Tables 3.3.7. and 3.3.8).

1. When seabed variations are mild (and current velocity is small), and the diffraction and reflection (by boundaries or structures) is negligible, it is sufficient to use models that consider refraction and shoaling, more specifically, phase-averaged models based on ray theory. These conditions occur on the outer continental shelf and the inner continental shelf for dissipative beaches.
2. Phase-averaged models are formulated in terms of the harmonic potential function, and quantify the energy density function, which is an averaged quantity. Because of their computational efficiency and the good results obtained, these methods are very effective for the study of oscillations on the inner continental shelf as well as for port areas where depths are not very shallow. They are particularly advisable when wave breaking is not a determining factor in wave transformation. The parabolic approximation of the MSPE (Ref-Dif model) can be applied in domains where the radiation-diffraction processes are negligible, and when the rotation of wavelength fronts is not greater than 40^o.
3. However, when variations in the seabed and boundaries can cause wave diffraction, the model applied should take this process into account. If in a wavelength, wavefronts rotate more than 40^o, the reflection from the structures and boundaries, or the radiation from the inlet aperture or port entrance and structural alignment changes are significant, then elliptic MSPE-type models should be applied. However, in the case of shallow water, Boussinesq models are preferable.
4. In light of the previously mentioned restrictions (and as long as the interaction of components because of bottom and boundary effects, and wave generation due to wind are not relevant), it is possible to obtain the transformations of each component of the spectral function by applying phase-averaged models, formulated in linear theory.
5. When wave generation processes in the work domain are relevant, a SWAN-type model should be applied (see section 3.6.3.1). This type of model shows the spatio-temporal evolution of the spectrum, which includes shoaling, refraction, and diffraction processes. It also considers source terms of three-wave interactions, bottom friction, and depth-induced breaking. The limitations of this model should be taken into account when quantifying diffraction.
6. A SWAN-type model should be applied when diffraction, reflection, and wind processes are significant. The limitations of this model should be taken into account when quantifying diffraction.

Table 3.3.7. ***Pysical processes of numerical wave models***

Phase-averaged models (linear theory)				
Transformation processes	Ray/Refraction*	Stokes I-MSPE		
		Elliptic	Hyperbolic	Parabolic
Refraction/shoaling	Yes	Yes	Yes	Yes
Diffraction	No	Yes	(Yes)	If $\theta < 40^o$
Reflection/radiation	(No)**	Yes	(Yes)	No
Bottom friction	No	No	Yes/No	Yes
Bottom-induced breaking***	Yes/No	Yes/No	Yes	Yes

(*) Ray theory should be applied to non-linear wave trains, particularly when studying shoaling and refraction.
(**) Ray theory can be applied to inverse refraction processes, and take into account the reflection from a fixed reflector.
(***) The majority of models implement wave breaking in the dissipation regime or by means of the surface roller concept. In both cases, it is implicitly assumed that reflection is zero at this boundary. For the sake of coherence, this should be specified when applying an elliptic model.

Table 3.3.8. ***Physical processes of non-stationary wave transformation numerical models***

Phase-resolving models		
Transformation processes	Stokes (lineal) MSP-t	Boussinesq (no lineal) various aprox.
Refraction/shoaling	Yes	Yes
Diffraction	Yes	Yes
Reflection	Yes	Yes
Bottom friction	No	Yes/No
Bottom-induced breaking*	No/Yes	Yes
Free-surface evolution	Yes	Yes

(*) In these models, wave train breaking is usually based on the surface roller concept.

3.4.5.1 Basic and instantaneous state variables in numerical models

The variables that resolve the different models are listed in Table 3.3.9. Generally speaking, once the problem is resolved, the kinematic and dynamic characteristics resulting from the presence of the wave train should be obtained with a wave theory according to the recommendations (see section 3.4.1.5 and Table 3.3.9).

Table 3.3.9. ***Calculation variable in wave transformation models***

Application variables				
	$\eta(\vec{x},t), u(\vec{x},t)$	H, T, θ	$H_{rms,0}, \overline{T_z}\ \overline{\theta}$	$S(f, \theta), S(\vec{k})$
Ray	No	Yes	Yes	Yes
Stokes (MSPE)	No	Yes	Yes	Yes
MSP-t	(Yes)	Yes	Yes	(Yes/Fourier)
Boussinesq	Yes	(No)	(No)	(Yes/Fourier)
N-S and others	Yes	(Yes)*	(Yes)*	No

(*) In practice, these models can be applied to wave train specifying its basic variables and proposing and initial wave profile (linear and no linear) $\eta(t, \vec{x})$.

3.4.5.2 Boundary conditions in boundary problems

Oscillatory motion in a fluid depends on the conditions imposed by the boundary movements of the domain in which the fluid is located and on the forces acting upon it.

3.4.5.2.1 Kinematic boundary condition

Generally speaking, the fluid follows the boundary movement, whether fixed or free. Conditions that refer to the kinematics of the movement are known as kinematic boundary conditions, and express the compatibility that should exist between boundary movements and the fluid in contact with the boundary. Any free or fixed surface can be represented by a mathematical expression of the type $F(x,y,z,t) = 0$. The boundary condition specifies that particles in contact with this surface should follow its movement, which is mathematically expressed as:

$$\frac{DF}{Dt} = 0 \Leftrightarrow -\frac{\partial F}{\partial t} = u \cdot \nabla F \text{ in } F(x,y,z,t) = 0 \tag{3.62}$$

This condition can also be written in function of the vector perpendicular to the surface $\vec{n} = \nabla F / |\nabla F|$:

$$-\frac{\partial F}{\partial t} = \vec{u} \cdot \vec{n} |\nabla F| \text{ en } F(x,y,z,t) = 0 \tag{3.63}$$

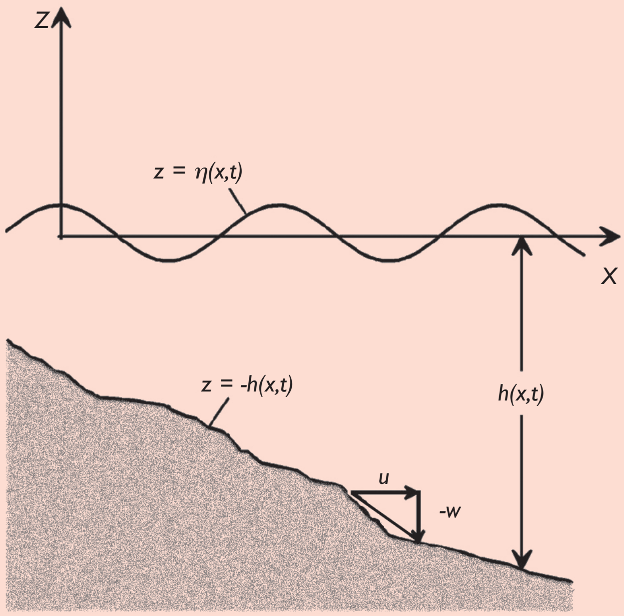
If the boundary is stationary (i.e. without varying in time), the boundary condition is reduced to:

$$\vec{u} \cdot \vec{n} = 0 \text{ in } F(x,y,z,t) = 0 \tag{3.64}$$

In other words, the velocity component normal to the boundary has to be zero.

In the case of the seabed surface, the cinematic boundary condition depends on whether the model considers it to be the boundary layer. Generally, speaking, the oscillatory boundary layer is very small (on the order of millimeters), and thus, is usually not considered. In these circumstances, and if the boundary is impermeable, the boundary condition on the seabed is known as a sliding boundary condition. More specifically, the water particle slides over the seabed, moving parallel to it, and the velocity component perpendicular to the boundary should be zero (see Figure 3.3.16). When it is necessary to consider the boundary layer, the cinematic boundary condition is that the water particle velocity moves with the seabed. If the seabed is fixed, the velocity of the fluid in contact with the boundary should be zero.

Figure 3.3.16. Cinematic boundary condition on a sloping impermeable seabed



3.4.5.2.2 DYNAMIC BOUNDARY CONDITION

The dynamic boundary condition specifies the distribution of pressures (and when relevant, tangential forces) at a given boundary. Generally speaking, it establishes a relation between acting forces, the cinematic characteristics of the fluid, and the distribution of pressures and tangential stresses at the boundary. There is no expression for this condition since it depends on the characteristics of the problem being analyzed.

3.4.5.3 Types of boundary conditions in analytical and numerical models of wave transformation

Generally speaking, there are three types of boundary conditions in these models: (i) the Dirichlet boundary condition that specifies the value of the potential or of the free surface of the boundary; (ii) the Neuman boundary condition that refers to the flow through the boundary; (iii) the mixed boundary condition which establishes a relation between inflowing and outflowing fluids in the boundary and the local velocity potential. Open boundary conditions (artificial boundaries), partially reflecting boundary conditions, or open sea boundary conditions (outer boundary) are usually defined as mixed boundary conditions, and are obtained from the parabolic approximation of the exact radiation condition (except in the case of one-dimensional problems).

Parabolic models should specify the characteristics of the wave train at the entrance of the domain. Elliptic models require the specification of boundary conditions in the entire domain perimeter. This should be closed. When waves reach a breakwater, beach, or coastline, one part of their energy dissipates because of breaking and transmission, another part is reflected, and still another part is transmitted to the other side of the domain. This partition of incident energy should satisfy the energy conservation equation. Consequently, when there is no transmission, the energy dissipated at a boundary should be the same as the energy reflected from that boundary.

However, it is important to remember that the reflected energy in an MSPE model depends on the angle of incidence. Thus, there is always a certain ambiguity in the establishment of boundary conditions in elliptic models. For this reason, it is advisable to obtain results with slight modifications in boundary conditions, analyze the variability of these results, and mark the possible deviations produced by this effect. It is also necessary to underline the possible appearance of spurious reflections due to the parabolic approximation of the radiation condition. In order to control these undesirable effects, at least second-order approximations should be applied, and optimal coefficients selected, according to the minimax approximation [(10)].

3.4.6 Hydraulic regimes in wave breaking and calculation models

In deep water, wave breaking occurs when wave crests steepen as the amplitude increases to the point at which the crest of the wave actually overturns or spills. This can happen with or without the help of the wind and the interaction of the waves (and current) during the process of generation and development. The maximum steepness of a gravity wave (Stokes regime) in deep water is limited.

$$\left(\frac{H}{L_0}\right)_{lim} \lesssim \frac{1}{7} \tag{3.65}$$

In medium and shallow depths, waves break because of one or both of the following factors:

1. The progressive reduction of depth during wave propagation (progressive wave trains) with or without the help of the wind;
2. The interaction between: (a) waves with a different propagation direction (stationary and partially stationary wave trains); (b) the wave and other long waves; (c) waves and currents.

In the first case, breaking occurs because of the gradual steepening of the progressive wave train. In the second case, breaking occurs because of one of the following:

- Case (a): local steepening (in the antinodes) of the stationary wave train or local steepening (in the pseudo-antinodes) of the partially stationary wave train.
- Case (b): temporal changes of water depth because of long wave oscillations.
- Case (c): wave steepening because the wave encountered a current flowing in the opposite direction.

3.4.6.1 Breaking criteria

To obtain the height, depth, and type of breaking of the wave train, it is necessary to adopt a breaking criterion. It is customary to consider that breaking occurs (i.e. Stokes conjecture) either when the particle velocity in the crest exceeds the celerity of the wave, or when the acceleration underneath the crest exceeds the acceleration of gravity.

(10) The minimax approximation is a minimization of the maximum error of given number of terms.

3.4.6.2 *Wave breaking because of decreasing depth*

The characteristics of the breaking wave depend on the slope, wave steepness, and the relative depth, or a combination of all of these factors.

$$f\left(\frac{H}{L},\frac{h}{L},tg\beta\right)=0 \quad (3.66)$$

$$f\left(I_r,\frac{h}{L}\right)=0 \quad (3.67)$$

$$I_r=\frac{tg\beta}{\sqrt{\frac{H}{L}}} \quad (3.68)$$

On a horizontal seabed, when one of the previous breaking criteria is fulfilled, the wave spills. When the bottom slope is very slight, the wave gradually evolves until it finally breaks, thus continuing until its total extinction. When the bottom slope is steeper, the wave not only breaks, but is also reflected. Its cinematic characteristics and form of breaking depend on the incident and reflected wave trains (phase and reflection coefficient).

Note. *The seabed slope influences how waves break. Generally speaking the slope depends on the nature of the seabed. Mild slopes often correspond to soft muddy bottoms; medium slopes correspond to sandy beds (fine and coarse sand); and the steepest slopes to beds of very coarse sand, gravel, and to protective breakwaters. In these last two cases, there is usually a change in slope between the shoaling zones and the breaking zone of the wave train.*

3.4.6.2.1 *Types of breakers*

Generally, when breaking occurs because of a gradual or sudden change in the seabed, wave breaking falls into the categories shown in Table 3.3.10,

Table 3.3.10. Wave breaking types and their respective Iribarren number

Ir	Breaker types
< 0.5	Spilling breakers
$0.5 < I_{r,I} < 2.3$	Plunging breakers
> 2.3	Collapsing/surging breakers

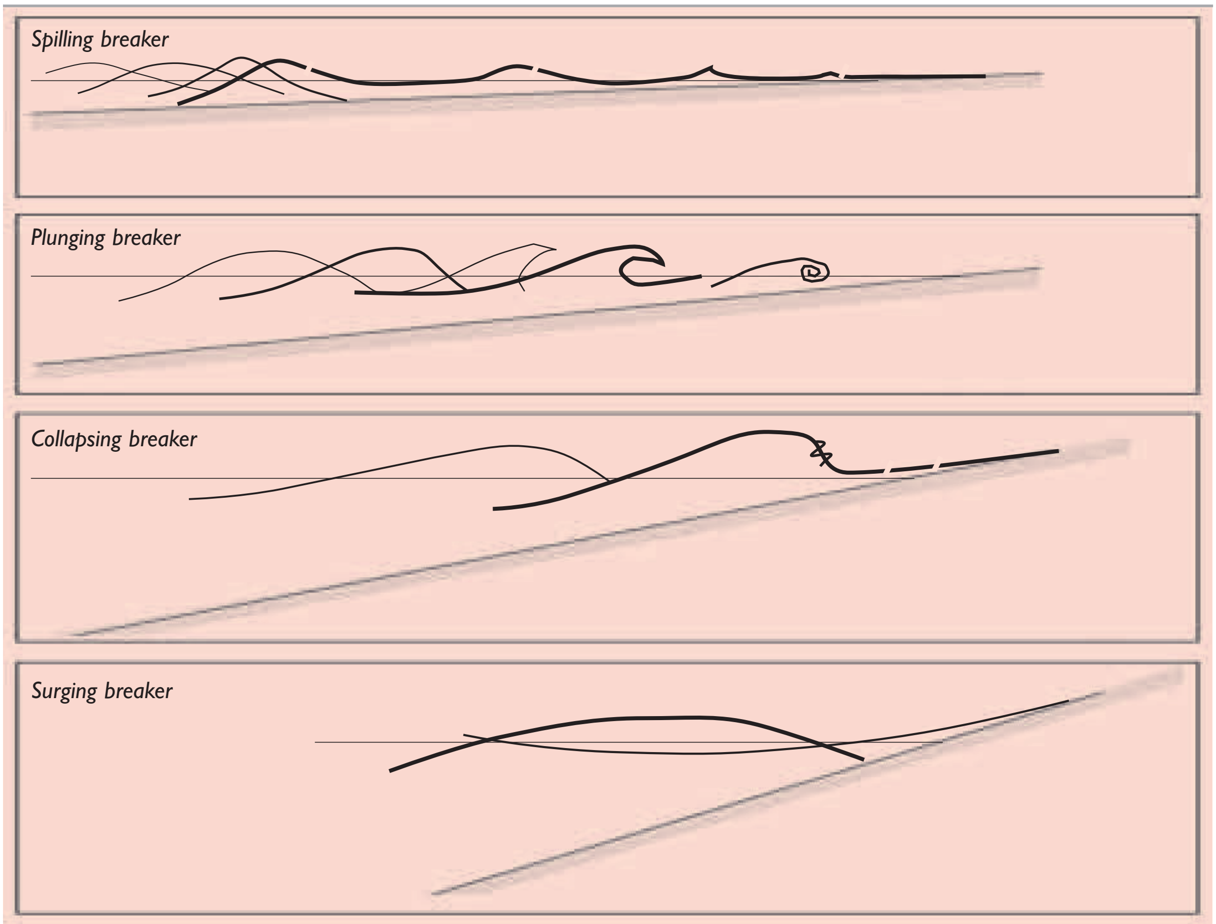
I_r is defined as the quotient of the seabed slope and the relative wave steepness.

$$I_r=\frac{\tan\beta}{\sqrt{\frac{H_b}{L_{z0}}}}\approx 1.25T_z\frac{\tan\beta}{\sqrt{H_b}}$$

where L_{z0} is the wavelength in deep water [11], and tanβ is the representative seabed slope in the breaking zone, and H_b, the wave height at the breaking point. Spilling breakers are most commonly associated with dissipative beaches, whereas plunging, collapsing, and surging breakers most frequently occur on reflecting beaches and sloping breakwaters (see Figure 3.3.17).

(11) The definition of the Iribarren number can also be applied to the state variables: significant wave height and mean zero-crossing period.

Figure 3.3.17. **Types of breaker on an impermeable slope**



3.4.6.3 Hydraulic regimes of breaking waves

For wave trains on the wave band, there are four breaking regimes, which depend on the seabed slope. A functional relation between adimensional monomials is recommended for each regime. The wave is said to break when its steepness or relative height exceeds the value of the functional relation (see Table 3.3.11).

Table 3.3.11. **Hydraulic regimes of breaking waves**

$tg\beta = m$	Approximation	Breaking criterion	Breaking regime
$m \leq 1/120$	$tg\beta \Rightarrow 0$	$(H/L)_b > f(h/L)$	Miche
	$(tg\beta,\ h/L) \Rightarrow 0$	$(H/h)_b \geq cte$	Breaker index
$1/120 < m \leq 1/40$	$I_r \lesssim 0.5$	$(H/h)_b = f(I_r)$	Dissipative
$1/40 < m \leq 1/7$	$0.5 \lesssim I_r \lesssim 3.0$	$(H/L)_b > f(I_r,\ h/L)$	Breaking-reflection I, II
$1/7 < m \leq 1/1.5$	$1.5 \lesssim I_r \lesssim 7.0$	$(H/L)_b > f(I_r,\ h/L)$	Protective slope

3.4.6.3.1 MICHE WAVE-BREAKING CRITERIA, BREAKING INDEX, AND DISSIPATIVE REGIMES

The evolution of the wave before and after entering the surf zone is assumed to be gradual, and there is no interaction between different wave trains. If the seabed slope does not change, once the wave has broken, it is said to satisfy the condition that the relative wave height is constant in the surf zone. This is equivalent to a uniform dissipation rate per unit of horizontal surface in the surf zone. The surf zone is thus said to be saturated.

Simple numerical solutions have been proposed that incorporate the parametrization of a body of water (roller) that travels with the wave, and represents the breaking wavefront (bore). These solutions are useful in order to model the circulation system induced by processes related to spilling and small plunging breakers. However, as yet, they have not been used to evaluate the pressure field on a sloping vertical plane, and to obtain the horizontal and vertical forces acting on it.

Miche regime: $m \leq 1/120$ $(I_r < 0.1)$:

$$\left(\frac{H}{L}\right)_b \lesssim \alpha_{Mb} \tanh(kh) \qquad \alpha_{Mb} \approx 0.14 \tag{3.69}$$

Breaking index regime: $m \leq 1/120$ $(I_r < 0.5)$:

$$\gamma_b = \left(\frac{H}{h}\right)_b \approx 0.8 \tag{3.70}$$

Dissipative regime: $1/120 < \text{m} \lesssim 1/40$ $(0.1 < I_r < 2.3)$:

$$\gamma_b = \left(\frac{H}{h}\right)_b \approx b_1 I_r^{b_2} + b_3 \qquad b_1 \approx 1, b_2 \approx 0.17, b_3 \approx 0.08 \tag{3.71}$$

3.4.6.3.2 BREAKING-REFLECTION REGIMES

A wave breaks because of reflection, and the phase determines the breaking point of the wave, as occurs in sudden slope changes of the seabed or at the toe of sloping breakwaters. If the breaking point is known, and in addition, the seabed slope does not change, the evolution of the wave after breaking can be represented by a bore model for overall equilibrium, which assumes that once the wave has broken, it becomes stable with a constant mean energy flux per unit of horizontal surface in the surf zone.

Non-violent plunging breakers can be analyzed by the roller model. In the case of violent plunging breakers, the problem can be numerically resolved by integrating complete sets of Navier-Stokes or Reynolds-averaged equations by means of advanced numerical techniques (e.g. finite volumes).

For collapsing and surging breakers, the movement of the wave over the slope is perceived as a swift ascent of the water and its subsequent descent. Its adimensionalized run-up and run-down in regards to wave height depends on breaker type and seabed characteristics (slope, roughness, and permeability). This type of breaker can also be analyzed by numerically integrating complete sets of Navier-Stokes or Reynolds-averaged equations.

When the slope is impermeable and the reflection can be assumed to be almost perfect (surging breaker), the resolution of non-linear long-wave [12] equations (Airy equations) on the slope provides information regarding the flow kinematics and dynamics of water on the slope. The numerical solution of these equations accurately specifies the wavefront or bore, considering collision conditions and incorporating terms of localized energy dissipation.

(12) See, for example, *Nonlinear long waves in shallow water,* (Mei 1989, Chapter 11)

Breaking-reflection regime I: $1/40 \lesssim m \leq 1/15$ **(plunging, collapsing, and surging breakers):**

$$\left(\frac{H}{L_0}\right)_b \approx 0.0277 \tanh\left[\frac{\beta_{lim}}{2\pi}\left(\frac{h}{gT^2}\right)^{\gamma_{lim}}\right]$$
$$\beta_{lim} = 17.7 \quad \left(m < \frac{1}{15}\right) \tag{3.72}$$
$$\gamma_{lim} = 0.90 \quad \left(m < \frac{1}{10}\right)$$

Breaking-reflection regime II: $1/15 < m < 1/7$ **(plunging, collapsing, and surging breakers):**

$$\left(\frac{H}{L_0}\right)_b \approx 0.0277 \tanh\left[\frac{\beta_{lim}}{2\pi}\left(\frac{h}{L_0}\right)^{\gamma_{lim}}\right]$$
$$\beta_{lim} = 152m + 6.6 \quad \left(m \geq \frac{1}{15}\right) \tag{3.73}$$
$$\gamma_{lim} = 1.92m + 0.72 \quad \left(m \geq \frac{1}{10}\right)$$

Protective breakwater regime: $1/7 < m \leq 1/1.5$ **(plunging and surging breakers):**

$$\left(\frac{H_I}{L}\right)_{lim} = \left(0.11 + 0.33\frac{1-|R|}{1+|R|}\right)\tanh kh \tag{3.74}$$

Equation (3.74) is an analytical approximation with a certain experimental basis, but it does not include the reflection phase. For this reason, it does not provide information concerning the point at which the breaking occurs.

Note. *In the case of collapsing breakers, the period of wave run-up and run-down on the slope is approximately equal to the wave period. This causes a resonant condition that Iribarren identified as critical for the stability of the components of the main layer of breakwaters.*

3.4.6.4 Wave train evolution in the surf zone

A flat seabed with only a mild slope produces spilling breakers. In this case, the surf zone is said to be saturated, in other words, the relative height is constant:

$$\gamma(x) = \frac{H(x)}{h(x)} = \text{cte} \tag{3.75}$$

The mean sea level progressively increases as the distance to the coast becomes smaller. It thus goes from a negative value at the breaking point to a positive value at the coastline. This significantly contributes to the flood line.

3.4.6.4.1 Evolution of wave train energy in the irregular surf zone

In these conditions, the wave after breaking should be represented by the superposition of the oscillating part of the wave upon which the roller rides. The roller extracts energy from the wave by rotating on it. Wave train propagation in the surf zone can be obtained by resolving the equation system composed of mass and momentum conservation equations (including the roller contribution), and the middle equation.

Sometimes, the bathymetry shows a gradual change in the seabed as its slope becomes gentler. In the case of spilling breakers or non-violent plunging breakers, the breaking wave experiences a rapid transformation, and then its height becomes more stable, rapidly reaching a state of equilibrium. In this case the variation in energy flow dissipation) in the surf zone depends on the difference in incident flows and stabilization:

$$\frac{dF_I(x)}{dx} \approx -\frac{\kappa}{h(x)}\left[F_I(x) - F_e\right] \qquad 0.15 \lesssim \kappa \lesssim 0.20 \tag{3.76}$$

where κ is an empirically evaluated constant.

Note. This approximation can be used to obtain the energy reduction of each of the components of the wavebreaking frequency spectrum, uniformly distributing the dissipation among frequency components. It can also be applied to obtain the probability function of the wave height at a point in the surf zone.
With the required conditions, this problem can be resolved by applying the models included in the Sistema de Modelado Costero *(SMC), a public domain software program available at: www.smc.unican.es.*

3.4.6.5 Sea level variation because of wave breaking

The mean sea level due to wave action can be defined by the following expression:

$$\overline{\eta_w}(x) = \frac{1}{T_z}\int_t^{t+T} \overline{\eta_w}(x,t)\,dt \tag{3.77}$$

When the seabed slope is mild and approximately constant, the drop in sea level before the breaking point can be calculated by the following expression:

$$\overline{\eta_{w,sd}} = -\frac{1}{8}\frac{kH^2}{\sinh(2kh)}, \quad x \geq x_b \tag{3.78}$$

And at the breaking point by:

$$\left(\overline{\eta_{w,sd}}\right)_b = (\gamma_I)^2_{lím}\frac{h_b}{16}, \quad x = x_b \tag{3.79}$$

where h_b is the depth at the breaking point; and x_b is the distance of the breaking point from the coastline, where the source of the coordinates is located. Downdrift from the breaking point, the sea level progressively begins to rise until reaching the coastline $(x = 0)$. For a flat seabed, a mild slope, and a saturated surf zone, this increase can be calculated by the following expression:

$$\overline{\eta_{w,su}}(x) = \left(\overline{\eta_{w,sd}}\right)_b + \frac{3\frac{(\gamma_I)^2_{lim}}{8}}{1+3\frac{(\gamma_I)^2_{lim}}{8}}\left[h_b - h(x)\right], \quad x \leq x_b \tag{3.76}$$

It is customary to also use these expressions for plunging breakers though their prediction capacity decreases as the plunge affects a larger portion of the water column.

Note. The horizontal line over the variable indicates that locally $\eta_{su}(x)$ is caused by the movement of the wave train. The magnitude slowly varies in the state, according to the evolution in wave height. This evolution can be statistically analyzed by sampling the data records of the mean sea level every Δt, and using this sample to infer a probability model. In the surf zone, the mean sea level rises. However, when wave breaking is forced by reefs or submerged breakwaters, downdrift from them, the waves stop breaking and the mean sea level falls. This difference in sea level generates a water circulation above the reef, which makes the water flow parallel to the coastline.
The equation that governs middle movement is the conservation equation for the horizontal momentum averaged in a period. The force per surface unit that causes this movement is the horizontal gradient of the radiation stress.

3.4.6.5.1 Sea level transformation models

When waves propagate in medium and shallow depths, they are transformed. As a result, elements that vary are their averaged quantities in a period or wavelength, their total energy E, and the radiation tensor S_{ij}. Spatial variations of these magnitudes can be calculated based on the results obtained from wave propagation models. Since these are averaged magnitudes of the second order, generally speaking, if there is no wave breaking, it is sufficient to analyze wave transformation with linear models (e.g. MSPE or Ref-dif if reflection is negligible). In any case, such variations cause changes in sea level. This process can be studied by applying depth-integrated and depth-averaged equations in a period mass and momentum conservation. Generally speaking, the solution should be obtained numerically.

Note. This can be done, for example, by applying the COPLA numerical code (SMC, University of Cantabria).

3.4.6.6 Wave breaking in crossed seas

When the wave train is formed by incident and reflected wave trains, the breaking process is somewhat different from the previous description. When both wave trains travel in the same direction, but in opposite senses and both crests meet, there is in an upwards vertical movement of the water to a height equal to the sum of the individual wave heights. This results in a slight reduction in the wave height of each of the wave trains and a lag in their propagation.

The role played by each wave is to act as a vertical wall for the other. If the seabed has a mild slope, it is advisable to apply the wave breaking criterion associated with stationary wave steepening (See Annex on wave train transformation in the surf zone). This can be done by estimating the representative reflection coefficient for one of the wave trains, either because its source is known (e.g. a breakwater) or because it is directly estimated. When the encounter is oblique, the amplitude of the vertical moment decreases with the angle between wave crests. The breaking criterion can be relaxed with the cosine of the angle between crests.

3.4.6.7 Wave breaking in the presence of a current

The effect of the current depends on its forward direction in regards to the direction of the wave train propagation. If the current and the wave are traveling in the same direction, the action of the current reduces the steepening of the wave, and thus, causes a delay in wave breaking. In contrast, when the wave and the current are traveling in opposite directions, this generally intensifies wave steepening, and makes the wave break earlier.

When the seabed has a very mild slope, it is possible to apply the boundary steepness criterion as long as this is calculated by using the wavelength obtained from the dispersion equation. For this calculation, factors to be taken into account are the current and the wavelength in the presence of the current, which usually will be greater than when the current is not present. The wave height in the presence of the current can be approximated by applying the energy conservation equation (without dissipation):

$$(EC_g)_0 = (EC_g)_c \qquad (3.81)$$

where the subscript 0 represents the energy flow of the wave train before it meets the current, and the subscript c defines the energy flux in the field of the current. $C_{g,c}$ is the propagation velocity of the energy,

$$C_{g,c} = \frac{1}{2} C_c \left[1 + \frac{2k_c h}{\sinh 2k_c h} \right] \qquad (3.82)$$

where C_c and k_c are the celerity of the wave train and the wavenumber in the presence of the current, calculated from the dispersion equation with the current (see section 3.4.1.2.1).

Since the energy flux is a vectorial quantity, in the dispersion equation, the current velocity should appear projected onto the propagation direction of the wave train.

3.4.6.8 Selection of a wave breaking model

When the operating conditions of the port and shoreline area or the net sediment transport rate depends on the wave breaking process, it is necessary to determine the characteristics (e.g. probability model and spectral density function, or by means of state descriptors) in the surf zone. In these cases a wave breaking regime should be adopted in accordance with Table 3.3.12.

Table 3.3.12. Selection of a wave breaking model

Breaking regime	Post-breaking evolution	Boundary
Miche	Wave train	Horizontal seabed
Breaking index	Saturation, $\gamma(x) = cte$	Dissipative
Dissipative	Estabilizing energy flux	Dissipative/medium
Breaking-reflection I, II	Movement up/down slope	Intermediate/reflecting
Protective slope	Movement up/down sslope	Sloping breakwater

Breaking regime	Breaker type	Reflection	Boundary
Miche	Ocassional	0	Horizontal seabed
Breaking index	Spilling breaker	~ 0	Dissipative
Dissipative	Spilling/plunging	$K_R < 0.15$	Dissipative/medium
Breaking-reflection I, II	Plunging/surging	$K_R > 0.25$	Medium/reflecting
Protective slope	Plunging/surging	$K_R > 0.35$	Sloping breakwater

Although wave breaking is a non-linear process, in most port and harbor area applications, it is sufficient to use a phase-averaged propagation model based on linear theory. Since there will always be some degree of uncertainty in the results, it is thus advisable to adopt a certain safety margin, especially when the relative depth is very small. In such cases and in shoreline areas in which circulation dynamics is forced by wave breaking, a non-linear model of wave transformation should be applied.

3.5 SHORT-TERM DESCRIPTION OF WAVE ACTION

The most basic description of the irregular wave train is performed in the time interval known as *sea state* (of short duration). This state is made up of a set of state descriptors (i.e. wave height, wave period and propagation direction). These descriptors are representative of its statistical variability, for example, significant wave height, zero-crossing period, and mean direction. When the wave height periodically varies in time, the wave train is said to form a wave group. This section recommends methods and tools for describing and characterizing sea states. See the section of the Annex pertaining to wave action for a summary of fundamental premises of sea state description, the definition of basic and instantaneous variables, and the definition of state descriptors.

3.5.1 Introduction

In a sea state, the values for wave height and wave period (H_z, T_z) are random variables with a joint probability distribution, whose parameters depend on state descriptors, which also depend on the 'age' (13) of the waves and relative depth, and thus on the sea level.

(13) The "age" of waves is defined by the quotient of their celerity and the velocity of the generating wind (C/U_{10}) which, among other things, depends on the duration of the generation and fetch. During the generation phase, the sea is said to be partially, and $(C/U_{10}) < 1$, whereas at the end of this phase, the sea is said to be fully developed $(C/U_{10}) \gtrsim 1$.

3.5.1.1 Importance of the relative depth and the oscillatory regime

The probability and wave energy density functions significantly depend on the relative depth of the oscillatory regime. Generally speaking, the Gaussian model can provide an adequate description of the sea state in the open sea. As waves steepen due to shoaling, and the spatial gradients contributing to refraction, reflection, diffraction, and breaking are no longer negligible, the Gaussian model loses validity. Nevertheless, this model continues to be a useful tool, and sufficient for the project design of many maritime structures. In fact, in many cases, it is the only available tool.

However, with a view to specifying the approximation margin and determining the advisability of expanding deep water theory to other depths, it is necessary to identify four regions for wave generation and transformation, depending on the extension of oscillatory motion to the bottom and the eventuality of wave breaking,

- **Open sea or deep water**, $h/L > 1/2$: oscillatory motion does not reach the seabed, and even if this happens, this causes short-wave [14] breaking, which is associated with the generation process of waves with smaller periods.
- **Outer continental shelf, medium depths** $1/10 < h/L < 1/2$: oscillatory motion reaches the seabottom at a speed significantly lower than at the surface, where short wave breaking occurs, associated with the generation process of waves with smaller periods.
- **Inner continental shelf, medium/shallow depths**, $1/20 < h/L < 1/10$: oscillatory motion reaches the seabed at a slower velocity than at the surface. It is possible to find short wave breaking associated with short wave periods, and occasionally, bottom-induced breaking of waves reaching critical steepness, with longer periods and great height.
- **Surf zone in general**, $h/L < 1/20$: oscillatory motion reaches a velocity approximately equal to that of the surface, and the majority of the waves break due to bottom effects associated with critical steepness or critical wave height.

3.5.1.2 Wave generation and transformation processes and spatial domains

When there is not sufficient information about the project location, it is necessary to use the available data pertaining to the atmospheric and maritime agents for the open sea, and adjust them so that they are valid for the site.

In medium and shallow depths, statistic and spectral characteristics of the sea states depend on the following: (1) water depth and its stratification; (2) the form, nature and relative variation kh of the seabed; (3) velocity and direction of the current and wind: (4) the obstacles and shape of the boundaries encountered by the waves in their propagation.

The spectral density and probability density functions change in respect to those for the open sea. In some cases, their functional structure is known, and it is thus possible to apply specific expressions in the same way as in deep water. This section describes some of the expressions that can be used in preliminary or pre-project studies.

The selection and application of wave generation and transformation models should be carried out, depending on the relevance of the processes involved. Some of the most significant are listed in Table 3.3.13.

3.5.1.3 Models and domains

Table 3.3.13 defines five spatial work domains: *(I)* ocean and open sea; *(II)* outer continental shelf; *(III)* inner continental shelf and shoreline areas, including the surf zone; *(IV)* port and harbor areas; *(V)* confined relatively shallow seas. The importance of relevant wave generation and transformation processes in each domain is determined by the quotient of the characteristic wave length, which is necessary for there to be significant change in the wave field and the dimensions of the domain.

(14) Short wave breaking is the process by which short-period waves in the sea state break either because of wind action or because they interact wwith waves of a larger period. They are characterized by their whitecaps that are easily detected on the sea surface.

Table 3.3.13. ***Physical processes of numerical wave transformation models***

Process	Ocean	Platform	Shoreline area	Ports
Wind acttion	Yes	Yes	No/Yes	No
Our-mode interaction	Yes	Yes	No/Yes	No
Three-mode interaction	No	No	Yes/No	No/Yes
Current refraction	No	Yes/No	Yes/No	No
Bottom refraction	No	Yes	Yes	Yes/No
Bottom friction	No	Yes	Yes	No/Yes
Obstacle-induced diffraction	No	No	No/Yes	Yes
Steepness-induced breaking	Yes	Yes	No/Yes	No
Bottom-induced breaking	No	No/Yes	Yes	No

3.5.2 Description of the open sea state (deep water)

A representative sample of the wave oscillations at a certain point in the sea can be analyzed in the time domain or frequency domain. This is known as a statistical or a frequency description, respectively, of the sea state. It is based on mathematical-statistical wave theory (see Annex). These are short-duration descriptions or characterizations of the sea state. Within this context, it is possible to admit that the variability of statistical or frequency state descriptors is not significant from an engineering perspective, and that the instantaneous variables, η, and the basic variables (H_z, T_z, θ) follow probability models whose statistical parameters are state descriptors, and which conserve the shape of the directional and frequency spectrum.

Generally speaking, descriptive models of the sea state in the open sea can also be applied to the outer continental shelf, where the relative depth satisfies $h/L_p > 1/2$.

3.5.2.1 *Probability function of* η

The vertical displacement of free surface with respect to a fixed level of reference is generally regarded as a Gaussian process. Once the reference level is selected so that $\overline{\eta} = 0$, η follows a Gaussian probability model with a zero mean and a typical deviation, σ_η, in other words, $N(0, \sigma_\eta)$. σ_η^2 is the variance of the process, and also quantifies its energy content,

$$p(\eta) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left[-\frac{\eta^2}{2\sigma_\eta^2}\right] \qquad \sigma_\eta^2 = \eta_{rms}^2 \tag{3.83}$$

where η_{rms} is the mean square displacement.

Note. *This statistical-mathematical model is no longer adequate when wave action becomes asymmetrical with respect to the mean sea level, as occurs in shallow water and in the surf zone. In this context, the process is no longer Gaussian. However, in the majority of practical applications, the Gaussian model provides a sufficient approximation.*

3.5.2.2 *Probability functions of individual wave height*

Zero-crossing wave height is defined as the arithmetic sum of the wave crest amplitude and the trough amplitude. In deep water and when there are swell waves (narrow-band spectrum, theoretically zero), the correlation between the consecutive crest and trough amplitudes is perfect, and the wave height density function is the Rayleigh function:

$$p\left(\frac{H}{H_{rms}}\right) = \frac{2H}{H_{rms}^2} \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] \qquad (H \geq 0) \tag{3.84}$$

and its distribution function is:

$$\Pr\left(\frac{H}{H_{rms}}\right) = 1 - \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right], \quad (H \geq 0) \tag{3.85}$$

This function can also be expressed by the mean wave height or the significant wave height as distribution parameters. The probability q of the wave height exceeding a specific value H_q is:

$$q = 1 - F\left(\frac{H_q}{H_{rms}}\right) = \exp\left[-\left(\frac{H_q}{H_{rms}}\right)^2\right], \quad (H_q \geq 0) \tag{3.86}$$

When the correlation between crests and troughs is zero, then according to certain hypotheses the wave height distribution function can be approximated by,

$$\Pr\left(\frac{H}{\sqrt{m_0}}\right) = 1 - \left[\exp\left(-\frac{H^2}{2m_0}\right) + \frac{\sqrt{\pi}}{2}\frac{H}{\sqrt{m_0}}\exp\left(-\frac{H^2}{4m_0}\right) erf\left(\frac{H}{2\sqrt{m_0}}\right)\right] \tag{3.87}$$

$$erf(X) = \frac{2}{\sqrt{\pi}} \int_0^X \exp\left(-u^2\right) du \tag{3.88}$$

Note. *In practice, the correlation between the values of crests and troughs will be in the interval* $[0,1]$. *For this reason, its joint distribution can be a bivariate Rayleigh distribution. This model is used to derive the joint distribution function of consecutive wave heights, as applied to the analysis of wave groups (see section 3.8).*

3.5.2.2.1 RELATION BETWEEN WAVE HEIGHT DESCRIPTORS

Under the assumption that wave heights follow a Rayleigh distribution function, Table 3.3.14 shows the relations between wave height descriptors, based on the Rayleigh distribution.
m_0 is the zero-order spectral moment.

Table 3.3.14. Relations between Rayleigh distribution statistics

Height	H/H_{rms}	$H/\sqrt{m_0}$	H/H_S
H_{rms}	1.0	$2\sqrt{2}$	0.706
Mode, $\hat{H}$	$1/\sqrt{2}$	2	0.499
Median, $\tilde{H}$	$(\ln 2)^{1/2}$	$(8 \ln 2)^{1/2}$	0.588
Medium, $\bar{H}$	$\sqrt{\pi}/2$	$\sqrt{2\pi}$	0.626
Significant, H_S	1.416	4.005	1.00
$H_{1/10}$	1.80	5.091	1.271
$H_{1/100}$	2.359	6.672	1.666

3.5.2.3 Probability function of the r-order statistic r

Given a sample of N values in ascending order, the r^{th} element of this new sequence is called an r-order statistic. The probability function of the number of elements or r-order statistics, which in an ascending set of storm peaks of size N are less than or equal to a given value, is a binomial.

$$\Pr\left(H_{r,N}\right) = \sum_{k=r}^{N} \binom{H}{r} F^k(H)\left[1 - F(H)\right]^{N-k} \tag{3.89}$$

where $F(H)$ is the waveheight distribution (generally the Rayleigh function, see section 4.10.6.3.).

Note. The distribution function $Pr(H_{r,1000})$ of the statistics of a sample of 1000 waves is:

$$\sum_{k=r}^{1000}\binom{1000}{r}\left[1-\exp\left(-\frac{H^2}{H_{rms}^2}\right)\right]^k \exp\left[\left(-H^2/H_{rms}^2\right)(1000-k)/2\right] \tag{3.90}$$

If an operational stoppage mode occurs in a certain subset of a structure when $H/H_{rms} \geq 4.8$, the probability of a stoppage not occurring in a set of 1000 waves is:

$$p = \Pr_{1000,1000}(4.8) = 0.99$$

If the subset of the structure withstands five waves with a wave height greater than $H/H_{rms} \geq 3.5$, the probability that it will be able to withstand 1000 waves is:

$$p = \Pr_{996,1000}(3.5) = 0.92$$

3.5.2.3.1 PROBABILITY FUNCTION OF MAXIMUM WAVE HEIGHT, $H_{max,N}$

In a sea state composed of H waves with a mean square wave height H_{rms}, the distribution function of the maximum wave height, $H_{max,N}$, or greatest height of the N waves (order statistic where $r = 1$) is:

$$\Pr\left(\frac{H_{max,N}}{H_{rms}}\right) = \left[\Pr\left(\frac{H}{H_{rms}}\right)\right]^N \tag{3.91}$$

where $\Pr(H/H_{rms})$ is the Rayleigh distribution function. The mode or most probable value of the maximum wave height depends on the number of waves and the mean square wave height:

$$\frac{\bar{H}_{max,N}}{H_{rms}} \simeq \sqrt{\ln N} \tag{3.92}$$

The mean wave height of the maximum wave height of N waves is:

$$\frac{\bar{H}_{max,N}}{H_{rms}} \simeq \sqrt{\ln N} + \frac{\gamma_E}{2}\frac{1}{\sqrt{\ln N}} \tag{3.93}$$

where $\gamma_E = 0.5772$. The largest waves can occur with a period approaching the significant period, in other words, the mean value of the top third of the larger wave periods.

If the probability of exceedance of p_{Hmax} of a wave height, H_0 (e.g. the design wave height) is defined by:

$$Pr[H_{max} > H_0] = p_{Hmax} \tag{3.94}$$

then this fulfills the following relation between wave height, H_0, the number of waves, and the probability of exceedance:

$$H_0 \simeq H_{rms}\sqrt{\ln\frac{N}{\ln\frac{1}{1-p_{H_{max}}}}} \tag{3.95}$$

Note. When the Rayleigh function is regarded as the waveheight distribution function, the wave height is thus assumed to be twice the wave amplitude, and that each of the waves are statistically independent events. When this is not acceptable, it is necessary to define the wave distribution as the joint distribution of two amplitudes separated by a certain time interval. For this case in particular, these two amplitudes, considered to be statistically independent, should be separated by the mean semi-period of the process.

The Raleigh distribution usually overestimates the probabilities of occurrence of the largest and smallest wave heights in the data record. The reasons for this deviation can be attributed to the non-fulfillment of the initial hypotheses. This refers to the spectral width, the statistical independence of successive waves, and the non-linearity and asymmetry of waves. Generally speaking, the Rayleigh distribution function does not fit well to experimentally obtained histograms for values of $\varepsilon > 0.5$. However, the statistical descriptors obtained by applying the Rayleigh distribution can be used with a high degree of reliability.

3.5.2.4 Wave height of multiple wave trains

When the waves that reach a point are formed by the linear superposition of different wave trains, the joint free surface can be expressed by:

$$\eta_* = \eta_I + \eta_{II} \tag{3.96}$$

If η_I and η_{II} are zero-mean random Gaussian variables, the vertical displacement of the joint free surface η_* is also a zero-mean random Gaussian variable.

3.5.2.4.1 Density function of the total wave height

In addition, if both oscillations have a narrow-band spectrum, then the resulting height of the wave train can be distributed according to a Rayleigh function of parameter $H_{rms,*}$. The parameter value is local, in other words, it depends on the location. Its value can be calculated by the following:

$$H^2_{rms,*} = H^2_{rms,I} + H^2_{rms,II} + 2H_{rms,I}H_{rms,II}\cos\varphi \tag{3.97}$$

where $H_{rms,I}$ and $H_{rms,II}$ are the mean square wave height of the waves, and is the lag between the φ irregular wave trains. The area under the total wave action spectrum (see section 3.5.2.12) is the variance of the process, which is related to $H^2_{rms,*}$,

$$\sigma^2_{\eta_*} \approx 8H^2_{rms,*} \tag{3.98}$$

Note. *Since all breakwaters are reflective, and all alignment changes and harbor entrances radiate a significant part of the incident energy, the distribution function of the total wave height in the area surrounding the breakwater can be obtained by this approximation (section 3.5.3.2).*

3.5.2.5 Probability function of a single wave

In the open sea, the distribution of the square of wave period T_z^2, defined by the upcrossing method, more or less follows a Rayleigh function.

$$p\left(\left[\frac{T_z}{\overline{T}_z}\right]^2\right) = 2.7\frac{T_z^3}{\overline{T}_z^4}\exp\left[-0.675\left(\frac{T_z}{\overline{T}_z}\right)^4\right] \tag{3.99}$$

where $\overline{T}_z$ is the mean period of the sample. This distribution of periods is only based on empirical data.

3.5.2.6 Joint probability function of the wave height and wave period

To provide a complete description of a sea state, it is necessary to jointly consider wave heights, wave periods, and propagation directions. However, the height and period of single waves are not statistically independent variables. The dependence is greater for a young sea state in its initial generation phase, and the dependence decreases as the sea state increases in age or becomes older. In a fully developed sea state (FDS), wave height and period can be regarded as statistically independent.

3.5.2.6.1 WAVE HEIGHT AND PERIOD OF INDEPENDENT WAVES

In such cases, wave heights H and T_z^2 can be assumed to follow Rayleigh distributions, and the joint density function of their wave height and period is:

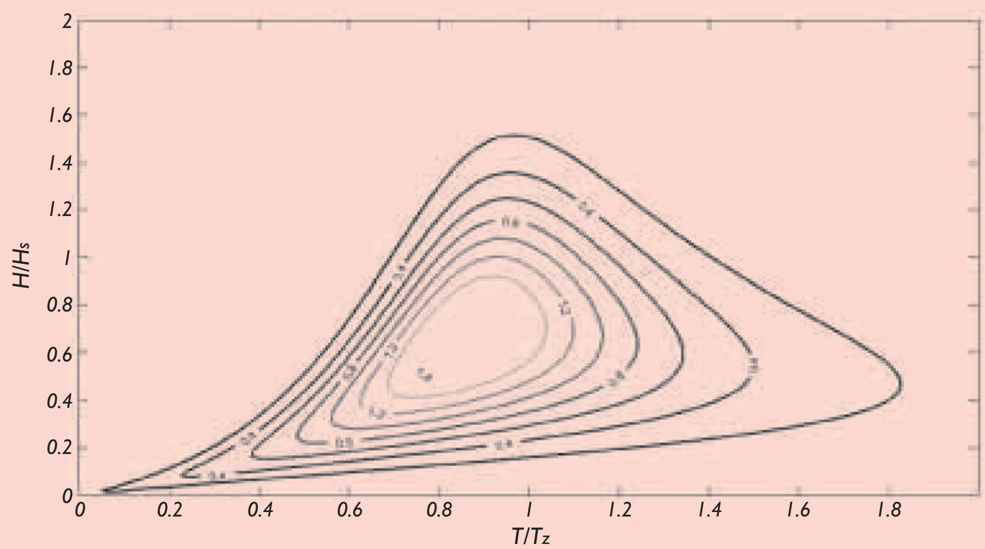
$$p\left(\frac{H}{H_{rms}},\frac{T_z}{\bar{T}_z}\right)=p_1\left(\frac{H}{H_{rms}}\right)\cdot p_2\left(\frac{T_z}{\bar{T}_z}\right) \tag{3.100}$$

where p_1 is the Rayleigh function of the wave heights and p_2 is the Rayleigh function of the wave periods (see sections 3.5.2.2 and 3.5.2.5, respectively). Both density functions refer to waves defined by the zero upcrossing method.

3.5.2.6.2 WAVE HEIGHT AND PERIOD OF DEPENDENT WAVES

When wave height and period are not independent (i.e. a narrow-band process), a joint density function of two random variables is the following (see Figure 3.3.18):

Figure 3.3.18. Joint distribution of wave heigh and period in the open sea



$$p\left(\xi,\frac{T_z}{T_{0,1}}\right)=\frac{2L(\nu)}{\nu\sqrt{2\pi}}\left(\frac{\xi}{\frac{T_z}{T_{0,1}}}\right)^2\exp\left\{-\xi^2\left[1+\frac{1}{\nu^2}\left(1-\frac{T_{0,1}}{T_z}\right)^2\right]\right\} \tag{3.101}$$

$$L(\nu)=\frac{1}{2\left[1+\frac{1}{\sqrt{1+\nu^2}}\right]}\approx\frac{1}{2}\left(1+\frac{\nu^2}{4}\right) \tag{3.102}$$

$$\nu=\left(\frac{m_0 m_2}{m_1^2}-1\right)^{1/2}$$
$$0<\xi<\infty;\quad 0<\frac{T_z}{T_{0,1}}<\infty \tag{3.103}$$

$$T_{0,1} = 2\pi \frac{m_0}{m_1} \tag{3.104}$$

where v is the spectral width; $T_{0,1}$ is the mean spectral period $(0,1)$: and ξ is the adimensional amplitude,

$$\xi = \frac{p_{ev}}{\sqrt{m_0}} \approx \frac{H_z}{2\sqrt{m_0}} \tag{3.105}$$

where p_{ev} is the amplitude (15) measured with respect to a certain reference level.

The density function mode lies in the following value pair:

$$H_z \approx \frac{2\sqrt{2m_0}}{\sqrt{1+\nu^2}} \quad \overline{T}_z = \frac{T_{0,1}}{1+\nu^2} \tag{3.106}$$

for which the value is:

$$p_{max}\left(H_z, T_z\right) = \sqrt{\frac{2}{\pi}} \frac{1}{e}\left(1+\frac{\nu^2}{4}\right)\left(\frac{1+\nu^2}{\nu}\right) \tag{3.107}$$

Once the joint density function is defined by $p(H_z,T_z)$, the marginal density function of the wave height is (16):

$$p\left(H_z\right) = \int_0^\infty p\left(H_z, T_z\right) dT_z \approx \xi \exp\left[-\frac{\xi^2}{2}\right] \tag{3.108}$$

and the density function of the wave period:

$$p\left(T_z\right) = \int_0^\infty p\left(H_z, T_z\right) dH_z = \frac{L(\nu)}{2\nu}\left(\frac{T_{0,1}}{T_z}\right)^2 \frac{1}{\left[1+\frac{1}{\nu^2}\left(1-\frac{T_{0,1}}{T_z}\right)^2\right]^{\frac{3}{2}}} \tag{3.109}$$

The mean value of this distribution is infinite. For this reason, it is advisable to work with $T_{0,1} \approx \overline{T}_z$

3.5.2.7 *Probability function of wave steepness*

Wave steepness is a measurement of the non-linearity of the wave, and intervenes in the Iribarren number to define the type of wave breaking on a slope. The density function of wave steepness can be directly obtained, based on the joint density function (H_r, T_z) by applying the derived function method (see the section in the Annex on random variables and probability).

3.5.2.7.1 *Significant wave steepness*

Significant wave steepness is defined by:

$$s_s = \frac{H_{m_0}}{L_{z,0}} \tag{3.110}$$

(15) In effect, p_{ev} is the envelope of the process. For narrow band processes, the envelope is approximately equal to the wave amplitude (i.e. half of the wave height.

(16) The marginal function of wave height should include the multiplying term $[1 + (v^2/4)\Phi(\xi/v)]$, whose value for $v = 0$ is identically equivalent to the unit, and in the interval $0 < v^2 < 0.36$, it approaches one. Thus, the distribution function of the wave heighs can be said to follow a Raleigh distribution.

This can vary with the age of the waves, in other words with their fetch F and time of generation t_g. In the Jonswap spectrum, significant wave steepness in deep water for a limited fetch and duration is, respectively:

$$s_s = 0.1081\left(\frac{\overline{U}_{10}}{F}\right)^{1/10} \tag{3.111}$$

$$s_s = 0.1717\left(\frac{\overline{U}_{10}}{t_g}\right)^{1/7} \tag{3.112}$$

where F is expressed in m, $\overline{U}_{10}$ in m/s and t_g in hours.

3.5.2.7.2 Limit wave steepness in the open sea

The waves in an irregular wave train break with somewhat lower values than those obtained for a regular wave train in deep water. For practical purposes, the following inequality (H_z in meters and T_z in seconds) should be adopted for individual waves in an irregular wave train,

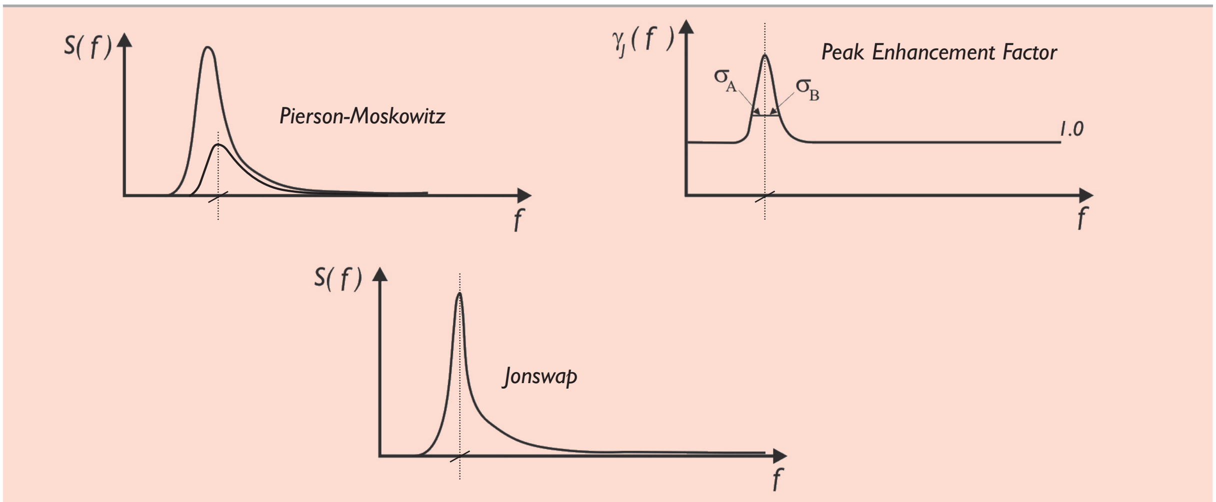
$$\left(H_z\right)_{lim} \lesssim 0.20T_z^2 \tag{3.113}$$

It is advisable to impose this maximum steepness condition when applying the density function $p(H_z,T_z)$.

3.5.2.8 Theoretical-experimental spectra in the open sea

Whenever possible, the spectral shape will be determined by specific measurements performed at the project site. If this type of data is not available and depending on the age of the waves ς and the relative depth, one of the following frequency spectra [17], $S_w(f)$, should be used (see Figure 3.3.19):

Figure 3.3.19. Spectral density function and peak enhancement function



- Pierson-Moskowitz Spectrum (P-M): for a totally developed sea and deep water;
- Jonswap Spectrum (J): for a partially developed sea and deep water;

(17) These spectra can be expressed in terms of spectral paramater or state descriptors.

- TMA Spectrum; for a developing sea and medium-depth and deep water. It can also be applied to sea states generated in medium and shallow depths.

Note. *The PM spectrum was obtained as an "average" of sea state spectra for fully developed seas. In contrast, the Jonswap spectrum was derived from initially developing or partially developed seas, including conditions of dynamic equilibrium in fully developed seas. Thus, information concerning the energetic over-saturation of its components is maintained, and as a result, of energy transfer processes. The age of the sea can be expressed as the adimensional monomials: fetch,* $\tilde{F}$ *and time* $\tilde{t}$ *:*

$$X = X\left(\tilde{F} = \frac{gF}{\left(u_{10}\right)^2}, \tilde{t} = \frac{gt}{\left(u_{10}\right)}\right) \tag{3.114}$$

where F *is the fetch or length of the generation field in the predominant wind direction and* t *is the time during which the wind is acting on the fetch. Generally speaking, these magnitudes are in* SI *units (in* m *and* s*).*

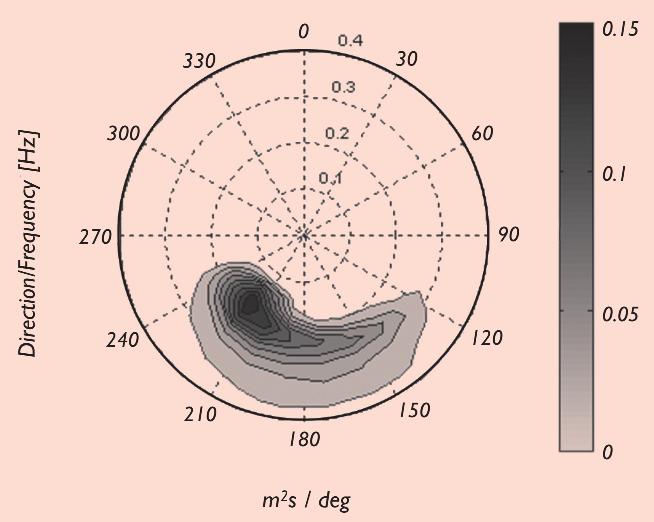
3.5.2.9 General expression of the spectral density function

In recent years, various spectral density functions have been proposed. They are based on dimensional analysis, wave generation theories, and numerous measurements of the sea in various phases of development. All of the spectra have the same basic structure and can be formulated as the product of at least four functions.

The first is the scalar spectral density function, previously defined by $S(f)$. It has two frequency ranges, a (higher) frequency range, $f > f_p$, proportional to f^{-5}; and another (lower) frequency range, $f < f_p$, proportional to f^{-4}. Another function describes the directional distribution of the energy by means of the directional spectrum, $D_w(\theta, f)$. The third function $\gamma(f)$ is known as the peak enhancement or oversaturation function, and quantifies the energy increase that a given component is capable of transporting when it is developing. In this way (see Figure 3.3.20), it is possible to distinguish the dependence of the frequency and of the direction. The spectral function can be generally expressed as:

$$S(f, \theta) = S(f)\, D_w(\theta, f)\, \gamma(f) \tag{3.115}$$

Figure 3.3.20. Frequency and directional distribution of the spectral density function



Note. *This functional structure allows the energy spectrum to incorporate other effects related to the wave generation, transformation, and extinction, which are significant at the construction site, as long as they can be described in the framework of linear wave theory.*

3.5.2.9.1 PM FREQUENCY SPECTRAL DENSITY FUNCTION

This spectral density represents fully developed sea (FDS) with a wind sea and in deep water (see Figure 3.3.21),

$$S_{PM}\left(f\right)=\alpha g^{2}\left(2\pi f\right)^{-5}\exp\left\{-0.74\left[\frac{2\pi f U_{*}}{g}\right]^{-4}\right\} \qquad (3.116)$$

where U_* is the wind speed measured at a height of 19.5 meters over the free sea surface. The modal frequency can be obtained as:

$$f_p=\frac{0.87}{2\pi}\left(\frac{g}{U_*}\right) \qquad (3.117)$$

For this reason, this spectrum only depends on one parameter, which is one speed. If it is assumed that the spectrum represents a narrow-band process, then the following simplified relations can be obtained:

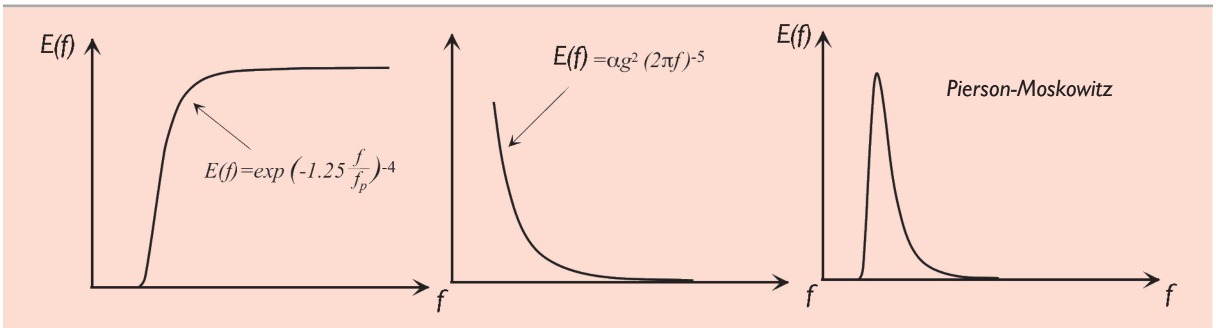
$$H_{m_0}=4\sqrt{m_0}=0.21\left(\frac{U_*^2}{g}\right) \qquad (3.118)$$

$$f_p=\frac{0.2}{\pi}\sqrt{\frac{g}{H_{m_0}}} \qquad (3.119)$$

$$S_{PM}\left(f\right)=\frac{\alpha g^{2}}{\left(2\pi f\right)^{4}}f^{-5}\exp\left\{-1.25\left[\frac{f}{f_p}\right]^{-4}\right\} \qquad (3.120)$$

where $\alpha = 0{,}0081$, $g = 9{,}81$ m/s^2; and f is the oscillation frequency in $1/s$. The spectral units of $S_{PM}(f)$ are m^2s (in SI).

Figure 3.3.21. Decomposition of the PM spectral density function in the open sea



Note. *In neutral atmospheres, approximately,* $u_{19,5} = 1{,}1u_{10}$.

3.5.2.9.2 JONSWAP FREQUENCY SPECTRAL DENSITY FUNCTION

Nevertheless, for all practical purposes, the Jonswap (J) frequency spectrum is regarded as valid for waves in the generation zone (sea), whether fully or partially developed, as well as for those that have left this zone (swells). This spectrum can be defined as shown in the following expression:

$$S_J\left(f\right)=\alpha g^{2}\frac{f^{-5}}{\left(2\pi\right)^{4}}\exp\left\{-\frac{5}{4}\left[\frac{f}{f_p}\right]^{-4}\right\}\gamma_J^{a} \qquad (3.121)$$

where γ_J is the peak enhancement factor, and can reach values in the interval $[1,7]$. Its mean value is $\overline{\gamma_J} = 3.3$, and its variance is $\sigma_{\gamma J} = 0.62$. The exponent $a(f/f_p)$ is defined by

$$a = \exp\left\{\frac{-\left(\frac{f}{f_p}-1\right)^2}{2\sigma_J^2}\right\} \quad (3.122)$$

$$\sigma_J = \overline{\sigma_a} = 0.07;\ \left(f < f_p\right) \quad \sigma_J = \overline{\sigma_b} = 0.09;\ \left(f > f_p\right)$$

where $\overline{\sigma_a}\,\overline{\sigma_b}$ are mean values obtained from a large number of data records. Parameters α and the peak frequency f_p depend on the age of the waves. For this reason, they should be expressed in terms of the adimensional fetch, as follows (also included are functional relations of the peak enhancement parameters of the spectrum with the peak frequency):

$$\tilde{F} = \frac{gF}{\overline{u_{10}}^2} \quad (3.123)$$

$$\tilde{f}_p = \frac{f_p \overline{u_{10}}}{g} = 2.18\tilde{F}^{-0.27} \quad (3.124)$$

$$\alpha = 0.0317\tilde{f}_p^{0.67} \quad (3.125)$$

$$\gamma_J = 5.870\tilde{f}_p^{0.86} \quad (3.126)$$

$$\sigma_a = 0.0547\tilde{f}_p^{0.32} \quad (3.127)$$

$$\sigma_b = 0.0783\tilde{f}_p^{0.16} \quad (3.128)$$

Note. *The adimensonal fetch units should be expressed in the international system units, for example, F in meters, $\overline{u_{10}}$ m/s, and g in m/s^2. It is not fair to compare the J and PM spectra since each was obtained in different conditions and sea development. Each type of spectrum is also based on different criteria. The PM spectrum was obtained as the mean value of spectra measured in fully developed seas. The J spectrum was obtained as representative of the main processes of wave generation in partially developed seas.*
In deep water, the spectral shape for high frequencies, $f > f_p$ fits the function f^{-5}, which is obtained by a simple dimensional argument. Admitting that the maximum energy per component (units m^2s) is limited by the breaking process, and basically depends on gravity, then:

$$S(f) \sim g^m f^n;\quad m = 2;\quad n = -5 \quad (3.129)$$

3.5.2.9.3 JONSWAP SPECTRAL PARAMETRIZATION WITH STATISTICAL DESCRIPTORS

The Jonswap spectrum can also be formulated in terms of statistical wave descriptors of the (H_s, T_s) where T_s is the significant period (mean of the highest third of the periods). The relation of T_s with the peak period T_p of the spectrum is given in 3.129, which defines the corresponding frequency spectrum for any sea state that has been characterized by its statistical descriptors (e.g. associated with extreme work conditions or in order to specify extreme sea states for physical model experiments). In these terms, the J spectrum is the following:

$$S_J(f) = \beta_J \frac{H_s^2}{T_p}\left(\frac{f}{f_p}\right)^{-5} \exp\left[-1.25\left(\frac{f}{f_p}\right)^{-4}\right]\gamma_J^a \quad (3.130)$$

where,

$$\beta_J \frac{0.0624\left(1.094 - 0.01915\ln\gamma_J\right)}{0.23 + 0.0336\gamma_J - \frac{0.185}{1.9+\gamma_J}} \quad (3.131)$$

$$T_p = \frac{T_s}{1 - \frac{0.132}{(\gamma_J + 0.2)^{-0.559}}} \tag{3.132}$$

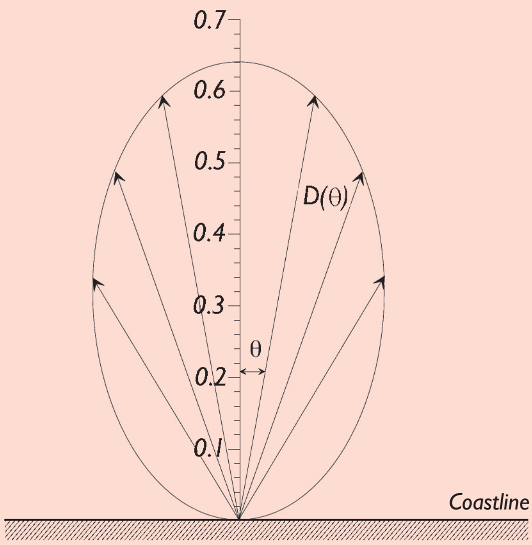
$$H_s \approx 4\sqrt{m_0} \approx 4\sqrt{\int_{f_{inf}}^{f_{sup}} S_J(f)\,df} \tag{3.133}$$

where the expressions of the adimensional parameters γ_J are in the previous section. With $S_J(f)$ in units of (m^2S), the significant wave height, H_s, is obtained in meters and the peak period in seconds.

3.5.2.10 Directional spectral density function

Whenever possible, the directional spectrum should be based on specific measurements. However, if instrument measurements are not available, the function $D_w(\theta, f)$, which describes the directional dispersion of the spectrum can be generically expressed by the following function (see Figure 3.3.22):

Figure 3.3.22. Angular distribution of the wave energy



$$D_w(\theta, f) = D_w(\theta, s) = G_0 \cos^{2s}(\tau) \tag{3.134}$$

where τ is the semi-angle in respect to the main propagation direction of the peak frequency, θ_0

$$\tau = \left(\frac{\theta - \theta_0}{2}\right), -\pi \le \theta \le \pi \tag{3.135}$$

and s is an empirical function that depends on the frequency,

$$s = s_{max}\left(\frac{f}{f_p}\right)^5 \quad f < f_p \tag{3.136}$$

$$s = s_{max}\left(\frac{f}{f_p}\right)^{-2,5} \quad f \ge f_p \tag{3.137}$$

and s_{max} depends on the age of the waves, and in shallow waters on wave steepness. For all practical purposes, the following values should be used:

s_{max} = 10, *waves in generation zone (sea)*
s_{max} = 25, *steep waves (swell)*
s_{max} = 75, *not very steep waves*

The function G_0 can be expressed as follows:

$$G_0\left(\theta_{min},\theta_{max}\right)=\frac{1}{\int_{\theta_{min}}^{\theta_{max}}\left[\cos\left(\frac{\theta}{2}\right)\right]^{2s}d\theta} \tag{3.138}$$

$$G_0\left(-\pi,\pi\right)=\frac{2^{2s-1}}{\pi}\frac{\Gamma^2\left(s+1\right)}{\Gamma\left(2s+1\right)} \tag{3.139}$$

where Γ is the gamma function, and θ_{min}, θ_{max} are the angles marking the sectors on either side of the main propagation direction where the energy content is significant.

Note. *The value of s_{max} depends on the age of the waves. For this reason its value can be estimated on the basis of a "wave age parameter", which includes the peak frequency of the spectrum f_p (1/s) and generating wind speed $\overline{u_{10}}$(m/s), according to the following equation ($g = 9.81 m/s^2$),*

$$s_{max}=\frac{11.5}{\left(\frac{2\pi f_p \overline{u_{10}}}{g}\right)^2} \tag{3.140}$$

3.5.2.11 Relations between statistical and spectral descriptors in the open sea

In the open sea, it can be generally supposed that spectral descriptors (H_{m_0}, T_m, θ_0) and statistical descriptors (H_s, $\overline{T_z}$, $\overline{\theta_w}$) are approximately equal,

$$H_s \approx H_{m_0}$$
$$\overline{T_z} \approx T_m = T_{0,2}$$
$$\overline{\theta_w} \approx \theta_0$$

Note. *It should be remembered that these relations between descriptors depend on relative depth and spectral shape. In Spain, whenever possible, the functional relations between state descriptors should be based on local measurements or data measurements from the network of* Puertos del Estado *(www.puertos.es).*

3.5.2.12 Spectrum of two or more sea states

On certain parts of the Spanish coastline, it is not infrequent for two sea states to simultaneously occur: (i) rough sea (or swell); (ii) wind sea (or sea). In these cases, the spectral frequency can be represented by the superposition of two frequency spectra:

$$S_w^*\left(f,\theta\right)=S_{w,sea}\left(f,\theta\right)+S_{w,swell}\left(f,\theta\right) \tag{3.141}$$

If no instrument measurements are available, both spectra can be described by the P-M or J functions.

3.5.2.12.1 BIMODAL SPECTRA

Besides the sea and swell spectra, there are other situations in which the spectrum can have more than one peak. For example, this occurs in those cases when the waves come from more than two directions or are generated in zones with fetch configuration effects. In the absence of instrumental measurements, the spectrum of total oscillation can be estimated in one of two ways: (i) by linearly superposing the energy in each frequency and direction of each of the generating sources (as described in the previous section); (ii) by adopting the Ochi-Hubble parametric bimodal spectrum. According to section 3.5.2.4, the area under the spectrum of the total sea is related to its mean square total wave weight:

$$m_{0,*} = \int_{f_{\inf}}^{f_{\sup}} S_T^*(f)df \tag{3.142}$$

$$H_{rms,*} \approx \sqrt{8m_{0,*}} \tag{3.143}$$

$$H_{s,*} \approx \sqrt{2}H_{rms,*} \tag{3.144}$$

Note. *Ochi and Hubble revised and expanded the source of spectral data used to formulate the Pierson-Moskowitz spectrum, and included bimodal and partially developed seas. They used this database to formulate a bimodal spectrum (with high- and low-frequency bands) of six parameters, even though all of them ultimately depend on one sole parameter, which is significant wave height.*

3.5.2.13 Extension to other kinematic and dynamic variables

The description of fluid movement can include other instantaneous variables, such as the velocity and acceleration as well as pressures and tangential stresses, whose temporal variability contains the same information as the oscillations that cause it. More specifically, if it is caused by the presence of wave action, it can be described by following the same scheme as for the sea state, in other words, by first defining the basic fast variable, instantaneous variable, basic cycle period, and afterwards its value as a slow variable or state descriptor. This expansion can be based on instrument measurements.

However, since in maritime engineering, it is usually sufficient to work with linear wave theory, probability functions and the frequency spectra of any of the kinematic or dynamic variables can be directly obtained from the results for instantaneous variables, free surface displacement, and associated basic variables. In the time domain, it is sufficient to apply statistical methods to obtain the derived probability functions, whereas in the frequency domain, it is sufficient to evaluate the frequency transfer function, which, when multiplied by the sea spectrum will provide the frequency spectrum of the derived random variable, in the same way as it is expanded in the two following subsections:

Note. *Similarly, this work scheme can be applied to other agents (e.g. vessel movement), as long as the vessel is located very confined and restricted waters. It can also be applied to any other variable that maintains a linear relation with the vertical displacement on the free surface.*

3.5.2.13.1 PROBABILITY DENSITY FUNCTIONS

The density function of any of the kinematic variables (velocity and acceleration) or dynamic variables (pressure) of the fluid in the presence of a sea state, characterized by the mean square wave height H_{rms} and mean period $\overline{T_z}$, are obtained by applying the procedure recommended in the following section for the horizontal velocity under the wave crest. The crest amplitude a_c and the maximum velocity underneath it u_c are related in linear theory by:

$$u_c(z) = a\frac{gk}{\sigma}\frac{\cosh k(h+z)}{\cosh kh} \tag{3.145}$$

whose density functions should satisfy:

$$p(a)\,da = p(u_c)\,du_c \tag{3.146}$$

in other words,

$$p(u_c) = du_c \frac{1}{(du_c / da_c)} = p(a)\,J^{-1} \tag{3.147}$$

$$J = \frac{du_c}{da} = \frac{gk}{\sigma}\frac{\cosh k(h+z)}{\cosh kh} \tag{3.148}$$

where $J(u_c, a)$ is the Jacobian of the transformation, and $p(a)$ according to the narrow-band hypothesis, is the Rayleigh density function of the mean square amplitude descriptor $a_{rms} = H_{rms}/2$:

$$p(a) = \frac{2a}{a_{rms}^2}\exp\left(-\left(\frac{a}{a_{rms}}\right)^2\right) \tag{3.149}$$

The velocity under the crest or under the trough (i.e. the amplitude of the velocity) at depth z follows the Rayleigh function of the mean square velocity statistical descriptor u_{rms}:

$$p(u_c(z)) = \frac{\sigma}{gk}\frac{\cosh kh}{\cosh k(h+z)}\frac{2a}{a_{rms}^2}\exp\left(-\left(\frac{a}{a_{rms}}\right)^2\right) = \frac{2u_c(z)}{u_{rms}^2}\exp\left(-\left(\frac{u_c(z)}{u_{rms}}\right)^2\right) \tag{3.150}$$

3.5.2.13.2 Spectral transfer functions

The transfer functions for each variable (velocity, pressure, etc.) can be obtained by applying linear theory. The horizontal velocity at depth z of the free surface, due to an irregular wave train, is obtained by the linear superposition of all orbital velocities due to each spectral component. In the case of one-directional waves:

$$u(z) = \sum_{n=1}^{\infty}\frac{a_n g k_n}{\sigma_n}\frac{\cosh k_n(h+z)}{\cosh k_n h}\cos(\sigma_n t - \varepsilon_n) \tag{3.151}$$

where $(a_n, \sigma_n, k_n, \varepsilon_n)$ are the amplitude, angular frequency, wavenumber, and random phase of each of the n spectral components. The orbital velocity u is a Gaussian variable and its spectral function $S_u(f, z)$ is related to the displacement spectrum $S_\eta(f)$ by the transfer function,

$$S_u(f,z) = \left|H_{\eta u}(f)\right|^2 S_\eta(f)$$
$$H_{\eta u}(f,x,y) = \frac{a(f)\,gk}{\sigma}\frac{\cosh k(h+z)}{\cosh kh} \tag{3.152}$$
$$\sigma = 2\pi f;\quad \sigma^2 = gk\tanh kh$$

$a(f)$ is the amplitude of the spectral frequency component f. Generally speaking, for frequency and directional spectra, the transfer function are vectorial, and the energy spectrum is the following:

$$S_u(f,\theta) = \left|H_{\eta u}(f)\right|^2 S_\eta(f,\theta) \tag{3.153}$$

Spectral descriptors are obtained by applying, along with their definition (see the section of the Annex on basic premises of sea state description):

$$m_{0,u} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{f_{\inf}}^{f_{\sup}}\left|H_{\eta u}(f)\right|^2 S_\eta(f,\theta)\,df\,d\theta \tag{3.154}$$

and the mean square velocity

$$u_{rms} = \sqrt{m_{0,u}} \tag{3.155}$$

3.5.2.13.3 EXTENSION TO OTHER AGENTS AND NON-LINEAR PROBLEMS

For those cases in which the variables are not linearly related (e.g. the drag force of a pier and the horizontal wave velocity), it is also possible to obtain the density function of the force from the density function of the square horizontal velocity. However, it is not possible to calculate the spectral density function from the drag force by applying the transfer function to the vertical displacement spectrum of the surface.

3.5.3 Description of medium and shallow water depths

In medium and shallow water depths, the probability density function and the spectral density function change in respect to the functions for the open sea or deep water. In certain cases, their functional structure is known, and thus, it is possible to apply specific expressions in the same way as for deep water. This section provides a description of certain expressions that can be used for this purpose. However, generally speaking, it is advisable to have information available for the project site (see the section on instrument measurements).

When no measurements are available for the project site, it is necessary to calculate the transformations of the probability and density functions based on descriptions obtained for the open sea and the continental shelf. These calculations can be performed by applying the transformation methods recommended in section 3.6.

When long-period sea level oscillations produce significant changes in water depth, which affect the wave transformation process, it is necessary to jointly describe and characterize sea states and sea level, as explained in section 3.5.6.

3.5.3.1 Probability density function of wave height

Wave propagation in medium and shallow water depths causes waves to steepen and their profile to become asymmetric. Thus, their crests become more sharply peaked, and their troughs flatten. This time series is rather different from the Gaussian model applied to deep water. Based on experimental data, it is possible to establish a theoretical boundary for the validity of the Gaussian wave model, based on significant wave height [(18)], according to the significant wave height relative to the point:

$$\frac{H_s}{h} \gtrsim 0.25$$

At medium and shallow depths, the distribution function begins to differ from the Rayleigh model. When instrument data are not available, and as long as there is no wave breaking, the following expression can be applied:

$$p\left(\frac{H}{\overline{H}}\right) = \frac{\beta_H}{\overline{H}}\left(\frac{H}{\overline{H}}\right)^{\frac{1+\delta_H}{1-\delta_H}} \exp\left[-\alpha_H\left(\frac{H}{\overline{H}}\right)^{\frac{2}{1-\delta_H}}\right] \tag{3.156}$$

where $\overline{H}$ is the mean wave height, α_H and β_H are two parameters that depend on the relation between the mean wave height and the water depth δ_H,

$$\delta_H = \frac{\overline{H}}{h}, \quad \leq \delta_H \leq 0.5 \tag{3.157}$$

$$\alpha_H = \frac{\pi}{4\left(1+\dfrac{\delta_H}{\sqrt{2\pi}}\right)} \tag{3.158}$$

(18) This criterion is similar to the boundary criterion of the profile bias, $\lambda_3 \approx 0.20$, used to analyze a time record by applying the Gram-Charlier probability density function.

$$\beta_H = \frac{2\alpha_H}{1-\delta_H} \tag{3.159}$$

where $\delta_H = 0$, the previous function coincides with the Rayleigh function.

However, as long as the waves do not break or the water is very shallow, it is not strictly necessary to apply non-Gaussian models to the project design of maritime structures.

In such cases, spectral and density functions for the open sea can be applied, thus incorporating the corresponding boundaries for larger breakers.

3.5.3.2 *Wave height of multiple wave trains: reflected, diffracted or irradiated*

When the wave action at a certain point is formed by the linear superposition of wave trains coming from different directions (e.g. incident and reflected wave trains),

$$\eta_* = \eta_I + \eta_r \tag{3.160}$$

If η_I and η_r are zero-mean Gaussian random variables, the vertical displacement of the free surface η_* is also a zero-mean Gaussian random variable.

3.5.3.2.1 *Density function of the total wave height*

In addition, if both oscillations have a narrow-band spectrum, the resulting wave height of the wave train can be distributed according to the Rayleigh function of parameter $H_{rms,*}$. The parameter value is local, and depends on the site. The mean square wave height of the total wave train can be obtained by:

$$H^2_{rms,*}(\vec{x}) = H^2_{rms,I}\left(1 + K_r^2 + 2K_r \cos\left[2\vec{k}\vec{x} + \varphi\right]\right) \tag{3.161}$$

where $H_{rms,I}$ is the mean square wave height of the incident waves and K_r, φ are the module and phase of the reflection coefficient (square mean), respectively (19). The area under the spectrum of the total wave train is the variant of the process, which is related to $H^2_{rms,*}$,

$$\sigma^2_{\eta_*} \approx 8H^2_{rms,*}$$

3.5.3.3 *Spectral density functions*

In shallow water, it is advisable to apply a modified version of the Jonswap spectrum by means of the function $\Psi(k, h)$. In addition, the spectral function should be adapted to depth h and k used as the wavenumber,

$$\Psi(k,h) = \tanh^2(kh)\left[1 + \frac{2kh}{\sinh 2kh}\right] \tag{3.162}$$

where k and f are related by means of the linear dispersion equation,

$$(2\pi f)^2 = gk \tanh kh \tag{3.163}$$

(19) If the spectrum is resolved with the Fourier Tranform in the complex field, the module and the quotient phase of the incident and reflected spectral components are, respectively, the module and phase of the reflection coefficient of the particular component (see section of wave groups).

Generally speaking, the spectral function in medium and shallow water depths can be expressed as follows:

$$S(f,h,\theta) = \Psi(f,h)\cdot S_J(f)\cdot D(\theta,f)\cdot \gamma_J(f) \qquad (3.164)$$

where $S_J(f)$ and $\gamma_J(f)$ are Jonswap functions (see section 3.5.3.9.2), and $D(\theta,f)$ is the directional spectrum. The depth has the effect of reducing the energy of each of the spectral components as compared to the spectrum for the open sea.

3.5.3.3.1 ONE-DIRECTIONAL TMA SPECTRUM

A one-directional frequency spectrum, which is representative of the sea state in medium and shallow water depths, when there is no bottom-induced wave breaking, is the spectrum known as the TMA [20]. This spectrum is obtained from the Jonswap spectrum (Figure 3.3.23),

$$S_{TMA}(f,h) = S_J(f) * \Psi(f_h,h) \qquad (3.165)$$

where f_h is an adimensional parameter of depth and frequency,

$$f_h = f\sqrt{\frac{h}{g}} \qquad (3.166)$$

The depth function $\Psi(f_h, h)$ takes the following forms:

$$\begin{aligned} \Psi(f_h,h) &= 2\pi^2 f_h^2, & f_h &\le \frac{1}{2\pi} \\ \Psi(f_h,h) &= 1-\frac{1}{2}(1-2\pi f_h)^2, & f_h &> \frac{1}{2\pi} \end{aligned} \qquad (3.167)$$

The spectral moments, and thus, the spectral descriptors are determined in the same way as for open sea spectra.

3.5.3.3.2 GENERALIZED JONSWAP SPECTRUM

The Jonswap spectrum can be applied to any water depth by adopting the following expressions of its spectral shape coefficients, α_J and γ_J:

$$\alpha_{Jh} = 0.0078\left(\frac{2\pi\overline{u_{10}^2}}{gL_p}\right)^{0.49} \qquad (3.168)$$

$$\gamma_{Jh} = 2.47\left(\frac{2\pi\overline{u_{10}^2}}{gL_p}\right)^{0.39} \qquad (3.169)$$

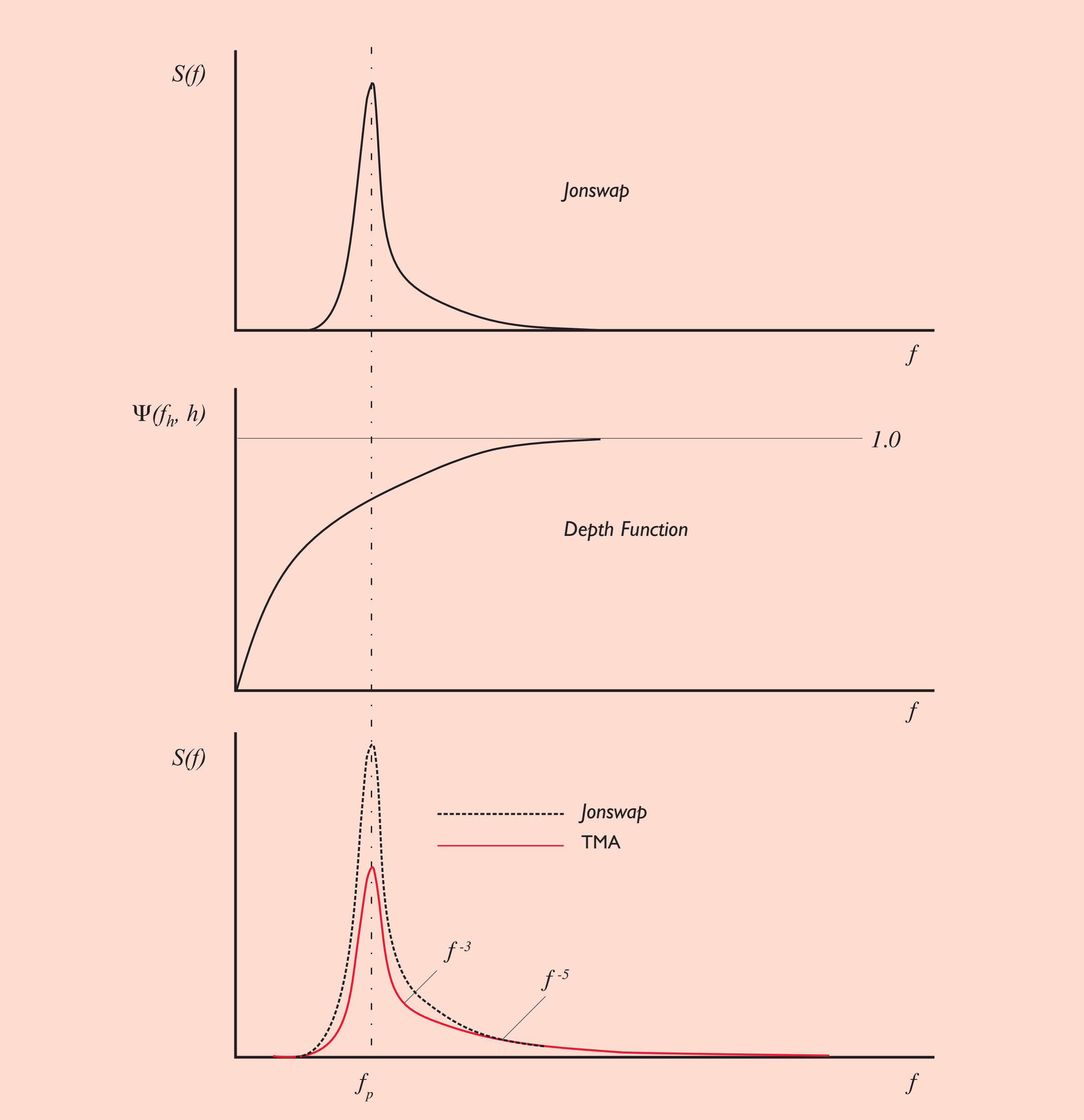
where L_p is the wavelength associated with peak frequency f_p, (L_{p0} in deep water, L_{ph} in depth h).

Once the Jonswap spectrum for the open sea is known with shape coefficients $(\alpha_{J0}, \gamma_{J0})$, the frequency spectrum for medium depths (h, for example) has the following spectral shape coefficients:

$$\alpha_{Jh} = \alpha_{J0}\left(\frac{L_{p0}}{L_{ph}}\right)^{0.49} \qquad (3.170)$$

(20) TMA stand for Texel-Marsen-Arsloe, the data collection sites used for the elaboration of the spectrum.

Figure 3.3.23. **Spectral density function in medium and shallow water depths**



$$\gamma_{Jh} = \gamma_{J0} \left(\frac{L_{p0}}{L_{ph}} \right)^{0,39} \qquad (3.171)$$

3.5.3.3.3 MULTIPLE WAVE SPECTRUM

When waves come from more than two directions (see section 3.5.2.10) or are generated in zones with fetch configuration effects, the total oscillation spectrum can be estimated by linearly superposing the energy in each frequency and direction of the generation sources.

When one of the irregular wave trains is the result of breakwater reflection (or of the radiation or diffraction from the breakwater head or of alignment changes), the reflected spectrum at a certain point can be obtained by:

$$S_T^*\left(f,\theta,\vec{x}\right)=\left|K_r\left(f,\theta,\vec{x}\right)\right|^2 S_I\left(f,\theta,\vec{x}\right) \tag{3.172}$$

$$\left|K_r\left(f,\theta,\vec{x}\right)\right|^2=1+\left|R\left(f,\theta,\vec{x}\right)\right|^2+2\left|R\left(f,\theta,\vec{x}\right)\right|\cos\left[2\vec{k}\left(f\right)\vec{x}+\varphi\left(f,\theta\right)\right] \tag{3.173}$$

where $R(f, \theta, \vec{x})$ and $\varphi(f, \theta, \vec{x})$ are the module and phase reflection coefficient of each of the spectral components. These spectra usually have various peaks.

When the seabed is almost horizontal and the breakwater is very long in respect to the wavelength, the reflection coefficient can be assumed to be constant at the face of the breakwater, and the area under the spectrum can be calculated by:

$$m_{0,*}\left(\vec{x}\right)=\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{min}}^{\theta_{max}} S_T^*\left(f,\theta,\vec{x}\right)dfd\theta \tag{3.174}$$

$$m_{0,*}=m_{0,I}+m_{0,r}+2m_{0r}\left(\vec{x}\right) \tag{3.175}$$

$$m_{0,I}=\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{min}}^{\theta_{max}} S_I\left(f,\theta,\vec{x}\right)dfd\theta \tag{3.176}$$

$$m_{0,r}=\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{min}}^{\theta_{max}}\left|R\left(f,\theta\right)\right|^2 S_I\left(f,\theta,\vec{x}\right)dfd\theta \tag{3.177}$$

$$m_{0,r}\left(\vec{x}\right)=\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{min}}^{\theta_{max}}\left|R\left(f,\theta\right)\right|\cos\left[2\vec{k}\left(f\right)\vec{x}+\varphi\left(f,\theta\right)\right]S_I\left(f,\theta,\vec{x}\right)dfd\theta \tag{3.178}$$

which is related to the mean square total wave height by:

$$H_{rms,*}\left(\vec{x}\right)\simeq\sqrt{8m_{0,*}\left(\vec{x}\right)}=\sqrt{m_{0,I}+m_{0,r}+2m_{0r}\left(\vec{x}\right)}$$
$$H_{s,*}\approx\sqrt{2}H_{rms,*} \tag{3.179}$$

Note. When the waves are reflected by a finite breakwater, and the seabed is not horizontal, an elliptic numerical model (e.g. MSPE) must be applied to calculate the reflection coefficient of each spectral component (see section 3.4.3.1).

3.5.4 Description of the surf zone

In shallow water, generally because of shoaling processes, refraction by the seabed, and when relevant, reflection by the boundaries and the seabed, individual waves become progressively steeper until they finally break. Generally speaking, open sea models cannot be applied to this context, especially if many waves are breaking, as happens in the surf zone.

When individual waves break because of bottom friction, this is reflected in a variation of significant wave height H_s and a modification of the spectral shape. Furthermore, the water depth is different from the water depths in waveless conditions, as described in the following sections.

In the design, construction, and exploitation of a port area, it is necessary to determine at least the following:

(1) whether a point in the sea with depth h and a sea state $(H_{rms}, \overline{T_z}, \theta)$ is located in the surf zone;
(2) height of the largest wave that travels without breaking at a point of the surf zone;
(3) for a certain meteorological state, the depth at which waves begin to break;
(4) probability and spectral density functions of the waves.

When there are no instrument data available for the project site, the probability and spectral density functions in the surf zone should be calculated on the basis of their descriptions for the open sea and continental shelf. The methods used for these calculations are the transformation methods recommended in section 3.6.

3.5.4.1 Probability density function of the wave height

When the instantaneous variable η stops being a Gaussian process, the wave crests and troughs, both in their number and amplitude, are no longer symmetrical, and the correlation between consecutive wave crests and troughs becomes distorted. Then, the wave amplitudes no longer fit a Rayleigh model, and are closer to a Weibull biparametric model.

For wide-band spectra and energy concentrated around the mean wave frequency $\bar{f}$, a probability density function of the limit wave height in the surf zone is:

$$p\left(\frac{H}{H_{rms}}\right) = \frac{H}{H_{rms}} * I\left(\frac{H}{H_{rms}}, \alpha_T\right) \qquad 0 \leq \frac{H}{H_{rms}} \leq \sqrt{\alpha_T} \tag{3.180}$$

$$p\left(\frac{H}{H_{rms}}\right) = \frac{H}{H_{rms}} * I\left(\frac{H}{H_{rms}}, \alpha_T\right)\left[1 - \frac{4}{\pi \cos \frac{\sqrt{\alpha_T}}{\frac{H}{H_{rms}}}}\right] \qquad \sqrt{\alpha_T} \leq \frac{H}{H_{rms}} \leq \sqrt{2\alpha_T} \tag{3.181}$$

$$I\left(\frac{H}{H_{rms}}, \alpha_T\right) = \int_0^\infty x J_0^{\alpha_T}\left(\frac{x}{\sqrt{\alpha_T}}\right) J_0\left(\frac{xH}{H_{rms}}\right) dx \tag{3.182}$$

$$\alpha_T = \frac{\pi}{7\sqrt{2}} \frac{\tanh\left(\bar{k}h\right)}{\bar{k}\sqrt{2m_0}} \tag{3.183}$$

where x is a mute integration variable; J_0 is the zero-order Bessel function; $\bar{k}$ is the wavenumber associated with the mean frequency of the waves, and which are related by means of the dispersion equation for depth h,

$$\left(2\pi\bar{f}\right)^2 = g\bar{k}\tanh\left(\bar{k}h\right) \tag{3.184}$$

When $\alpha_H \rightarrow \infty$, the density function approximates the Rayleigh function.

3.5.4.1.1 Truncated Rayleigh function

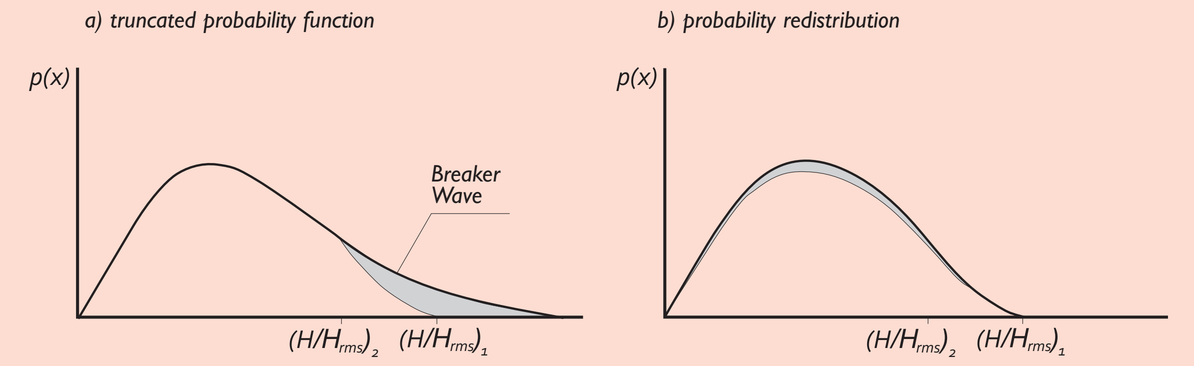
For swells, the wave height distribution in the surf zone can be simply calculated by truncating the Rayleigh probability function in wave height H_b, which satisfies the breaking criteria,

$$p^*\left(\frac{H}{H_{rms}}\right) = \frac{2H}{H_{rms}^2} \exp\left(-\frac{H}{H_{rms}}\right)^2 \qquad \left(0 \leq H \leq H_b\right) \tag{3.185}$$

by (uniformly) redistributing the excess probability among the wave heights that do not break in such a way as to fulfill (see Figure 3.3.24),

$$\int_0^{\frac{H_b}{H_{rms}}} p^*\left(\frac{H}{H_{rms}}\right) d\left(\frac{H}{H_{rms}}\right) = 1 \tag{3.186}$$

Figure 3.3.24. Truncated probability density function in the surf zone and probability distribution



3.5.4.2 Breaking criteria for individual waves in a irregular wave train

Generally speaking, all waves that locally satisfy one of the wavebreaking criteria (see section 3.4.6) either are in the process of breaking or have broken. However, for practical purposes, it is advisable to slightly lower the values of the coefficients proposed for the regular wave train, as shown in the following table:

$$Horizontal\ bottom:\ \left(\frac{H_z}{L}\right)_{\lim,irregular} \lesssim c_1 \tanh\frac{2\pi h}{L}$$

$$0.12 \le c_1 \le 0.14;\quad c_{1,regular} \approx 0.14$$

$$Mild\ slope:\ \left(\frac{H_{z,I}}{L_0}\right)_{\lim} \lesssim a_1\left\{1-\exp\left[-a_2 k_{z0} h\left(1+a_3\left(\tan\beta\right)^{\frac{4}{3}}\right)\right]\right\}$$

$$0.12 \le a_1 \le 0.18;\quad a_{1,regular} \approx 0.17$$

$$a_{2,regular} \approx 0.75;\quad a_{3,regular} \approx 15$$

$$All\ slopes:\ \left(\frac{H_z}{h}\right)_{\lim,irregular} = \gamma_{\lim,irregular} \lesssim b_1 I_{r,I}^{b_2} + b_3$$

$$0.90 \le b_1 \le 1.10;\quad b_{1,regular} \approx 1.0$$

$$b_{2,regular} \approx 0.17;\quad b_{3,regular} \approx 0.08$$

Depending on the way in which they break, breakers will evolve or simply dissipate. After that point, there will be no waves above the steepness or the limit relative height. Statistically speaking, their occurrence is not possible, and, therefore, the probability density function should be truncated. Moreover, this process can be represented in the form of a frequency spectrum, and in state value descriptors, such as H_{rms} or H_s, which are bounded.

3.5.4.3 Evolution of the relative wave height

At any point in the surf zone, it is possible to obtain a sample of the values of the relative height of individual waves,

$$0 \le \frac{H_z(x)}{h(x)} \le \gamma_{\lim,irregular}$$

3.5.4.3.1 PROBABILITY DENSITY FUNCTION OF THE RELATIVE WAVE HEIGHT

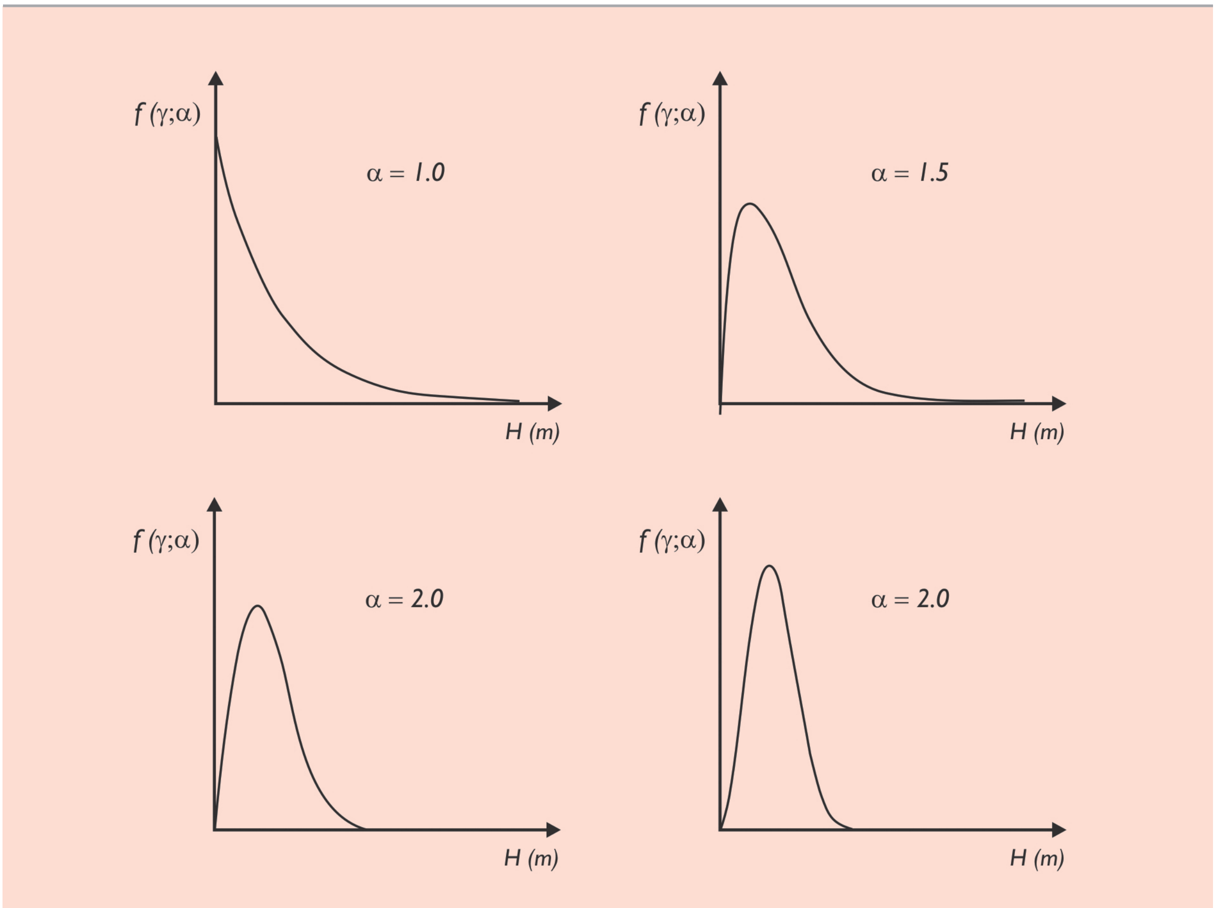
The probability density function of the random variable for relative wave height $\gamma = H/h$ is (see Figure 3.3.25),

$$p(\gamma;\alpha(x)) = \frac{\ln 2}{\left(\gamma_{rms}\ln\sqrt{2}\right)^{\alpha}}\alpha_w \gamma^{\alpha_w - 1}\exp\left[-\left(\frac{\gamma}{\gamma_{rms}\ln\sqrt{2}}\right)^{\alpha_w}\right] \qquad (3.187)$$

$$\alpha_w > 0$$

The probability model depends on two statistical parameters, γ_{rms} and α_w. When $\alpha_w = 2$, this is a Rayleigh function. The value of parameter $\alpha_w(x)$ mainly depends on the relative depth h/L. When the waves begin to break, this function should be truncated for the domain $\gamma \geq \gamma_{lim}$ and the area under the function should be homogenized to the unit. The wave height can be directly obtained as $H(x) = \gamma h(x)$.

Figure 3.3.25. Evolution of the density function of the relative wave height in the surf zone



3.5.4.4 State descriptors in the surf zone

In the same way as for the regular wave train, breaking criteria can also be defined for an irregular wave train in terms of the limit mean square relative steepness, $(H/L)_{rms,\lim}$, or the limit mean square relative wave height, $\gamma_{rms,\lim}$. If the value of the descriptor is higher than the limit value,

$$\left(\frac{H(x)}{L(x)}\right)_{rms} \geq \left(\frac{H_{rms}}{\bar{L}_z}\right)_{\lim} \qquad (3.188)$$

$$\gamma_{rms}(x) \geq \gamma_{rms,\lim} \qquad (3.189)$$

it can be concluded that some of the waves in the sea state are in the process of breaking, or have already broken. Beyond this point, the state descriptors γ_{rms}, $[H(x)/L(x)]_{rms}$ evolve in the surf zone by decreasing (though usually not monotonically) as the waves near the coastline. The state descriptor, mean square relative wave height γ_{rms}, will not be the greatest value in the interval. Rather, it will verify the following:

$$0 \leq \gamma_{rms}(x) \leq \gamma_{\lim,irregular} \qquad (3.190)$$

3.5.4.4.1 *Mean square breaking index*

For seabed slopes and relatively shallow depths, $[\tan\beta < 1/40,\ h/L < 1/20]$, and if the boundary reflection is negligible, then it can be assumed that the largest waves in the sea state are breaking when the value of the mean square breaking index fulfills the following:

$$\gamma_{rms,\lim} = \frac{H_{rms}}{h_k} > 0.30 \qquad (3.191)$$

When this quotient is larger than value $\gamma_{rms,\lim} \geq 0.40$, a wide range of waves are breaking [(21)].

When this index remains constant all along the profile, the surf zone is said to be saturated. In such cases,

$$\gamma_{rms}(x) \approx 0.35 \qquad \forall x \in \text{(surf zone)}$$

Although in the surf zone, the ratio between the significant wave height and the mean square wave height is not that of the Rayleigh model, this limit can be initially calculated in terms of the significant wave height by applying the relation derived from the Rayleigh model, namely, $H_s/H_{rms} \sim \sqrt{2}$.

The depth h_k should be in the depth interval $[h_{min,k} \leq h_k \leq h_{max,k}]$, taking into account all of the contributions to the sea level that are simultaneous and compatible with the sea state.

3.5.4.4.2 *Mean square limit steepness*

It is advisable to place an upper limit on the steepness value of the mean square wave for a horizontal seabed or one where the slope is negligible,

$$\left(\frac{H_{rms}}{\overline{L}_z}\right)_{lim} \lesssim 0.08 \tanh \frac{2\pi h_k}{\overline{L}_z} \qquad (3.192)$$

where $\overline{L}_z$ is the mean wavelength corresponding to the mean period of the waves, and calculated by means of the dispersion equation,

$$\left(\frac{2\pi}{\overline{T}_z}\right)^2 = g\overline{k}_z \tanh \overline{k}_z h_k \qquad (3.193)$$

$$\overline{k}_z = \frac{2\pi}{\overline{L}_z} \qquad (3.194)$$

and h_k is the depth associated with the sea state, in other words, the depth that is simultaneous and compatible with it. When the seabed slope is mild, but not negligible, the following equation [(22)] can be applied:

(21) $\gamma_{rms,lim}$ is known as the breaking index and the letter s, indicates that it is applied to the mean square wave height.
(22) In this case the reflection is said to be negligible.

$$\left(\frac{H_{rms}}{\bar{L}_{0,z}}\right)_{lim} \lesssim 0.10\left\{1-\exp\left[-a_2\,\bar{k}_{z0}h_k\left(1+a_3\left(\tan\beta\right)^{\frac{4}{3}}\right)\right]\right\} \tag{3.195}$$

3.5.4.4.3 Maximum wave height limited by breaking and probability of presentation

When the initial breaking conditions are fulfilled,

$$\gamma_{b,s} = \frac{H_s}{h} \approx 0.42 \tag{3.196}$$

it is possible to estimate the maximum wave height (assuming that the irregular wave action is a sequence of independent waves) by applying the breaking criteria of a regular wave train *(H,T)* (see section 3.4.6). It is also necessary to take the seabed slope into account, according to the following.

Once depth h_c and sea state $H_{s,c}$ are selected, the most probable wave height $\tilde{H}_{\max,N}$ is a random variable that depends on N, the number of waves in the state. In other words, it depends on the duration of sea state $t_{d,c}$ By applying the corresponding inequality to the breaking regime $f(H_b,\ h_c,\ m) \leq 0$, the breaking wave height H_b is obtained. Then, the probable maximum wave height is compared with the maximum breaking height

$$\tilde{H}_{max,N} \lesseqgtr H_b \tag{3.197}$$

The total probability that the maximum wave at a certain point will be greater than or equal to that of the maximum breaking height, taking into account the simultaneous occurrence of the depth, sea state, duration, and maximum wave is,

$$p_{b,\max} = \Pr\left[\tilde{H}_{\max,N} \geq H_b \left|(h \geq h_c)\cap\left(H_s \geq H_{s,c}\right)\cap\left(t_d \leq t_{d,c}\right)\right.\right] \tag{3.198}$$

When waves are beginning to break, unless the depth is very shallow, the probability density function of the wave heights can be said to be the Rayleigh function truncated in value H_b/H_{rms}. The probability of exceedance,

$$p_{H_b} = \Pr\left[H \geq H_b\right] \tag{3.199}$$

will be uniformly distributed in the Rayleigh density function in the domain,

$$0 \leq \frac{H}{H_{rms}} < \frac{H_b}{H_{rms}} \tag{3.200}$$

in order to guarantee that the total area under the density function is equal to the unit.

3.5.4.5 Fraction of breaking waves, Q_b

If Q_b is the fraction of waves that are breaking, and $H_{mnb}(h(x,y))$ is the maximum height of a wave that does not break at that depth $h(x,y)$, and assuming that the waves that do not break $H < H_{mnb}$ follow a truncated Rayleigh distribution function, the fraction of breaking waves can be calculated by resolving the equation,

$$\frac{1-Q_b(x,y)}{Q_b(x,y)} = -\left(\frac{H_{rms}(x,y)}{H_{mnb}\left(h(x,y)\right)}\right)^2 \tag{3.201}$$

$$H_{rms}(x,y) = 2\sqrt{2}\left[\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}} S_\zeta(f,\theta)\,df d\theta\right]^{\frac{1}{2}} \tag{3.202}$$

The limit breaking wave height H_{mnb} can be obtained by applying any of the breaking criteria given in section of these Recommendations pertaining to individual waves. Nevertheless, this is usually done by using the relative wave height to define a breaking index value. Accordingly, it is necessary to calculate the depth at the breaking point $h(x,y)$, and include all of the contributions that are simultaneous and compatible with the sea state (see section 3.4.6).

The limit value of the state descriptors provides information concerning the number of waves that are breaking. When this breaking index criterion (adjusted to H_s) is strictly satisfied,

$$f\left(H_b, h_c, m\right) \le 0$$
$$m \le \frac{1}{20}; \quad \gamma_{b,s} = \frac{H_s}{h} \approx 0.42 \tag{3.203}$$

the largest waves of the irregular wave train are breaking. Generally, this ratio increases as waves reach and advance into the surf zone. When this happens, the number of breaking and broken waves also increases. If it follows that,

$$m \le \frac{1}{20}; \quad \gamma_{b,s} = \frac{H_s}{h} \approx 0.55$$

it can be assumed that all waves, whose height exceeds the significant wave height, are either breaking or have broken, and that approximately, at least the top third of the waves (i.e. the largest ones) are either breaking or have broken.

The variable m should be representative of the seabed slope in the zone where the waves experience the sharpest steepening and breaking. This is approximately between two and four wavelengths towards the sea from the breaking point.

Note. *A rough assessment of the number of waves breaking on a saturated beach can be obtained by assuming that the waves follow the Rayleigh distribution (when they begin to break), and that all of the waves exceeding the limit are either breaking or have broken,*

$$N_{or} \approx \frac{D}{\overline{T}_z} p \tag{3.204}$$

$$P = \Pr\left[H > H_b \,|\, H_{rms}\right] \tag{3.205}$$

where D is the duration of the sea state; $\overline{T}_z$ is the mean period in the state; and p is the probability that the waves will exceed the probability of breaking. In saturated surf zones, the breaking index remains constant, and at any point in the zone, at least one third of the propagating waves (i.e. the largest ones) are either breaking or have broken. If the seabed slope of the surf zone is steeper than the slope required for it to remain saturated, the number of breaking waves becomes greater as they get closer to the coastline. Near the coastline, all of the waves are breaking or have broken.

3.5.4.6 Hydraulic regimes in wave breaking

The transformation of waves because of bottom effects can be analyzed by means of the state descriptors $(H_{rms}, \overline{T}_z, \overline{\theta})$ by applying breaking criteria similar to those applied to wave trains, as shown in table 3.3.15. It should be underlined that the coefficients of the various formulas were obtained with flat seabeds (constant slope).

Table 3.3.15. Wave breaking criteria and regimes

$tg\beta = m$	Approximation	Breaking criterion	Breaking regime
$m \le 1/120$	$tg\beta \Rightarrow 0$	$H_{rms}/\overline{L}_z < f(h/\overline{L}_z)$	Miche
	$(tg\beta,\ h/L) \Rightarrow 0$	$H_{rms}/h < cte$	Breaking Index
$1/120 < m \le 1/40$	$I_r \lesssim 1.5$	$H_{rms}/h = f(I_r)$	Dissipative
$1/40 < m \le 1/7$	$1.5 \lesssim I_r \lesssim 6.0$	$H_{rms}/\overline{L}_z > f(I_r,\ h/\overline{L}_z)$	Breaking-reflection, I, II
$1/7 < m \le 1/1.5$	$1.5 \lesssim I_r \lesssim 10.0$	–	Protective slope

- **Miche regime:** $m \leq 1/120$ **(spilling breakers)**

$$\frac{H_{rms}}{L_z} \lesssim \alpha_{M,rms} \tanh(kh) \qquad \alpha_{M,rms} \approx 0.06 \tag{3.206}$$

- **Breaking index regime:** $m \leq 1/120$ **(spilling breakers)**

$$\gamma_{b,rms} = \frac{H_{rms}}{h} \approx 0,30 \tag{3.207}$$

- **Dissipative regime:** $1/120 < m \lesssim 1/40$ **(spilling and plunging breakers)**

$$\gamma_{b,rms} = \left(\frac{H_{rms}}{h}\right)_b \approx b_1 I_{r,rms}^{b_2} + b_3 \qquad b_1 \approx 1, b_2 \approx 0.17, b_3 \approx 0.08 \qquad I_{r,rms} = \frac{tg\beta}{\sqrt{\dfrac{H_{rms}}{L_z}}} \tag{3.208}$$

- **Breaking-reflection regime I:** $1/40 \lesssim m \leq 1/15$ **(plunging and collapsing breakers)**

$$\left(\frac{H_{rms}}{L_{z,0}}\right)_b \approx 0.0277 \tanh\left[\frac{\beta_{\lim}}{2\pi}\left(\frac{h}{gT_z^2}\right)^{\gamma_{\lim}}\right] \qquad \beta_{\lim} = 17.7 \quad \left(m < \frac{1}{15}\right) \qquad \gamma_{\lim} = 0.90 \quad \left(m < \frac{1}{10}\right) \tag{3.209}$$

- **Breaking-reflection regime II**: $1/15 < m < 1/7$

$$\left(\frac{H_{rms}}{L_{z,0}}\right)_b \approx 0.0277 \tanh\left[\frac{\beta_{\lim}}{2\pi}\left(\frac{h}{L_{0,z}}\right)^{\gamma_{\lim}}\right] \qquad \beta_{\lim} = 152 \cdot m + 6.6 \quad \left(m < \frac{1}{15}\right) \qquad \gamma_{\lim} = 1.92 \cdot m + 0.72 \quad \left(m < \frac{1}{10}\right) \tag{3.210}$$

- **Protective slope regime:** $1/7 < m \leq 1/1.5$ **(collapsing and surging breakers)**

$$\left(\frac{H_{rms}}{L_z}\right)_b \approx x \frac{1 - K_{R,rms}}{1 + K_{R,rms}} + y \tanh(kh) \tag{3.211}$$

3.5.4.6.1 INITIAL BREAKING DEPTH

Once an open water sea state is selected $(H_s, \bar{T}_z, \bar{\theta})$, it is possible to obtain the depth at which large waves begin breaking by applying the inequation of the hydraulic regime for wave breaking, representative of the work conditions, to each of the values in the grid of the numerical model of wave transformation.

After marking the boundaries of the initial breaking zone, it is advisable to apply the individual wave criteria (see section 3.4.6) to the largest waves in the irregular wave train, always taking into account their probability of presentation. If no information is available, these waves are assumed to have the mean period of sea state $\bar{T}_z$.

3.5.4.7 Frequency spectrum of waves in the surf zone

There is no spectral shape that is representative of any surf zone because this shape depends on the processes that have intervened in wave transformation and breaking. In shoaling conditions, the evolution of the spectrum is similar to that of the wave trains. Thus, energy increases in low frequencies at the cost of energy in high frequencies. Therefore, it follows the same pattern as in deep water with the incorporation of triad interaction in the energy transfer. However, there is one important difference. In deep water, the upper tail of the spectrum remains proportional to f^{-5} during the entire process, whereas in shallow waters, the upper tail changes its shape to f^{-3}.

When the construction work is located in the surf zone, it is advisable to obtain the structure and spectral shape of the waves based on measurements taken at the location, and to use these results to calibrate one of the numerical methods recommended in the corresponding section.

Note. According to the hypothesis of Kitaigorodskii, the shape of the energy spectrum is universal if it is expressed in wavenumber terms. Thus, the upper tail f^{-5} in deep water is proportional to k^{-3}. Consequently, this would also be the spectral shape at medium and shallow depths, as well as in the surf zone. By applying the dispersion equation, it is possible to obtain an expression of the spectral shape in the frequency domain under these conditions,

$$S(f,h) = S(f,h=\infty)\,\phi(f,h) \tag{3.212}$$

$$\phi(f,h) = \left(\frac{k_0}{k_h}\right)^3 \frac{\left(\frac{dk}{df}\right)_h}{\left(\frac{dk}{df}\right)_\infty} = \frac{1}{2n}\tanh^2 kh = \left(\frac{C}{C_0}\frac{C_{g0}}{C_g}\right)^3 \tag{3.213}$$

$$n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{3.214}$$

3.5.4.8 Evolution of the mean sea level in the surf zone

The mean sea level evolves all along the surf zone, which signifies that it affects the evolution of H_{rms}. Therefore, the wave propagation and sea level equations should be resolved at the same time. Accordingly, the middle equation (associated with the mean square wave height, and thus identified by $\overline{\eta_{Hrms}}$ is the following:

$$\frac{d\overline{\eta_{H_{rms}}}}{dx} = -\frac{1}{\left(\overline{\eta_{H_{rms}}} + h\right)}\frac{d}{dx}\left[\frac{1}{8}H_{rms}^2\left(\frac{1}{2} + \frac{\frac{4\pi h}{L}}{\sinh\left(\frac{4\pi h}{L}\right)}\right)\right] \tag{3.215}$$

which should be resolved with the open-sea boundary condition $\overline{\eta_{Hrms}} = 0$. Thus calculated, $\overline{\eta_{Hrms}}$ is a statistic or state descriptor (mean square sea level) associated with the mean square wave height.

3.6 SEA STATE GENERATION AND TRANSFORMATION

When there is not sufficient data available pertaining to the project site, wave generation models should be applied, which are based on wind fields over the ocean. Such models are used to predict future sea states as well as to analyze past ones. In other words, they are used for the retroanalysis or systematic analysis of sea states produced by previous atmospheric conditions (see section 3.1.5).

This section first presents wave generation methods. It then analyzes wave transformation methods and models applicable to the outer and inner continental shelf as well as port and shoreline areas (domains III, IV and V).

3.6.1 Spatial domains and selection of models

The selection and application of the models of wave generation and transformation should be based on their relevance. For this purpose, five spatial work domains are defined (see section 3.1.2.3): *(I)* ocean and open sea; *(II)* outer continental shelf; *(III)* inner continental shelf and shoreline areas including the surf zone; *(IV)* port areas; *(V)* shallow confined seas. Table 3.3.16 summarizes the relevant wave generation and transformation processes for each domain.

Table 3.3.16. ***Physical processes of numerical models of wave generation and transformation***

Process	I	II	III	IV	V
Wind interaction	Yes	Yes	No/Yes	No	Yes
Four-mode interaction	Yes	Yes	No/Yes	No	No
Three-mode interaction	No	No	Yes/No	No/Yes	Yes
Current refraction	No	No/Yes	Yes/No	No/Yes	Yes
Refraction-induced breaking	No	Yes	Yes	Yes	Yes
Bottom-induced friction	No	No	Yes	Yes	Yes
Obstacle-induced diffraction	No	No/Yes	Yes	Yes	Yes
Steepness-induced breaking	Yes	Yes	Yes	Yes	Yes
Bottom-induced breaking	No	No	Yes	No/Yes	Yes

3.6.1.1 Slow-moving and fast-moving processes

When the wave field is forced by one of the processes listed in Table 3.3.16, the changes that it undergoes can be either fast or slow. In the case of a slow change, its characteristic magnitudes (e.g. wave height, propagation direction, etc.) vary only slightly in a fraction of a wavelength. In the case of a fast change, the wave's cinematic and dynamic characteristics are very important, as occurs in the area near the breaker and intermediate beaches. This behavior means that it is necessary to work with two types of models: (i) models that resolve wave generation and transformation at the wavelength scale; (ii) models that are capable of working with very small spatial increases, comparable to the wavelength.

In the first type of model, the wavelength is assumed to be almost stationary. Wave fields can be said to vary slowly, and processes can be evaluated by applying cinematic propagation equations in terms of the wavenumber, wave group celerity, and an energy balance equation. These models are known as phase-averaged models.

Wave generation models in the open sea and irregular wave train transformation during the propagation of waves over seabeds with slowly varying bathymetry use the phase-averaged description. Examples of wave generation models are WAM, WAVEWATCH, and SWAN. Wave transformation models include Irribaren's wave plans, ray models based on Snell's Law, and the mild-slope equation.

In contrast, in the case of fast changes, the focus is on rapidly varying wave fields and local properties related to the phase average (e.g. energy flux), which vary significantly in a fraction of the wavelength, as occurs in the surf zone. In order to adequately evaluate that behavior, models should be applied that resolve the processes in the phase, without finding the average, by describing instantaneous oscillatory motion, either in the time domain or in the frequency domain. These models are known as phase-resolving models, among which are time-dependent wave transformation models, such as Boussinesq and MSEP-t, as well as advanced numerical models that resolve Navier-Stokes equations.

3.6.1.2 Selection of models

The following models can be applied in the work domains (see Tables 3.3.17, 3.3.18, and 3.3.19).

- Generation models (phase-averaged)

Table 3.3.17. Models of wave generation in the open sea

Domain	WAM	WAVEWATCH
I	×	×
II	×	×

- Models of wave generation, transformation, and breaking (phase-averaged)

Table 3.3.18. Models of wave generation, transformation and breaking

Domain	SWAN
II	(×)
III	×
IV	×
V	×

- Models of wave translation and breaking

Table 3.3.19. Models of wave transformation and breaking

Domain	MSPE-t/Bousssinesq (1)	MSPE,Ref-dif/H2D (2)	N-S y RANS (3)
III	×	×	×
IV	×	No	×
V	×	×	×

Note. *(1) Phase-resolving models; (2) Phase-averaged models; (3) Models based on general equations of fluid movement (phase-resolving).*
None of these models is valid for all the work domains. In this sense, SWAN is most widely applied model. Generally speaking, for practical applications, it is necessary to use models whose domains overlap in order to convey information concerning the open sea to the project site.

3.6.2 Wave generation models in the open sea

The use of parametric, graphical or numerical models depends on the importance of the construction project, on the available data, and on the generation conditions, identified by the geometry of the generation surface, uniformity of wind speed and direction, and its temporal evolution.

3.6.2.1 *Parametric generation models: homogeneous field*

When the wind field is homogeneous, it is possible to apply parametric models of state descriptors or spectral density functions.

3.6.2.1.1 *State descriptors*

Parametric models provide the values of state descriptors and their evolution in time and space by applying functional relations between the adimensional significant wave height $\widetilde{H}_s$ and the adimensional peak period $\widetilde{T}_p$ with the adimensional fetch equation term $\widetilde{F}$, the adimensional time $\widetilde{t}$, and, when relevant, the adimensional depth $\widetilde{h}$, where

$$\tilde{H}_{m_0} = \frac{gH_{m_0}}{u_{10}^2}; \quad \tilde{F} = \frac{gF}{u_{10}^2}; \quad \tilde{T}_p = \frac{gT_p}{u_{10}}; \quad \tilde{t} = \frac{gt}{u_{10}}; \quad \tilde{h} = \frac{gh}{u_{10}^2};$$

$$\tilde{H}_{m_0} = f_1(\tilde{F}, \tilde{t}); \quad \tilde{T}_p = f_2(\tilde{F}, \tilde{t})$$

These models have the virtue of being quick and simple. Furthermore, they provide initial data regarding meteorological states during the passing of storm events in terms of the fetch length and uniform windspeed. The functions were obtained from visual and instrumental information collected in the majority of seas and oceans over the last sixty years.

Their application should be limited to the fulfillment of a set of ideal generation conditions. Such conditions include the absence of geographic obstacles in the fetch and a homogeneous wind blowing over it, as occurs in well-defined synoptic situations, such as tropical and extratropical storms.

In these circumstances, the following relations give the evolution of the spectral significant wave height and the peak period a long the fetch, taking into account the generation depth,

$$\tilde{H}_{m_0}(h) = \tilde{H}_{m_0}(\infty)\left[\tanh\left(\alpha_3 \tilde{h}^{\beta_3}\right)\tanh\left(\frac{\alpha_1 \tilde{F}^{\beta_1}}{\tanh\left(\alpha_3 \tilde{h}^{\beta_3}\right)}\right)\right]^{\lambda_1} \tag{3.216}$$

$$\tilde{T}_p(h) = \tilde{T}_p(\infty)\left[\tanh\left(\alpha_4 \tilde{h}^{\beta_4}\right)\tanh\left(\frac{\alpha_2 \tilde{F}^{\beta_2}}{\tanh\left(\alpha_4 \tilde{h}^{\beta_4}\right)}\right)\right]^{\lambda_2} \tag{3.217}$$

Table 3.3.20 shows the values of the constants,

Table 3.3.20. Parametric sea state models: parameters

α_1	α_2	α_3	α_4
4.14 * 10^{-4}	2.77 * 10^{-7}	0.343	0.100

β_1	β_2	β_3	β_4	λ_1	λ_2
0.79	1.45	1.14	2.01	0.572	0.187

When the sea state is fully developed, the following equations can be applied (see Table 3.3.21):

Table 3.3.21. FDS model parameters

A_1	A_2	B_1	B_2
2.88 * 10^{-3}	0.459	0.45	0.27

$$\tilde{H}_{m_0}(\infty) = A_1 \tilde{F}^{B_1} \tag{3.218}$$

$$\tilde{T}_p(\infty) = A_2 \tilde{F}^{B_2} \tag{3.219}$$

where [$\tilde{H}_{m0}(\infty)$, $\tilde{T}_p(\infty)$] are the maximum state descriptor values. Therefore, these values represent what is known as a fully developed sea (FDS), and [$\tilde{H}_{m0}(h)$, $\tilde{T}_p(h)$] are the values during the generation all along the fetch. These relations take into account the water depth during wave generation. Consequently they can be applied to all relative depths. However, they do not consider wave propagation phenomena, and for this reason, their application is restricted to the ocean, to open or confined seas, and to the continental shelf.

3.6.2.1.2 Spectral density functions

All coefficients and parameters of the spectral functions, particularly the Jonswap spectrum, are defined in terms of fetch and adimensional time generation. It is assumed that the magnitude and direction of the generation wind speed is uniform. By applying these expressions (see section 3.5.2.9 as well as the following sections), it is possible to obtain the temporal evolution in front of the fetch or the spatial evolution in the fetch of the spectrum, and to use this to calculate $\widetilde{H}_{m_0}$ and $\widetilde{T}_p$.

3.6.2.1.3 Effects of the configuration of the wave generation area and non-uniform wind

Independently of fetch length or the generation time, if the width of the generation is bounded, the energy of the waves generated will be less than if there were no boundaries. Since the parametric functions do not consider this effect, it is necessary to incorporate growth limitation criteria. When the wind is not uniform, either because of a change in intensity or direction, or because of a change in both intensity and direction, parametric methods should not be applied. In these circumstances, and in the absence of numerical tools, graphical methods are a viable alternative. Nevertheless, a better option is the direct application of numerical methods.

3.6.2.1.4 Graphical methods

Graphical methods use the previously mentioned functional relations, and quantify the energy increase in an oscillatory energy "cell" throughout fetch and time. The cell is nourished by the energy of each spectral component defined in terms of amplitude, frequency, and direction.

Note. For many years, integrated and directional graphical methods were the only tool available to obtain wave and storm regimes on the shoreline, based on visual and instrumental data as well as synoptic wind data. Today, these methods are rarely used even though they provide a physical sense of the wave generation process that is not easy to achieve with numerical methods.

3.6.2.2 Numerical generation models

Numerical generation models provide the temporal evolution of sea states by means of the spectral density function at any point in the calculation grid. Based on any one of the spectral functions, it is possible to obtain the temporal evolution of the state descriptors (in the frequency domain), and by applying the appropriate equations, the temporal evolution of the statistical descriptors and the corresponding probability functions of the state descriptors, namely $p(H_z, T_z)$.

3.6.2.2.1 In the open sea: WAM

When there are no currents, the spectral evolution of the sea depends mainly on the action of the wind, non-linear resonant four-mode interaction, and deep-water wave breaking due to steepening. When the wind field varies in intensity as well as direction, as occurs in extratropical storms, the spatial and temporal evolution of the sea should be performed by calculating the evolution in each spectral component. A model that can be used for this is WAM, which can be accessed at the following website: http://www.puertos.es/es/oceanografia_y_meteorologia/index.html. This model uses the HIRLAM wind fields to calculate the evolution of the spectral density function without any assumptions regarding the shape of the function.

3.6.2.2.2 Seas and continental shelf: WAVEWATCH

WAVEWATCH is a WAM-type model with a more efficient computational algorithm that reduces numerical diffusion. Accordingly, it includes local variation terms (non-stationary terms) of water depth and current, namely,

$\Omega \neq 0$, and the source terms of whitecapping and breaking due to bottom friction. In the Mediterranean, the version of this model takes into account the wave dissipation and refraction caused by the seabed at the grid points with an eighth-degree resolution.

These models can be directly applied all the way up to the coast. Nevertheless, because of the rapid changes in water depth that are so characteristic of coastal zones, it is necessary to increase the spatial resolution. For this reason, these models are generally not used in shoreline areas, unless the calculations are carried out with separate models (e.g. from depths of 40-50 meters, as described in sections 3.4.3 and 3.4.4 on wave propagation models in the shoreline area).

3.6.3 Models of wave generation, transformation, and breaking

When waves propagate on the continental shelf, and generation and transformation processes are equally important, it is best to work with models that resolve the energy conservation equation for each of the spectral components (frequency and direction) in each of the grid cells. In this case, the equations formulated should be in a Eulerian coordinate system, and include shoaling, refraction, and diffraction processes as well as the terms of energy transfer induced by triad interaction, whitecapping, and breaking due to bottom friction.

Wave transformation in shoreline areas should not be analyzed with models of the WAM or WAVEWATCH type. Since the waters there are much shallower, the necessary grid size is an important constraint. Furthermore, the possibility of incorporating whitecapping and breaking due to bottom friction because of depth reduction is limited. In addition, these models apply linear wave theory, and do not take either diffraction or reflection into account. For this reason, these models should not be applied to areas near the coast, the surf zone, or places where there are coastal defense structures, cliffs, etc. SWAN is the best model to apply in such cases. When generation processes are not important, it is even possible to directly apply transformation models.

3.6.3.1 Domain II-V Models: SWAN

SWAN is a numerical model that is used to quantify wave propagation in medium and shallow water depths. It maintains the open sea source terms, and adds those for friction-induced dissipation and for breaking due to wave-bottom interaction. SWAN provides the temporal evolution of sea states by means of the spectral density function at any point in the calculation grid. Based on each local spectral function, it is possible to obtain the temporal evolution of state descriptors (in the frequency domain). When the appropriate equations are applied, the model also gives the temporal evolution of statistical descriptors and the corresponding probability functions of the state variables, namely, $p(H_z, T_z)$.

This model is an extension of the WAM model, and includes phenomena that occur in shallow water. The transport equation can be formatted in terms of σ, the intrinsic or relative frequency and the propagation direction, θ, the direction normal to the crest of each of the spectral components.

Generally speaking, the diffraction effect is not completely incorporated. For this reason, this model should not be used in situations in which the diffraction will foreseeably be important. In reality, SWAN includes an evaluation of the diffraction parameters in terms of the energy of each component. It ignores the wave phase. In other words, it uncouples refraction and diffraction, the two processes that change the direction of the wave. When this occurs, the SWAN model produces an acceptable result.

In coastal areas, it is crucial to model the source (sink) term of energy dissipation due to breaking due to wave-bottom interaction. Although SWAN incorporates these effects, when wave energy dissipation in the radiation tensor gradients is important, specific wave transformation models should be used, regardless of whether they are coupled with circulation models in the surf zone.

Note. SWAN is a public domain model (www.swan.tudelft.nl) which uses wind fields from HIRLAM or other sources to simultaneously analyze wave generation and transformation. This model can also be applied to shallow waters because it includes the source terms for dissipation by whitecapping and bottom friction besides those for deep water,

3.6.4 Transformation models in medium and shallow water depths

The application of wave transformation models requires the specification of the sea level. If the analysis is performed by states, and if for each state, a sea level is determined (for long and medium-period oscillations), this results in the transformation of the meteorological state (i.e. simultaneous description of sea and sea level). By sequentially applying the models (at each sea state and associated sea level), a curve is obtained of the meteorological states at any point of the domain. Based on these states, loading cycles and calm cycles are identified. This makes it possible to calculate the regimes of the meteorological states at the project site. Generally speaking, these models are derived from wave train transformation models (see 3.4.3.). Depending on their origin, they can be classified in the same way as for wave train models: phase-integrated models, phase-resolving models, and advanced models.

3.6.4.1 Phase-integrated linear models

With the constraints specified in sections 3.4.3 and 3.4.5, phase-integrated linear models can be applied to:

- Basic variables: individual waves in an irregular wave train, represented by basic variables, $H_{z,0}$, T_z, θ_0 (where the subscript 0 refers to the initial reference point). The simplest version is ray theory (Snell's Law and the hypothesis of the conservation of energy flux between orthogonals, which is valid when there are no currents).
- Irregular wave train represented by state descriptors, $(H_{rms,0}, \overline{T_z}\ \overline{\theta_0})$.
- Spectral components defined in terms of their amplitude, frequency, and direction, $a_n(f, \theta_{n,m})$, formulated in the frequency domain $S(f, \theta)$ or their equivalent in the wavenumber domain $S(\vec{k}, \theta)$.

3.6.4.2 Results of linear models and their applications

With the previously expressed constraints, linear models, thanks to their simplicity of calculation and efficiency, can be applied to the propagation of individual waves in an irregular wave train, state descriptors, and spectral components.

3.6.4.2.1 Transformation of individual waves

As long as the interaction between components and the breaking of individual waves are not the determining processes, linear transformation models can be applied to each set of values $(H_{z,0}, T_{z,0}, \tilde{\theta}_0)$ of the irregular wave train. This gives the pair of transformed values $(H_z, T_{z,0}, \tilde{\theta})$ (the wave period remains the same) at any point in the sea, where $\tilde{\theta}_0$ is a representative direction of the irregular wave train in deep water, and $\tilde{\theta}$ is the propagation direction at that point. The result is a transformation coefficient, as though it were a wave train, which only depends on the period and initial direction.

$$K_{Tr,z}\left(\vec{x},T_z,\tilde{\theta}_0\right)=\frac{H_{z,t}\left(\vec{x},T_z,\tilde{\theta}\right)}{H_{z,0}}=\Phi\left(\vec{x},k_z h,\tilde{\theta}\right) \tag{3.220}$$

3.6.4.2.2 State descriptors

When the state descriptors, are applied, as though to a regular wave train, the transformation coefficients obtained for the descriptors are a rather imprecise approximation of the transformation of the irregular wave train,

$$K^{*}_{Tr,rms}\left(\vec{x},T_p\right)=\frac{H_{rms,t}\left(\vec{x},T_p\right)}{H_{rms,0}}=\Phi\left(\vec{x},k_p h,\overline{\theta}\right) \tag{3.221}$$

where k_p is the wavenumber associated with the peak period of maximum propagation energy.

The value of this coefficient at a point in the sea can be used to obtain the local mean square wave height. With only a few exceptions, it can be used as a statistical descriptor of the Rayleigh function. In this case, it should be remembered that $K_{Tr,rms}$ only quantifies the transformation of the spectral components with the greatest energy content (peak period). This approximation should only be applied to swells (narrow-band process), and when the bathymetry of the continental shelf is fairly straight and parallel with only slight variations.

3.6.4.2.3 PROBABILITY DENSITY FUNCTION

The previous method provides the probability density function of the wave heights at any point as long as certain conditions are satisfied and there is a swell, such that the waves have the same period and are traveling in the same direction. Under these assumptions, the Rayleigh density function $p_0(H_0/H_{rms,h})$ in deep water is transformed into the density function $p_h(H/H_{rms,h})$, such that,

$$p_h\left(\frac{H}{H_{rms}}\right)_h d\left(\frac{H}{H_{rms}}\right)_h = p_0\left(\frac{H}{H_{rms}}\right)_0\left(\frac{H}{H_{rms}}\right)_0 \tag{3.222}$$

$$p_h\left(\frac{H}{H_{rms}}\right)_h = p_0\left(\frac{H}{H_{rms}}\right)_0\left|\frac{\partial\left(\frac{H}{H_{rms}}\right)_0}{\partial\left(\frac{H}{H_{rms}}\right)_h}\right| \tag{3.223}$$

where the term on the right is the Jacobian of the transformation.

When there is a sea, and the wave period is not constant, the transformation of each individual wave depends on its period and direction, such that various waves of different directions and periods can arrive at the same point traveling in the same direction. Even though the method may seem tedious, the density function for each direction can be calculated so that it satisfies the following equality:

$$\sum p_0\left(H_0,T_0,\theta_0\right)\Delta H_0\Delta T_0\Delta\theta = \sum p_h\left(H_h,T_z,\theta_h\right)\Delta H_h\Delta T_z\Delta\theta \tag{3.224}$$

where the sum reflects the contribution of all the value pairs, which in deep water have direction θ_0 and which arrive at the point, traveling in direction θ_h. Assuming that the wave height, period, and direction are independent, the previous condition is the following:

$$p_0(H)\,p_0(\theta) = p_h(H)\,p_h(\theta)\frac{\partial H_0}{\partial H_p}\frac{\partial\theta_0}{\partial\theta} \tag{3.225}$$

3.6.4.2.4 TRANSFORMATION OF SPECTRAL COMPONENTS

Except when nonlinear phenomena and the interaction between components are important, linear transformation models, elliptic MSPE, or its parabolic approximation can be applied to each of the frequency and directional components of the spectrum $f(\theta)$ to carry out the propagation of the spectrum.

When the bathymetry is straight and parallel and with little variation, the shape of the spectral density function can be directly obtained as explained in the section of the Annex on basic premises of sea state description. Generally speaking, it is advisable to use a numerical model. All of these models can be formulated by assuming that the energy associated with (narrow) band frequency remains on that band during the transformation process, and that the intervening processes can be obtained by means of the linear superposition of each of them. For each frequency band, the quantity of energy contained, which is quantified by the square of the free surface displacement, does not vary during the transformation. Consequently, the laws and methods used to calculate the transformation of monochromatic wave trains can be directly applied to each spectral component, though it is necessary to always take into account the fact that its energy travels along its corresponding ray at the group velocity.

The spectral results can be used to obtain state frequency and statistical descriptors for any point in the domain, and the corresponding probability functions of the basic variables $p(H_z, T_z)$ the conditioned variables $p(H_z \mid \overline{T_z})$, $p(H_z)$, and the density functions of order statistics (e.g. $H_{\max,N}$).

Effective coefficients of the irregular wave train. The amplitude of the component at a point of the project site is related to its amplitude in the open sea at:

$$S_{Tr}\left(f,\theta^{*},\vec{x}\right)=K_{Tr}^{2}\left(\vec{x},f,\theta\right)S_{0}\left(f,\theta\right) \tag{3.226}$$

where $K_{Tr}^{2}(\vec{x}, f, \theta)$ is the transformation coefficient of the component. When they are applied to each of the spectral components with their frequency and direction, this gives the effective coefficient of the irregular wave train,

$$K_{Tr,ef}^{2}\left(\vec{x}\right)=\frac{\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{\inf}}^{\theta_{s\sup}}S_{Tr}\left(f,\theta,\vec{x}\right)dfd\theta}{\int_{f_{\inf}}^{f_{\sup}}\int_{\theta_{\inf}}^{\theta_{s\sup}}S_{0}\left(f,\theta\right)dfd\theta} \tag{3.227}$$

The three individual transformation coefficients of each component, of each state descriptor, and the effective coefficient,

$$K_{Tr,ef}\neq K_{Tr,rms}^{*}\neq K_{Tr}\left(\vec{x},f,\theta\right) \tag{3.228}$$

The state descriptor coefficients and the effective coefficient represent the overall (average) behavior of the irregular wave train, which differs from that of each individual wave in the train.

3.6.4.2.5 *SPECTRAL TRANSFER FUNCTIONS*

The spectral description of the transformed waves can be directly obtained from the incident spectrum, and the transfer function applied to each of the spectral components.

Vertical displacement of the free surface, η. The multiplication of the transfer function by the spectrum of the free surface displacement in the open sea,

$$S_{\eta}\left(f,\theta,x,y,h\right)=\left|H_{\eta\eta}\left(f,\theta,x,y\right)\right|^{2}S_{0}\left(f,\theta\right) \tag{3.229}$$

provides the spectral shape at point $P(x,y,h)$. Based on this shape, the spectral descriptors can be calculated at the point as well as the probability models. Equation 3.229 is applied to this state in stationary conditions, and consequently, time is not an intervening factor.

The transfer function is specific of each of the transformation processes, namely, shoaling, refraction, reflection, and diffraction. The subscript η indicates that the spectrum and the transfer function refer to the free surface displacement variable, which is assumed to be a Gaussian variable.

Horizontal velocity, u. Similarly, it is possible to obtain the spectrum of the velocities in the seabed, of the pressures at the toe of a vertical wall, etc. In such cases, the transfer function is constructed by applying linear theory. For example, the horizontal velocity at depth z of the free surface is obtained by the linear superposition of all orbital velocities, due to each of the spectral components. In the case of unidirectional waves,

$$u\left(x,y,z\right)=\sum_{n=1}^{\infty}\frac{a_{n}\left(x,y\right)gk_{n}}{\sigma_{n}}\frac{\cosh k_{n}\left(h+z\right)}{\cosh k_{n}h}\cos\left(\sigma_{n}t-\varepsilon_{n}\right) \tag{3.230}$$

which is a Gaussian variable, and whose spectral function is $S_u(f,x,y,z)$, which is related to the displacement spectrum by the transfer function,

$$S_u(f,x,y,z)=\left|H_{\eta u}(f,x,y)\right|^2 S_\eta(f,x,y) \tag{3.231}$$

$$\left|H_{\eta u}(f,x,y)\right|=\frac{a(f,x,y)\,gk}{\sigma}\frac{\cosh k(h+z)}{\cosh kh}$$
$$\sigma=2\pi f;\quad \sigma^2=gk\tanh kh \tag{3.232}$$

In the context of linear theory, the acceleration $a(\vec{x}, z)$ and pressure $p(\vec{x}, z)$ are also Gaussian processes. Once the energy spectrum is known for the vertical displacement of the free surface $S\eta(\vec{x})$, it is possible to calculate:

$$S_a(\vec{x},f)=\left|H_{\eta a}(\vec{x},f)\right|^2 S_\eta(\vec{x},f) \tag{3.233}$$

$$S_p(\vec{x},f)=\left|H_{\eta p}(\vec{x},f)\right|^2 S_\eta(\vec{x},f) \tag{3.234}$$

where $S_a(\vec{x}, f), S_p(\vec{x}, f)$ are the energy spectra of the velocity fields, accelerations, and pressures caused by the waves at the project site. In addition, $H_{\eta a}(\vec{x}, f)$, $H_{\eta p}(\vec{x}, f)$ are their transfer functions, which depend on the frequency. These are local functions, and thus, are specific of the location.

Spectral transfer function of the vertical displacement, ζ. The transfer function can be applied to the calculation of the energy spectrum, transformed by the refraction, shoaling, reflection, and diffraction of each component to be expressed (see the section of the Annex on wave train transformation in medium and shallow water depths):

$$S_\zeta(f,\theta,x,y)=\left|H_{\eta\zeta}(f,\theta,x,y)\right|^2 S_\eta(f,\theta) \tag{3.235}$$

where $S\eta(f, \theta)$ is the directional spectrum in deep water; $S\zeta(f, \theta, x, y, z)$ is the total wave spectrum at point $P(x, y)$, ζ is the generic vertical displacement variable of the free surface due to transformed waves; and $H_{\eta\zeta}(f, \theta, x, y)$ is the transfer function at the point,

$$\left|H_{\eta\zeta}(f,\theta,x,y)\right|^2=1+\left|K_m(f,\theta,x,y)\right|^2+2\left|K_m(f,\theta,x,y)\right|\cos\left(kx\alpha_x+ky\alpha_y+\varphi(f,\theta)\right) \tag{3.236}$$

For processes of mild slope refraction and shoaling $\zeta=\eta$, the previous expression can be simplified as follows:

$$\left|H_m(f,\theta,x,y)\right|^2=\left|K_m(f,\theta,x,y)\right|^2;\quad m=S,R \tag{3.237}$$

For reflection and diffraction, the transfer function should include the oscillatory term $(\cos(kx\alpha_x+ky\alpha_y+\varphi(f, \theta))$ due to the propagation from the source of diffraction and reflection, as well as to the lag resulting from the process itself. The zero-order moment of the transformed wave ζ is,

$$m_{0,\zeta}(x,y)=\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}} S_\zeta(f,\theta,x,y)\,df\,d\theta=m_{0,\eta}+m_{0,m}+2m_{0,m}(X) \tag{3.238}$$

where,

$$m_{0,m}(X)=\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}} S_\zeta\left|K_m(f,\theta,x,y)\right|\cos(X)\,S_\eta(f,\theta,x,y)\,df\,d\theta \tag{3.239}$$

$$X=kx\alpha_x+ky\alpha_y+\varphi(f,\theta) \tag{3.240}$$

$$m_{0,\eta}=\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}} S_\eta(f,\theta,x,y)\,df\,d\theta \tag{3.241}$$

$$m_{0,\zeta}=\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}}\left|K_m(f,\theta,x,y)\right|^2 S_\zeta(f,\theta,x,y)\,df\,d\theta \tag{3.242}$$

For cases of reflection and diffraction, $m_{0,m}(X)\neq 0$. In contrast, for mild-slope refraction and shoaling, $m_{0,m}(X)=m_{0,\eta}(X)=0$.

Note. *When the spectrum propagates in a medium that typically causes reflection and diffraction, as occurs when there is a breakwater, the interaction between incident, reflected, and diffracted wave trains becomes especially relevant because it produces the non-uniformity of wave heights around the breakwater as well as in the harbor area.*

3.6.4.2.6 VALIDITY OF THE LINEAR APPROXIMATION AND APPLICATION FIELD

The validity of the results obtained by applying wave transformation models to individual waves and spectral components is not any better than the validity of the transformation models themselves, and particularly, of the linear approximation of the wave action on which they are based. Nevertheless, linear theory is the basis for the description of the sea. It uses the concept of the mean energy per unit of the horizontal surface of each component. This is a phase-averaged quantity, and precisely for this reason, the application of phase resolving methods is rarely justifiable in the description of wave action at a specific point.

Furthermore, it is crucial to remember that state descriptors are magnitudes that express the overall (average) behavior of the sample of individual waves of the irregular wave train. Consequently, their transformation because of refraction, shoaling, reflection or defraction processes should produce results that, generally speaking, are fewer than those obtained for individual waves, particularly for the larger waves in the irregular wave train.

3.6.4.3 Phase-resolving models of an irregular wave train

According to sections 3.4.3 and 3.4.4, when the steepness of a wave influences the celerity of its propagation, it is advisable to apply a non-linear transformation model, either in the Stokes regime or in the Boussinesq regime. Such models are best for shallow water and the surf zone.

They are necessary to quantify the medium-period oscillations generated by the spatial and temporal variations of waves and groups of waves that can force the oscillations, regardless of whether they are resonant, in wharfs in port areas as well as in the surf zone. They are formulated by adopting the approximations derived from the Stokes or Boussinesq regime. They resolve conservation equations formulated in the instantaneous variable of the vertical displacement of the free surface η and the (horizontal) velocity field u.

When there is a time record of wave action for a certain point, phase resolving models can provide the transformation of that record to any point in the domain. Assuming that conditions pertaining to the stationarity and homogeneity of the process are satisfied, the result obtained is the record of the transformed wave train at the point which, when analyzed in terms of statistics and frequency, provides the corresponding state descriptors and the density functions of probability and spectral density, respectively.

3.6.4.4 Resolving models of general equations

In recent years, thanks to scientific advances and the increase in computational power with the implementation of numerical schemes in finite volumes (among other methods), great progress has been made in the direct integration of Navier-Stokes and Reynolds equations (RANS) without having to assume any type of fluid movement, They are applied to instantaneous variables, free surface η and horizontal velocity u. In the same way as in phase-resolving methods, when there is a time record of waves action at a certain point, they provide the transformation of that record at any point in the domain.

3.6.4.5 Selection of wave transformation models (without breaking)

The selection of a wave transformation model (except wave breaking) for Domains III-V should be adapted to the criteria and recommendations presented in section 3.4.5.

3.6.5 SURF ZONE MODELS

From the perspective of the project design and management of port and shoreline areas, it is vitally necessary to know within the context of each sea state when waves begin to break; how many of them break; at what height they break; and how they evolve in the surf zone. This information is specified in the probability and spectral density functions of the waves which, because of their spatial evolutionary nature are functions of their location. Furthermore, when the waves are spilling or plunging breakers, they force the sea level to vary in the surf zone, and their influence in the depth of the water column is not negligible. In such circumstances, the evolution of the waves in the surf zone and the evolution of the sea level should be calculated simultaneously.

3.6.5.1 Basic premises for surf zone models

When the operating conditions of the port area (long waves in the wharf), the floodline of the coast, or the net sediment transport depends on the wave breaking process, it is necessary to determine the characteristics of waves (i.e. probability density function and spectral density function) in the surf zone and the number of breakers. Because of the dynamics of wave breaking, these functions are local. When there are no instrumental measurements available of density functions, it is necessary to apply numerical wave transformation methods, which include criteria to determine when each wave in the train breaks, and how it evolves in the surf zone.

For this purpose it is best to apply wave transformation models that incorporate a breaking criterion as well as a dissipation-evolution model of wave breaking. In addition, these models should resolve equation systems that govern sea level movements associated with the variation of the radiation tensor of the waves. The models derived from the application of the mild slope equation (MSPE) and its approximations provide information for the sequential, but not simultaneous resolution of sea level evolution and the circulatory system generated. Examples of such models include Ref-Dif and Copla in the SMC. Far away from the shore, in the zone where waves break, this approximation is often sufficient. However, for areas near the coastline, where the floodline is mainly governed by the variation in sea level, it is advisable to apply time-dependent transformation models (i.e. phase-resolving models) or advanced numerical models. Moreover, in the presence of wave groups, the radiation tensor varies over time. Accordingly, in these conditions, the problem should be resolved by applying MSPE-t and Boussinesq phase-resolving models.

3.6.5.2 Broken wave and energy dissipation models

The simplest criterion is to admit that the surf zone is saturated since its depth integrally limits the local wave height. These conditions are adequate for mild, regular (dissipative) slopes with spilling breakers and incipient plunging breakers. When the hypothesis is not applicable, for example, when there is a sharp variation in depth or when the bottom is very irregular, it is best to apply the hypothesis of stable broken wave energy flow (see the section in the Annex on wave train transformation in the surf zone).

However, most numerical models that include wave breaking assume that the when the wave breaks, it forms a rolling volume of water (and becomes a roller). The roller rides on the face of the wave with its celerity, and extracts energy from the wave. The roller model is adequate for spilling and plunging breakers, and has the virtue of making the mean wave quantities, mean mass flows, momentum compatible with the energy with the energy dissipation per unit of horizontal surface. Since this is a phase-averaged model, it adapts itself very well to these wave transformation models. Nevertheless, the roller also can be modeled in terms of instantaneous variables, and thus, it can be implemented in phase-resolving models.

3.6.5.2.1 DISSIPATION BECAUSE OF BOTTOM FRICTION

The mean dissipation per unit of horizontal surface because of irregular waves can be evaluated by considering the mean square velocity at the sea bottom,

$$\overline{D^*_{bottom}} = \rho_w C_D u^3_{rms,bottom} \tag{3.243}$$

where C_D is a drag coefficient, and $u_{rms,bottom}$ can be obtained by means of the wave energy spectrum,

$$u^2_{rms,bottom}(x,y) = \int_{\theta_{min}}^{\theta_{max}} \int_{f_{\inf}}^{f_{\sup}} \left| H_m(f,\theta,x,y) \right|^2 S_\eta(f,\theta)\, df d\theta \tag{3.244}$$

where the transfer function is,

$$\left| H_m(f,\theta,x,y) \right| = \frac{2\pi f}{\sinh kh} \tag{3.245}$$

The mean dissipation per unit of horizontal surface for each spectral component is obtained by uniformly distributing $\overline{D^*_{bottom}}$ among all the spectral components,

$$\overline{D^*_{bottom}} = \rho_w C_D \left\{ \int_{\theta_{min}}^{\theta_{max}} \int_{f_{\inf}}^{f_{\sup}} \left| H_m(f,\theta,x,y) \right|^2 S_\eta(f,\theta)\, df d\theta \right\} u_{rms,bottom} \tag{3.246}$$

$$S^*_{bottom}(f,\theta,x,y) = -\frac{C_D}{g} \left[\frac{2\pi f}{\sinh kh} \right]^2 S(f,\theta,x,y)\, u_{rms,fondo}(x,y) \tag{3.247}$$

$$\overline{D^*_{bottom}(x,y)} = \rho_w g \left\{ \int_{\theta_{min}}^{\theta_{max}} \int_{f_{\inf}}^{f_{\sup}} S^*_{bottom}(f,\theta,x,y)\, df d\theta \right\} u_{rms,bottom}(x,y) \tag{3.248}$$

Even though the coefficient C_D depends on the Reynolds number and the relative roughness of the bed, an approximate value of 0.015 can be adopted.

Note. *This method of calculating dissipation because of bottom friction has many inconsistencies, but the relatively little importance of the source term, except in a small number of cases, means that there is no need to increase its complexity.*

3.6.5.2.2 Dissipation during bottom-induced breaking

Although there are other possibilities, two solutions can be adopted here. The first, which is in the time domain, evaluates dissipation in individual waves in the wave train. The second solution remains in the frequency domain and distributes the dissipated energy among the spectral components after evaluating the dissipated energy by means of a state descriptor.

Dissipation in the time domain. When calculating dissipation because of bottom effects, it is generally assumed that there is no reflection, and that the surf zone is saturated. For this purpose, the dissipation produced by breakers is quantified, and uniformly distributed among spectral components.

$$S^*_{breaker}(f,\theta,x,y) = \frac{\overline{D^*_{breaker}(x,y)}}{m_o(x,y)} S(f,\theta,x,y) \tag{3.249}$$

where dissipation from breaking is obtained by,

$$\overline{D^*_{breaker}(x,y)} = -\frac{1}{4} \alpha_{breaker} Q_b(x,y) \frac{1}{\overline{T_{z,breaker}}} H^2_{max} \tag{3.250}$$

where $\overline{T_{z,breaker}}$ is the mean period of the breaking waves; Q_b is the fraction of the breaking waves; and H_{max} is the maximum height of the waves that do not break at that water depth $h(x,y)$. Assuming that the non-breaking waves $H < H_{max}$ follow a truncated Raleigh distribution function, the fraction of breaking waves can be calculated by resolving the following equation:

$$\frac{1 - Q_b(x,y)}{Q_b(x,y)} = -\left(\frac{H_{rms}(x,y)}{H_{max}(h(x,y))} \right)^2 \tag{3.251}$$

$$H_{rms}(x,y) = 2\sqrt{2}\left[\int_{\theta_{min}}^{\theta_{max}}\int_{f_{\inf}}^{f_{\sup}} S_{\xi}(f,\theta)\,df d\theta\right]^{\frac{1}{2}} \tag{3.252}$$

The maximum wave height, H_{max}, can be obtained by applying any of the breaking criteria explained in section 3.5.4.6, though in this case, the relative wave height is generally used to define the breaking index. For this purpose, it is necessary to calculate the depth at breaking point $h(x,y)$, thus including all contributions that are simultaneous and compatible with the sea state.

Dissipation in the frequency domain. Assuming that the dissipation of each component is proportional to the complex amplitude, $A_{i,j}$, where subscripts i, j stand for the frequency component, f_i, and the direction θ_j,

$$\overline{d_{breaker,ij}} \approx \alpha_{ij} A_{ij} \tag{3.253}$$

the energy conservation equation (i.e. stationary and one-directional energy, $A_{i,j} = A_i$),

$$C_{gi}\frac{\partial A_i}{\partial x} + \frac{1}{2}A_i\frac{\partial C_{gi}}{\partial x} = -\alpha_i A_i \tag{3.254}$$

and when terms are grouped,

$$\frac{\partial}{\partial x}\left[C_{gi}\left|A_i\right|^2\right] = -2\alpha_i\left|A_i\right|^2 \tag{3.255}$$

The integration of this equation into the frequency domain (and when relevant, in the directional domain) results in the following equation:

$$\frac{\partial}{\partial x}(EC_g) = -\overline{D^*_{breaker}(x)} = -\frac{1}{2}\alpha_D H^2_{rms}(x) \tag{3.256}$$

These calculations require the specification of the dissipation coefficient for each spectral component, and the coefficient, α_D, which affects the total dissipation in the state. Generally speaking, the coefficient, α_i, is said to be the same for all the components.

Thus, the total dissipation can be calculated, among other methods, by the time-domain formulation, described in the previous section; by a specific formulation of the dissipation in the sea state (proportional to H^7_{rms}); or by an equation in which the dissipation is proportional to the energy flux deficit to achieve stabilization,

$$\overline{D^*_{breaker}(x)} \sim \left[E(x)C_g(x) - (EC_g)_s\right] \tag{3.257}$$

where the energy flux necessary to reach stabilization $(EC_g)_s$ is assumed to be proportional to the square depth $h^2(x)$.

These model be applied to each of the spectral components. In this case, the spectral shape should be limited at reduced depths, according to the Kitaigorodskii hypothesis. In this assumption, the upper tail of the spectrum $(f \geq f_p)$ is proportional to f^{-3} instead of f^{-5}, as occurs in deep water. Since each of the spectral components is said to behave the same way as an individual wave in the irregular wave train, the energy reduction per unit of horizontal surface in the breaking process can be calculated by,

$$\Delta S(f_n) = \frac{\Delta E(f_n)}{\rho_w g} \approx \frac{1}{2}\left[a^2_{n,b} - a^2_{n,e}\right] \tag{3.258}$$

The stabilization amplitude of each of the components, $a^2_{n,e}$, is the value that corresponds to the spectral shape in shallow water. In other words, it is proportional to f_n^{-3}.

In any case, it should be taken into account that these methods produce a spectrum with a low frequency cut-off. To bring this spectrum closer to reality, its lower tail can be completed by means of the functional structure of the TMA or FRF spectrum, since both spectra are based on the Kitaigorodskii hypothesis, and allow a transition from shape f^{-5} to shape f^{-3}, according to relative depth kh.

3.6.5.3 Numerical transformation methods that include breaking

According to previous sections, wave action in the surf zone can be described by applying phase-integrated numerical models, phase-resolving numerical models, or advanced numerical models in accordance with the following criteria.

3.6.5.3.1 PHASE-INTEGRATED MODELS

Phase-integrated models can be applied to medium and fairly shallow water depths, to areas far away from the boundaries, to gently sloping beaches of fine or medium-grained sand, or to river mouths and estuaries with soft, muddy bottoms, where the interaction between consecutive waves and the boundary reflection are negligible. These models can also be applied to the analysis of wave evolution in the surf zone, especially, swell wave systems and at depths at which refraction has made the propagation direction uniform. These methods are reasonably reliable to determine the zones in which the waves will predictably break during severe loading cycles, as occurs in port areas and navigation channels. These models can be applied to the transformation of individual waves, state descriptors, and spectral components, Generally speaking, the way that they analyze bottom-induced breaking and the evolution of the broken wave is the following (see Table 3.3.22):

Table 3.3.22. Wave transformation process in the surf zone in phase-averaged models

Phase-averaged models (linear theory)				
Processes of transfomation	Ray* and Refrac*	Elliptic**	Stokes I-MSP Hyperbolic***	Parabolic***
Bottom-induced breaking	Yes	Yes/No	Yes	Yes
Evolution of the broken wave	Yes/No	Yes/No	Yes	Yes

(*) The ray theory applied to the individual wave or to the stated descriptor can include a breaking criteria for a horizontal seabed or a mild slope. The evolution of the broken wave can be incorporated when, for example, the surf zone is said to be saturated.
(**) The elliptic MSPE model can be used to model bottom-induced breaking as long as this process occurs without reflection. When this is not the case, the calculations should be based on the proportion of reflected energy during the breaking process, and reflection and dissipation processes should be uncoupled. The same constraint holds if the evolution of the broken wave involves reflection processes. Otherwise, it is sufficient to say that the surf zone is saturated.
(***) The hyperbolic and parabolic approximations of the MSPE incorporate horizontal bed and mild slope breaking criteria. They model the evolution of the broken wave, by assuming that the surf zone is saturated or by using the surface roller concept.
All models can be applied to spectral components by incorporating criteria to distribute the dissipated energy among spectral components in the wave breaking process, and also after the wave has broken.

3.6.5.3.2 PHASE RESOLVING MODELS

Boussinesq or MSPE-t models and shallow-water models resolve equations of free surface movement $\eta(\vec{x},t)$ by determining all the wave transformation processes, including breaking. Generally speaking, they model the roller or water volume that simulates the behavior, propagation and evolution of the tidal bore. This approximation is valid for breaking regimes in which the predominant factor is gradual dissipation (i.e. the Miche breaking index and dissipative regime). When the variables, $\eta(\vec{x},t)$, $u(\vec{x},t)$, are substituted by the Fourier expansion, transformation models of the spectral components are obtained (see Table 3.3.23).

Table 3.3.23. Wave transformation processes in the surf zone in phase-resolving methods

Phase-resolving models		
Processes of transfomation	Stokes (linear) MSP-t*	Boussinesq (non-linear) various approximations**
Bottom-induced breaking	Yes/No	Yes
Evolution of the broken wave	Yes/No	Yes

(*) The same constraints hold as for stationary elliptic MSPE models.
(**) They assume dissipative wwave breaking.

3.6.5.3.3 Advanced numerical models

Two-dimensional (2DV) Reynolds-averaged Navier–Stokes (RANS) equations are averaged on the turbulence scale by means of the finite volume technique, unlike numerical models based on Boussinesq equations or nonlinear equations for shallow water. Their numerical resolution does not require the imposition of any initial wave theory, and the quantification of wave breaking is based on a turbulence model. The variables, $\eta(\vec{x},t)$, $u(\vec{x},t)$ are applied, and in principle, can incorporate any breaking criteria applied to the kinematics or dynamics of water particles. When relevant, reflection is also considered.

Note. *Currently, advanced numerical models are limited by their two-dimensional (2DV) formulation, the computational time involved, and the cost of their user license.*

3.6.5.4 Numerical models for the mean sea level

In the Stokes regime, the mean sea level can be calculated at each grid point by resolving mass and momentum conservation equations, which are vertically integrated with bottom-friction and surface-friction terms and forced by radiation tensor gradients. These can be calculated from the state descriptor results obtained with the phase-integrated methods (ray, refrac, and MSPE). The solution can be obtained by numerical methods, as explained in the *Sistema de Modelado Costero* [Coastal Modeling System] (www.smc.unican.es). The phase-resolving methods, and particularly, those based on Boussinesq equations, directly calculate vertical displacement on the free surface, and this result can be used to directly obtain variation in sea level.

3.6.5.5 Model calibration

For maritime structures, whose nature is $[IRE > 6, ISA > 5]$, and for those at which the greatest foreseeable wave heights in their useful life is not limited to bottom-induced breaking, it is advisable to collect instrument measurement data taken at the project site, and use these data to calibrate the models, especially probability models.

3.6.6 Wave transformation in zones of regular bathymetry

On the inner continental shelf, tidal platforms and beaches with a regular bathymetry and when the wave breaking regimes are either the breaking index, dissipative, or reflection-breaking, the evolution of waves before breaking, while breaking, and after breaking is assumed to be gradual, and that there is no interaction with other waves. In these conditions, it is possible to calculate the probability density function and the spectral density function of the waves in the surf zone, assuming that they are functions that refer to that particular point, independently of their characteristics previous to the surf zone. In other words, locally, they should satisfy hydrodynamic limit stability criteria.

3.6.6.1 Transformation of the probability function $p(H,T_z)$

In these circumstances, the transformation of the probability function can be determined by applying a wave energy transformation model (phase-averaged model) to each of the value pairs (H,T_z) of the joint distribution function $p(H,T_z,\overline{\theta};h)$, conditioned to direction $\overline{\theta}$ and to sea level h, until the breaking criteria is satisfied. After this point, it will be necessary to adopt a broken wave model (saturation regime, roller model, or energy flux stability of the broken wave) and a local variation equation of the mean sea level, due to wave breaking. In inner areas of the surf zone, this model can be of the same order as the calm water depth.

Assuming that the broken waves behave like a bore, at each point in the surf zone, it is possible to calculate the following: *(1)* waves that break at each point; *(2)* waves that have broken previously, and which propagate at that point in the form of broken waves (bores); *(3)* waves that have not broken. The evolution of a bore can be determined on the basis of the following two criteria:

The surf zone is saturated either by the breaking index of each wave or by the local descriptor, the mean square wave weight.

1. The difference between the local energy fluxes of the broken wave and a stabilization value.
2. With these data it is possible to locally obtain the density function of the wave heights and wave period.

3.6.6.2 Rayleigh distribution

The previously described method can be simplified by assuming that the waves, before entering the surf zone, follow a Rayleigh distribution. After the initial breaking point, the Raleigh function is no longer representative of the wave height distribution, and the density function is truncated. Given that the area under the density function should be equal to the unit, the excess probability density can be accumulated in the breaking height or can be uniformly distributed among the other wave heights (see section 3.5.4.1.1). Both methods are usually sufficiently approximated for their application in the surf zone.

3.6.6.3 Evolution of the relative wave height

The density function of the monomial H/h is a Rayleigh function outside of the surf zone, However, inside the surf zone, it follows a Weibull distribution (see section 3.5.4.1). In surf zones that are moderately uniform, the relative depth variation and γ_{rms} can be calculated by applying phase-integrated methods, particularly ray theory, and by adopting simplified expressions, based on long wave theory.

3.6.6.4 Transformation of the spectral density function

The evolution of the spectral density function can be obtained by applying a phase-averaged model (ray theory, parabolic or hyperbolic MSPE, selected for its adequation to the project site conditions) to each of the spectral components. In each step, the total energy under the spectrum should be calculated as well as its state descriptors. Assuming that the wave heights follow a Rayleigh distribution, it is necessary to do the following:

1. Discover whether the state descriptors satisfy the breaking criteria and if individual wave heights are breaking. When this is the case, it is then necessary to calculate the value of the state descriptor that corresponds to the new spectral shape. In the wave height domain, a breaking criteria should be applied to individual waves; the distribution function should be truncated; and its parameter recalculated.
2. Recalculate the spectrum with new state descriptor values, determine the energy deficit from dissipation due to breaking by both methods, and keep the worst result.
3. Adopt a distribution criteria among components of the overall energy deficit due to breaking, which was obtained in the previous step. Generally speaking, the uniform distribution of the energy deficit among components usually is a good fit for experimental data
4. Verify that in any of the ways used to calculate the spectrum, the spectrum, expressed in accordance with the wavenumber, maintains shape.

3.7 MEDIUM AND LONG-TERM DESCRIPTION OF WAVE ACTION

Useful life can be understood as an experiment whose final result shows the following: *(1)* whether a subset of the structure has experienced one or more of the failure modes assigned to the ultimate and serviceability limit states; *(2)* whether a subset of the structure has experienced a stoppage because of one or more of the stoppage modes assigned to the operational limit states. This result depends on the specific sequence of meteorological states that will occur throughout the useful life of the structure. If the experiment could be repeated, the result of each experiment would be different, given the random nature of the occurrence of these agents and their manifestations. However, the occurrence, magnitude, and duration of climate agents is subject to their respective probability models, and consequently, the results of the experiment can be anticipated from a probabilistic perspective. For this reason, it is a good idea to be aware of the probability models of climate agents as well as those pertaining to the response of the structure. Generally speaking, this type of statistical description is known as a long-term description.

The previous sections explained methods for the description of waves (height, direction, and individual wave period) in the sea state. This section provides criteria and methods for the medium and long-term description of climate agents, mainly, sea oscillations, including waves. Such agents can be organized in loading cycles and cycles of use and exploitation as a random sequence of states; in meteorological years as a random sequence of loading cycles and cycles of use and exploitation; and finally, in their useful life as a deterministic sequence of meteorological years. In middle latitudes, meteorological years are usually regarded as statistically independent tests.

3.7.1 Statistical description of sea states: annual regimes

The meteorological state is the joint manifestation of the atmospheric and marine agents, and includes at least, windspeed and atmospheric pressure, sea state and wave group level η_{NM}, and sea level due to long waves, tsunamis and meteo-tsunamis ζ_{mm}, meteorological tide η_{MM}, and astronomical tide η_{NA}, defined by the following state descriptors:

- sea state: H_s, $\overline{T_z}$, $\overline{\theta_w}$
- wind state: $\overline{u_{10}}$, $\overline{\theta_u}$, $\nabla \overline{p_a}$
- sea level state: η_{NM}, η_{MA}, η_{MM}, ζ_{mm}

The duration of each state depends on the temporal variability of the process at the point being considered although, generally speaking, it can be said that a representative duration of maritime climate agents can be found in a time interval of 30-45 minutes, and of atmospheric climate agents in a time interval of 5-15 minutes. For this reason, although a curve is mentioned, in reality, the information is a discrete sequence of histograms of a given duration.

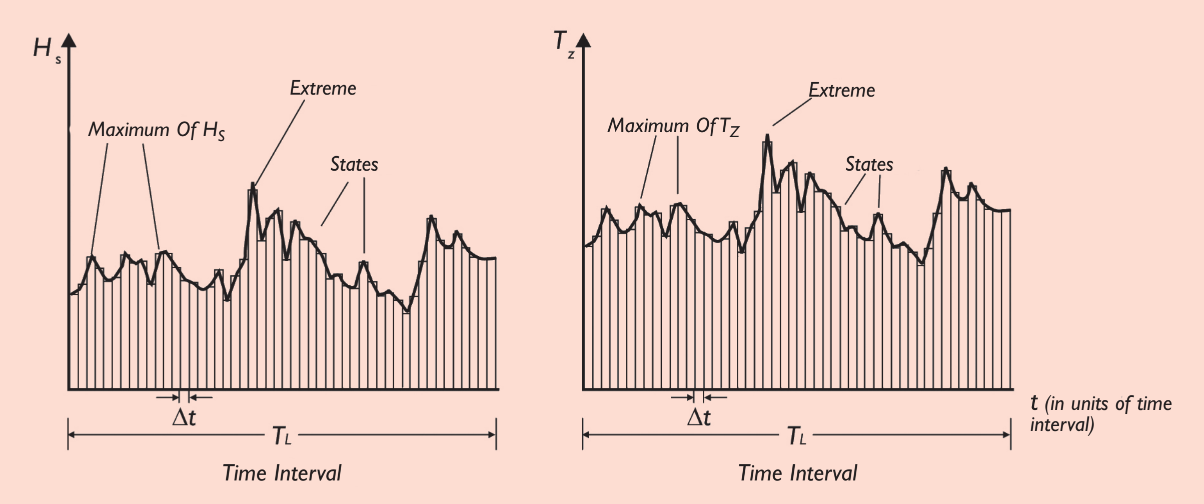
3.7.1.1 State curves

The state curve is a representation of the temporal evolution of one of the meteorological state descriptors at a certain point in the sea (see Figure 3.3.26). The function (22) $H_s(t) > 0$ (significant wave height or of any other climate agent descriptor) is random and non-stationary due to its seasonal and hyperannual variability. Furthermore, these curves are often asymmetrical with respect to the annual mean. The selection of a threshold value $H_{s,cr}$ makes it possible to identify loading cycles as well as use and exploitation cycles. Loading cycles are relevant states in the behavior of a subset of the structure in relation to safety and serviceability. In other words, they are extreme work conditions. Use and exploitation cycles are relevant states in the performance of a subset during its use and exploitation in the context of normal work and operating conditions. Of course, apart from these conditions, other exceptional and unexpected conditions may also occur. This may happen because of the unusual exceedance of the values of the agents either because of an accident or predictable events resulting from the exceptional activity.

Note. Exceptional work conditions can be treated in the same way as extreme work conditions. However, in order to be considered in the project, it is first necessary to specify, a priori, the joint failure probability in exceptional conditions. This can be done by applying the same procedure recommended for extreme work conditions, which consists of the following steps: (i) determine the principal failure modes; (ii) calculate the social and economic repercussion index due to the worst failure mode that can occur in those work conditions; (iii) determine the nature of the construction work, and based on this information, determine its joint failure probability.

(22) The following sections deal with H_s and refer to the sea state. However, any other agent can be described in the same way.

Figure 3.3.26. State curves



3.7.1.1.1 TECHNICAL STOPPAGES, INSTRUMENTAL BREAKDOWNS, AND GAP FILLING TECHNIQUES

The time series obtained with instrument measurements usually have time intervals in which, either because of maintenance work or technical breakdowns, there are information gaps. In such cases, it is necessary to apply gap-filling techniques to complete the data in the time series. These include statistical techniques, correlations with other state variables, or duly verified physical relations between variables.

3.7.1.2 Relative frequency of sea states in Υ

If the time is calculated during which sea state curves exceed a given value $H_s > H_{s,i}$ is computed, this value is then divided by Υ, the total duration of the analysis, this gives the relative frequency in Υ.

$$F\left(H_s > H_{s,i};\Upsilon\right) = \frac{\sum \tau_{s,i}}{\Upsilon} \tag{3.259}$$

where $\tau_{s,i}$ is the duration of the exceedance i of value $H_s > H_{s,i}$. If the calculation is performed with data from various meteorological years (e.g. $\Upsilon = M$), the exceedance regime is representative of the exceedances in time intervals of M years.

3.7.1.2.1 RELATIVE FREQUENCY OF SEA STATES IN THE METEOROLOGICAL YEAR

Furthermore, if meteorological years are regarded as independent events, then the sample of M years has M occurrences of the event $H_s > 0$, one for each meteorological year.

Consequently, the previous calculation can be applied by considering the meteorological year as the time interval. In other words,

$$F\left(H_s > H_{s,i};year\right) = \frac{\sum_i \tau_{s,i}}{M\,year} \tag{3.260}$$

is representative of the exceedances in an average year.

The exceedance in other time intervals (e.g. seasons, winter-summer, monsoons, etc.) can be defined in the same way.

3.7.1.3 Sea state regime (Wave regime)

There is no clear scientific reason to select one probability model in preference to another to fit to exceedance data. Models used for this purpose include the log-normal model, modified log-normal model, Weibull bi-parametric and triparametric models, modified exponential model, etc. Generally speaking, the log-normal model is well fitted to the central values of the sample, while the Weibull tri-parametric function more accurately reproduces the behavior of upper-tail values. In the absence of more precise data, the following generalized gamma function model is recommended for the H_s data set because it combines the advantages of the lognormal model for the central values and the advantages of the Weibull model for the upper tail,

$$f_{H_s}\left(H_s;\Upsilon\right)=\frac{c}{\Gamma(m)}\lambda^{cm}H_s^{cm-1}\exp\left[-\left(\lambda H_s\right)^c\right] \quad 0 < H_s < \infty \tag{3.261}$$

where $\Gamma(m)$ is the gamma function; m, λ and c are the three parameters of the function, which can be estimated by equaling the sample and theoretical moments $j = 2, 3$ *and* 4, to thus obtain a system of three equations with the three unknowns. The theoretical moments are expressed as follows:

$$E\left[H_s^j\right]=\frac{1}{\lambda^j}\frac{\Gamma\left(m+\frac{j}{c}\right)}{\Gamma(m)} \tag{3.262}$$

When the time interval for which the probability model is estimated is the meteorological year, this is known in Spain as the *régimen de oleaje* (sea state regime) in the average year.

3.7.1.4 Regime of exceedance duration

The exceedance duration is the time (meteorological year or useful life), during which the state descriptor surpasses or exceeds the selected value. Generally speaking, this random variable follows a Weibull parametric model, whose parameters are functions of the exceedance level (H_s).

Note. The ROM 0.4 recommends the Weibull biparametric distribution as the probability model of the exceedance duration regime of the mean windspeed $\overline{u_{10}}$.

3.7.1.5 Sea state regime during the useful life of a structure

The definition of the distribution function or regime can be applied to any time series measured for a certain time interval (e.g. the meteorological year or useful life). For the analysis of the functionality of the subset (in terms of the serviceability limit states), it is necessary to evaluate the joint failure probability during the useful life of the structure. This analysis of subset performance includes failure modes pertaining to durability, accumulated deformation, and fatigue, among other factors, and should include all states that exceed a certain threshold value, whose occurrence can significantly affect the functioning of the structure. Generally speaking, the necessary data for this type of analysis is usually not available since measurements generally correspond to time periods of shorter duration than the useful life V. For example, such a period could be K years, where $K < V$. One possible solution for this problem is to admit the statistical independence of meteorological years. In that case, the data can be said to make up a sample series of K occurrences, one for each meteorological year. From this sample, it is possible to obtain an annual regime of sea states $F_{H_s}(x)$. Under such an assumption, the exceedance regime $\Phi_{H_s}(x)$ over the threshold value of the useful life V is the following:

$$\Phi_{H_s}\left(x;V\right)=\left\{F_{H_s}\left(x;year\right)\right\}^V \tag{3.263}$$

In order to obtain the function, $F_{H_s}(x)$, it is advisable to have data corresponding to a period of various years. The minimum number of years should be equal or greater than $V/3$. It is evident that the more representative $F_{H_s}(x)$ is and the better it satisfies the statistical independence of the meteorological years, the more representative $\Phi_{H_s}(x;V)$ will be of the probability of occurrence of limit states during the useful life of the structure.

Note. The regime of exceedances without a threshold, and with the year as the time interval, coincides with the sea state regime or annual mean regime. The regime of exceedances during the useful life regardless of whether a certain threshold is exceeded, is similar to the long-term wave height distribution or long-term statistics.

For the construction and repair phases, it is best to work with seasonal regimes. However, in middle latitudes, it is preferable to work with time intervals of less than three months, given the meteorological variability, characteristic of the beginning and end of the seasons. Otherwise, one of the methods described in the following section can be used.

Finally, the over-threshold exceedance regime in the V-year time interval should not be confused with the V-year extreme regime. The former is used for all over-threshold states, whereas the latter is only used for peaks or the annual maximum value.

3.7.1.6 Fitting marginal probability models to the data

Various methods can be applied to fit a probability model to the data series for one random variable, for example, the significant wave height. All of them are based on the premise that the distribution function is known, and what is being estimated are the distribution parameters. These methods, which differ in the parameters used, include the graphical method (by means of probability paper), the moment method (equalization of sample moments and theoretical moments of the distribution), and method of maximum likelihood (maximization of the logarithm of the likelihood function, as applied to sample values).

Any of these methods can be applied, but the most advisable are those that guarantee consistency, efficiency, and an absence of bias [23].

Note. If there is long-term sea state curve for a project site period, increasing values of threshold $H_{s,i}$ can be selected, and the data can be plotted on probability paper for the corresponding probability function. In the case of the Wiebull biparametric function, there is (H_s in meters),

$$\begin{aligned} X &= 100 \quad \ln\left(2.5 H_{s,i}\right) \\ Y &= 100 \quad \ln\left\{\ln\left(\frac{1}{\Pr\left[H_s > H_{s,i}\right]}\right)\right\} \end{aligned} \tag{3.264}$$

If the data fit the Weibull probability function, then they should remain on a straight line when they are represented on a system of Cartesian axes with the variation interval of the reduced variables,

$$\begin{aligned} &Y = c + dX \\ &\left[100 < X < 350; 50 < Y < 250\right] \end{aligned} \tag{3.265}$$

Once coefficients c, d of the straight line are determined, the Weibull parameters are the following:

$$\begin{aligned} &a = d \\ &b = \frac{1}{2.5}\exp\left[-\frac{c}{100d}\right](meters) \end{aligned} \tag{3.266}$$

3.7.1.7 Limitations due to the non-stationarity of the random process

All of the methods recommended in the previous sections consider the random process $H_s(t)$ to be stationary. Consequently, when these methods are applied, attention should be paid to the sampling interval, the stationarity

(23) See *Notas sobre Métodos Estadísticos Aplicados a la Ingeniería Marítima y de Costas* [Notes on Statistical Methods applied to Coastal and Maritime Engineering]

by time intervals (year), and the statistical independence of consecutive sea states. In order to obtain the probability function of H_s, it is advisable to have a sample of sea state curves for one-hour time intervals. The rhythm of measurements should then be increased (even to continuous measurements) when the significant wave height is over a certain threshold.

Statistical moments of certain data series have been found to have a monthly (or seasonal) variability. Since the occurrence of sea states is said to have an annual statistical periodicity (pulse of the planet), their seasonal variability can be quantified by subtracting their mean annual H_s from each annual data series, and then calculating the mean value and the standard deviation of the remainder over time. The Fourier analysis of these time series provides the information required to evaluate seasonal variability, if it exists.

A similar procedure can be used to quantify hyperannual variability though this requires time series of data that have at least three hypercycles, whose periodicity in middle latitudes is assumed to be 11-15 years. In such cases, seasonal and hyperannual variability should be simultaneously analyzed by expanding the data set in a Fourier series, and working with the corresponding remainders.

3.7.1.8 Joint regime (H_s, $\overline{T_z}$)

The joint information (H_s, $\overline{T_z}$) is either collected on a contingency or danger table, or represented on a dispersion diagram of the significant wave height and mean wave period (see Figure 3.3.27). This information can be used to elaborate joint probability models $[H_s, \overline{T_z}]$ by following the method described in the previous section to obtain the annual exceedance regime as well as that corresponding to the useful life.

For this purpose, it is important to bear in mind the explanations given in the previous sections regarding the time interval and the non-stationarity of the process.

Since the process of fitting bivariate probability models to the data is often somewhat complex, it is best to use one of the following three models:

- Method 1: method based on the marginal and conditioned functions, and which is composed of the following steps:

  1. Obtain the marginal density function $f(H_s)$, (e.g. the generalized gamma function);
  2. Obtain the conditioned density function, $f(\overline{T_z} \mid H_s)$ from the available data for each H_s value (e.g. log-normal);
  3. Calculate the bivariate density function, $f(\overline{T_z}, H_s)$.

$$f\left(\overline{T_z}, H_s\right) = f\left(\overline{T_z} \middle| H_s\right) * f\left(H_s\right) \qquad (3.267)$$

  When the generalized gamma function for the marginal function of H_s is combined with a log-normal for the mean period conditional function, the joint density function can be written as follows:

$$f\left(\overline{T_z}, H_s\right) = \frac{c}{\Gamma(m)} \lambda^{cm} H_s^{cm-1} \exp\left[-\left(\lambda H_s\right)^c\right] * \frac{1}{\sqrt{2\pi}\sigma\left(H_s\right)\overline{T_z}} \exp\left\{-\frac{\left[\ln \overline{T_z} - \mu\left(H_s\right)\right]^2}{2\left[\sigma\left(H_s\right)^2\right]}\right\} \qquad (3.268)$$

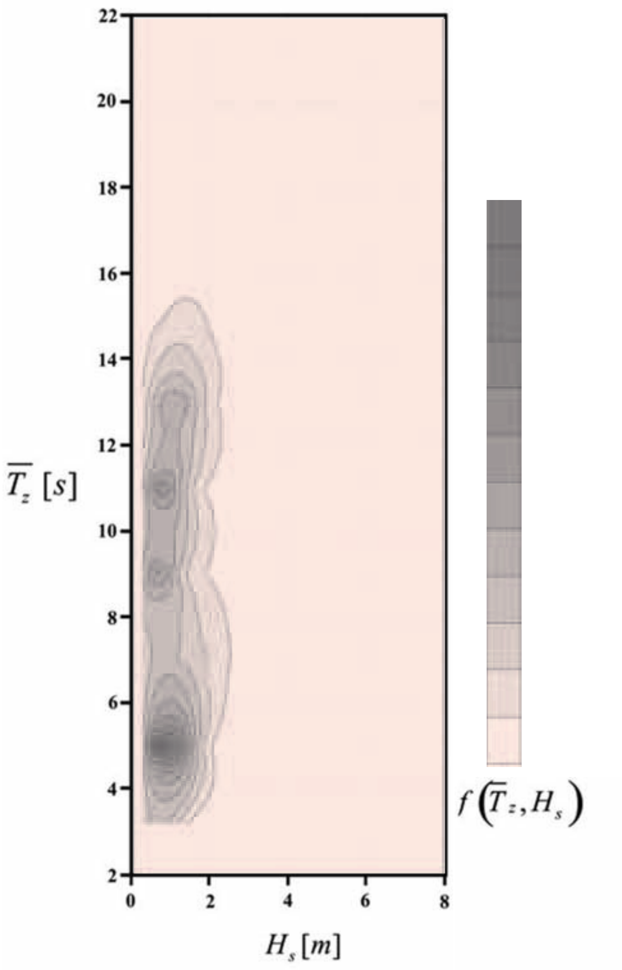
  where parameters σ and μ are functions of H_s as shown below:

$$\mu\left(H_s\right) = a_1 + a_2 H_s^{a_3} \qquad (3.269)$$

$$\sigma\left(H_s\right) = b_1 + b_2 \exp\left[b_3 H_s\right] \qquad (3.270)$$

  and which are estimated on the basis of the data in the fit of the conditional density function.

Figure 3.3.27. **Mean regime of the significant wave height and the zero-step mean period, Cadiz, h ≈ 50m**



- Method 2: method based on marginal functions.

If the marginal density functions $f(\overline{T}_z)$, $f(H_s)$ have been obtained, the joint density function can be obtained by means of the following expression:

$$f\left(\overline{T}_z, H_s\right) = \Psi * f\left(\overline{T}_z\right) * f\left(H_s\right) * \Phi \quad (3.271)$$

where Ψ is the contingency or dangerousness coefficient given by,

$$\Psi = \frac{P_1 P_4}{P_2 P_3} \quad (3.272)$$

These are the probabilities accumulated in the four quadrants in which the diagram $(\overline{T}_z, H_s)$, can be divided by the (arbitrary) point $(H_{si}, \overline{T}_{zi})$,

$$P_1 = \Pr\left[H_s < H_{si}, \overline{T}_z < \overline{T_{z,i}}\right] = F\left(H_s, \overline{T}_z\right) \quad (3.273)$$

$$P_2 = \Pr\left[H_s < H_{si}, \overline{T}_z > \overline{T_{z,i}}\right] = F\left(H_s\right) - F\left(H_s, \overline{T}_z\right) \quad (3.274)$$

$$P_3 = \Pr\left[H_s > H_{si}, \overline{T}_z < \overline{T_{z,i}}\right] = F\left(\overline{T}_z\right) - F\left(H_s, \overline{T}_z\right) \quad (3.275)$$

$$P_4 = \Pr\left[H_s > H_{si}, \overline{T_z} > \overline{T_{z,i}}\right] = 1 - F(H_s) - F(\overline{T_z}) - F(H_s, \overline{T_z}) \tag{3.276}$$

and function Φ is calculated by means of,

$$\Phi = \frac{(\Psi - 1)\Lambda + 1}{\left\{S^2 - 4\Psi(\Psi - 1)F(H_s)F(\overline{T_z})\right\}^{\frac{3}{2}}} \tag{3.277}$$

where,

$$\Lambda = \left[2 - (P_2 + P_4) - (P_3 + P_4)\right] - 2\left[(P_2 + P_1)(P_3 + P_1)\right] = F(H_s) + F(\overline{T_z}) - 2F(H_s)F(\overline{T_z}) \tag{3.278}$$

$$S = 1 + (\Psi - 1)\left[2 - (P_2 + P_4) - (P_3 + P_4)\right] = 1 + (\Psi - 1)\left[F(H_s) + F(\overline{T_z})\right] \tag{3.279}$$

Between the contingency coefficient Ψ and the correlation coefficient of the sample ρ, there is the following functional relation:

$$\rho = \frac{\Psi + 1}{\Psi - 1} - \frac{2\Psi}{(\Psi - 1)^2} \ln \Psi \tag{3.280}$$

which can be used to verify goodness of fit.

- Method 3: method involving the application of copula functions.

Copulas are multivariant functions with uniform marginal variables. In other words, they are functions whose arguments are not the initial variables, but rather their corresponding empirical or theoretical distribution functions. The one-parameter functions most frequently used for climate variables are the Gumbel copula and the Plackett copula. The calculation of the copula function parameters can be carried out by maximum likelihood. This can be done either in a general way by leaving all of the (marginal and copula) parameters free. However, what is usually done is to previously fit the marginal parameters, and once this is done, regard them as representative, and only estimate the dependence parameter.

Gumbel Copula

$$C_\delta(u, v) = \exp\left\{-(\tilde{u}^\delta + \tilde{u}^\delta)^{\frac{1}{\delta}}\right\} \tag{3.281}$$

$$u = F(x); v = F(y) \tag{3.282}$$

$$u = -\log u; \tilde{v} = -\log v \tag{3.283}$$

when δ is the dependence parameter that varies between $\delta \to 1$, in the case of independence, and $\delta \to \infty$ when both variables tend to simultaneously increase (dependence in the positive quadrant). The joint density function is,

$$c_\delta(u, v) = \frac{C_\delta(u, v)}{uv} \frac{(\tilde{u}\tilde{v})^{\delta - 1}}{(\tilde{u}^\delta + \tilde{v}^\delta)^{2 - \frac{1}{\delta}}} \left[(\tilde{u}^\delta + \tilde{v}^\delta)^{\frac{1}{\delta}} + \delta - 1\right] \tag{3.284}$$

Plackett Copula

$$C_\delta(u, v) = \frac{1}{2\eta}\left\{1 + \eta(u + v) - \left[(1 + \eta(u + v))^2 - 4\delta\eta uv\right]^{\frac{1}{2}}\right\} \tag{3.285}$$

where u, v are the distribution functions of the representative variables, $\eta = \delta - 1$; and δ is the dependence parameter. Its value is $\delta \to 1$, in the case of independence; its value is $\delta \to \infty$ when both variables simultaneously increase; and it is $\delta \to 0$ when both variables tend to jointly decrease.

The density of the copula is the following:

$$C_\delta(u,v) = \frac{\delta\left[1+\eta\left(u+v-2uv\right)\right]}{\left[\left(1+\eta\left(u+v\right)\right)^2 - 4\delta\eta uv\right]^{\frac{3}{2}}} \tag{3.286}$$

3.7.1.9 Sea state regime conditioned to the propagation direction

In most projects for port as well as for shoreline areas, it is necessary to apply H_s regimes conditioned to the propagation direction. If sufficient data is available, it is best to proceed in the same way as in the case of $(H_s, \overline{T_z})$. This means using the marginal and conditional functions to do the following:

1. Obtain the marginal density function $f(H_s)$, (e.g. the generalized gamma function);
2. Obtain the conditioned density function, $f(\overline{\theta}\,|\,H_s)$ based on the data available for each H_s value;
3. Calculate the bivariate density function, $f(\overline{\theta}, H_s)$

$$f\left(\overline{\theta},H_s\right) = f\left(\overline{\theta}|H_s\right) * f\left(H_s\right) \tag{3.287}$$

Evidently, when both descriptors are independent, it holds that,

$$f\left(\overline{\theta},H_s\right) = f\left(\overline{\theta}\right) * f\left(H_s\right) \tag{3.288}$$

A simpler way of doing the same thing is to divide the data series in sectors of an angle of 11.25^{o} or $22{,}50^{o}$ (depending on the information available), and to calculate the probability of H_s for each,

$$F\left(H_s;\overline{\theta_i} < \overline{\theta} < \overline{\theta_{i+1}}\right) = \Pr\left[H_s > H_{s,i};\overline{\theta_i} < \overline{\theta} < \overline{\theta_{i+1}}\right] \tag{3.289}$$

By definition, the probability of exceedance is less than the probability of scalar exceedance. In the open sea, this function is usually a good fit to the difference of two Weibull functions of the same exponent,

$$F\left(H_s;\overline{\theta_i} < \overline{\theta} < \overline{\theta_{i+1}}\right) = \exp\left[-\left(\frac{H_s}{b_1}\right)^a\right] - \exp\left[-\left(\frac{H_s}{b_2}\right)^a\right] \tag{3.290}$$

$$b_1 > b_2$$

3.7.1.10 Sea state regime conditioned to the sea level

It is best to work with a sea state regime conditioned to the sea level ζ_{NM} when one or more of the following conditions are satisfied:

- Bottom-induced wave breaking is a process that is relevant to the wave height magnitude at the site.
- The amplitude of the astronomical tide significantly influences wave breaking.
- The relation between the amplitudes of the meteorological tide, η_{NM}, and other processes that influence the sea level, tsunamis, and meteo-tsunamis, ζ_{mm}, and of the astronomical tide, η_{MA}, is $O(1)$. This is particularly pertinent to the occurrence of loading cycles.

If sufficient information is available, it is advisable to proceed in the same way as in the case of $(H_s, \overline{T_z})$. This means using the marginal and conditioned functions to do the following:

1. Obtain the marginal density function $f(H_s)$, (e.g., the generalized gamma function).
2. Obtain the conditioned density function, $f(\zeta_{NM}\,|\,H_s)$, based on the data available for each H_s value.
3. Calculate the bivariate density function, $f(\zeta_{NM}, H_s)$.

$$f(\zeta_{NM}, H_s) = f(\zeta_{NM} | H_s) * f(H_s) \tag{3.291}$$

Obviously, when both descriptors are independent, it holds that,

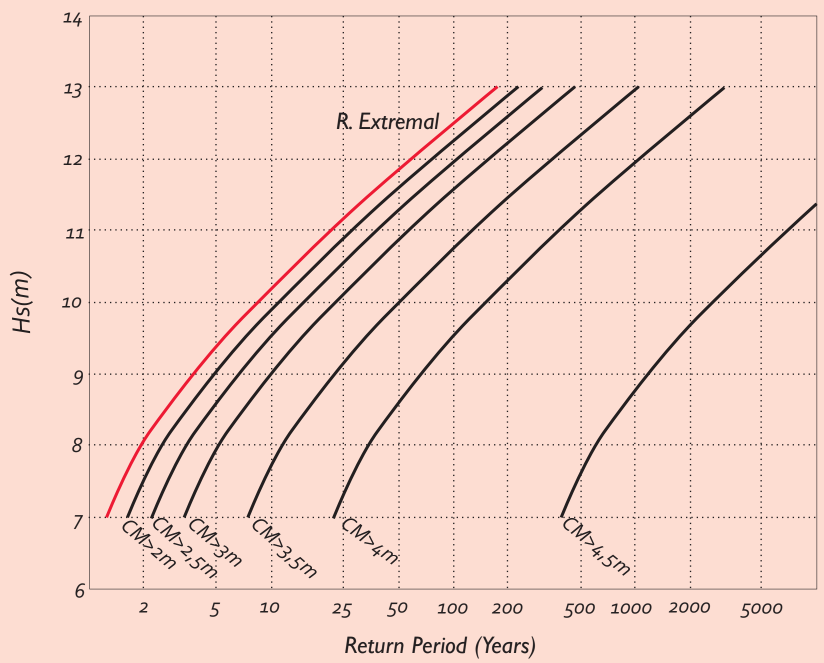
$$f(\zeta_{NM}, H_s) = f(\zeta_{NM}) * f(H_s) \tag{3.292}$$

As in the case of the direction variable, the data series can be divided into level intervals (depending on the available data and the interval of variability), and the probability of threshold exceedance can be calculated for each of them. The distribution function of the exceedance of H_s, conditioned to an interval of level $[\zeta_{NM,i} < \zeta_{NM} < \zeta_{NM,i+1}]$ is,

$$f(H_s; \zeta_{NM,i} < \zeta_{NM} < \zeta_{NM,i+1}) = \Pr[H_s > H_{s,i}; \zeta_{NM,i} < \zeta_{NM} < \zeta_{NM,i+1}] \tag{3.293}$$

where ζ_{NM} is the height of the water column from the bottom at the instant of the measurement, and represents all possible contributions to it (see Figure 3.3.28).

Figure 3.3.28. ***Extreme regime of sea states conditioned to the sea level***



3.7.2 Statistical description of operational and calm cycles

An operational cycle is a continuous sequence of meteorological states in which all the state descriptors are lower than their respective threshold values. The meteorological cycle can also be classified as a calm from an operational perspective though this does not imply that the state descriptors are cancelled out. The operational cycle is defined by the non-exceedance of the threshold value. In the case of the waveheight descriptor, $\Upsilon_{op} = H_{s,u}$

$$\Pr[H_s(t) < H_{s,u}] = 1 - \Pr[H_s(t) \geq H_{s,u}] = 1 - F(H_{s,u}) \tag{3.294}$$

In middle latitudes, the operational cycle is far from being a rare event. Rather, it occurs quite frequently. From the perspective of maritime and atmospheric climate agents, a way of defining normal operating conditions is when the meteorological state descriptors do not exceed any of the threshold values. In these circumstances, the port area is operational although it might not be in use.

3.7.2.1 Persistence, regimes

The persistence of the operational cycle is the time passed by the threshold value of the state descriptor between a downwards step and the following upwards step. In the same way as non-exceedance, persistence is a random variable. In port areas, because of the special nature of harbor conditions, there is no need to fit it to a specific distribution function. In the open sea and at various other locations, thanks to visual data, persistence is a good fit for a biparametric Weibull distribution.

Generally speaking, there is no reason for there to be a functional relation between the probability model parameters of the persistences of loading and operational cycles or their numbers since they have different threshold values.

3.7.2.1.1 Mean persistence of the operational cycle

The mean persistence of $\overline{\tau_{op}}$ of the threshold value in Υ_{op},

$$\overline{\tau_{op,cr}} = \frac{\sum_i \tau_{op,u,i}}{\overline{n_{op,\Upsilon_{op}}}} \tag{3.295}$$

where $\overline{n_{op,\Upsilon}}$ is the mean number of non-exceedances of threshold, Υ_{op}, in other words, the mean number of loading cycles in the time interval (e.g. a meteorological year).

3.7.2.2 Regimes of minimum values

Sometimes, it may be necessary to determine very strict operating conditions so that the threshold value is very small. This means that it will no longer belong to the class of centered values, but will belong to the lower tail class of values. In this case, the persistence regimes of minimum values should be treated in much the same as those of maximum values. In other cases, it may be necessary to specify the minimum or maximum persistences of the operational cycles. Once more, this situation becomes a problem of minimums and maximums. It is thus best to follow the data fitting techniques recommended in the Annex. The random variable, maximum persistence or minimum persistence in the time interval (meteorological year), can be defined for this purpose, and a probability model can be fit to the sample of these values.

The probability that the maximum persistence will not be exceeded, or that the minimum persistence will be exceeded in the time interval, can be obtained by the conditioned probability. In other words, this is the probability that there will be a persistence lower than the minimum conditioned to the occurrence of the operational cycle.

$$\Pr\left[x < x_{min} \middle| \Upsilon \le \Upsilon_{op}; year\right] = \frac{\Pr\left[\left(x < x_{min}\right) and \left(\Upsilon \le \Upsilon_{op}\right)\right]}{\Pr\left[\Upsilon \le \Upsilon_{op}; year\right]} \tag{3.296}$$

3.7.3 Statistical description of loading cycles

A loading cycle is formed by a random sequence of meteorological states. In each of these states, the probability of a failure mode affecting the safety and serviceability is significant. Therefore, the statistical description of loading cycles is the key to calculating the safety or serviceability of the maritime structure in extreme work conditions.

In the case of sea states, the principle descriptors of the cycle are their duration or persistence, the maximum value of the state or maximum peak, and the number of loading cycles in the useful life. The three descriptors are random variables with their respective probability functions, as described in what follows. The duration or persistence of the state of a random process is related to its capacity to change over time.

Generally speaking, marine oscillatory processes require a minimum time for the action of generating or damping forces in order to change. This involves having a minimum sample size in order to arrive at a reliable statistic. For this reason, the duration of the state can be fixed or determined. Nevertheless, the same is not true of the duration of the loading cycle, $\tau_{s,cr}$, which is random. It depends, among other things, on the threshold value, and quantifies the time that the state curve remains above this value between two consecutive upward steps.

When the threshold is sufficiently high, the maximum peak of the cycle belongs to the family of rare events, and complements the sample data, composed of the annual maximum peaks (one per year). Its use allows an approximation to the function of the annual extreme value, based on all available information.

The curve (24) $H_s(t) > H_{s,cr}$ (significant wave height or of any other climate descriptor) provides the necessary information to be able to do the following: (1) calculate the relative frequency of sea states over a threshold value and based on this frequency, fit it to a probability function that represents its variability in the time interval analyzed; (2) calculate the number of loading cycles and their duration; (3) calculate a sample of maximum peaks; (4) calculate a sample of annual maximums.

Note. *In each of the sea states of the cycle, the value pairs of the wave height and period have a joint distribution function, whose parameters depend on the state descriptors as well as on the age of the waves. This distribution function provides the corresponding marginal and conditioned distributions of the wave heights and periods as well as the maximum wave height distributions and other order statistics.*
Moreover, in each of the sea states of the cycle, the waves can be described by means of the spectral density function and its corresponding moments as well as by state descriptors. In addition, there are functional and statistical relations between the statistic and frequency descriptors of both. These relations can be applied to one another to obtain a description of sea states and to verify the quality of the data.

3.7.3.1 Relative frequency of threshold exceedance values

If there is a sample of persistences, $\tau_{s,cr,i}$ of the $i = 1, \ldots, I$ loading cycles in a time interval, Υ, (e.g. meteorological year or useful life), this sample can be used to obtain probability models of the event (i.e. threshold exceedance) and of the random variable (i.e. duration of that exceedance).

If on the sea state curve, measured in various meteorological years, different values of $H_s > H_{s,cr}$ are taken, and the accumulated time is calculated when the curve is over the value of H_s, and then divided by the duration $\Upsilon_{H_{s,cr}}$ of the exceedance of threshold $H_{s,cr}$, this gives the frequency of the threshold exceedance in the time interval analyzed, in other words,

$$\Pr\left(H_s > H_{s,i} \,\middle|\, H_{s,i} > H_{s,cr};\Upsilon\right) = \frac{\sum_i \tau_{s,i|cr}}{\Upsilon_{H_{s,cr}}} \tag{3.297}$$

where $\tau_{s,i\,|\,cr}$ is the duration of the exceedance of value $H_s > H_{s,i}$ conditioned to the threshold value exceeding critical value $H_{s,i} > H_{s,cr}$.

By definition the conditioned function should satisfy the following:

$$\Pr\left(H_s > H_{s,i};\Upsilon\right) = \overset{*}{\Pr}\left(H_s > H_{s,i} \,\middle|\, H_{s,i} > H_{s,cr}\right) * \Pr\left(H_{s,i} > H_{s,cr}\right) \tag{3.298}$$

If data can be regarded as belonging to *M* statistically independent occurrences, one for each meteorological year, the fit probability model is the annual regime of over-threshold sea states,

$$\overset{*}{\Pr}\left(H_s > H_{s,i} \,\middle|\, H_{s,i} > H_{s,cr}; year\right) = \frac{\sum_i \tau_{s,i|cr}}{M \square years_{H_{s,cr}}} \tag{3.299}$$

(24) The following sections deal with H_s and refer to the sea state. However, any other agent can be described in the same way.

3.7.3.2 Probability model of over-threshold exceedances: GPD

The conditional probability of random variable H_s when it is known that $H_{s,i} > H_{s,cr}$, is the following:

$$\Pr\left[\left(H_s < H_{s,i} \middle| H_{s,i} > H_{s,cr}\right); \Upsilon\right] = \frac{F\left(H_{s,i}\right) - F\left(H_{s,cr}\right)}{1 - F\left(H_{s,cr}\right)} \quad (3.300)$$

$$H_{s,i} > H_{s,cr}$$

where F is the distribution function of H_s.

For sufficiently high values of $H_{s,cr}$, this probability adequately approaches the generalized Pareto distribution, $GPD(\sigma_P, \xi_P)$

$$F^*\left(H_s > H_{s,i} \middle| H_{s,i} > H_{s,cr}; \Upsilon\right) = \left[\frac{1 - \xi_P\left(H_{s,i} - H_{s,cr}\right)}{\sigma_P}\right]^{\frac{1}{\xi_P}} \quad (3.301)$$

$$H_{s,i} > H_{s,cr}$$

where σ_P is the scale parameter $(\sigma_P > 0)$ and ξ_P is the shape parameter. If $\xi_P = 0$, then that gives the exponential distribution function. This probability model can also be applied when the meteorological year is the time interval used.

Note. In some open sea applications, it often happens that $F(H_s)$ fits a biparametric Weibull function, such as,

$$F^*\left(H_s\right) = \exp\left[-\left(\frac{H_s}{b}\right)^a\right] \quad (3.302)$$

where a and b are two parameters that depend on the site. More specifically, b is directly proportional to the threshold value, H_s, and is thus a scale parameter. In contrast, a provides the growth rate of H_s when the probability of exceedance decreases, and is thus a shape parameter.

3.7.3.3 Regime of over-threshold exceedance conditioned to the propagation direction

In most of the projects of ports as well as shoreline areas, it is necessary to apply exceedance regimes of H_s conditioned to the propagation direction.

If there is sufficient information available, the procedure described in section 3.7.2.9 should be followed.

3.7.3.4 Persistence regimes

The persistence of the loading cycle is the time that the state descriptor value remains continuously over the threshold value. Like exceedance, persistence is also a random variable. In the open sea, this variable follows a biparametric Weibull model, whose parameters depend on the threshold value $H_{s,cr}$.

3.7.3.4.1 Mean persistence and mean number of loading cycles

The mean persistence $\tau_{s,cr}$ of the threshold value in Υ is,

$$\overline{\tau_{s,cr}} = \frac{\sum_i \tau_{s,cr,i}}{\overline{n_{cr,\Upsilon}}} \quad (3.303)$$

where $\overline{n_{cr,\Upsilon}}$ is the mean number of over-threshold exceedances in Υ, in other words, the mean number of loading cycles in that time interval.

3.7.3.5 Probability model of the number of loading cycles in Υ

If the over-threshold exceedance $H_s > H_{s,cr}$ of an event is a rare event in a given time interval (e.g. the quotient of the number of events and the number of days in Υ is $O(10^{-1})$ or less), then the number of loading cycles $n = n_{cr,\Upsilon}$ in Υ is a random variable, which generally fits a Poisson probability model, whose density function is the following:

$$f(n) = \frac{v^n e^{-n}}{n!} \tag{3.304}$$

where $v = \overline{n_{cr,\Upsilon}}$ and $n = 0,1,2,\ldots$ are the possible number of loading cycles (over threshold exceedances) that can occur in time interval Υ.

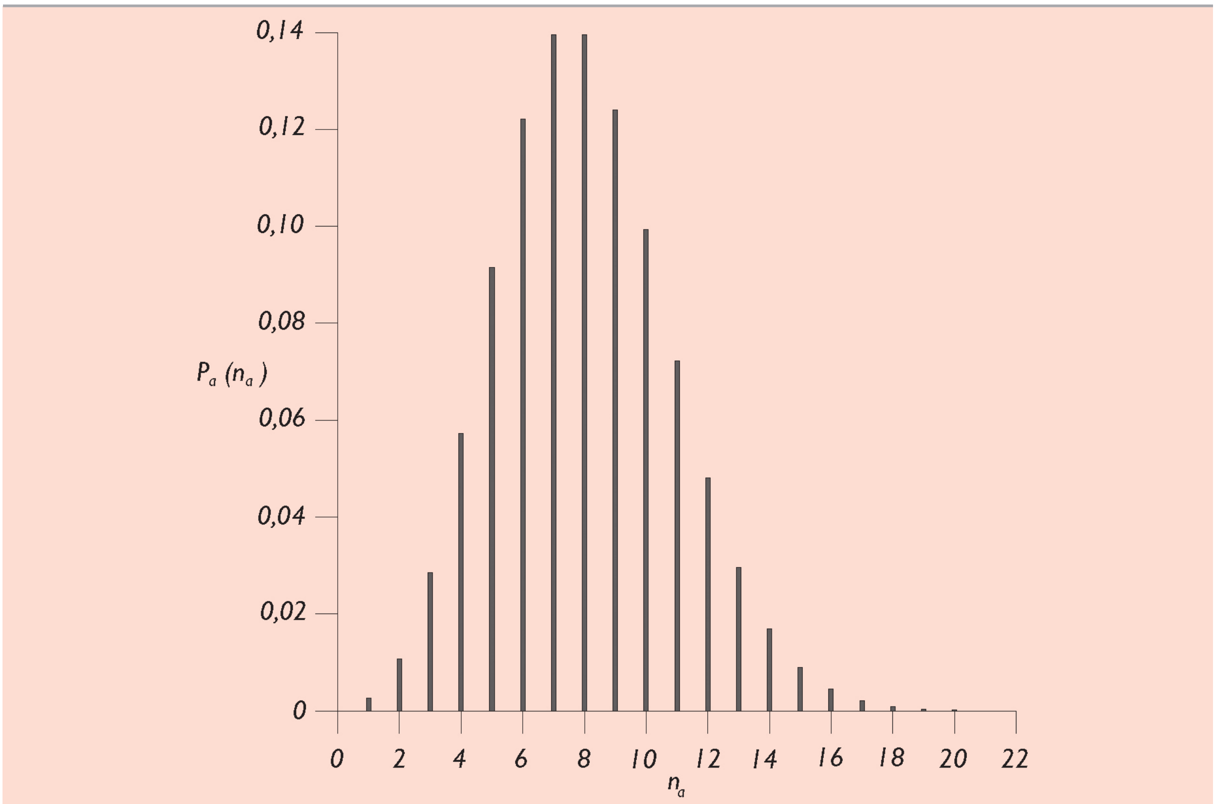
3.7.3.5.1 NUMBER OF LOADING CYCLES IN THE METEOROLOGICAL YEAR AND IN THE USEFUL LIFE OF A STRUCTURE

The number of observations (i.e. loading cycles) during the year n_a or useful life n_V are discrete random variables (the same as the wavenumber in a sea state or the zero-crossing period of individual waves), whose statistical descriptors and probability model can be estimated on the basis of measurements and retroanalysis. Generally speaking, the accumulated distribution of rare events (defined as over-threshold) in a time interval can be fit to a Poisson Model (see Figure 3.3.29).

$$P_a(n_a) = \frac{v_a^n e^{-v_a}}{n!}, \quad n = n_a = 0,1,2,3\ldots \text{ in the metoreological year} \tag{3.305}$$

where the distribution parameter is $v_a = \overline{n_a}$ mean number of loading cycles (mean number of maximum peaks) in the meteorological year. Similarly, the distribution function of the number of observations in the useful life can be defined as follows:

Figure 3.3.29. **Poisson density function of the number of loading cycles in the meteorological year**



$$P_V\left(n_V\right) = \frac{v_V^n e^{-v_V}}{n!}, \quad n = 0,1,2,3\ldots \text{ in the useful life} \tag{3.306}$$

where $v_V = \overline{n_V}$ mean number of loading cycles (mean number of maximum peaks) in a structure's useful life. Generally speaking, no information is available regarding useful life. However, assuming that years are statistically independent tests, the mean number of loading cycles in a structure's useful life can be estimated by

$$\overline{n_V} = \overline{n_a} * V \tag{3.307}$$

Once the distribution is estimated for the meteorological year, the distribution during the useful life can also be derived for the useful life.

Note. *When extreme data are scarce, the extreme regime should be calculated from the sample of the maximum peaks in the loading cycle. For this reason, it is necessary to know the mean value of the number of cycles in a year. If a sample of values of V is available, and if in the sample, there are n_V peaks, under the hypothesis of the statistical independence of each meteorological year, the mean number of cycles per year $\overline{n_a}$, in other words, the Poisson parameter, $v = \overline{n_a}$ can be estimated by,*

$$v = \overline{n_a} = \frac{n_V}{V} \tag{3.308}$$

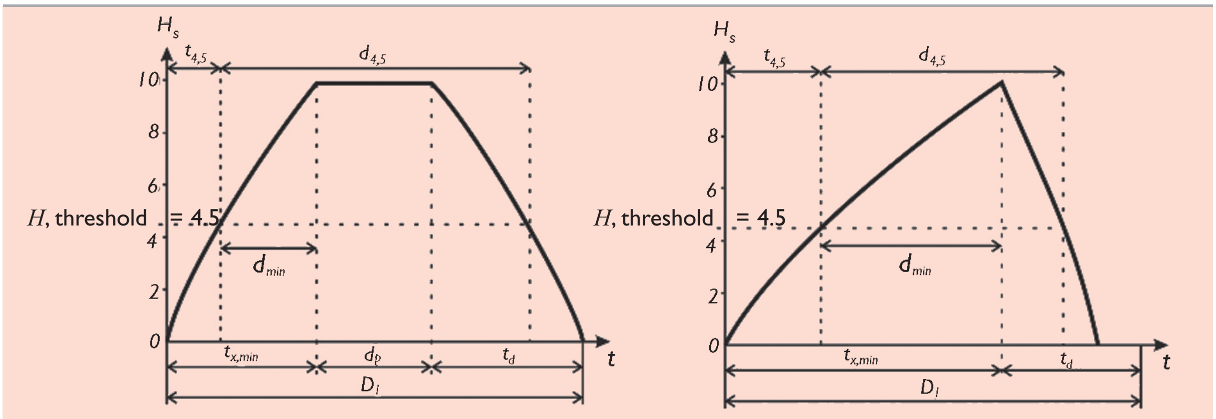
3.7.3.6 Loading cycle models

In order to obtain the distribution functions of the descriptors associated with the occurrence of loading cycles, it is necessary to take their shape into account. This contains information regarding the generation and evolution of the sea state. In middle latitudes, loading cycles are generally associated with the passage of extratropical storms, and thus the shape of meteorological state evolves according to the following pattern. At the beginning of the state, there is a swell sea without wind. As the swell intensifies with the approximation of the storm, the sea spectrum shows two peaks going in different directions. These two peaks finally merge into one peak in the dominant direction. After the storm has passed, the energy of the spectrum is considerably reduced in all frequencies, without any one frequency dominating the others.

3.7.3.6.1 Temporal evolution

The temporal evolution of the sea state curves $(H_s - t)$, $(\overline{T}_z - t)$, $(\overline{\theta} - t)$ provides the shape of the storm (see Figure 3.3.30). In each of the coastal areas in Spain, the threshold sea state of the loading cycle generated by an

Figure 3.3.30. Synthetic shapes of storm evolution



extratropical storm can be initially fixed at a certain value $H_s > H_{s,u}$. The shape of this curve can be decomposed into three sections: (1) generation and growth; (2) maximum; (3) dissipation. The first section of the curve depends on the windspeed, its uniformity over the fetch, and the dimensions of the fetch, which in turn, depend on the size and dimensions of the storm, the pressure at the center of the storm, and the pressure gradient.

The section surrounding the maximum and its duration depends on the rotation speed of the storm and how fast it is moving. Both factors are constrained by the synoptic boundary conditions, which finally are those that regulate the decline of the curve, along with angular and dispersion processes as well as the shape of the coastline. Therefore, the shape of the loading cycles is random.

3.7.3.6.2 *Shape parameters of the loading cycle*

Once the loading cycle is divided into meteorological states, the failure probability as opposed to the safety of the breakwater can be calculated on the basis of the failure probabilities in each of the states. The initial states of the generation section of the curve, as well as the final states of the decline, before the threshold value is crossed make a slight contribution to the maximum section of the curve. For this reason, the calculation of the failure probability during the loading cycle can be approximated by working with loading cycles of equivalent shape, depending on whether the storm circulates rapidly in the zone or moves and evolves more slowly. In the first case, the shape of the cycle is similar to a triangle, whereas in the second case, its shape is trapezoidal or even rectangular.

The parameters that define the equivalent shape of the loading cycle are the threshold value and the exceedance duration, $H_{s,u}$, τ_u, the maximum value, and its duration, $H_{s,\max}$, $\tau_{\max}$ and a representative value of the intensity of the storm and the duration of its exceedance, $H_{s,r}$, τ_r, for example, about 75% of the value of the maximum. It is advisable to adopt a triangular or rectangular shape when the following relations are approximately fulfilled, Table 3.3.24, (Figure 3.3.30)

Table 3.3.24. Shape and parameters of the loading cycle

	Triangular	Rectangular
$\tau_{\max}$ (hours)	≤ 3	> 3
$H_{s,r}$ / $H_{s,u}$	≈ τ_u / τ_r	0.75 - 1.0
$H_{s,r}$ / $H_{s,\max}$	< 0.75	≥ 0.750
τ_{eq}	–	area ≈ $\tau_{eq} * H_{s,\max}$

where τ_{eq} is the equivalent duration of a rectangular storm that is obtained by approximately conserving the area under the real state curve and the rectangular one.

When the evolution of the storm cannot be suitably reproduced with one of the two previously mentioned shapes, it is best to try using trapezoidal shapes or two rectangles, one for $(H_{s,u} \le Hs(t) < H_{s,r})$ and the other for the rectangular triangular section $(H_{s,r} \le Hs(t) < H_{s,\max})$. This can be done by adopting for the base of each one the criterion of area conservation.

A more objective way of classifying the magnitude and intensity of the storm is by means of the parameter M_T which quantifies its total energy content with the following expressions:

$$M_T$$

$$\text{triangular}: \frac{(H_{s,max} - H_{s,u})\tau_u}{2}; \quad \text{rectangular}: (H_{s,max} - H_{s,u})\tau_u; \quad \text{trapezoidal}: \frac{(H_{s,max} - H_{s,u})(\tau_u + \tau_{max})}{2}$$

Note. The failure probability against the safety during the useful life of a structure does not vary significantly with the shape of the loading cycle. It is thus not necessary to meticulously adjust the shape of the loading cycle.

3.7.3.7 Meteorological tide associated with the loading cycle

Once the shape of the loading cycle and the characteristics of the storm that generated it are known, the resulting meteorological tide can be calculated by applying one of the models described in section 3.12.1, or more simply by using a functional relation with the significant wave height of each state. These relations are specific of each zone of the project site, and before they are applied, they should be verified.

3.7.4 Analysis of maximum and extreme values

Generally speaking, the term *extreme value* designates the highest value that a random variable can take in a given number of observations. The largest variable is in turn a random variable that depends on the size of the sample. This definition can be applied to the maximum wave height in (1) a sea state with a duration of one hour; (2) a loading cycle of random duration; (3) a meteorological year; (4) or in the useful life.

Therefore, in order to predict the maximum value of the wave height and its probability of exceedance in one of the time intervals, one must know the expected number of waves that can occur in the corresponding time interval. Furthermore, the definition is valid for the descriptor (e.g. signification wave height) of the maximum state in: (1) loading cycle of random duration; (2) a meteorological year; (3) the useful life of a structure. Similarly, to predict its maximum value and probability of exceedance, it is necessary to know the expected number of observations in the corresponding time interval. In the case of the annual maximum or extreme sea state, one per year, the number of observations is equal to the number of years. It is thus already determined.

However, the peak is the largest sea state in the loading cycle and the maximum peak refers to the maximum sea state of the loading cycles. In this case, the number of observations is equal to the number of cycles that can occur in the time interval, for example, the meteorological year. However, the number of peaks or loading cycles that can occur in the year is random.

This situation is paradoxical. The distribution function of the highest value of a sample of *n* observations is obtained by means of a asymptotic approximation to its real shape when the number of observations is very large. Consequently, the most logical method to obtain the annual extreme regime or storm regime is to fit the probability model to a sample of extreme values, one per year. Nevertheless, in the majority of cases, this sample is finite and very small, and for this reason, the result obtained is uncertain.

On the other hand, the extreme regime can be approximated on the basis of the extreme distribution of the exceedances over the threshold value as long as they satisfy the requirement of statistical independence and rare (infrequent) value. The maximum peak of the loading cycles satisfies both these criteria, and it can be demonstrated that the distribution function of the largest maximum peak value asymptotically approaches the extreme regime or the distribution of the annual maximum sea state. To make this calculation, one must know the number of peak values or loading cycles that occur during the year, which, as previously mentioned, is a random variable. Fortunately, for rare events, this variable is a good fit for the Poisson distribution of the parameter for the mean number of peaks (or loading cycles during the year). The following sections provide the information needed to obtain the extreme regime of sea states, according to one of the two previously described procedures.

3.7.4.1 Distribution of the maximum wave height in a sea state

The waves measured in a sea state are an sequential sample of *n* heights that are generally not independent. However, if these wave heights are ordered according to size from the smallest to the largest, $[H_1, \ldots, H_n]$, where H_1 is the smallest height, and H_n is the largest, the elements in the sample are independent, and each has its own probability function.

If the density function and distribution function of a random variable H are $f_H(x)$, $F_H(x)$, then the probability function of the n number of elements is a binomial. This is the case of a sequence of random variables (e.g. the wave height) ranked according to increasing size, which are less than or equal to a given value (see section 4.10.6.3 of the Annex). This binomial expression can be used to obtain the no-exceedance probability of any element or order statistic. For the largest value, H_{max}, this is the following:

$$G_{H_{max}}(x) = \left[\left\{F_H(x)\right\}^n\right] \quad (3.309)$$

$$g_{H_{max}}(x) = n\left\{F_H(x)\right\}^{n-1} f_H(x) \quad (3.310)$$

where n is the size of the sample or number of observations, which depends on the duration of the sea state. Although the sea state has a random duration, for physical as well as statistical reasons, the upper and lower boundary of the duration should be clearly marked. For this reason, $\Delta t \approx O(1\ hour)$ a given duration is generally adopted, for instance, $\Delta t \approx O(1\ hour)$. If the zero-crossing mean period of the sea state is $\overline{T_z}$, the number of observations is the following:

$$n = \frac{\Delta t}{\overline{T_z}} \quad (3.311)$$

$F_H(x)$ is the Rayleigh distribution function of the significant wave height parameter, or the mean square wave height, of the initial variable, which is the wave height (see the section in the Annex on random variables and probability).

3.7.4.2 Probability of exceedance of the maximum wave height

The preceding section described how to obtain the distribution function of the maximum wave height in a sea state. The following sections discuss how to obtain the distribution function of the largest wave in a loading cycle, in the random sequence of loading cycles in the meteorological year and in the useful life of a structure.

3.7.4.2.1 In a loading cycle

Once the shape of the loading cycle is known, the probability of exceedance of a given maximum wave height in the loading cycle is the following:

$$F_{H_{max}}(H > x; cycle) = 1 - \prod_{n_e=1}^{N_e}\left\{1 - \Pr\left[H > x; H_s = H_{s,n_e}\right]\right\}^{\frac{\Delta t_{n_e}}{\overline{T_{z,ne}}}} \quad (3.312)$$

where Δt_{ne} is the duration of the state; $n_e = 1, \ldots, N_e$ is the number of sea states in the loading cycle; and $\overline{T_{z,ne}}$ is the mean period of each of the *ne* sea states. Another way of expressing this equation is:

$$F_{H_{max}}(H > x; cycle) = 1 - \exp\left\{\sum_{n_e=1}^{N_e}\frac{\Delta t_{n_e}}{\overline{T_{z,n_e}}}\ln\left[1 - \Pr\left(H > x; H_s = H_{s,n_e}\right)\right]\right\} \quad (3.313)$$

where $\eta_H = \Delta t_{ne} / \overline{T_{z,ne}}$ is the number of waves in the sea state. If the shape of the cycle can be analytically expressed, then it is possible to write:

$$F_{H_{max}}(H > x; cycle) = 1 - \exp\left\{\int_0^{\Gamma}\frac{\ln\left[1 - \Pr\left(H > x; H_s = H_{s,n_e}\right)\right]}{\overline{T_{z,n_e}}}dt\right\} \quad (3.314)$$

and the mean maximum wave height in the loading cycle is:

$$\overline{H_{max}} = \int_H^{\infty} F_{H_{max}}(H > x)dx \quad (3.315)$$

This calculation does not take into account the sequence of occurrence of the successive sea state. It also assumes that their duration or the number of sea states in the loading cycle is known. When this duration is random, but the shape of the loading is known, the number of sea states in the cycle $N_e / \Delta t_{ne}$ and the mean number of waves in each sea state is $\Delta t_{ne} / \overline{T_{z,ne}}$.

The joint distribution function of the duration of the cycle and of the maximum wave height is the following:

$$F_{H_{max},D_c}\left(H > x; D_c > z\right) = F\left(H_{max} > x \middle| D_c\right) * F_{D_c}\left(z\right) \tag{3.316}$$

where $F_{D_c}(z)$ is the marginal distribution function of the durations or persistences of the loading cycles where

$$D_c = \sum_{n_e=1}^{N_e} n_e * \Delta t_{n_e} \tag{3.317}$$

and the number of waves in each of the cycles is

$$n_H = \sum_{n_e=1}^{N_e} n_e * \frac{\Delta t_{n_e}}{\overline{T_{z,n_e}}} \tag{3.318}$$

3.7.4.2.2 During the year and in the useful life

The same procedure can also be used to calculate the distribution function of the maximum wave height in a meteorological year in which a random number *n* of loading cycles of random duration occur. The distribution function of the maximum wave height in the meteorological year is the following:

$$F_{H_{max}}\left(H > x; year\right) = \sum_{n=0}^{\infty} P_N\left(n\right) * F_{H_{max},D_c}\left(H > x; D_c > z\right) \tag{3.319}$$

in other words, this distribution function is obtained by weighting the previously obtained value, representing the probability of there being $n = 1, 2, \ldots, N_e$ loading cycles in the year.

If each meteorological year is assumed to be a statistically independent event, the distribution function of the maximum wave height in the useful life of a structure is

$$G_{H_{max}}\left(H > x; V\right) = \left[F_{H_{max}}\left(H > x; year\right)\right]^V \tag{3.320}$$

3.7.4.3 *Distribution function of annual peaks (Peaks over Threshold Regime)*

The maximum state descriptor value in the loading cycles $H_{s,\max,cycle}$ is a random variable. The sequence of values, $H_{s,\max,cycle,j}$ where $j = 1, 2\ldots$, occurring in the time interval are a statistically independent sample of elements. When a sufficiently high threshold sea state value, $H_{s,cr}$, is selected, the distribution function, $F_{H_{s,\max,cycle}}(H_s)$, generally fits the distribution function of the Generalized Pareto Distribution (GPD)(σ_P, ξ_P),

$$F_{H_{s,max,cycle}}\left(x \middle| x > H_{s,cr}\right) = \left[\frac{1 - \xi_P\left(x - H_{s,cr}\right)}{\sigma_P}\right]^{1/\xi_P}; \; x > H_{s,cr} \tag{3.321}$$

where σ_P is the scale parameter $(\sigma_P > 0)$ and ξ_P is the shape parameter. If $\xi_P > 0$, this gives the exponential distribution function. When sufficient data is available, it is advisable to divide the meteorological year in at least two periods: (i) one period when storm events frequently occur; (ii) another period when storm events are less frequent. This same information can be used to obtain the distribution function of the number of loading cycles (or maximum peaks) $P_a(n_a)$ in the meteorological year and their mean value, $\overline{n_a}$.

Note. *Strictly speaking, the peak regime is not an extreme regime. For this reason, there are various theoretical functions that can be fit to it. However, it can be fit by means of the generalized extreme value function (GEV).*

3.7.4.4 Distribution function of the maximum number of N annual peaks (annual maximum peak regime)

In most cases, there is only a small amount of data available to fit the extreme value distribution. One way to solve this problem is to work with the maximum peak of the loading cycles, namely, $H_{s,\max,cycle}$. When a sufficiently high value is defined for the exceedance threshold, this guarantees that the peak values correspond to statistically independent loading cycles. This means that the independence requirement for sample elements is fulfilled. However, unlike the annual extreme values, the peak values are not equally spaced, and the sample is composed of a finite, but random number of elements. The sample of peak values, $H_{s,peak,j}$ where $j = 1, 2\ldots, n$, makes up a set of random variables with the same distribution function $F_{Hs,peak}(x)$, where x is an observed value, and n is a random number of peaks that occur in a given time interval (e.g. meteorological year).

If $P_N(n)$ is the accumulated distribution function of N, the number of peaks in a given time interval, the distribution function of the maximum of loading cycle peaks can be calculated by weighting the function $[F_{Hs,peak}(x)]^n$ by means of $P_N(n)$ for all possible values n of N,

$$G_{H_{s,extreme}}(x) = \sum_{n=0}^{\infty} P_N(n) * \left[F_{H_{s,peak}}(x)\right]^n \tag{3.322}$$

When the occurrence of loading cycles is assumed to be a Poisson process, this gives the following:

$$G_{H_{s,extreme}}(x) = \sum_{n=0}^{\infty} P_N \frac{v^n e^{-v}}{n!} * \left[F_{H_{s,peak}}(x)\right]^n \tag{3.323}$$

where v is the mean number of loading cycles in the time interval (e.g. meteorological year). The previous expression (3.323) can also be written as follows:

$$\begin{gathered} G_{H_{s,extreme}}(x) = e^{-v\left[1-F_{H_{s,picoeak}}(x)\right]} \\ 1 - p_e = e^{-vp_p} \approx 1 - vp_p \end{gathered} \tag{3.324}$$

where

$$\begin{aligned} p_p &= \Pr\left[H_{s,peak} > x\right] = 1 - F_{H_{s,peak}}(x) \\ p_e &= \Pr\left[H_{s,extreme} > x\right] = 1 - G_{H_{s,extreme}}(x) \end{aligned} \tag{3.325}$$

$G_{Hs,extreme}(x)$ is the probability of exceedance of the annual maximum peak or extreme value. For small values of vp_p, the following equation holds:

$$p_e = 1 - e^{-vp_p} \approx vp_p \tag{3.326}$$

This expression can be used to evaluate the return period of the annual maximum sea state, based on the distribution function of the loading cycle peaks, $F_{Hs,peak}(x)$, and the mean number of peaks per year, v.

The following table provides the extreme function, $G[\max H_s, annual]$ which corresponds to a given $F_{Hs,peak}(x)$. (See Table 3.3.25.)

Table 3.3.25. Distribution function of annual peaks over threshold

$F_{Hs,peak}(x)$	$G_{Hs,extreme}(x)$
Exponential: $1 - exp[-\lambda(x - \varepsilon)]$	Gumbel: $exp[-ve^{-\lambda(x-\varepsilon)}]$
Pareto: $1 - (a/x)^{\theta}$; $x > a,\ \theta > 0$	Frechet: $exp[-v(a/x)^{\theta}]$
GPD: $1 - [1 - \zeta_P(x - H_{s,cr} / \sigma_P)]^{1/\xi_P}$	GEV: $1 - exp\{-([1 - \delta_E(x - \mu_E)] / \sigma_E)^{1/\delta_E}\}$

λ, a, σ_P and σ_E are scale parameters; θ, ξ_P and δ_E are shape parameters; and ε, $H_{s,cr}$ and μ_E are localization parameters. If $\xi_P = 0$, the GPD function reverts to the exponential function with displacement, $\varepsilon = H_{s,cr}$, whose extreme function is the Gumbel function of scale parameter, $\lambda = \sigma_P$ and localization parameter, $\varepsilon + \ln\nu / \lambda = H_{s,cr} + \sigma_P \ln\nu$. The relations between the GPD parameter (σ_P, ξ_P, $H_{s,cr}$) and the GEV parameter (σ_E, δ_E, μ_E) are the following:

Localization parameter

$$\mu_E = \frac{H_{s,cr} + \sigma_P \left(nP_{cr}\right)^{\xi_P} - 1}{\xi_P} \tag{3.327}$$

Scale parameter $(\sigma_P > 0)$

$$\sigma_E = \sigma_P \left(nP_{cr}\right)^{\xi_P} \tag{3.328}$$

Shape parameter

$$\delta_E = \xi_P \tag{3.329}$$

If the threshold value is sufficiently high, the distribution tail of the annual maximums and the maximum peaks of the loading cycles fit closely enough in order to accept that

$$G_{H_{s,max\ cycle}}\left(x\right) \approx G_{H_{s,max\ annual}}\left(x\right) \tag{3.330}$$

Note. *The observations of the loading cycle peaks are known as* peaks over threshold *(POT) or* partial distribution series *(PDS).*
If the sea state curve for K years is available, the work scheme consists of the following steps:

1. *Select loading cycles, based on the definition of a threshold value. Each loading cycle should belong to independent storms. The number of cycles in the sample N is then counted.*
2. *Select the maximum peak values per loading cycle, $H_{s,\max,cycle}$*
3. *Calculate the relative frequency, in other words, the number of observations that exceed different levels of $H_{s,\max,cycle} = x$, and fit the distribution function, $F_{H_{s,\max,cycle}}(x)$. The fit can be carried out by using probability paper or by applying GPD formulas.*
4. *Calculate the mean number of loading cycles during the year, ν. This is done by counting the total number of cycles (maximum peak values), and dividing this number by the number of years of data collection: $\nu = N/K$*
5. *Apply the following expression:*

$$G_{H_{s,extreme}}\left(x\right) = e^{-\nu\left[1 - F_{H_{s,peak}}\left(x\right)\right]} \tag{3.331}$$

6. *Calculate the distribution parameters, $G_{H_{s,\max,peak}}(x) \approx G_{H_{s,\max,annual}}(x)$.*

3.7.4.5 Annual maximum sea state regime (storm regime)

The annual maximum (or extreme) regime is the distribution function of the highest value in each meteorological year of the sea state descriptor, which has the greatest likelihood of coinciding with the peak of one of the loading cycles $(X_{\max} \approx \max X_{peak})$. If there is a sample of maximum sea state observations, $H_{s,\max,annual}$, one per year, of size N, and the storm events of each year regarded as statistically independent, then the sample is composed of N equally spaced, independent elements. The distribution function of the maximum value of the N observations is $G_{H_{s,\max,anual}}(x)$. In Spain, this distribution function is known as the *Régimen de Temporales* (Storm Regime).

Data is fit to a probability model by ordering the *N*-value sample from the lowest to the highest value, and assigning the accumulated frequency (non-exceedance frequency).

$$P_i = \frac{i}{N+1}, \quad i = 1,2,\ldots,N \tag{3.332}$$

It is customary to fit the data by using the maximum likelihood method as well as a graphical representation on probability paper. This procedure assumes that the density function of extreme values is uniform. In other words, it assumes that all values are equally probable, and since one value is taken per year, that all data are equally spaced.

The distribution function of the maximum value of the n-size data series, where $n \to \infty$ asymptotically approaches the generalized extreme value function (GEV) (σ_E, δ_E, μ_E). If X_{max} is the random variable (maximum value when $n \to \infty$), then its distribution function is

$$G_{X_{max}}(x) = \Pr[X \le x] = \exp\left\{1 - \left[\frac{1-\delta_E(x-\mu_E)}{\sigma_E}\right]^{\frac{1}{\delta_E}}\right\} \tag{3.333}$$

where the three distribution parameters are the following: μ_E is the localization parameter; $\sigma_E(\sigma_E > 0)$ is the scale parameter; and δ_E is the shape parameter. For the fit function, it is best for the sample to contain a sufficient number of years, $(n > 10)$.

According to the previous section, there is an underlying distribution function $F_{H_s}(x)$ such that,

$$G_{H_{s,max\,annual}}(x) = G_{X_{max}}(x) = \left[F_{H_s}(x)\right]^N \tag{3.334}$$

even though in the procedure to determine G, it is not usual to obtain $F_{H_s}(x)$.

The return period, T_R refers to the mean number of years that should pass between two consecutive exceedances of this value. It is expressed by the following expression:

$$T_R(x) = \frac{1}{G_{H_{s,max\,annual}}(x)} \tag{3.335}$$

3.7.4.6 *Work conditions*

As long as the probability of exceedance of the value of the predominant agent inducing the failure is negligible for all sea states less than the extreme value, the verification of failure modes assigned to ultimate limit states for extreme or exceptional work conditions will be carried out by applying the extreme regime.

Otherwise, for ultimate limit states as well as for serviceability limit states, it is necessary to work with the distribution function of annual over-threshold exceedances and their extension to the useful life of a structure. This distribution can be obtained from a sample of various years at the project site location or from the maximum peak regime combined with an estimate of the generation and damping curves of the loading cycles, as described in the following sections.

Generally speaking, these conditions are rare or fairly improbable events. For thus reason, the probability models for these conditions can be estimated by adopting the work conditions in the section that describes treating rare events as a Poisson event. Special attention should be given to the time interval of calculation (e.g. meteorological year, useful life, etc.). In these circumstances, except for the agents that are generated by the same cause, all the other agents can be regarded as independent.

3.7.5 Prediction of the sea state on the coasts of Spain

In Spain, *Puertos del Estado* along with the *Agencia Estatal de Meteorología* (AEMET) publish wind and sea forecasts twice a day for the North Atlantic and the western Mediterranean Sea. The source of the wind forecast, used as a forcing agent for wave generation models, is the high-resolution limited-area model, known as HIRLAM, which is by AEMET. The prediction horizon is 72 hours for the Atlantic Ocean and Mediterranean Sea with wind fields predicted every three hours. The system of sea state prediction begins each day at 5:00 and at 17:00. The results become available approximately one hour later, when they can be accessed on the server. This prediction system is based on the application of various wind generation models and sea generation models, forced by wind fields, which are predicted by the HIRLAM model at a height of ten meters, and the WAM wave prediction model.

The version of the model applied to the Atlantic Ocean is for deep water. Consequently, bottom-induced phenomena are not taken into account. In the case of the Mediterranean sea, the shallow depth version of the model is used This model takes into account the attenuation and refraction caused by the seabed at grid points regarded as shallow water areas. In the same way as for the Atlantic Ocean and Mediterranean Sea, specific applications have been developed for the Cantabrian Sea, the Gulf of Cadiz, and the Canary Islands.

Furthermore, the WAVEWATCH model has been developed in the Strait of Gibraltar with the WAM boundary conditions for the Atlantic Ocean and Mediterranean Sea. This third-generation model is activated twice a day to predict wave action. The *Sistema de Predicción en las Autoridades Portuarias* (SAPO) [Prediction System of the Port Authorities] is made up of local sea state prediction systems every 72 hours, specifically developed for port areas and their immediate surroundings. The system is based on the SWAN model, and takes into consideration the transformations of waves as they approach the coastline. In certain cases, it includes a prediction module of the turbulence inside the port.

3.8 WAVE GROUPS

This section describes short-period oscillatory movements (sea) and long-period oscillatory movements (long waves) related to the wave group. Short oscillatory movement is characterized by wave sequence, number, and amplitude that surpasses a certain threshold. The wave group is thus described in terms of its amplitude and period as well as its spatial variability. In both cases, descriptors of the wave group and of the associated long wave are related to the underlying sea state descriptors.

3.8.1 Introduction

Generally speaking, in a sea state, the consecutive waves are not independent, and the consecutive wave heights are positively correlated in that they are composed of larger waves followed by smaller ones. Accordingly, the wave height slowly varies over time with a period several times higher than the mean period of the waves. Wave group formation is related to generation and interaction processes between components, and with the angular and frequency dispersion of the waves once they abandon the area of generation (fetch).

The number of waves N in each of the sequences depends on the age of the waves and on the generating wind speed, though the number is generally in the interval $[3 < N < 6]$. Consequently, a complete sequence of high and low waves has wave number $[6 < N_t < 12]$, and the duration or mean period of group $\overline{T_g}$ is in the interval $[6\overline{T_z} < \overline{T_g} < 12\overline{T_z}]$, in other words, in the interval $[30 < \overline{T_g}(s) < 300]$. In a wind sea (wide-band function), the correlation between consecutive waves is low; in contrast, in a swell (narrow-band function), the correlation between consecutive waves is high.

Generally speaking, the following are regarded as valid for wave groups: (1) Gaussian description of the instantaneous variable $\eta(\vec{x}; t)$; (2) probability models of wave height and period; (3) frequency description by means of the spectral energy function $S_\eta(\vec{x}; f, \theta)$. However, the structure of the wave group can modify the occurrence of consecutive waves.

3.8.1.1 Importance of wave groups

The presence of wave groups does not substantially modify the probability of occurrence of larger waves. However, it makes their action repeat itself various times, which increases the dangerousness of construction work. The worst group is the monochromatic wave train, whose wave heights are all higher than or equal to the threshold value, and whose number is theoretically infinite.

Furthermore, a wave, which is associated with the wave group, is propagated with the same period of the group. This wave is also transformed. Its amplitude is amplified and free wave modes are liberated, which can force oscillations in wharfs, floating structures, and the surf zone.

These oscillations can contribute to long-period movements inside rubble-mound breakwaters, modify breakwater overtopping, and contribute to the accumulation of interstitial pressures in the soils, and fill material, which, if not reduced over time, can significantly increase the probability of liquefaction.

Therefore, in maritime structures where wave group action can be a determining factor and the nature of the structure is $[ERI > 5,\ SERI > 6]$, it is advisable to perform a statistical and frequency analysis of sea states in loading cycles and determine the joint probability function of consecutive waves and the shape of the energy spectrum in the frequency band of waves groups, according to the criteria described in the following sections.

3.8.1.2 Instantaneous variables

The instantaneous variable is the vertical displacement of the free surface with respect to a static reference level. This variable contains three "signals": one signal is related to the waves; another signal η_{LB}, is related to the long waves (which make them a group); and another, η_{HB}, is related to the higher harmonics,

$$\eta(\vec{x};t) = \eta_{wave}(\vec{x};t) + \eta_{LB}(\vec{x};t) + \eta_{HB}(\vec{x};t) \qquad (3.336)$$

3.8.2 Description in the state

The occurrence of wave groups can be estimated by evaluating the joint probability that two consecutive waves (H_1, H_2) will exceed a threshold value.

This probability depends on the correlation between consecutive waves. Generally speaking, the threshold value is usually the mean wave height or the significant wave height.

3.8.2.1 Joint probability of consecutive wave heights, $f(H_1, H_2)$

Once the correlation coefficient between two consecutive waves is defined by

$$\rho_{HH} = \frac{1}{\sigma_H^2}\frac{1}{N-1}\sum_{i=1}^{N-1}\left(H_i - \overline{H}\right)\left(H_{i+1} - \overline{H}\right) \qquad (3.337)$$

where σ_H is the standard wave height deviation and N is the wave number in the record, the joint probability model of these two wave heights depends on their correlation. Generally speaking, the consecutive wave heights are positively correlated. Depending on the correlation of the two consecutive waves, the following probability models are obtained.

3.8.2.1.1 Independent wave heights

If the individual waves are independent, each follows a Raleigh distribution with a mean square wave height, and consequently:

$$f(H_1, H_2) = f(H_1) * f(H_2) \qquad (3.338)$$

3.8.2.1.2 Correlated wave heights

In these conditions, consecutive wave height pairs, H_1, H_2, are generally said to follow a bivariate Rayleigh distribution with a mean square wave height of the sea state H_{rms},

$$f\left(H_1,H_2\right)=\frac{\pi^2}{4\left(1-\kappa^2\right)}\frac{H_1H_2}{H_{rms}^4}\exp\left[-\frac{\pi}{4}\frac{1}{1-\kappa^2}\frac{H_1^2+H_2^2}{H_{rms}^2}\right]I_0\left[\frac{\pi}{2}\frac{\kappa}{\left(1-\kappa^2\right)}\frac{H_1H_2}{H_{rms}^2}\right] \tag{3.339}$$

where κ is a correlation parameter and I_0 is the modified zero-order Bessel function. The relation between the correlation parameter and linear correlation coefficient is the following:

$$\rho_{HH}=\frac{E(\kappa)-\frac{1}{2}\left(1-\kappa^2\right)K(\kappa)-\frac{\pi}{4}}{1-\frac{\pi}{4}} \tag{3.340}$$

where $K(\kappa)$, $E(\kappa)$ are complete elliptic integrals of the first and second kind, respectively. This equation is numerically resolved for a given value of the correlation coefficient. A sufficiently accurate approximation to be applied in engineering (truncation error less than 1% for $0 \le \kappa < 0.95$) is the following:

$$\rho_{HH}\approx\frac{\pi}{16-4\pi}\left[\kappa^2+\frac{\kappa^4}{16}+\frac{\kappa^6}{64}\right] \tag{3.341}$$

3.8.2.1.3 Expected value of correlation parameter $\bar{\kappa}$

This parameter can be obtained by the following expression:

$$\bar{\kappa}=\frac{1}{m_0}\left\{\left[\int_0^\infty S_\eta(f)\cos\left(2\pi fT_{0,2}\right)df\right]^2+\left[\int_0^\infty S_\eta(f)\sin\left(2\pi fT_{0,2}\right)df\right]^2\right\}^{\frac{1}{2}} \tag{3.342}$$

where,

$$T_{0,2}=\sqrt{\frac{m_0}{m_2}} \tag{3.343}$$

and m_0, m_2 are the zero-order spectral moment and the second-order spectral moment, respectively (see section of the Annex on the basic premises of sea state description).

Note. In practice, the correlation coefficient ρ_{HH} varies from 0.2 for wind-wave conditions to 0.65 for swell conditions. ρ_{HH} and the value of $\bar{\kappa}$ are obtained for a given time series, whereas is the expected value of a large number of sea state realizations.

3.8.2.2 Wave number probability

The evaluation of wave numbers, j_1 and j_2, can be carried out in two ways. The first method is direct, and involves working with a discrete random wavenumber variable. The other method involves working with a continuous random variable, which is a waveheight envelope. In the first case, the result is obtained directly, whereas the second analysis provides an intermediate result, which is the time over the threshold. On the basis of this result, it is possible to obtain, for example, the mean wave number by dividing this time by the mean wave period in the sea state.

3.8.2.2.1 Probability function of j_1

The probability function of the wave number depends on whether the consecutive waves are correlated. What is defined is the probability that a wave H_0 exceeding the threshold height is $H*$, on the condition that the previous $H\_ > H_*$ is,

$$p_+ = \Pr\left\{H_0 > H_* \middle| H_- > H_*\right\} \tag{3.344}$$

And the probability that neither of the two exceeds the threshold is expressed by,

$$p_- = \Pr\left\{H_0 < H_* \middle| H_- < H_*\right\} = 1 - p_+ \tag{3.345}$$

Independent consecutive waves. If the consecutive waves in the wave train are not correlated, the probability p_+ and probability p_- are respectively given by

$$p_+ = \exp\left[-\frac{H_*^2}{H_{rms}^2}\right] \qquad p_- = 1 - p_+ \tag{3.346}$$

A group of j_1 waves is a process in which the first wave exceeds the threshold value, H_*. The subsequent waves $(j_1 - 1)$ also exceed the threshold. However, the following wave, number $(j_1 + 1)$ does not exceed the threshold value. Consequently, the probability of the group having j_1 number of waves is,

$$P(j_1) = p_+^{j_1-1}(1-p_+) = \left\{\exp\left[-\frac{H_*^2}{H_{rms}^2}\right]\right\}^{j_1-1}\left\{1-\exp\left[-\frac{H_*^2}{H_{rms}^2}\right]\right\} \tag{3.347}$$

Correlated consecutive waves. When the Rayleigh bivariate probability function is applied,

$$p_+ = \frac{\int_{H_*}^{\infty}\int_{H_*}^{\infty} f(H_-,H_0)\,dH_-dH_0}{\int_{H_*}^{\infty}\int_{0}^{\infty} f(H_-,H_0)\,dH_-dH_0} \tag{3.348}$$

$$p_- = \frac{\int_{0}^{H_*}\int_{0}^{H_*} f(H_-,H_0)\,dH_-dH_0}{\int_{0}^{H_*}\int_{0}^{\infty} f(H_-,H_0)\,dH_-dH_0} \tag{3.349}$$

The probability that the wave number in the wave group exceeds j_1 is:

$$P(j_1) = p_+^{j_1-1}(1-p_+) \tag{3.350}$$

The mean and variance of the distribution function are the following:

$$\mu_1 = \overline{j_1} = E[j_1] = \sum_{j=1}^{\infty} jP(j_1) = \frac{1}{1-p_+} \tag{3.351}$$

$$\sigma_1^2 = \left\{E[j_1^2] - (E[j_1])^2\right\} = \frac{p_+}{(1-p_+)^2} \tag{3.352}$$

Mean number of waves in group $\overline{j_1}$. If in a record, there are N_w wave heights of which N_{H*} exceed the threshold height, and N_g groups are in the record, the mean number of waves in the group in the sea state is:

$$\overline{j_1} = E[j_1] = \frac{N_{H_*}}{N_g} \tag{3.353}$$

If the end of each group is identified by the sequence of wave heights higher than the threshold height, followed by a wave height lower than the threshold,

$$N_g = N_w \Pr\left[(H_i > H_*)\Lambda(H_{i+1} < H_*)\right] \tag{3.354}$$

and the number of waves in the record which are higher than the threshold height is

$$N_{H_*} = N_w \Pr\left[H_i > H_*\right] \tag{3.355}$$

then, the mean number of waves in the group is:

$$\bar{j}_1 = \frac{N_{H_*}}{N_g} = \frac{\Pr\left[H_i > H_*\right]}{\Pr\left[H_i > H_* \Lambda H_{i+1} < H_*\right]} = \frac{1}{1 - \Pr\left[H_i > H_* \middle| H_{i+1} < H_*\right]} \tag{3.356}$$

Therefore, the mean number of waves in a group is directly proportional to the correlation coefficient between consecutive wave heights, $\rho_{HH,t}$ and depends on the correlation between non-consecutive wave heights.

3.8.2.2.2 PROBABILITY FUNCTION OF THE NUMBER OF WAVES, j_2

Similarly, it is possible to calculate the distribution function of the total number of waves in the group as well as their mean and variance,

$$P(j_2) = \frac{(1-p_-)(1-p_+)}{p_- - p_+}\left(p_-^{j_2-1} - p_+^{j_2-1}\right) \tag{3.357}$$

$$\mu_2 = \bar{j}_2 = \frac{1}{1-p_+} + \frac{1}{1-p_-} \tag{3.358}$$

$$\sigma_2^2 = \frac{p_+}{(1-p_+)^2} + \frac{p_-}{(1-p_-)^2} \tag{3.359}$$

3.8.2.3 *Density function of the envelope of the group*

For narrow-band spectra, the variation of $A(t)$ in time is slow compared with the carrier wave. In addition, wave crests and troughs faithfully follow the shape of the envelope. This means that if amplitude $A(t)$ and its velocity of change $dA/dt = \hat{A}(t)$ are statistically independent, they follow Rayleigh distribution and a normal distribution, respectively,

$$f(A) = \left(\frac{A}{m_0}\right)\exp\left[-\frac{A^2}{2m_0}\right] \tag{3.360}$$

$$f_{H_s}(H_s;\Upsilon) = \frac{c}{\Gamma(m)}\lambda^{cm} H_s^{cm-1}\exp\left[-(\lambda H_s)^c\right] \quad 0 < H_s < \infty \tag{3.361}$$

3.8.2.3.1 MEAN NUMBER OF WAVES IN GROUP Π

From these two distributions, the mean wave number in a group can be obtained by dividing the mean envelope exceedance duration of amplitude $H_*/2$ by the zero-crossing mean period. This results in the following expression:

$$\Pi = \sqrt{\frac{2m_0}{\pi}}\frac{\sqrt{1+v^2}}{v}\frac{1}{H_*} \tag{3.362}$$

where v is the spectral width parameter, defined by

$$v^2 = \frac{m_2 m_0}{m_1^2} - 1 \tag{3.363}$$

This result can be applied to narrow-band spectra. Alternatively, available data can be filtered so that they can be fitted to this situation. In any case, Π differs from $\overline{j_1}$ and does not include any information concerning the correlation between consecutive wave heights.

3.8.2.4 Energy spectrum of the envelope amplitude

For waves with a narrow-band spectrum, the spectral density function of envelope amplitude can be approximated by

$$S_A(f) \approx \left(2 - \frac{\pi}{2}\right) m_0 \Gamma_\eta(f) \tag{3.364}$$

where $\Gamma_\eta(f)$ is the spectral density function of the envelope defined by

$$\Gamma_\eta(f) = \frac{2}{m_0^2} \int_0^\infty S_\eta(f^*) S_\eta(f^* + f)\, df^* \tag{3.365}$$

and m_0 is the zero-order moment of the spectral density function of the sea state, $S_\eta(f)$. The standard deviation σ_A of $A(t)$ is defined from the zero-order moment of the spectrum $S_A(f)$,

$$\begin{aligned} m_{0A} &= \int_0^\infty S_A(f)\, df \\ \sigma_A &= \sqrt{m_{0A}} \end{aligned} \tag{3.366}$$

3.8.2.4.1 Mean amplitude value

The mean amplitude value of the envelope can be approximated by

$$\overline{A(t)} \approx \frac{1}{2}\sqrt{2\pi m_0} \tag{3.367}$$

3.8.2.4.2 Mean wave length and period of the wave group

The mean period of the wave group is the following:

$$\overline{T_g} = \sqrt{\frac{m_{0A}}{m_{2A}}} \tag{3.368}$$

The mean wave length of the wave group can be approximated by:

$$\overline{L_g} = \frac{n L_{0,2} \overline{T_g}}{T_{0,2}} \tag{3.369}$$

$$n = \frac{C_g}{C} \tag{3.370}$$

$T_{0,2}$ is a spectral estimate of the mean period of the sea state,

$$T_{0,2} = \sqrt{\frac{m_0}{m_2}} \tag{3.371}$$

and $L_{0,2}$ is the mean wavelength of the sea state, obtained from the dispersion equation with period $T_{0,2}$,

$$\left(\frac{2\pi}{T_{0,2}}\right)^2 = g\frac{2\pi}{L_{0,2}}\tanh\left(\frac{2\pi h}{L_{0,2}}\right) \tag{3.372}$$

3.8.2.4.3 Spatial and temporal steepness of the wave group

The steepness of the wave group in the spatial domain is

$$S_{gx} = \frac{\sigma_A}{L_g} \tag{3.373}$$

and the steepness of the wave group in the time domain is

$$S_{gt} = \frac{\sigma_A}{g\overline{T}_g^2} \tag{3.374}$$

Both parameters have information concerning the mean amplitude and length or period of the wave group (of the envelope).

3.8.2.5 Frequency description of associated waves

Wave transformation because of bottom and boundary effects can generate subharmonic associated waves that cause sea level oscillations, whose mean period is related to the mean period of the wave group. It can also generate superharmonic waves that steepen and intensify the asymmetry of the wave group profile. The transformation processes depend on the frequency and directional spectrum of the waves and the nature of the boundaries. For this reason, the spectral shape is a local attribute of the port or shoreline area.

The local spectrum can be obtained by applying transfer functions, G^+, G^-, (see Figures 3.3.31 and 3.3.32). The spectrum of subharmonic waves can be obtained from the local sea state spectrum $S_\eta(f)$,

$$S_{\eta_{LB}}(f) = 2\int_0^{\frac{f}{2}} S_\eta(f)S_\eta\left(f+f^*\right)\left[G^-\left(f,f^*\right)\right]^2 df^* \tag{3.375}$$

In shoreline areas where long-period oscillations can be a factor that determines operationality, it is advisable to perform a series of specific measurements to calibrate, contrast, and compare transfer functions. Special attention should be paid to the dependence of transfer functions on the direction of approach. When instrument measurements are not available, it is best to evaluate the frequency spectrum of subharmonic waves by applying theoretical transfer functions and a frequency spectrum of the sea state that most clearly reflect local conditions.

3.8.3 Transformation of the wave group and the associated sea level

The study of wave group transformation is more complex than the study of individual waves since wave height locally varies over time. The transformation of each individual wave is reflected in the structure of the group. Accordingly, the wave group experiences refraction, diffraction, reflection, and reduction of its amplitude because of the breaking of individual waves, and thus, the spatial and temporal variation of the radiation tensor.

3.8.3.1 Importance of the temporal and spatial variation of sea level

The spatial and temporal variation of the radiation tensor causes the spatial and temporal variation in the amplitude of associated long waves and the generation of free waves. This process occurs in the diffraction of the wave group in the area surrounding the head of a breakwater or in the sudden changes in depth in a navigation channel. The main period of free waves and the associated long wave is that of the forcing term, namely, the wave group.

Free waves travel with the celerity of semi-enclosed water body oscillations, such as wharfs in port areas, inlets, and estuaries. If both celerities coincide, then the water body oscillates in resonance conditions. In this case, the amplitude of the oscillation can increase without limit. If the forcing is not resonant, then the oscillation is limited. Once the cause stops, the oscillation also stops. Similarly, free waves can force the oscillation of moored vessels and floating structures.

This mechanism is the same one that generates infragravity waves and edge waves in the surf zone. The spatial and temporal variation of the breaking point and of the wave magnitude and residual structure of the broken wave group inside the surf zone are factors that control the infragravity oscillatory regime.

Note. Factors associated with the spatial gradients of the radiation tensor are a variation in the mean sea level $\overline{\eta}$ *and a mean current with components* (U, V) *integrated in the depth. Apart from the mass conservation equation, the conservation of linear momentum equation in a control volume (which considers the waves propagating in the direction of the* x*-axis, towards the beach) is:*

$$\frac{\partial U}{\partial t} + g\frac{\partial \overline{\eta}}{\partial x} = -\frac{1}{\rho h(x)}\frac{\partial S_{xx}}{\partial x} \tag{3.376}$$

This equation is equal to the shallow water equation with a forcing term that varies in time and space. If the wave envelope is $H(x,t) = 2A(x, t)$*, and given that y* S_{xx} *depends on* $H^2(x,t)$*,* U *and* $\overline{\eta}$ *are functions that slowly vary with* x *and* t*. Thus,* $\overline{\eta}(x, t) = \eta_{LB}(x, t)$ *is a wave that travels at the propagation speed of the envelope, in other words, with the celerity of wave group* C_g*, and which is associated with or joined to the group. The solution of the homogeneous problem is superposed on the solution to the previous equation (3.354), which provides the response to the forcing of the independent term. In other words, when the forcing is zero, they are long waves that travel with long wave celerity* $C = \sqrt{gh(x)}$*, independently of the wave group, and for this reason, they are free waves.*

3.8.3.2 Transformation models

For simple bathymetries and mild slopes in the Stokes regime, the local spectrum (at the project site) can be calculated by including the subharmonic and superharmonic components. This means applying the transfer functions, $G^+(kh)$, $G^-(kh)$. The solution to the problem is composed of first-order (linear) oscillations, the second-order component, associated subharmonic $(f_n - f_m)$ and superharmonic $(f_n + f_m)$ waves, and transfer functions G^+ and G^-, (see Annex).

In other cases, the wave group transformation can be performed by using one of the following procedures:

1. Transformation of the wave group envelope by applying a linear propagation model, forced by the gradient of the radiation tensor. However, a phase-integrated wave height transformation model should first be applied, and the value of the radiation tensor should be obtained. It is not necessary to apply phase-resolving models.
2. Transformation of the time series of the record by using models that account for its temporal evolution, for example, those belonging to the Stokes regime (MSPE-t) or the Boussinesq regime (Funwave). The result at each point provide the information needed to quantify the wave group transformation and that of the associated sea level.
3. Transformation of the spectrum by using a wave generation and transformation model, valid in the domains IV and V, and which includes the interaction of components and bottom-induced friction and breaking.

The result of procedure 2 is a time series, and the result of procedure 3 is an energy spectrum. In both cases, the result obtained at each point provides necessary and sufficient information to quantify the wave group transformation and the associated sea level.

3.8.4 The physical model testing of wave groups

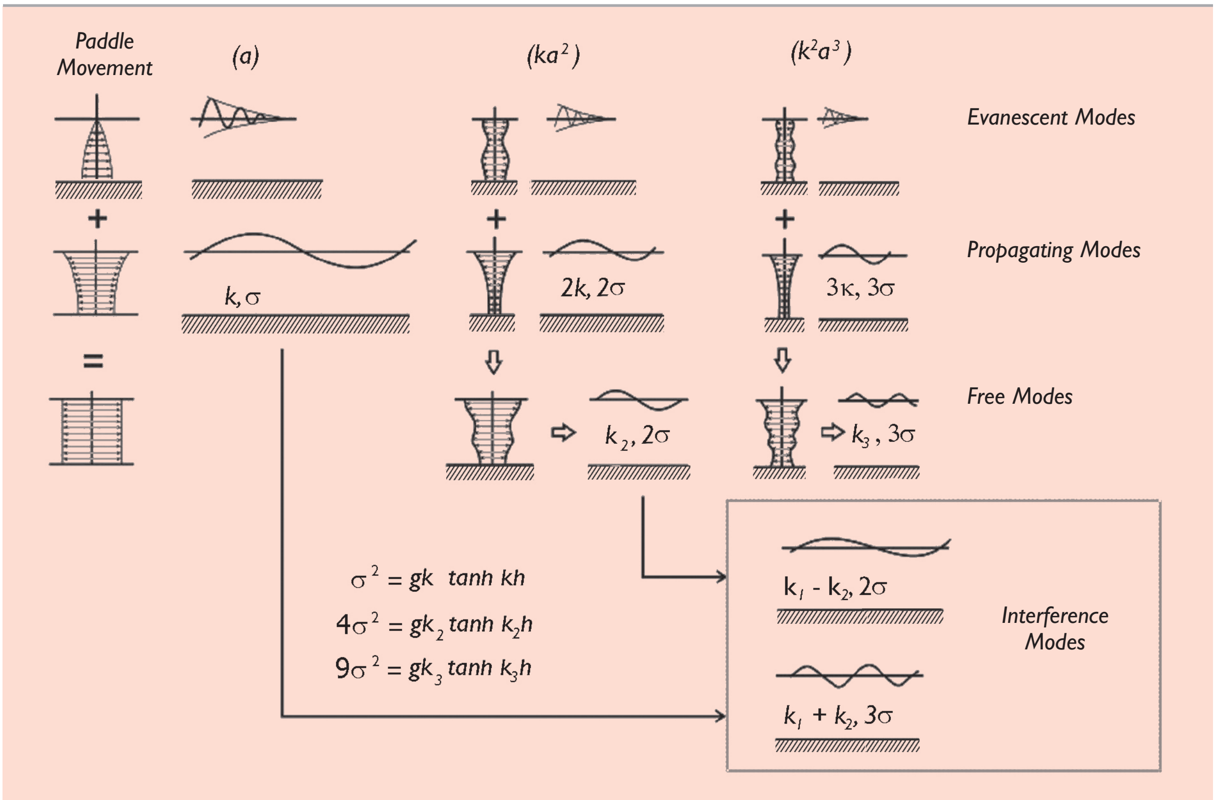
In Maritime Engineering, it is frequently necessary to experimentally verify the performance of breakwaters against wave action. This verification can be carried out by means of experiments in a wave tank or wave channel with a scale-model of the structure. Sea oscillations in the domain are usually reproduced by paddle movement. The validity of the experiment depends on whether the processes of the prototype are accurately reproduced in the experiment. Evidently, for the test results to be useful, the cinematic and dynamic characteristics of the model waves must be similar to those of the prototype.

Regardless of whether the necessary and sufficient conditions for this similarity are satisfied, the generation mechanism of the oscillatory motion, and the boundary conditions of the spatial domain of the experiment are the source of experimental noise that must be taken into account.

3.8.4.1 Wave generation limitations: free and forced waves

Waves are generated in the laboratory by paddle motion. This motion may involve rotation, strokes, or a combination of the two. However, the paddle motion should have the same cinematic characteristic as the linear wave. The velocity field of the water particles in contact with the paddle should also be the same as that of the wave. However, the vertical profile of the waves follow a hyperbolic cosine function. This lack of fit between motion types is compensated by the generation of evanescent waves, which rapidly disappear (see Figure 3.3.31).

Figure 3.3.31. Waves generated by a piston-type wave paddle



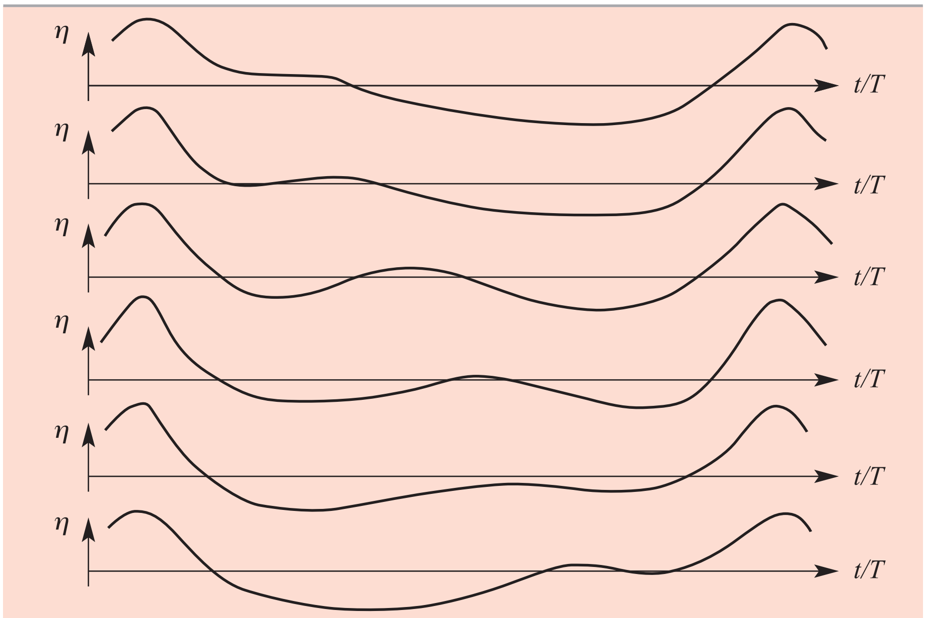
Fuente: Y. Goda.

Furthermore, when wave generation is carried out in waters of medium and shallow depths, and the wave profile is fitted to that of a second-order Stokes wave with energy content in frequencies, f and $2f$, this generates a wave associated with the wave train and the free waves necessary to compensate the associated wave's lack of fit to the paddle motion. This results in a parasite wave (secondary crest) that propagates through the wave channel, and which distorts the wave profile (see Figure 3.3.32). This effect is more pronounced as the steepness of the generated waves increases.

Finally, when irregular waves are generated in shallow waters, there is an interaction of neighboring components that generates associated long waves with a frequency $(f_n - f_m)$, which can affect experimental results.

These effects are more pronounced when the wave height and the non-linearity of the signal are greater. For these reasons, in the case of more severe wave action, the generation of regular or irregular wave trains in the laboratory should include paddle motion that compensates the spurious signal (free waves) that alter the wave train profile, its kinematics, and dynamics. If such compensation is not feasible, then it is necessary to include an analysis of the extent to which such interactions have affected experimental results.

Figure 3.3.32. Profile of the secondary crest during wave generation in a wave channel



3.8.4.2 Boundary limits

A further consideration is the fact that the boundaries of the wave channel or tank as well as those of the construction work and maritime structure cause the transformation of the incident wave train, which depends on the morphology and typology of the obstacle. The quality of the experimental results depends on the possibility of controlling boundary effects on the representative zone of the experiment. All experimental work should include an analysis of such circumstances, and how they have been accounted for in the evaluation of results and recommendations.

3.8.5 Joint description of wave groups and sea level

In middle latitudes, loading cycles are generated by the passage of extratropical storms. In these conditions, before the arrival of the worst of the storm, the initial sea states contain groups of long-period waves of small amplitude. In maximum sea states, the structure of the wave group can become masked by the dynamics of wind-induced generation, but in most cases, these groups of long-period waves continue to exist. The period of the wave group is shortened in the initial phase of growth, and its amplitude increases because of the higher energy content of maximum sea states. As the storm abates, the wave groups lose intensity, and thus, their wave period once again increases.

The operationality and safety of port installations are determined, among other failure modes, by short-period oscillatory motion (waves) and long waves (generically known as resonances), which affect each of the elements and subsets of the port area. The transition of periods and amplitudes of the wave groups that can occur during the loading cycle make up a wide range of work conditions that should be considered during the verification process.

3.8.5.1 Recommendations for the joint description

When describing sea oscillations, it is advisable to analyze the presence of wave groups, their transformation by the boundaries, and changes in the sea bed, and also the generation of subharmonic and superharmonic waves. This analysis should be carried out for extreme work conditions as well as for normal work conditions. In the first case, the analysis of the structure of the wave group should be linked to each of the states of the loading cycle. In this sense, it is necessary to consider the phases of growth, development, and dissipation, as well as the evolution of the respective frequency and directional spectra.

These phases should be correlated with the trajectory and evolution of the storm that generates the loading cycle. It is also advisable to analyze the spectral shape, the evolution of the energy frequency peaks, the generation of subharmonic oscillations, and the temporal depth variations because of the meteorological tide. In seas in which the astronomical tide is significant, the passage of the storm and its associated processes should be analyzed according to the different phases of the astronomical tide. Therefore, the passage of the storm should be initiated with each of the following four sea states: low tide, rising mid tide, high tide, and falling mid tide. Based on these states, the passage of the storm can be described at the same time as the tide sequence.

The tidal range should be selected on the basis of its probability of occurrence during the year. This should be done in such a way that the joint probability of the simultaneous occurrence of the loading cycle and the astronomical tidal range is clearly established. The calculation of the joint probability of both events is based on the assumption that both events are statistically independent.

3.9 MEDIUM-PERIOD OSCILLATIONS

Medium-period oscillations are made up of barometric pulses, meteo-tsunamis, and tsunamis. They produce oscillations on the free sea surface in the period interval ($5\ min < T < 4\ hours$). These are large-scale atmospheric fluctuations, stronger than gusts, whose period of fluctuation has a duration of tens of seconds, and which are due to mechanical turbulence produced on the surface. Oscillations in port and shoreline areas are also medium-period oscillations, such that,

$$\zeta_{PI}(\vec{x},t) = \zeta_{ms}(\vec{x},t) + \zeta_{ma}(\vec{x},t) + \zeta_{area}(\vec{x},t) \qquad (3.377)$$

In the same way as other sea oscillations, tsunamis and meteo-tsunamis can be described as a sequence of monochromatic long waves, or as an irregular train of long waves in bands of frequencies and amplitudes. In this case, the signal, $\zeta_{mt}(\vec{x}, t)$, can be separated by using spectral techniques.

3.9.1 Tsunamis

A tsunami on the coast (shoreline and port areas) is a sequence of vertical displacements of the free sea surface, $\zeta_{ms}(\vec{x}, t)$, composed of a small number of wave groups (i.e. 5-10), preceded by sudden fall in sea level, and followed by an oscillatory tail of waves with a shorter period than the normal period of such wave groups. The duration of a tsunami episode or event (loading cycle) is 2-5 hours.

3.9.1.1 Introduction

A tidal wave is commonly known by the Japanese term, *tsunami* (*tsu*, port + *nami*, wave). The wavelength of such a wave group is on the order of hundreds of kilometers, $L \sim O(100\ km)$; the celerity in the open sea on the order of a hundred of meters per second, $C \sim O(100\ m/s)$; and the wave period is on the order of tens of minutes, $T \sim O(100\ min)$. The number of waves in a tsunami and their amplitude depends on the distance to the coastline, and the water depth at the generation point. It also depends on the intensity of the seaquake and the dynamics of propagation related to the shape of the continental shelf.

The oscillation of the tsunami at a point in the sea depends on its magnitude in the generation zone, and its transformation by the topography in the propagation zone. In the same way as other gravity waves, the transformation of a tsunami includes shoaling processes, refraction, reflection, diffraction, and dissipation, due to bottom-induced friction and breaking.

Topographic effects can be divided into global, regional, and local, depending on spatial gradients of the water depth. Global topographic effects correspond to the open sea, whose depth varies in the range of 1000-10000 m over a distance of 500-10000 *km*. Regional topographic effects are those produced by the slope and continental shelf, where the water depth varies 100-1000 *m* over a distance of 50-500 *km*.

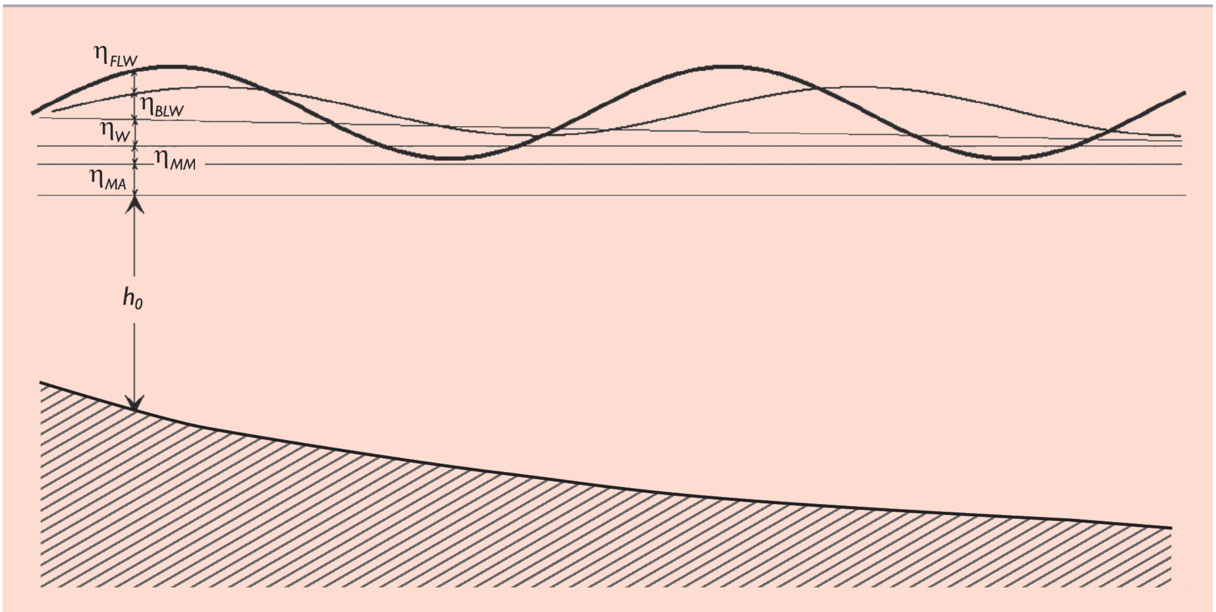
Finally, local topographic effects occur in shoreline areas (inlets, estuaries, and the surf zone) as well as port areas where the water depth varies 1-100 *m* over a distance of 0.5-50 *km*.

Generally speaking, the tsunami behaves like a train of linear long waves, which are non-dispersive in frequencies and amplitudes. This means that their celerity does not depend either on their period or amplitude since it is small when compared to the propagation depth. During their transformation and approach to the coastline, some tsunamis become significantly larger to the extent that their celerity begins to depend on the amplitude until they finally break. Depending on their initial shape, the waves can even divide into solitary waves, and eventually break.

3.9.1.2 Basic and instantaneous variables

The instantaneous variable of a seismic tsunami is the vertical displacement of the free sea surface with respect to a reference level sampled at regular time intervals on the order of a minute of duration (see Figure 3.3.33). In this way, the part of the signal related to the waves is filtered. The basic variable is the amplitude of each wave in the tsunami sequence.

Figure 3.3.33. Variables in a tsunami



3.9.1.2.1 Description as a long wave

It is advisable to describe the tsunami in the Boussinesq regime by modeling the initial oscillation as a solitary wave, or as a train of cnoidal or solitary waves. For this purpose, one should define the volume of water underneath the shape of the wave, which depends on the characteristics of the generating seaquake.

3.9.1.3 State description

The persistence of the tsunami is thirty minutes to four hours, approximately. Once the tsunami is defined as a sequence of waves that occur in this time interval, the process can be said to be quasi-stationary and homogeneous. Thus, their time and frequency can be described by applying the linear Gaussian theory of signals (i.e. the instantaneous variable is Gaussian). The manifestation of the tsunami that satisfies these conditions represents the tsunami state, which in terms of its persistence, lasts over three or four meteorological states.

3.9.1.4 Statistical description of the tsunami

Generally speaking, the occurrence of the tsunami is superposed on the occurrence of other long waves related to atmospheric activity. For this reason, the instantaneous variable ζ_{obs} measured during the occurrence of a (seismic) tsunami on a point on the coast, P($\vec{x}$), can be represented by,

$$\zeta_{obs}(\vec{x},t) = \zeta_m(\vec{x},t) + \zeta_b(\vec{x},t) + \varepsilon(t) \qquad (3.378)$$

where $\zeta_m(\vec{x}, t)$ is the signal due to the tsunami, and $\zeta_b(\vec{x}, t)$, the base signal generally related to atmospheric activity. $\varepsilon(t)$ is the noise of the instrument which, generally speaking, can be neglected when the signal of the tsunami is important. These signals can be separated by means of spectral techniques, as described in the following section.

The effects of the continental shelf and the local topography are reflected on the narrowing of the tsunami spectrum. In the area near the coastline, the process can be said to be a narrow-band process. Consequently, the amplitudes of the tsunami (basic variable) follow a Rayleigh distribution, whose parameter is the area under the spectral density function. The maximum amplitude, as long as sufficient statistical conditions exist regarding the minimum size of the sample, follows a Rayleigh function raised to the N power.

The wave periods of the tsunami depend on the conditions in the generation zone. However, the band of the periods spreads (approximately 2-500 *minutes*). The principal modes of the tsunami waves can be estimated by the following:

$$T_n = \frac{2l}{n\sqrt{gh}} \quad n = 1,2,\ldots \qquad (3.379)$$

where l is a representative length of the dimensions of the generation area; h is the wave depth in the epicenter zone; and n is the number of nodes.

Note. If the generation area is a rectangle of 100×60 *km, and the depth is* 1100 *m, the periods obtained are* $T_1 = 36$ *min*, $T_2 = 18$ *min.*

3.9.1.5 Spectral description of the tsunami

In the frequency domain, the spectrum of the observed signal can be written as follows:

$$\zeta_{obs}(\vec{x},\omega) = S_m(\vec{x},\omega) + S_b(\vec{x},\omega) + S_\varepsilon(\omega) \qquad (3.380)$$

where S_m and S_b are, respectively, the spectra of the tsunami and of the atmospheric base signal in the vicinity of the port and shoreline areas, and $S_\varepsilon(w)$ is the noise spectrum of the instrument. The frequency band of the seismic tsunamis is $0.002–0.5$ cpm [(25)], in other words, a wave period band of approximately 2-500 *minutes*.

The spectrum of the tsunami in the port or shoreline area, $S_m(\vec{x}, w)$, can be determined by the following:

$$S_m(\vec{x},\omega) = W_m^2(\vec{x},\omega)\, Z_m(\omega) \tag{3.381}$$

where $Z_m(w)$ is the spectrum of the tsunami in the zone of origin, and $W_m^2(\vec{x}, w)$ is the topographic spectral transfer function, which can be divided into three parts,

$$W_m^2(\vec{x},\omega) = Y_m(\omega)\, Q_m(\omega)\, P_m(\omega) \tag{3.382}$$

where Y_m, Q_m, P_m are, respectively, the topographic transfer functions of the ocean (open sea), continental shelf, and the shoreline and port area. In the frequency band of the tsunami, this function does not depend on the time, but rather on the site location, $\vec{x}$. In contrast, the spectral density function of the tsunami, $Z_m(w)$, is constant in the generation area.

Similarly, the spectrum of the atmospheric base signal, $S_m(\vec{x}, w)$, can be determined by:

$$S_b(\vec{x},\omega) = W_b^2(\omega)\, S_{b0}(\omega) = Q_b(\omega)\, P_b(\omega)\, S_{b0}(\omega) \tag{3.383}$$

where Q_b, P_b are, respectively, the topographic transfer functions of the continental shelf and the port and shoreline area; and $S_{b0}(w)$ is the base spectrum of the atmospheric signal in the ocean. Fortunately, the shape of this spectrum is a monotonic function without any peaks, and is universal. The occurrence of an atmospheric event only changes its energy content, but not its shape,

$$S_{b0}(\omega) = \frac{1}{2\pi} E_0 \frac{1}{\omega^2} \tag{3.384}$$

where E_0 depends on the atmospheric constant. On the open sea, its value is within an interval of $10^{-3} – 10^{-4}$) cm^2/cpm.

3.9.1.5.1 Consequences for the calculation and spectral application

The peaks that can appear in the energy spectrum of the atmospheric signal are due to the resonant coupling of the atmospheric signal with the continental shelf and the port and shoreline areas. Similarly, the peaks in the signal of the tsunami, which are different from those of the source tsunami are the result of its interaction with the continental shelf and shoreline areas.

Consequently, the spectral shape of the tsunami at a site should be the same (with the same peaks though different energy content) for tsunamis produced in nearby generation zones as long as the propagation direction is also the same. In contrast, the spectral shape of the same tsunami at different locations should be different (i.e. with other peaks) since the topographic conditions and spectral transfer functions at those locations differ.

Given that the seismic tsunami spectrum and the spectrum of the base atmospheric signal have the same frequency band, the topographic transfer functions of the continental shelf and shoreline areas can be said to be equal:

$$\begin{aligned} Q_m(\omega) &= Q_b(\omega) \\ P_m(\omega) &= P_b(\omega) \end{aligned} \tag{3.385}$$

(25) Cpm stand for cycles per minute. The period of the tsunami is $T = 1/f$, where f is the frecuency.

As long as the origin of the tsunami is not far from the coast and there is a recording device, such as a tide gauge located near the shoreline, the spectrum of the tsunami at its point of origin can be calculated by assuming the following $S_\varepsilon \ll S_m$,

$$Z_m(\omega) = \left[\frac{S_{obs}(\vec{x},\omega)}{S_b(\vec{x},\omega)} - 1\right]\frac{\alpha_m}{2\pi} E_0 \frac{1}{\omega^2} \tag{3.386}$$

where α_m is an adimensional constant that permits the correction (calibration) of the signal by taking into account the angular and radial dispersion of the signal, stemming from the distance between the tsunami's point of origin and the measurement point.

Based on this result, or once the spectrum of the tsunami in the generation zone is known, it is then possible to obtain the spectrum of the tsunami at other points of the coastline. Once the topographic transfer function of the point under consideration is known, it is then possible to obtain the spectrum of another tsunami generated in another area, as long as the direction of the propagation is the same.

3.9.1.6 Long-term description

On the Spanish coastline, tsunamis (or meteo-tsunamis) are rare events. They generally have a small amplitude, but occasionally, a tsunami with a large amplitude can have devastating consequences. Long-term description can be performed in the same way as the description for the loading cycles of sea states. In this case, the variables of the loading cycle (tsunami) are the volume of wave beneath the initial wave or maximum amplitude, the number of waves in the cycle (or mean wave period), and the duration.

3.9.1.6.1 Distribution of the annual maximum amplitude (extreme tsunami regime)

It is advisable to follow the method described in section 3.8.2, namely, the peak-based extreme distribution. In order to obtain a distribution function of the maximum amplitude, it is necessary to previously establish a threshold amplitude $a_{m,u}$, which depends on the area under study, and to know the number of tsunamis, n_m, which have exceeded the threshold in the time interval of M years. Assuming that the occurrence of tsunamis is a rare event, and that their number follows a Poisson distribution of parameter, $v_m = n_m/M$, then the distribution function of the maximum amplitude or annual extreme regime of the amplitude of the tsunami can be obtained by means of the following expression:

$$F_{A_{max}} = \exp\left[-\frac{n_m}{M}\left(1 - F_A(a)\right)\right] \tag{3.387}$$

where $F_A(a_m) = \Pr[A > a_m]$ is the distribution function of the amplitudes that exceed threshold amplitude, $a_{m,u}$.

3.9.1.7 Transformation models

When the long wave propagates over the continental shelf and is forced by the local topography , dispersion mechanisms come into play, which affect the amplitudes and frequency of the initial wave. The result, which depends on the initial shape of the wave, involves the steepening, decomposition, and eventually the breaking of the originally wave into new waves and solitary waves.

3.9.1.7.1 Time domain models

The displacement of the free surface at the project site because of the occurrence of a tsunami (or a meteo-tsunami) can be obtained by applying phase resolving models of the MSPE-t or Boussinesq type. It can also be obtained by resolving Airy non-linear long-wave equations with an entry signal in the generation point, measurement, or a theoretical wave with the characteristics of the tsunami.

3.9.1.8 Verification for tsunamis

Generally speaking, tsunamis on the Spanish coastline are rare events (Poisson processes with a very small parameter). For this reason, their occurrence in the useful life of the port or shoreline area can be regarded as an agent that is independent of the other climate agents in the physical environment, both atmospheric agents and marine agents.

When the probability of exceedance in the structure's useful life $F_{A_{max}} = \Pr[A > a_m] > 10^{-4}$ has a tsunami amplitude a_m, which can significantly affect the safety of the structure and the terrain, the tsunami should be included in the verification of extreme work conditions as a predominant agent as well as another agent. Otherwise, it is necessary to follow the recommendations of the ROM 0.0, and include the verification of the area for the tsunami in exceptional work conditions.

3.9.1.8.1 The tsunami as an extraordinary agent

The tsunami should be considered an extraordinary agent when in the useful life of the structure, the probability of a tsunami whose magnitude is significant for the safety of the structure is at least two orders of magnitude less than the joint probability of the failure modes applied to the ultimate limit states, as shown in Table 2.2.8 (see Chapter 2). For example, this applies when the probability of a tsunami in the useful life of the structure is $p_m \leq 10^{-4}$ for a structure with a social and environmental repercussion index $[SERI = 20\text{-}29]$, whose joint probability is $p_{f,ELU} \leq 10^{-2}$.

If these requirements are met, the tsunami need not be considered when calculating the joint probability for the safety of the structure, or when calculating the probability of stoppage modes pertaining to its use and exploitation. If the developer of the construction work deems it necessary, the possible occurrence of a tsunami could be regarded as an exceptional chance work condition of the physical environment $C_{T,3,1,1}$ (section 4.5.2.3 of the ROM 0.0).

3.9.1.8.2 The tsunami as an extreme agent

The tsunami should be regarded as an extreme agent when in the useful life of the structure, the probability of the occurrence of a tsunami, whose magnitude is significant for the safety of the structure, is at least one order of magnitude less than the joint probability of all the failure modes assigned to the ultimate limit state according to Table 2.2.8 (see Chapter 2). For example, this applies when the probability of a tsunami in the useful life of the structure is $p_m \leq 10^{-3}$ for a construction work with a social and environmental repercussion index $[SERI = 20\text{-}29]$, whose joint probability is $p_{f,ELU} \leq 10^{-2}$.

If these requirements are met, the tsunami must be considered when calculating the joint probability for the safety of the structure in both normal as well as extreme work and operating conditions. In this case, the tsunami will be regarded as the sole predominant agent, and the combination of factors will be carried out according to the recommendations in section 4.9 of the ROM 0.0.

3.9.1.9 Warning network

In addition, if the probability of the significant tsunami in any year of the useful life of the structure is greater than 10^{-2}, there should be a warning system that alerts the people to its occurrence, and allows sufficient time for the suspension of port activity as well as for the actions necessary to protect people as well as equipment.

3.9.2 Tsunami generated by a landslide

Landslides on the sea coast as well as underwater landslides in the ocean are agents that can cause tsunamis. Both processes are associated with the geotechnical instability of the terrain or fill material. They can also be indicative of the instability induced by sea oscillations, waves, wave groups, or seaquakes.

3.9.2.1 Verification

When the probability of exceedance in the useful life of the structure, $F_{A_{max}} = \Pr[A > a_m] > 10^{-4}$ corresponds to a tsunami amplitude *am*, which is significant for the safety of the structure and the terrain, it is advisable to analyze the characteristics of the tsunami, amplitude and period of the landslide. Its propagation towards the site should be studied, and when relevant, the safety and operationality of the structure should be verified in the face of the tsunami.

The verification of the structure against these tsunamis should be performed according to the recommendations concerning tsunamis of seismic origin.

3.9.3 Meteo-tsunamis

Meteo-tsunamis are rapid (26) pressure pulses and medium winds that produce oscillations on the free sea surface, $\zeta_{ma}(\vec{x}, t)$, formed by a number of small wave groups in respect to a reference level. Some of them are related to the dynamics of the storm; others are produced by the interaction of atmospheric dynamics with the local topography; still others are thermal. All of them, however, have periods of 5-30 minutes. In these Recommendations, these pulses are all known as a meteo-tsunami. The duration of a meteo-tsunami episode or event (loading cycle) is 2-5 hours.

3.9.3.1 Local barometric pulses

On shoreline areas, due to the orographic influence, buildings, large installations, and docked and anchored vessels, local barometric pulses are produced, which are related to the emission of whirlwinds and the readaptation of the airflow leeward of the obstacle, or because of local thermal gradients associated with spatial gradients of the albedo.

These pulses can generate edge waves on neighboring beaches, which are propagated at the opening of the port area, and which can also cause oscillations in the wharf itself. The oscillation can be resonant either because the barometric pulse is stationary (attached to the obstacle) or because the component of its propagation speed in the direction parallel to the coast is equal to the celerity of one of the edge wave modes or to the longitudinal or transversal mode of the oscillation of the wharf.

In both cases, the process for the identification and quantification of a meteo-tsunami is very similar to that of atmospheric pulses in the open sea.

3.9.3.2 Short, medium, and long-term description

The medium-term and long-term temporal and frequency description of a meteo-tsunami should follow the recommendations for the description of tsunamis.

3.9.3.3 The meteo-tsunami as an extreme agent

When the magnitude and frequency of oscillations can significantly affect the safety and the use and exploitation of the area, and when this cannot be remedied by modifying the geometry of the structure, the meteo-tsunami should be regarded as a marine climate agent. This means that its occurrence should be considered in extreme work conditions as well as normal work and operating conditions. Furthermore, its failure and stoppage modes should be evaluated as well as its contribution to the joint failure mode in relation to safety and loss of operationality.

---

(26) *Rapid* should be understood in regards to the evolution time of an extratropical storm, which is a question of hours.

3.9.3.4 Joint description of meteo-tsunamis

Because of their origin, meteo-tsunamis are linked to the simultaneous occurrence of certain atmospheric processes with compatible values. Accordingly the description and characterization of meteo-tsunamis should be jointly carried out with the atmospheric climate agents that force them. Their probability of exceedance should be compatible with the probability of exceedance of the values of these agents.

3.9.4 Other sources of medium oscillations

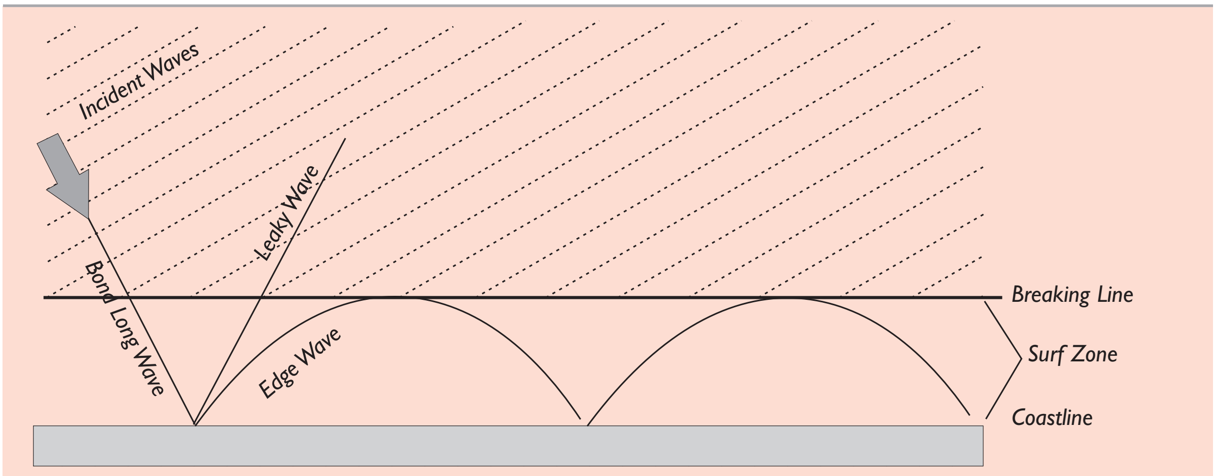
Apart from seismic tsunamis and meteo-tsunamis, other agents can generate oscillations in port and shoreline areas, which, depending on the geometry of the area, can be resonant. Such agents include edge waves or coastal-trapped waves, and shear waves. Both types of wave occur in shallow water areas, generally at a depth of less than 10 m.

When the probability of occurrence of these waves and their coupling with the harbor area is significant, then they should be included as marine climate agents, and their contribution to the joint probability of failure should be evaluated along with their loss of operationality. The forcing of the area by these oscillations can be approximated by applying an elliptic numerical model, such as the MSP-UGR (see section 3.6.7.2).

3.9.4.1 Edge waves

The edge wave is an oscillation that propagates along the coast. Its amplitude oscillates and decays perpendicularly to it, depending on the mode until its total extinction (see Figure 3.3.34).

Figure 3.3.34. ***Types of infragravity waves***



This is the reason why these waves are also known as coastal-trapped waves. The dispersion equation (simplified theory) relates the slope of the coast, $tg\beta \approx \beta$, the wave length (along the coast $L_y = 2\pi/k_y$), and the oscillation periods of each mode ($T_n = 2\pi/w_n$),

$$\omega_n^2 = gk_y \sin(2n+1)\beta$$
$$(2n+1)\beta \leq \frac{\pi}{2} \qquad (3.388)$$

The mode number n indicates the number of oscillations perpendicular to the coast. The periods of the different edge wave modes are within the interval $(5 < T_n(s) < 300)$.

They can be generated by various mechanisms, including local or open-sea barometric pulsations, the interaction of oblique wave trains with a different period, wave groups, etc. If the orientation, width, and depth of the port opening or of the wharf are suitable, the edge wave train can force its oscillation, and if the periods of the wharf itself and that of one of its modes coincide, the oscillation can be resonant.

The more spread out the shelf is, the greater the number of possible modes. Such conditions are more frequently found in marinas and fishing wharfs than in large commercial ports.

3.9.4.2 Shear waves

The shear wave is an instability of the current parallel to the coast generated by the oblique incidence of the wave. The wave period is usually in the range $(100 < T(s) < 600)$. The same as for edge waves, if the orientation, width, and depth of the port or wharf opening are adequate, the shear wave can force its oscillations, and if the periods of the wharf itself and that of one of its modes coincide, the oscillation can be resonant.

3.9.5 Forced oscillations in port and shoreline areas

Forced oscillations in port and shoreline areas are medium-period oscillations of the free sea surface $\zeta_{area}(\vec{x}, t)$. They are forced by atmospheric or maritime oscillations, especially by the free modes associated with wave groups, barometric pulses, meteo-tsunamis, and tsunamis.

The fundamental period of the area and of its harmonics depends on geometry and depth. If the period of the forcing agent is equal to the fundamental period or one of its harmonics, the forcing is said to be resonant. In this case, the amplitude of the oscillation can increase substantially.

The forces that are opposed to the oscillation of the area (apart from gravity, which is the main restorative force) are due to bottom friction, lateral boundaries, the aperture, turbulent movement, and the radiation of the oscillation through the opening.

3.9.5.1 Time-domain description

The oscillations of the area, $\zeta_{area}(\vec{x}, t)$, satisfy the Laplace equation. They can thus be described by the linear superposition of the principal mode and its harmonics, whose amplitudes satisfy the boundary conditions. If the signal $\zeta_{area}(\vec{x}, t)$ is irregular, it can be statistically analyzed. Furthermore, if the oscillation is resonant (wide-band spectrum), the envelope of the signal is a Rayleigh process. Therefore, the amplitudes of the oscillation at a certain point in the area (basic variable) also follow a Rayleigh distribution. The maximum amplitude, as long as the minimum size of the sample satisfies sufficient statistical conditions, follows a Rayleigh function raised to the N power.

3.9.5.2 Frequency-domain description

In the same way as other sea oscillations, with the corresponding theoretical limitations, the Fourier transform can be applied to the vertical displacement of the free sea surface at a point in the area, and the spectral density function obtained. The period band of shoreline areas is approximately 60-600 *seconds,*

3.9.5.3 Generation and transformation of oscillations into medium-period oscillations in the area

In the case of port areas whose nature is $[ERI > 5, SERI > 6]$, their regime of medium-period oscillations should be studied on the basis of possible external forcings, whether atmospheric or maritime. This can be done by doing one of the following: (1) solving the whole problem, in other words, the transitory portion of the port area oscillation, forced by the external agent, and expressed by the spatial gradients of the forcing term or an external oscillation (wave or spectrum); (2) determining the transfer function of the area in the interval of medium periods (60-600 seconds) and analyzing the possible forcing agents.

3.9.5.3.1 Time-domain models

The displacement of the free surface in port or shoreline area can be obtained by applying time-independent models (e.g. MSPE-t or Boussinesq models) or by resolving Airy non-linear long-wave equations. This includes the forcing terms, regardless of whether they are barometric pulses or wave groups. It is important for the models to resolve the energy radiation from the port opening. However, it is not essential for them to evaluate the friction dissipation, although in this case, the calculated resonant responses have an amplitude that, at least in theory, is infinite.

3.9.5.3.2 Frequency-domain models

The effect of the local geometry and topography is the transformation of the outer forcing spectrum. It is advisable to determine the spectrum for a point in the port area, based on the spectrum $Z_{ms}(w)$ on the outside (for port areas at the opening), by applying transfer function, $W^2_{ms}(\vec{x}, t)$,

$$S_{area}(\vec{x},\omega) = W^2_{area}(\vec{x},\omega) Z_{external}(\omega) \quad (3.389)$$

which can usually be obtained by applying the elliptic model, MSPE, to each of the spectral components. This method can be applied to the spectrum of the oscillations forced by the atmospheric components as well as to those resulting from wave group transformation, as long as there is a spectrum of the oscillation in the generation zone.

When the outer spectrum is not known, an MSPE model can be applied by sweeping the interval of periods (60-600 *seconds*) with intervals of two to five seconds. In this way, the fundamental period can be obtained as well as the harmonic periods that reflect the oscillatory behavior of the area and its possible forcings.

3.9.5.4 Long-term description

Generally speaking, the origin of medium-period oscillations in port areas is in the barometric pulses, meteo-tsunamis, tsunamis, and oscillations resulting from wave group transformation. The project design of the area should be in consonance with the fact that such oscillations are rare events, and have a small amplitude.

The long-term description should be performed in the same way as that recommended for other loading cycles.

3.9.5.5 Verification against wharf oscillations

When the probability of exceedance in the useful life of a structure, $F_{Amax} = \Pr[A > a_m] > 10^{-4}$, has an oscillation amplitude value, a_{area}, which is significant for the safety of the structure and the terrain, this oscillation should be included in the verification of extreme work conditions as a predominant agent, and when relevant, as another agent. When their forcing is due to loading cycles of sea states (wave groups), they should be included in the verification along with other oscillations of the sea level. In a Level I verification method, the compatibility criteria of terms should be applied, depending on the combination type (see section 5.3.7 of the ROM 0.0). In Level II and III verification methods, the joint probability with the sea states should be considered.

When relevant, and in accordance with the ROM 0.0, the area should be verified against tsunamis in exceptional work conditions.

3.10 LONG-PERIOD OSCILLATIONS

Long-period oscillations ζ_{PL} fall within period band $(T > 3\ hours)$. They mainly include the meteorological tide, ζ_{MM}, related to the passage of an atmospheric storm and the astronomic tide, ζ_{MA},

$$\zeta_{PL}(\vec{x},t) = \zeta_{MA}(\vec{x},t) + \zeta_{MM}(\vec{x},t) \tag{3.390}$$

where ζ is the instantaneous variable of the process, which defines the vertical displacement of the free sea surface with respect to the reference sea level (RSL) (see section 3.12).

3.10.1 Prediction of the sea level on the Spanish coastline

The Nivmar System is a set of applications that provides a short-term prediction of the sea level ($48\ hours$) in terms of the meteorological and astronomical tide. Nivmar is based on the harmonic prediction of the sea level, and on the use of the numerical models, HAMSOM (atmospheric circulation, *Puertos del Estado*) and HIRLAM (atmospheric circulation, *Instituto Nacional de Meteorología*). Additionally, Nivmar uses tide gauge data provided by REDMAR to correct systematic deviations that cannot be resolved by the circulation model.

3.10.1.1 HAMSOM Model

HAMSOM is a three-dimensional model of ocean circulation developed by the IFM (Institute für Meereskunde, Hamburg) and by Maritime Climate (*Puertos del Estado*, Madrid). The HAMSOM model is based on a set of seven differential equations in partial derivatives. The unknowns are the three velocity components, pressure, water density, salinity, and temperature. On the horizontal plane, complete Navier-Stokes equations are used, and on the vertical plane, the hydrostatic equation. These relations are used along with the continuity equation, the salt/temperature conservation equations and the equation of seawater state. The model is able to consider the forces of the wind, atmospheric pressure, tides, and baroclinic gradients. The most important restriction of this model pertains to the hydrostatic approximation, which limits the application range to long waves.

Note. *HAMSOM is the ocean circulation component of the ECAWOM model. ECAWOM was developed as part of the MAST project, and is still being experimentally tested. It integrates the physical processes of the atmosphere (resolved by HIRHAM), wave action (resolved by WAM), and ocean circulation (resolved by HAMSOM) into one model. It also takes into account how these elements interact with each other. Moreover, it is important to point out that any prediction of the sea level should resolve the variation of the reference level and its estimate, which in turn depends on the prediction itself. The method used by Puertos del Estado for this purpose is described on its web page.*

3.10.1.2 Prediction of the astronomical tide

Puertos del Estado provides the prediction of the astronomical tide, which is based on tide gauge data as well as data from numerical modeling.

3.10.2 Meteorological tide

Meteorological tides are the response of the sea level, due to tangential stresses on the water surface because of wind action and variations in the atmospheric pressure field above the water.

3.10.2.1 Introduction

The spatial and temporal variation of the atmospheric pressure causes the water column to rise or fall. Furthermore, the continuous action of the wind on the sea surface generates a drag. When this drag is confined by the coast or because of bathymetric variations, it also causes the free sea surface to rise or fall. This response becomes even more important as the depth, h, becomes shallower (and exceeds 20% of the water depth in the area $h < 10\ m$). Both are random mechanisms, and consequently, they should be described with probability models.

On the Spanish coastline, the highest meteorological tides are associated with the passage of an extratropical storm. In this type of event, the sea level rises, reaches one or various maximums, and then falls. Given that period T of the oscillation is long (in hours and days) and its wavelength L is large (in hundreds of meters), the relative depth h/L is very small, ($h/L < 1/20$), and the kinematics and dynamics of the wave is asymptotically that of a long wave, whose propagation celerity, when non-frictional linear theory is applied, can be estimated by $c = \sqrt{gh}$.

Note. *The temporal scale of the meteorological tide is much larger than the scale of barometric pulses, which ranges from a few minutes to four hours, approximately. Both processes can be simultaneous, but can also occur independently since barometric pulses can be generated by other causes apart from the passage of a storm.*

3.10.2.2 Basic and instantaneous variables

The instantaneous variable of the meteorological tide is the displacement (exceedance) of the free surface η_{MM} with regards to a fixed reference level, generally the MSWL or SLW. However, the meteorological tide occurs simultaneously with the astronomical tide. For this reason, it is measured with respect to a mobile reference level In such conditions, the meteorological tide is the remainder of the signal (27),

$$\eta_{MM}(\vec{x},t) = h_i(\vec{x},t) - \left[h_{0,i}(\vec{x}) + \Delta h + \eta_{MA,i}(\vec{x},t)\right] \tag{3.391}$$

where $\eta_{MM}(\vec{x}, t)$ is a random variable and its temporal evolution is a non-stationary process. The duration of the exceedance or the time during which the sea level consecutively exceeds the level of reference is also a random variable. Given its relation to the passage of the storm, this exceedance lasts approximately one to three days. A basic variable of the meteorological tide should be defined by the mean value of the instantaneous signal in a small time interval (e.g. 5-10 minutes).

Note. *In the remainder there may be other oscillations of five to ten minutes. These are related to barometric pulses and wind field fluctuations, and should be identified and filtered. Except in certain cases pertaining to confined and reduced water bodies or to resonance in wharfs (and vessels), they are not significant in maritime engineering.*

3.10.2.3 State descriptor

Because of its generation process, sea level variation due to the meteorological tide $\eta_{MM}(t)$ is usually a slow process that lasts several hours. As a result, it is possible to divide the seal level exceedance curve in time intervals of approximately one hour. This means that in a sea or meteorological state, the level of the meteorological tide is assumed to be constant, and equal to its mean value, $\overline{\eta_{MM}}$, in the time interval,

$$\overline{\eta_{MM}}(\vec{x}) = \frac{1}{d_t}\int_{t_i}^{t_i+d_t} \eta_{MM}(t)\,dt \tag{3.392}$$

where t_i is the time instant in which the meteorological state begins, and d_t is its duration. This descriptor is a slow variable.

Note. *When the meteorological tide is obtained by numerical models, the input information regarding wind fields and windspeed can condition the duration of the state and lengthen it to three hours.*

(27) The remainder of the signal also contains other oscillations, which should be identified and filtered.

3.10.2.4 Loading cycle and amplitude

The $\overline{\eta_{MM}}$ evolves over time, and its temporal sequence or state curve defines the loading cycle of the meteorological tide. Regarding the loading cycle, a generation and growth phase should be defined, as well as a peak, and a phase of dissipation and extinction. Their relation to the passage of the storm should also be made explicit. Generally speaking, these phases are correlated with the loading cycle phases of the sea state.

The peak of the cycle is defined by the amplitude or maximum displacement, $\overline{\eta_{MM,max}}(\vec{x}) = \max[\overline{\eta_{MM}}(\vec{x}, t)]$. This is sometimes also referred to as the tide amplitude, although, strictly speaking, it is not the correct term. A cycle can have various peaks. The maximum amplitude in the cycle, $\overline{\eta_{MM,max}}$ is correlated with the maximum significant wave height (peak) of the loading cycle, $H_{s,peak}$, although they do not necessarily have to coincide in the same state.

In the case of island coastlines in Spain, it is advisable to identify and relate meteorological tide, generation phase, maximum value, and dissipation or extinction phase with local meteorological process and their temporal evolution.

3.10.2.5 Generation and transformation models

The equations that govern the generation, propagation, transformation, and dissipation of the meteorological tide are those of long waves, $h/L < 1/20$. These equations are derived from the Navier-Stokes equations for turbulent flow. They are vertically integrated in all of the water column from the bottom at depth h to the free surface η, vertically displaced with regards to the mean sea level, and assuming that in the interphases of the fluid with the atmosphere and land, respectively, there may be normal and tangential forces acting on them. If the bottom is impermeable, its effect on the movement of the fluid is only present in tangential stresses.

Regarding the free sea surface, the atmosphere can act on the water with normal stresses (pressures). Except in very shallow waters, it is generally accepted that there is much less vertical movement than horizontal movement. Accordingly, vertical accelerations are regarded as negligible, such that the flow is considered to be flat, and the pressure is regarded as hydrostatic.

3.10.2.6 Numerical models

Generally speaking, the solution to long-wave equations can be found in numerical models. For this purpose, it is possible to use any of the usual algorithms with suitable boundary conditions with forcing terms, depending on the temporal evolution of the pressure gradients and the tangential actions of the wind on the water surface. This information is obtained from the meteorological data pertaining to the wind and pressure action on the sea and their evolution over time, which is provided by the HIRLAM model. This gives the temporal evolution of the state variables, $\overline{\eta_{MM}}(\vec{x}, t)$ and the horizontal variables $[U, V]$ averaged in the water column.

In very shallow water and with the astronomical tide, the calculation of the meteorological tide should be performed for various tide levels. Furthermore, it should be taken into account that the meteorological tide affects water depth. Accordingly, before analyzing wave transformation, it is necessary to determine the contributions of both types of tide.

3.10.2.7 Selection of numerical models

The recommendations in the Annex (section concerning long waves for the astronomical tide) should be followed. Generally, in port areas with a depth of over fifteen meters, convective terms can be ignored, and it is possible to work with the linear problem. When geometric conditions make the velocities $U \sim 1\ m/s$ and its gradients important, the convective terms also become important.

3.10.2.7.1 STATIONARY CONDITIONS

When the water depth at the site exceeds fifteen meters, or when the spatial velocity gradients are not significant, the maximum amplitude of the meteorological tide can be estimated by assuming stationary conditions of the maximum pressure gradient associated with the storm and by applying one of the analytical solutions described in the Annex. Special attention should be paid to the boundary conditions.

Furthermore, when the depth is less than ten meters, it is necessary to estimate the meteorological tide regime. If the shoreline area is small and similar to a channel, the equations in the Annex and their solutions can be applied.

3.10.2.8 Shape models of the meteorological tide

If there is insufficient statistical data pertaining to the meteorological tide and wind and pressure fields, the shape of the meteorological tide can be simulated on the basis of available amplitude and time duration values.

3.10.2.9 Medium-term and long-term description

Meteorological tide events which are relevant for the safety of the structure correspond to sea level states that exceed a certain level. Their treatment and medium and long-term description correspond to the recommendations for sea state loading cycles (see sections 3.7.3 y 3.7.4). For this reason, the following sections merely reproduce the content of those sections.

3.10.2.10 Over-threshold state curves

The state curve is a representation of the temporal evolution of the state of the meteorological tide at a point in the sea. The function, $\overline{\eta_{MM}}(\vec{x},\ t)$, is random and non-stationary, due to its seasonal and hyper-annual variability. After the selection of its maximum and minimum threshold values, $[\overline{\eta_{MM}}]_{cr}$, it is possible to identify its loading cycles and its use and exploitation cycles, respectively. The threshold values depend on the site.

3.10.2.11 Relative frequency of exceedances of the threshold value

If the time is calculated in which the state curve is over a threshold value, $\overline{\eta_{MM}}(\vec{x},\ t) > [\overline{\eta_{MM}}]_{cr}$, and it is divided by Υ, the total duration of the time interval under analysis, this gives the relative frequency in Υ

$$F\left(\overline{\eta_{MM}}(\vec{x},t) > \left[\overline{\eta_{MM}}\right]_{cr};\Upsilon\right) = \frac{\sum_i \tau_{s,i}}{\Upsilon} \tag{3.393}$$

where $\tau_{s,i}$ is the duration of exceedance i.

If the calculation is performed with data from various meteorological years $\Upsilon = M$ (e.g.), the regime of over-threshold exceedances is representative of exceedances in time intervals of M years.

3.10.2.12 Probability model of over-threshold exceedances: GPD

The conditional probability of the random variable, $\overline{\eta_{MM}}$, when it is known that $\overline{\eta_{MM}} > [\overline{\eta_{MM}}]_{cr}$, is the following:

$$\Pr\left[\left(\overline{\eta_{MM}} < \overline{\eta_{MM\ i}} \,\middle|\, \overline{\eta_{MM\ i}} > \overline{\eta_{MM\ cr}}\right)\right] = \frac{F\left(\overline{\eta_{MM\ i}}\right) - F\left(\overline{\eta_{MM\ cr}}\right)}{1 - F\left(\overline{\eta_{MM\ cr}}\right)}, \quad \overline{\eta_{MM\ i}} > \overline{\eta_{MM\ cr}} \tag{3.394}$$

where F is the distribution function of $\overline{\eta_{MM\,i}}$.

For sufficiently high values of $\overline{\eta_{MM}}_{cr}$, this probability is an adequate approximation to the Generalized Pareto Distribution (σ_P, ξ_P),

$$F^*\left[\left(\overline{\eta_{MM}} < \overline{\eta_{MM\,i}} \,\middle|\, \overline{\eta_{MM\,i}} > \overline{\eta_{MM\,cr}}\right)\right] = \left[\frac{1-\xi_P\left(\overline{\eta_{MM\,i}} - \overline{\eta_{MM\,cr}}\right)}{\sigma_P}\right]^{\frac{1}{\xi_P}} \quad \overline{\eta_{MM\,i}} > \overline{\eta_{MM\,cr}} \tag{3.395}$$

where σ_P is the scale parameter ($\sigma_P > 0$) and ξ_P is the shape parameter. If $\xi_P = 0$, then the exponential distribution is obtained. This probability model can also be applied when the meteorological year is the selected time interval.

3.10.2.13 Regime of the exceedance duration

The exceedance duration is the time interval (e.g. meteorological year, or useful life) in which the state descriptor remains over the selected value. Following the behavior of the sea state, it is advisable to begin the fit with a biparametric Weibull model, whose parameters are functions of the exceedance level.

3.10.2.14 Probability model of the number of meteorological time cycles Υ

If the event $\overline{\eta_{MM}}(\vec{x}, t) > [\overline{\eta_{MM}}]_{cr}$ over-threshold exceedance is a rare event in the time interval (e.g. the quotient of the number of events and the number of days in Υ is on the order of $O[10^{-1}]$ or less), then the number of loading cycles $n = n_{cr,\Upsilon}$, in Υ is a random variable that generally can be fit to a Poisson probability model, whose density function is,

$$f(n) = \frac{\nu^n e^{-n}}{n!} \tag{3.396}$$

where $\nu = \overline{n_{cr,\Upsilon}}$ and $n = 0, 1, 2, \ldots$ are the possible number of loading cycles (over-threshold exceedances) that can occur in time interval Υ.

3.10.2.15 Distribution function of the maximum amplitude of the meteorological tide cycle

The maximum amplitude in each meteorological tide cycle is a random variable. The sequence of its values in the time interval is a sample of statistically independent elements. If the threshold value is sufficiently high, the distribution function $F_{\overline{\eta_{MM,max}}}$ is an adequate approximation to the Generalized Pareto Distribution, $GPD(\sigma_P, \xi_P)$,

$$F_{\overline{\eta_{MM,max}}}\left(x \,\middle|\, x > \overline{\eta_{MM\,cr}}\right) = \left[\frac{1-\xi_P\left(x - \overline{\eta_{MM\,cr}}\right)}{\sigma_P}\right]^{\frac{1}{\xi_P}} \quad x > \overline{\eta_{MM\,cr}} \tag{3.397}$$

where σ_P is the scale parameter ($\sigma_P > 0$) and ξ_P is the shape parameter.

3.10.2.16 Distribution function of the annual maximum amplitude

If $P_N(n)$ is the accumulated distribution of N number of peaks in the time interval, the distribution function of the maximum peak of the meteorological tide cycles can be calculated by weighting the function $[F_{\overline{\eta_{MM,max}}}(x)]^n$ by means of $P_N(n)$ for all possible values of N,

$$G_{\overline{\eta_{MM,\max}}}(x) = \sum_{n=0}^{\infty} P_N(n) * \left[F_{\overline{\eta_{MM,\max}}}(x)\right]^n \tag{3.398}$$

If the Poisson parameter is $\nu_{MM} = n_{MM} / M$, then the distribution function is the maximum amplitude, or it can also be obtained by using the following expression:

$$G_{\overline{\eta_{MM,\max}}}(x) = \exp\left[-\frac{\eta_{MM}}{M}\left(1 - F_{\overline{\eta_{MM,\max}}}(x)\right)\right] \quad (3.399)$$

3.10.2.17 Extreme regime of the amplitude of the meteorological tide

The annual maximum or extreme regime is the distribution function of the highest value in each meteorological year of the maximum amplitude of each meteorological tide cycle, which has the highest probability of coinciding with the peak of one of the loading cycles ($X_{max} \approx \max X_{peak}$).

The data can be fit to a probability model by ordering the sample of N values from the smallest to the largest, and assigning the accumulated frequency (non-exceedance frequency),

$$P_i = \frac{i}{N+1}, \quad i = 1,2,\ldots,N \quad (3.400)$$

The distribution function of the maximum value of the data series of size n, when tends asymptotically to the generalized extreme value function, $GEV(\sigma_E, \delta_E, \mu_E)$. If X_{max} is the random value (maximum value when $n \to \infty$), then its distribution function is the following:

$$G_{X_{max}}(x) = \Pr[X \le x] = \exp\left\{-\left[\frac{1-\delta_E(x-\mu_E)}{\sigma_E}\right]^{\frac{1}{\delta_E}}\right\} \quad (3.401)$$

where μ_E, is the localization parameter; $\sigma_E(\sigma_E > 0)$ is the scale parameter; and δ_E is the shape parameter. For the sample to fit the function it should contain a sufficient number of years ($n > 10$).

3.10.2.18 Joint description with other agents

Because of its atmospheric origin during the passage of a storm, the occurrence of the meteorological tide is statistically correlated with the occurrence of waves and groups of waves, and in certain cases, with the oscillations induced by barometric pulses and meteo-tsunamis. For this reason, when possible, it is best to obtain the joint distribution functions of sea states and the meteorological tide.

3.10.2.19 The meteorological tide in the verification

When the probability of exceedance in a structure's useful life $F_{\overline{\eta_{MM,max}}}(a) = \Pr[\overline{\eta_{MM,max}} > a] > 10^{-4}$ has a maximum tide amplitude $\eta_{MM,max}$, which is significant for the safety of the structure and the land, it should be included in the verification of extreme work conditions. In this respect, it should be regarded as a predominant agent, and when relevant, as another agent.

For the Level I verification, it is sufficient to determine the mean amplitude in the loading cycle, assuming that it occurs simultaneously with the maximum significant wave height in the cycle. The compatibility of both values should be verified by taking into account their origin, depending on the combination type (see section 5.3.7 of the ROM 0.0).

For Level II and III verifications, it is necessary to use the joint distribution function of the maximum amplitude and the significant wave height of the storm peak as well as the correlations between their respective phases of generation, maximum, and dissipation.

Generally speaking, for meteorological states associated with normal work and operating conditions, the correlation between signals will be weak or non-existent. When this occurs and no other information is available, the sea level states, due to the meteorological and astronomical tides and the sea states, can be assumed to be statistically independent.

3.10.3 Astronomical tide

The astronomical tide is a set of regular movements of the sea or ocean surface, due to variations in the force of gravity. These variations are caused by the moments of heavenly bodies, mainly those of the moon revolving around the Earth, and those of the Earth, revolving around the Sun.

3.10.3.1 Introduction

In the open sea, the amplitude of the tide is around a meter, and the velocity of the tidal current is very small, and much less than the velocity due to the wind action. However, when the tide approaches the continental shelf and propagates over the outer continental shelf, two things occur that can cause the previous description to vary. The first is the Coriolis effect, and the second is its reflection on the continental shelf and the edge of the coastline.

Generally speaking, port and shoreline areas do not significantly modify the local components of the astronomical tide, more specifically, its range or period. However, when boundaries change, the velocity of the tidal current can undergo substantial variation. For this reason, in the project design of port areas, it is usually sufficient to represent the tide in terms of eight components. Since its amplitude and phase values are specific of each location, such a description should not be directly applied to other points on the coastline without a study of the propagation of the tide. It should thus be systematically measured over a period of at least 40 days so that its principal components can be calculated.

The possibility of approximating the time function, which locally describes oscillatory movement due to the astronomical tide by means of a Fourier series expansion of sines means that the vertical displacement of the sea surface over time at a certain location can be written as follows:

$$\eta_{MA}(t) = a_0 + \sum^{N} H_s \cos\left(\frac{2\pi t}{T_n} + \alpha_n\right) \qquad (3.402)$$

where N is the total number of harmonic components considered; H_n is the amplitude; T_n is the period; and α_n is the phase of the component n. In order to consider a certain displacement in respect to a reference level, a_0 is also included. Since the period of each of the tide components is known, and does not change with propagation *n,* the astronomical tide can be predicted at a certain point by determining the amplitudes and local phases of each one of them.

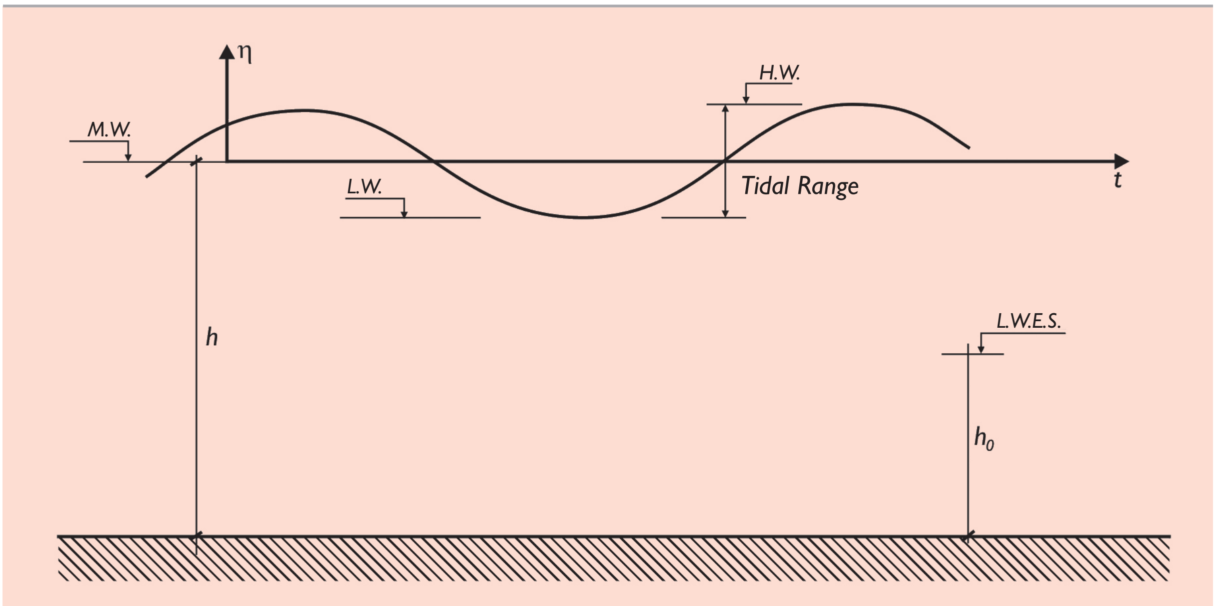
The astronomical tide is a long wave with a period of approximately twelve hours and half in most seas and oceans. Its propagation celerity is approximately $c = \sqrt{gh}$, where h is the depth, and g is the acceleration of gravity. Upon entering the continental shelf, the tide wave is transformed. Its amplitude and phase vary because of shoaling, refraction, bottom friction, lateral boundary effects, and reflection. For this reason, the characteristics of the tide wave (i.e. its amplitude, period, and daily, monthly, annual and hyperannual variability) are local, in other words, specific for each point.

3.10.3.2 Instantaneous and basic variables

The instantaneous variable of the tide is the vertical displacement of the free sea surface, $\eta_{MA}(\vec{x}, t)$, with respect to a reference level. This level is generally the lowest water level that has been reached because of the astronomical tide (see Figure 3.3.35). It is a fixed level that is known as port zero even though some ports have situated the zero a few centimeters below that level (see section 3.15).

The basic variable of the tide is its range or amplitude, defined by the vertical distance in a tide cycle between its crest or high tide level and its trough or low tide level. It varies over the days, depending on the temporal evolution of the relative movements of the Sun, Earth, and Moon. The astronomical tide is a modulated wave. In a lunar month, the tidal range has two maximums and two minimums, which are known as spring tides and neap tides, respectively. Twice a year the range of the spring tides will be higher and that of the neap tides will be lower than the other tides during the year. The range of the tide also varies over the years, reaching maximum values every 18 years.

Figure 3.3.35. **Parameters for the description of the astronomical tide**



3.10.3.3 State variable

The amplitude of the tide on the Spanish coastline varies, depending on the area in the interval [0.30-5.5] in meters. Consequently, it varies slowly over time, and $d\eta_{MA}(t) / dt$ is small. The astronomical tide can be accurately described by the vertical displacement, η_{MA}, determined at time intervals of approximately one hour. In any case, this slow variable can be regarded as a state descriptor. It is thus advisable to use the mean displacement $\overline{\eta_{MA}}$ of the values during the state,

$$\overline{\eta_{MA}} = \frac{1}{d_t} \int_{t_i}^{t_i+d_t} \eta_{MA}(t)\, dt \qquad (3.403)$$

where t_i is the instant in time when the meteorological state begins, and d_t is its duration.

This description can be applied to other variables related to the astronomical tide (e.g. the velocity of the current), although in certain locations the local acceleration of the velocity at mid-tide may require the reduction of the duration, d_t.

3.10.3.4 Loading cycle of the astronomical tide

Generally speaking, there are 3-4 consecutive tide cycles with a range exceeding 0.85, the range of a spring tide or a neap tide. It can be assumed that this consecutive sequence of cycles, in which the range of the tide belongs to the spring tide, constitutes a loading cycle of the astronomical tide. In this case, the maximum amplitude of the spring tides in the year are the peaks of the loading cycles and the mean number of peaks in the year is determined.

3.10.3.5 Velocity of the tidal current

The oscillatory movement of the tide involves the velocity and acceleration of the water. Generally, the local acceleration is very small. Notwithstanding, the convective acceleration can be locally important, especially during intermediate phases of flood tide and ebb tide. In the same way as for the vertical displacement, instantaneous and basic velocities are defined. This is the maximum value in the flood cycle or ebb cycle.

The state descriptor is the mean velocity of the values during the state,

$$\overline{U_{MA}} = \frac{1}{d_t}\int_{t_i}^{t_i+d_t} U_{MA}(t)\,dt$$
$$\overline{V_{MA}} = \frac{1}{d_t}\int_{t_i}^{t_i+d_t} V_{MA}(t)\,dt \qquad (3.404)$$

3.10.3.5.1 Tidal ellipse

For a certain component of the tide wave, defined by the angular frequency ω, the velocity ellipse can be easily determined. For this tide component, the velocity components can be expressed as:

$$U = A_1 \cos\omega t + B_1 \sin\omega t \qquad (3.405)$$

$$V = A_2 \cos\omega t + B_2 \sin\omega t \qquad (3.406)$$

For the harmonic component considered, the velocity components can be transformed to the principal axes of the ellipse in terms of which the velocities can be expressed as follows:

$$U' = a\cos(\omega t - \varepsilon) \qquad (3.407)$$

$$U' = b\sin(\omega t - \varepsilon) \qquad (3.408)$$

where a and b, the phase ε and the inclination of the axes can easily be calculated from the constants A_2, B_1 and B_2. The ellipse will be:

$$\frac{U'^2}{a^2} + \frac{V'^2}{b^2} = 1 \qquad (3.409)$$

Note. *In the case of a tide propagating in an inlet or estuary, the channel can be represented simply as the transversal dimension of an estuary or semi-enclosed water body where the tide is being analyzed. As the width of the channel becomes smaller, the ellipse tends to adopt the form of a straight line, whose ends represent the maximum and minimum velocities and whose longitude represents twice the maximum amplitude of the velocities at the point.*

3.10.3.5.2 Variation of the depth of the tide current

Generally speaking, the maximum amplitudes of tide currents are observed at depths close to the free sea surface. These amplitudes gradually decrease as the depth increases. When the velocity profile must be ascertained, a measurement of the mean current velocity *u* near the water surface (z_1) should be taken and the following potential velocity profile [(28)] assumed:

$$u(z) = u(z_1)\left(\frac{z+h}{z_1+h}\right)^{\frac{1}{7}} \qquad (3.410)$$

where h is the depth at the point of measurement. On the boundary layer adjacent to the bottom, it is customary to assume a law of logarithmic velocities, generally expressed as:

(28) When the current velocity is in capital letters, it refers to the velocity integrated in the water column. When it appears in lowercase letters, it represents the velocity at a specific depth.

$$u = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right) \quad (3.411)$$

where u_* is the friction velocity:

$$u_* = \sqrt{\frac{[\tau_x]-d}{\rho}} \quad (3.412)$$

$[\tau_x]_{-b}$ is the shear stress at the bottom; ρ is the density; $k = 0.41$ is the Van Karman constant; z is the distance to the bottom (direction perpendicular to the bottom); and z_0 is the mean roughness of the bottom (see Annex).

However, if there are relative changes in the water density with the depth, this will cause baroclinical or internal waves, which will influence the vertical structure of the currents. They should be quantified by completely resolving the problem of the propagation of the tidal wave, including the conservation of water temperature and salinity.

3.10.3.6 Subtidal currents

When, at a certain point, the mean of components U and V in a tide cycle is calculated, what is obtained is the resulting net amount of the current or the residual tide current. Even though its velocity is small, its effects on the transport of substances are important since for all practical purposes, it behaves like a permanent current. This deformation of the tidal ellipse has a topographic origin since it is the result of the interaction of the boundaries and the tide wave. This interaction can be evaluated by adding components of the tide to the harmonic series expansion.

As this circulation depends on the tidal amplitude, it varies with the spring and neap tides.

Note. When tidal currents are studied with numerical models, it is necessary to differentiate between residual currents due to the tide itself and spurious residual currents due to non-conservational properties of the variables in the model, This phenomenon should be verified by comparing residual currents calculated with real data. Apart from the greater temporal variability of currents in respect to sea levels, currents possess a much greater spatial variability than the position of the free sea surface.

3.10.3.7 Generation and transformation models

The generation and transformation of the astronomic tide is obtained by integrating the equations in shallow water, forced by gravity components per unit of mass or the forces generated in the direction of the coordinated axes. The solution of the equation system can be obtained by applying any of the usual techniques for solving shallow water equation systems.

3.10.3.7.1 The Coriolis effect on the inner continental shelf

In most cases, the Coriolis effect on the propagation of the tide over the inner continental shelf is negligible in comparison to the effects of bottom friction and local topography. In zones near the coastline, such as the inner continental shelf of the Cantabrian Sea or in the inlets and estuaries of the Spanish shoreline (all of which are very narrow in comparison to the length of the tide wave), the velocity perpendicular to the propagation direction of the tide wave V is very small, and thus, its contribution to the equation of the quantity of horizontal, $-f_cV \approx 0$. The equation corresponding to the axis is reduced to the equilibrium between the Coriolis effect and the gradient of the free sea surface, $f_cU + g(\partial\eta/\partial y) = 0$. In these latitudes (around parallel 40), $f_c \approx 10^{-4} rad/s$. For tidal current velocity, $U \approx 1 m/s,$ the gradient of the free surface is $\Delta\eta/\Delta y \approx 10^{-2}\, m,$ a negligible quantity, which reflects its incapacity (in a short distance) to put into motion a transversal current in the inner continental shelf or in the inlet.

3.10.3.8 Prediction of the tide wave

The instantaneous variable is the vertical displacement of the free sea surface, $\zeta_{MA}(\vec{x}, t)$, with respect to a fixed level of reference that is locally described by means of Fourier cosine series expansion.

$$\eta_{MA}(t) = a_0 + \sum^{N} H_n \cos\left(\frac{2\pi t}{T_n} + \alpha_n\right) \quad (3.413)$$

where N is the total number of harmonic components considered; H_n represents the amplitude; T_n is the period; α_n is the phase of component n, (see section 3.13.3.1); and a_0 determines a displacement of the level of reference. Since the period of each of the tidal components is known and do not change with the propagation, the prediction of the astronomical tide at a point can be performed by determining the local amplitudes and phases for each of them. The harmonic constants and their phase are specific to the location, and should be determined on the basis of a record of sufficient duration (approximately one month and a half). The astronomical tide is generated in oceans where the attraction of heavenly bodies is more effective. In the open sea, the principal semi-diurnal lunar and solar components and the principal lunar and lunar-solar components practically control over 95% of the oscillation of the free surface. In fact, the latter three components each contribute to the oscillation with approximately half of the signal of the principal lunar component. The principal lunar component, M_2, is usually taken as the reference component, H_{ref}, to define the amplitudes of the other components.

3.10.3.8.1 Principal harmonic components of the tide

The harmonic analysis calculates the amplitudes and phases of a certain number of sinusoidal functions of a given frequency. Given the fact that there are hundreds of tidal components, it is not practical to include all of them since many have negligible amplitudes. Regarding tidal components, the international nomenclature designates them with a letter corresponding to their origin. More specifically, S is for sun and M is for moon, with a subscript indicating the approximate period: the subscript 2 refers to a half day; 1 refers to one day; f refers to a fortnight or 15 days; m corresponds to a month, *sa* is semi-annual, etc. Components can be classified in three main categories:

- Semi-diurnal components: periods of approximately 12 hours. The principal components of this class are the mid lunar component M_2, and the main solar component, S_2.
- Diurnal components: periods of approximately 24 hours. The principal components of this category are the lunar-solar component, K_1, and the principal lunar component, O_1.
- Long-period components: Larger periods lasting days. The principal fortnightly component is the lunar fortnightly component, M_f, and the principal monthly component is the lunar monthly component, M_m.

Table 3.3.26 shows the most important components. The coefficient in the last column expresses the amplitude of each component in relation to a unitary amplitude of the semi-diurnal lunar component.

Table 3.3.26. Periods and amplitudes relative to the principal components of the astronomical tide

Category	Component, n	Symbol	Period, T_n	H_n/H_{ref}
Semidiurnal	Principal lunar	M_2	12.42 h.	1.00
Semidiurnal	Principal solar	S_2	12.00 h.	0.46
Semidiurnal	Lunar elíptica	N_2	12.66 h.	0.192
Semidiurnal	Lunar - solar	K_2	11.97 h.	0.127
Diurnal	Lunar - solar	K_1	23.93 h.	0.584
Diurnal	Principal lunar	O_1	26.87 h.	0.415
Diurnal	Principal solar	P_1	24.07 h.	0.194
Fortnightly	Fortnightly lunar	M_f	13.66 días	0.172
Monthly	Monthly lunar	M_m	27.55 días	0.091
Semiannual	Semiannual solar	S_{sa}	128.7 días	0.08

Note. In the open sea, the principal semi-diurnal lunar and solar components and the principal lunar and lunar-solar components practically control over 95% of the oscillation of the free surface. In fact, the latter three components each contribute to the oscillation with approximately half of the signal of the principal lunar component. The principal lunar component, M_2, is usually taken as the reference component, H_{ref}, to define the amplitudes of the other components.

3.10.3.8.2 Classification of the tide at a certain point

The harmonic constants of the tide can be used to make a basic classification of the dominant nature of the tide at a certain point by means of the shape factor. The shape factor F is defined as the quotient of the sum of the amplitudes of the two principal diurnal harmonics in the numerator and the two principal semidiurnal harmonics in the denominator:

$$F = \frac{K_1 + O_1}{M_2 + S_2} \qquad (3.414)$$

where each term is represented by the amplitudes of the components. Based on the local value of the shape coefficient, the astronomical tidal wave can be classified as follows:

- Semidiurnal: $0 < F < 0.25$. In one day there are two high tides and two low tides approximately the same tidal range. The interval between spring tides and neap tides is 14.8 days.
- Mixed with a predominance of the semi-diurnal tide: $0.25 < F < 1.5$. There are two high tides and two low tides daily, but their amplitude and duration are unequal.
- Mixed with a predominance of diurnal tide: $1.5 < F < 3.0$. Occasionally, there is one high tide and one low tide per day. Sometimes, there are two high tides per day, but their amplitude and duration are very unequal.
- Diurnal: $F > 3$. There is generally one high tide and one low tide per day, except in neap tides where two high tides and two low tides may occur. The interval between spring tides and neap tides is 13.7 days.

The chapter on data provides representative values of the shape factor on the Spanish coast.

3.10.3.8.3 Tide components generated in shallow waters and by boundaries

As previously explained, shallow waters and/or the presence of coastal boundaries generate new tidal components. Thus, these factors must be considered in a complete characterization of the tide at a certain point.

The distortion of the tidal profile in these conditions can be adequately represented by including additional components with components that double, triple, etc,. the frequency of the basic component. For example, in the case of the component M_2, the additional component with a double frequency is known as M_4; the additional component with a triple frequency is M_6, and so on.

The new components generated by the interaction of components with frequencies different from the original are designated by the initials of the source. For example, the four-times-daily component, MS_4, is generated by the semi-diurnal components, M and S.

3.10.3.9 Medium-term and long-term description

Astronomical tide events that are relevant for the safety of a structure generally correspond to spring tide cycles, defined as the tide level states that exceed a certain threshold. Its treatment and medium-term and long-term descriptions are the same as those recommended for the analysis of sea states (see sections 3.7.3 and 3.7.4). For these descriptions, it should be remembered that the number of spring tide cycles per year or in the useful life of the structure is a certain number that is already known.

3.10.3.10 State curves of the astronomical tide

The state curve is a representation of the temporal evolution of the astronomical tide, $\overline{\eta_{MA}}$, with respect to a fixed reference level. Based on this curve, it is possible to obtain for a given time interval (e.g. one year) the relative frequency and the distribution function of the exceedance of sea level by applying the same methods recommended for sea states.

Generally speaking, the log normal model is a good fit for the centered values of the sample, whereas the Weibull tri-parametric function more accurately reproduces the behavior of the upper tail values. It is advisable to begin the fit with the generalized gamma function.

3.10.3.11 Loading cycles and number of astronomical tide cycles: spring tides

The exceedance of a threshold near the values of spring tide amplitudes, $\overline{\eta_{MA}}(\vec{x}, t) > [\overline{\eta_{MA}}]_{cr}$, defines the loading cycles per astronomical tide. The number of cycles in the year is already determined. When the maximum amplitude of the cycle is considered to be a random variable, $\eta_{\overline{MM,max}}$, the sequence of its values in the time interval constitutes a sample of statistically independent elements. For this reason, the following distribution functions can be calculated.

3.10.3.12 Distribution function of the maximum range of the astronomical tide cycle

Since the selected threshold value is sufficiently high, the distribution function, $F_{\overline{\eta_{MM,max}}}$, is usually an adequate approximation of the Generalized Pareto Distribution, $GPD(\sigma_P, \xi_P)$,

$$F_{\overline{\eta_{MA,\max}}}\left(x \middle| x > \overline{\eta_{MM\ cr}}\right) = \left[\frac{1 - \xi_P\left(x - \overline{\eta_{MM\ cr}}\right)}{\sigma_P}\right]^{\frac{1}{\xi_P}}; \quad x > \overline{\eta_{MM\ cr}} \tag{3.415}$$

where σ_P is the scale parameter ($\sigma_P > 0$) and ξ_P is the shape parameter.

3.10.3.13 Distribution function of the annual maximum tidal range

Given the fact that tide ranges, such as spring tides (e.g. the spring tide) are rare events in the year, the distribution function of the maximum range can be obtained by the following expression:

$$G_{\overline{\eta_{MA,\max}}} = \exp\left[-\frac{n_{MA}}{M}\left(1 - F_{\overline{\eta_{MA,\max}}}\right)\right] \tag{3.416}$$

where n_{MA} is the number of spring tides in M years.

3.10.3.14 Extreme regime of the astronomical tidal range

The regime of the maximum annual or extreme tidal range is the distribution function of the largest value in each meteorological year of the maximum amplitude of each tide cycle. The fit of the data to a probability model is performed by ordering the sample of N values from the smallest to the largest and assigning the accumulated frequency (or of no exceedance),

$$P_i = \frac{i}{N+1}, \quad i = 1,2,\ldots,N \tag{3.417}$$

The distribution function of the maximum value of the data series of size n, when $n \to \infty$ asymptotically tends to the generalized extreme value function $(\sigma_E, \delta_E, \mu_E)$. If X_{max} is the random value (maximum value when $n \to \infty$), then its distribution function is the following:

$$G_{X_{max}}(x) = \Pr[X \le x] = \exp\left\{-\left[\frac{1-\delta_E\left(x-\mu_E\right)}{\sigma_E}\right]^{\frac{1}{\delta_E}}\right\} \quad (3.418)$$

where $X_{max} = A_{max}$; μ_E is is the localization parameter; $\sigma_E(\sigma_E > 0)$ is the scale parameter; and δ_E is the shape parameter. To fit the data to the function, it is advisable for the sample to have a sufficient number of years ($n > 10$).

3.10.3.15 Joint description with other agents

Because of its atmospheric origin during the passage of a storm, the occurrence of the meteorological tide is statistically correlated with the occurrence of waves and groups of waves, and sometimes with the oscillations induced by barometric pulses and meteo-tsunamis. For this reason, whenever possible, it is advisable to obtain the joint distribution functions of the sea states and the meteorological tide.

3.10.3.16 The astronomical tide in the verification

On the Spanish coastline, the astronomical tide is a marine climate agent that should be included in the project designs of all maritime and port structures because of its contribution to the determination of the sea level and the freatic level, the velocity and direction of the current, and its interaction with other sea oscillations, especially the waves.

In Level I Verifications, it is sufficient to determine the amplitude of the tidal range, which is simultaneous with the maximum significant wave height of the loading cycle of the sea state and sea level (including the meteorological tide) in the body of water and the freatic level. The compatibility of values should be verified by taking into account the origin and the combination type (see section 5.3.7 of the ROM 0.0).

For Level II and III verifications, it is necessary to use the distribution functions of the vertical displacement of the free sea surface, due to the astronomical tide and the annual amplitudes of the spring tide. This amplitude is regarded as statistically independent of the significant wave height of the peak of the storm (loading cycle).

3.10.4 Marine currents and oceanic circulation

The currents related to ocean dynamics, which are a part of the general oceanic circulation, are usually not important for dimensioning port areas. However, they can affect sediment transport on the inner continental shelf. On the continental shelf and near the seacoast, the current, independently of the long waves considered in this chapter, can be caused by estuarine circulation and fluvial discharges.

3.10.4.1 Instantaneous, basic, and state variables

The instantaneous variables of the current are the three components of its velocity and direction. These components fluctuate around a mean value, depending on the turbulent intensity. Both vary, depending on the distance to the seabed. Each of the instantaneous velocity components can be decomposed in a mean value $\overline{u_c(z)}$, and fluctuation velocity u_c' which generally fits a Gaussian process.

Within the context of these Recommendation, *current* refers to water movements in an almost stationary regime. In other words, it refers to water movements that vary slowly over time with the period of the generating agent, and are obtained as the time average in an interval of 10-20 minutes. The statistical descriptors of the state of the current are the three components of the mean velocity and a mean direction.

Consequently, except in the case of the current caused by the propagation of the astronomical tide, the magnitude and local direction of any other current associated with the propagation of long waves are slow random variables, and should be treated as such.

3.10.4.1.1 MEAN CURRENT INTEGRATED IN THE WATER COLUMN

Generally speaking, it is sufficient to work with (U, V), which are the horizontal components of the velocity field, according to axes x and y, respectively, averaged in the total depth of the water column, $h = h_0 + \eta$. This indicates the direction of the current in relation to the geographic north.

$$U = \frac{1}{h}\int_{-h_0}^{\eta} u dz \qquad V = \frac{1}{h}\int_{-h_0}^{\eta} v dz \tag{3.419}$$

where h_0 is the depth of the still water level, based on a horizontal reference plane; η is the rise in sea level, based on a reference plane due to long waves. (U, V) indicate the direction of the current with respect to the geographic north. $U(h_0 + \eta)$ and $V(h_0 + \eta)$ express the total volume of water per unit of width or water flow through the perpendicular planes of the *x* and *y* axes, respectively. With these values it is possible to calculate the mean direction of the current.

It should be remembered that the current, in module as well as in sense, can vary significantly in areas where there is hydraulic constriction on the flow of the generating agent (e.g. mouths, openings, accesses to wharfs), and that the presence of a structure can substantially modify the pattern of currents.

3.10.4.2 Models of ocean circulation

The prediction of ocean currents can be performed by using the tridimensional model of ocean circulation developed by the Institute für Meereskunde (IFM) in Hamburg and by *Clima Marítimo* (Puertos del Estado, Madrid), known as HAMSON. This model resolves a set of seven differential equations in partial derivatives of the three velocity components, pressure, water density, salinity, and temperature. On the horizontal plane, complete Navier-Stokes equations are used, and on the vertical plane, the hydrostatic equation. These relations are used along with the continuity equation, the salt/temperature conservation equations and the equation of seawater state. The model is able to consider the forces of the wind, atmospheric pressure, tides, and baroclinical gradients.

3.10.4.3 Other phenomena related to long wave propagation

Apart from the previously mentioned phenomena, port and shoreline activities can also be affected by other phenomena related to long wave propagation in general, and to the astronomical tide in particular. These include the transport of natural and contaminating substances in coastal, estuarine, and fluvial regions. In the case of estuaries, tidal inlets, and river ports, the phenomena associated with saline transport, such as the formation of salt wedges or salt fronts with non-uniform distributions in depth, and the changes in the salt distribution, affecting the flora and fauna, can be relevant In such cases, it is advisable to ascertain the conditions in which the floating plume is positive or negative. Also relevant is its relation with the formation of longitudinal and distal bars as well as other morphological structures, due to the interaction of the waves and the water circulation.

3.11 JOINT DESCRIPTION (CLIMATE AGENTS)

Maritime and port structures are constructed to facilitate normal operation of use and exploitation. Without suffering damage or deformation in structure or shape, such structures should be able to withstand extreme and even exceptional work conditions, resulting from the mutual interference of the structure and its immediate environment (ROM 0.0, section 4.5). Accordingly, the structure and its subsets, in all of its phases, should satisfy project requirements, and more specifically, the joint probability of failure modes and operational stoppage modes. The verification methods are procedures to verify that these requirements are satisfied.

To verify maritime and port structures, it is necessary to select the agents that act simultaneously, and select their values, always taking into account their compatibility. The majority of marine and atmospheric climate agents are random processes with the same forcing agents. For this reason, their occurrence, temporal evolution, and intensity at a certain point in the sea are correlated. Consequently, simultaneous climate agents and their compatibility values should be selected on the basis of their shared probability models. Whenever possible, this description should be based on instrument measurements performed at the site. If such measurements are not available, then for the Spanish coastline, data provided by *Puertos del Estado* (www.puertos.es) and the *Instituto Nacional de Meteorología* (www.inm.es) can be used. These data should be applied to the project site by using numerical models that have been properly verified.

3.11.1 Dynamics of climate agents

On the Spanish coastline, climate agents can be classified in terms of the dynamics by which they are generated. Accordingly, they can be atmospheric, seismic, or astronomical. These three types of dynamics are regarded as independent and their joint probability is equal to the product of their marginal probabilities.

3.11.1.1 Atmospheric dynamics

Atmospheric dynamics generates oscillations in the waves and wave group band. It also causes variations in sea levels due to meteo-tsunamis and the meteorological tide. It is necessary to consider at least the following manifestations of atmospheric dynamics: the passage of extratropical cyclones (storms, good water conditions and anticyclones, and local or regional atmospheric disturbances). On the Spanish coastline, the loading cycles of sea states and sea level states (broadly called meteorological states) are associated with the passage of frontal systems and storms. Calms (or operational cycles) are generally linked to good weather conditions and anticyclones.

During each of these manifestations, atmospheric climate agents, such as wind, pressure, barometric pulses and precipitation, and maritime agents, such as waves, wave groups, meteo-tsunamis, and the meteorological tide, are simultaneous agents. As a result, there are compatibility relations between their respective values.

3.11.1.2 Tsunami dynamics

Tsunami dynamics generates the loading cycles of tsunamis, which can be considered statistically independent of other loading cycles, Generally speaking, they are associated with exceptional and sometimes extreme work conditions. At certain locations along the Mediterranean coast, normal work and operating conditions should include the simultaneous occurrence of meteorological calm cycles and tsunamis of reduced magnitude.

3.11.1.3 Astronomical dynamics

Astronomical dynamics generates loading cycles and calm cycles of the astronomical tide. These cycles can be regarded as statistically independent of the other cycles. Loading cycles usually correspond to the monthly sequences of spring tides. The other tides are generally associated with cycles of calm. Both cycles should be considered in normal work and operating conditions as well as extreme work conditions.

3.11.1.4 Simultaneity of loading cycles

Depending on the work conditions, the joint description of climate agents should be based on the possible or probable simultaneities of the following loading cycles:

- *Extreme and normal work and operating conditions*
  Storm, local atmospheric disturbances, and astronomical loading cycles
  Anticyclonic (good weather) and astronomical loading cycles
- *Extreme and exceptional work conditions*
  Tsunamis, astronomical and anticyclonic (good weather) loading cycles
  Tsunamis, astronomical, and storm loading cycles

3.11.2 Shared regimes of loading cycles

In middle latitudes, the year is generally considered as the meteorological pulse of the planet. Therefore, the year is usually the statistical test. In any one year, there can be a random number of each of these loading cycles and calm cycles, related to atmospheric dynamics, seismic dynamics, and a given number of astronomical cycles.

The short-term, medium-term, and long-term statistical description of the cycles can be carried out by following the recommendations for the loading cycles of sea states. Consequently, any of the cycles can be defined by means of state descriptors, such as the threshold values of the beginning of the cycle, the duration of the persistence of the cycle and each of its phases, maximum and minimum values or combinations of these values (e.g. maximum value and duration).

Note. Strictly speaking, it is not necessary to define the cycle in terms of its state descriptors. It can also be done in terms of basic variables (e.g. wave heights). In this case, the threshold value would be a wave height, and the year would be a sequence of wave heights, grouped in loading cycles. This description can be useful to verify maritime structures whose failure modes occur when they are confronted with a wave greater than the calculation value. This is the case of vertical breakwaters, for example.

Although there is a lack of information, the climate data observed over recent years seem to point to the existence of sets or groups of meteorological years in which the intensity of the agents and the number of loading cycles are greater or smaller than in other meteorological groups. The mean duration of these groups is around 7-8 years. Thus, a complete hypercycle lasts around 14-16 years. This means that there should be on the order of 6-7 hypercycles during the century, and 3-4 hypercycles during a useful life of 50 years, or 2-3 hypercycles during a useful life of 25 years.

This evidence is in contrast with the underlying hypothesis in the construction of a sample in meteorological years, which assumes that what occurs in each year is statistically independent of what occurs in the previous or following years.

3.11.2.1 Simultaneity of work conditions

When the work conditions in which the failure is produced is due to the occurrence of a sequence of M state variables associated with one of the three possible loading cycles (atmospheric, seismic, or astronomical), then it is necessary to estimate the joint extreme regime. In these circumstances, three situations of different degrees of complexity can arise.

3.11.2.1.1 Situation I

All variables are associated with the same Poisson process. This is the case of the passage of a storm with simultaneous generation and with storm-dependent variables (i.e. sequence of sea states, sea level associated with the meteorological tide).

3.11.2.1.2 Situation II

Random variables are associated with different Poisson processes, which, generally speaking, are statistically independent (e.g. sequence of sea states and superposed level on the tidal range sequence of the astronomical tide, or the astronomical tide and a tsunami sequence).

3.11.2.1.3 Situation III

Random variables are associated with different Poisson processes, and have different temporal and spatial scales (sequence meteorological, astronomical, and tsunami states).

3.11.2.2 Random variables associated with the same Poisson process

Generally speaking, the extreme work conditions of breakwaters occur during the passage of extratropical storms. For this reason, all climate and soil agents induced by these conditions are associated with the same Poisson process. The distribution function of the maximum is calculated by following the procedure of the maximum peaks in the loading cycle, but conditioned to the sea level (or propagation direction), namely:

$$G_X(x;z) = \sum_{n=0}^{\infty} P_N(n) * \left[F_X(x;z)\right]^n \qquad (3.420)$$

3.11.2.2.1 Peak regime by direction

It is often possible to have the distribution function of the maximum peak of the loading cycle by direction. This means that the distribution parameter depends on each propagation sector. If the density function of the different propagation sectors is known (e.g. $f_\Theta(\theta)$), then the maximum peak function is $F_{Hs,peak}(x;\ \theta)$, which is equal to:

$$F_{H_{s,peak}}(x) = \int_0^\pi F_{H_{s,peak}}(x;\theta) * f_\Theta(\theta)\, d\theta \qquad (3.421)$$

The extreme distribution function for all directions is thus the following:

$$G_{H_{s,max}}(x) = \sum_{n_\theta=0}^{\infty} P_{N_\theta}(n_\theta) * \int_0^\pi \left[F_{H_{s,peak}}(x;\theta) * f_\Theta(\theta)\, d\theta\right]^{n_\theta} \qquad (3.422)$$

Regarding the extreme regime, the same process can be used for all sea levels.

3.11.2.3 Random variables associated with different Poisson processes

If the sequence of M Poisson variables are statistically independent, and each of them is associated to a Poisson process (loading cycle) whose parameter, $v_m,\ m = 1,\ \ldots,\ M,$ then the extreme distribution function is the following:

$$G_{X_1,X_2,\ldots X_m}(x_1,x_2,\ldots x_M) = \prod_{m=1}^{M} e^{-v_m\left[1-F_{X_m}(x_m)\right]} \qquad (3.423)$$

where $F_{Xm}(x_m)$ is the shared distribution function of each one of the random variables X_m, which are the peaks of the different loading cycles.

3.11.2.4 Random variables associated with different Poisson processes and different scales

In these cases, the statistical description of the different variables must be carried out in the same time interval (meteorological year), and with the same kind of values (peak values in the loading cycle). The loading cycles of the astronomical tide, namely, the cycles of the spring tides, whose number in the year (though not their amplitude) is already determined, can be adopted as the loading cycles during the year. The probability of occurrence during the year of the peaks of the two or three loading cycles (meteorological, seismic, and astronomical) is obtained by the product of the probabilities of occurrence of the two or three peaks, obtained from the annual distribution functions of each one.

Note. *This approximation provides a criterion for selecting the compatible values of the three loading cycles, depending on their joint probability of occurrence (simultaneity).*

3.11.2.5 General case

Random variables should be grouped in terms of their association with the different Poisson processes and the relevant formulas given in the two previous sections should be applied.

3.11.3 Joint regimes of seal level and sea states

It is best to work with the exceedance regime conditioned to the sea level in those cases when during a loading cycle (lasting 2-5 days), the sea level can vary by more than 10% of the still water depth, $h_0 + \Delta h$; when there is the possibility of wave breaking; and when at the project site, one or more of the following conditions is satisfied:

- Bottom-induced wave breaking is a relevant process in the magnitude of the wave height at the site.
- The amplitude of the astronomical tide significantly influences wave breaking.
- The relation between the amplitudes of the meteorological tide, η_{MM}, and other processes that influence the sea level and the astronomical tide, η_{MA} is $O(1)$.

If there is sufficient information available, it is advisable to proceed (as in the case of $(H_s, \overline{T}_z)$, on the basis of the marginal and the conditioned functions. Depending on the site, a joint probability model should be obtained in the following way.

The spatial variability of marine dynamics, particularly of the astronomical and meteorological tides along the Spanish peninsular and island coastline, permits certain simplifications in the calculation of joint regimes. Generally speaking, the coastal stretches fit one of the following three situations:

- Situation I: $\eta_{MA} \leq 1\ m$ and $\eta_{MA} \leq \eta_{MM}$ (Alboran Sea, Valencian Oval, Catalonian Sea, and the Balearic Archipelago) (beaches and inland ports in coastal inlets and estuaries);
- Situation II: $\eta_{MA} \geq 3\ m$ and $\eta_{MA} \geq \eta_{MM}$ (Cantabrian Sea, Atlantic Ocean, Gulf of Cadiz, Western Canary Islands);
- Situation III: $1 < \eta_{MA} < 3$ y $\eta_{MA} \approx \eta_{MM}$ (Strait of Gibraltar as well as both sides, Ceuta, Eastern Canary Islands) (beaches and ports in intermediate zones of coastal inlets and estuaries).

3.11.3.1 Situation I

During the passage of the storm, the sea states and the states of the meteorological tide are caused by the same force. Generally speaking, the peaks of the loading cycle are correlated, though they do not have to occur in simultaneous states. To fit the data to the bivariate probability model, $F(H_{s,peak}, \eta_{MM})$, it is advisable to subtract the astronomical tide from the available data series (simultaneous curves of sea states and sea level), and then use this to construct the sample of value pairs $(H_{s,peak}, \eta_{MM})$. When the wind direction is a determining factor in the sea state regimes and the meteorological tide level, it is best to work with the trivariate variable $(H_{s,peak}, \eta_{MM}, \overline{\theta})$.

If the dependence is complete, then, once the value of $H_{s,peak}$ (and its probability of exceedance) is known, that gives the value of η_{MM}, and the joint probability of $(H_{s,peak}, \eta_{MM})$ is equal to that of H_s,

$$f\left(\eta_{MM}, H_{s,peak}\right) = f\left(H_{s,peak}\right) \tag{3.424}$$

If the meteorological tide and the significant wave height are independent (a fairly improbable event though it can occur in confined shallow waters), it should hold that,

$$f\left(\eta_{MM}, H_{s,peak}\right) = f\left(\eta_{MM}\right) * f\left(H_{s,peak}\right) \tag{3.425}$$

The joint regimes of the sea states and the sea level (meteorological and astronomical tides) can be obtained by assuming the statistical independence of the loading cycles of the atmospheric and astronomical dynamics.

$$f\left(\eta_{MM}, H_{s,peak}\right) = f\left(\eta_{MM} \middle| H_{s,peak}\right) * f\left(H_{s,peak}\right) * f\left(\eta_{MA}\right) \tag{3.426}$$

where,

$$\eta_{NM} = \eta_{MA} + \eta_{MM}\left(H_{s,peak}\right) \tag{3.427}$$

The η_{MA} is the astronomical tidal range in its loading cycle, in other words, the spring tides. This is the case if $F(\eta_{MA})$ is the distribution function of the peaks of the tidal range, and $f(\eta_{MA})$ is its density function.

3.11.3.2 Situation II

The variability of the sea level is due to the astronomical tide, and the sea states are basically conditioned by this tide, especially the port areas. If η_{MM} and $H_{s,peak}$ are completely dependent, and if the data series is sufficiently large, it is best to work with time series of homogeneous data, grouping the data regarding

$H_{s,peak}$, according to levels of the astronomical tide: low tide, mid tide, and high tide (with a duration of approximately two hours). If at the site, the current induced by the tide is important $(U_c < 0.5\ m/s)$, this circumstance should be taken into account by including two levels related to mid tide, namely, the flood tide and the ebb tide.

The fit of the probability models provide the sea state regime conditioned to the astronomical tide level. Once the data is fitted to the probability models, the result is the joint regime of the peaks of the sea states and the sea levels induced by the astronomical tide,

$$f\left(\eta_{MA}, H_{s,peak}\right) = f\left(H_{s,peak} \middle| \eta_{MA}\right) * f\left(\eta_{MA}\right) \tag{3.428}$$

When η_{MM} and $H_{s,peak}$ are assumed to be totally independent, it can be accepted that,

$$f\left(\eta_{NM}, H_{s,peak}\right) = f\left(H_{s,peak} \middle| \eta_{MA}\right) * f\left(\eta_{MA}\right) \tag{3.429}$$

where,

$$\eta_{NM} = \eta_{MA} + \eta_{MM}\left(H_{s,peak}\right) \tag{3.430}$$

In shallow water, it is probable that η_{MM} and $H_{s,peak}$ are not perfectly correlated. In such cases, it is necessary to fit a bivariate probability model (η_{MM} y $H_{s,peak}$), conditioned to the astronomical sea level.

3.11.3.3 Situation III

The astronomical and meteorological tides are of the same order of magnitude, but the waves are the predominant agent. If the data series is broad (> 10 *years*), if the duration of the loading cycles includes two or more tide cycles, and if the mean number of loading cycles during the year is ten or more (approximately one or more per month), then it is advisable to proceed in the same way as in the previous case of shallow waters.

This means working with the bivariate variable (η_{MM} y $H_{s,peak}$) or, when relevant, the trivariate variable $(H_{s,peak}, \eta_{MM}, \overline{\theta})$, to do the following:

1. Obtain the conditioned density function, $f(\eta_{MM}, H_{s,peak} \mid \eta_{MA})$ for the different levels of the astronomical tide.
2. Calculate the conditioned density function,

$$f\left(\eta_{NM}, H_s\right) = f\left(\eta_{MM}, H_{s,peak} \middle| \eta_{MA}\right) * f\left(\eta_{MA}\right) \tag{3.431}$$

where,

$$\eta_{NM} = \eta_{MA} + \eta_{MM}\left(H_{s,peak}\right) \tag{3.432}$$

When the sea level is the predominant agent, one should proceed in the same way, but in this case, it is best to work with the bivariate variable $(\eta_{MM,peak}, H_s, \overline{\theta})$.

3.12 VERIFICATION AGAINST CLIMATE AGENTS

According to the ROM 0.0, the joint probability of all the failure modes against safety in the subset of the structure should not exceed the value given in Table 2.2.2. Depending on the nature of the structure, the verification of project requirements should be performed by applying some of the Level I and Level I and Level II or Level III methods.

Generally speaking, the occurrence of the loading cycles of climate agents causes extreme or exceptional work conditions. They are thus predominant agents that induce failure modes against the safety of the structure. The structure itself is usually operational during calm cycles of climate agents. Under such circumstances, threshold values of operationality limit the values of the combination of agents and actions in the verification of failure modes with other predominant agents. In the first case, the values of the predominant agents belong to the class of extreme or maximum values, whereas in the second case, they belong to the class of medium or minimum values.

3.12.1 Levels II and III

In the Level II and III verifications, joint probability models are used in which all loading cycles are taken into account. For each state, it is necessary to calculate whether failure modes occur whose predominant agents are climate agents. This process is effectively performed by applying numerical simulation techniques, such as the Monte Carlo method (Chapters 6 and 7 of the ROM 0.0).

3.12.1.1 Monte Carlo simulation

When failure modes belong to ultimate limit states, it is enough to know whether the failure mode occurs in each of the loading cycles of the useful life of the structure. The Monte Carlo method can be used to simulate a high number of useful lives as sequences of loading cycles and cycles of atmospheric, seismic, and astronomical calms (which, in turn, are formed by sequences of states), whose descriptors satisfy the joint probability models. The failure modes of the subset are then verified against each of the simulated useful lives. The procedure to be followed depends on whether the failure modes are assigned to the serviceability or ultimate limit states.

3.12.1.2 Ultimate limit states

After repeating the experiment (useful life) many times, the frequency of occurrence of the set of failure modes in the useful life of a structure is an approximation of the joint failure mode. The number of failures is counted in terms of loading cycles, always assuming that once a failure mode occurs, which is assigned to an ultimate limit state, the structure is irreparably damaged and cannot be recovered. In this case, an upper limit of the joint probability of failure is,

$$f[S_1] + f[S_2] + \ldots f[S_n] < p_{f,V} \qquad (3.433)$$

where $f[S_i]$ is the mean frequency of occurrence of failure mode i, calculated as follows:

$$f[S_i] = \frac{1}{M}\sum_{j=1}^{M} \frac{\text{number of loading cycles that induce failure}}{\text{number of loading cycles of the useful life}} \qquad (3.434)$$

and M is the number of simulated repetitions of the experiment (useful lives).

3.12.1.3 Ultimate and serviceability limit states

The same work method can be applied to obtain a sample of the multivariate variable for the safety margin of each of the n principal failure modes $S[S_1, S_2 \ldots, S_n]$. By fitting a multivariate probability model, the verification of the joint probability of failure is calculated as follows:

$$\Pr\left[S\left[S_1 < 0, S_2 < 0, \ldots S_n < 0\right]\right] < p_{f,V} \quad (3.435)$$

This solution is more precise than the previous one. The application of this technique to serviceability limit states requires a knowledge of the performance of the structure as it fails.

3.12.1.4 Operational limit states

This same procedure can be used to calculate the number of times that the structure stops being operational as well as the time that passes before it becomes operational again, in other words, the duration of the stoppage. These data can be used to estimate the structure's operationality, the mean number of operational stoppages in the year, and the maximum duration of the operational stoppage.

3.12.2 Level I

The three cycles of sea oscillations are statistically independent though not mutually exclusive. For this reason, in general, the three can occur simultaneously, and with any of the normal, extreme, and exceptional work conditions. By applying Level I methods, the requirement of the joint probability of failure against climate agents can only be approximately verified.

Note. *Chapter 3 of the ROM 2.0-08 gives an example of the procedure that is recommended in the following section to estimate the joint failure probability of docks and inner port structures.*

3.12.2.1 Verification of normal work and operating: WOC_1

In such work conditions, climate agents are not generally predominant. For this reason, climate agent values can be determined on the basis of the regimes of their respective cycles of calms (taking into the account, when relevant, the correlation between agents). These values will be related to the use and exploitation thresholds of the structure. It is necessary to bear in mind that each of the agents by itself can induce the stoppage mode.

3.12.2.2 Verification of extreme and exceptional work and operating conditions: WOC_2, WOC_3

In the verification of extreme and exceptional work conditions due to the occurrence of climate agents in the physical environment, it is necessary to apply the following criteria of simultaneity, compatibility, and of the combination of agents (actions).

3.12.2.3 Simultaneous and predominant cycles in the failure mode

Accordingly, for each failure mode, it must be determined which of the atmospheric cycles *(B)*, tsunamis *(M)*, and astronomical cycles *(A)* are simultaneous, and which can be predominant in the occurrence of the failure mode.

The complete collection of loading cycles is the following:

$$\Psi = \left\{B, M, A, BM, BA, MA, BMA, B^c M^c A^c\right\} \quad (3.436)$$

Since cycles that cannot occur simultaneously are excluded, it is necessary to analyze if simultaneity is physically possible at the site.

3.12.2.4 Compatible values of the agents of simultaneous cycles

For each set of simultaneous cycles, compatible values of the different agents are determined, taking into account their origin and dependence, and the probability assigned to the failure mode under consideration. In the most general case, the failure can be induced by one of the following contexts:

- **a.** the occurrence of an agent of an atmospheric loading cycle *(B)*; of a tsunami loading cycle *(M)*; or of an astronomic tide loading cycle;
- **b.** the occurrence of the agents of an atmospheric loading cycle and a tsunami loading cycle; of an atmospheric loading cycle and an astronomical tide loading cycle; or of a tsunami loading cycle and an astronomic loading cycle;
- **c.** the simultaneous occurrence of the agents of the atmospheric, tsunami, and astronomical tide loading cycles.

The probability that a failure will occur, at least once, because of one of the these cycle combinations is:

$$p_{f,\text{mode},1} = \left[p_{B,1} + p_{M,1} + p_{A,1}\right] - \left[p_{B,1} * p_{M,1} + p_{B,1} * p_{A,1} + p_{M,1} * p_{A,1}\right] + \left[p_{B,1} * p_{M,1} * p_{A,1}\right] \qquad (3.437)$$

where all terms are calculated, based on extreme regimes (regimes of maximums),

cycle of atmospheric dynamics	$\Pr[B] = G_{X_B}(x_B) = e^{-\nu_B\left[1-F_{X_B}(x_B)\right]}$
cycle of tsunami dynamics	$\Pr[M] = G_{X_M}(x_M) = e^{-\nu_M\left[1-F_{X_M}(x_M)\right]}$
cycle of astronomic tide dynamics	$\Pr[A] = G_{X_A}(x_A) = e^{-\nu_A\left[1-F_{X_A}(x_A)\right]}$

When in the loading cycle (as is the case of the atmospheric cycle) there is more than one agent with the same origin, $(X_1, X_2, \ldots X_m)$, generally, $\nu_m = \nu$ is equal for all agents. For this reason,

$$\Pr[B] = G_{X_1,X_2,\ldots X_m}(x_1, x_2, \ldots x_M) = \prod_{m=1}^{M} e^{-\nu_m}\left[1 - F_{X_m}(x_m)\right] \qquad (3.438)$$

3.12.2.5 Contribution of each possibility

The contribution of case (a), with its sign to the probability of failure of context (a),

$$p_{B,1} + p_{M,1} + p_{A,1} \qquad (3.439)$$

occurs when one or various agents of one cycle (*(B)*, *(M)*, or *(A)*) are predominant in causing the failure. The value of the agents is determined by analyzing the predominant loading cycle, based on its extreme distribution (distribution of maximum values). The values of the agents of the other two cycles should also be compatible with these values, and belong to the corresponding calm cycle, such that they do not contribute to the probability of failure (e.g. a frequent value (medium regime)). The contribution of context (a) to the failure mode probability is on the order of 10^{-1} o or less.

The contribution of context (b), with its sign, to the probability of failure,

$$p_{B,1} * p_{M,1} + p_{B,1} * p_{A,1} + p_{M,1} * p_{A,1} \qquad (3.440)$$

occurs when one or various agents of two loading cycles act simultaneously to induce the failure mode ((B) and *(M)*, or *(B)* and *(A)*, or *(M)* and *(A)*). The values of the agents are selected from their respective extreme or maximum regimes so that they fulfill the equation. Generally, two cases are considered. In each, the value of one of the agents has a small probability of occurrence, whereas the other has a greater probability of occurrence. The agent or agents of the non-simultaneous cycle have a value that is compatible with the two previous ones, and which belongs to the calm cycle, such that it does not contribute to the probability of failure (e.g. a frequent value (medium regime)).

The contribution of (b) to the probability of the failure mode is on the order of 10^{-2} or less.

The contribution of context (c), with its sign, to the probability of failure,

$$p_{B,1} * p_{M,1} * p_{A,1} \tag{3.441}$$

occurs when the three loading cycles act simultaneously to induce the failure. All of the agent values belong to the extreme or maximum regime. The values of the agents are selected to satisfy the condition of the probability of failure. In most cases, this condition is satisfied with values near to the threshold value of the cycle and its probability of exceedance (of extreme regimes). In other words, it is satisfied with values, whose probability of no-exceedance lies in the interval [0.2-0.5].

The contribution of case (c) to the probability of the failure mode is on the order of 10^{-3} or less.

An upper limit of the probability of the failure mode because of the occurrence of at least one of the loading cycles or of all three simultaneous is the following:

$$p_{\sup,\text{mode},1} > \left[p_{A,1} + p_{S,1} + p_{M,1}\right] + p_{B,1} * p_{M,1} * p_{A,1} \tag{3.442}$$

For subsets of the structure whose nature is such that the joint probability of failure for all the failure modes,

$$p_{f,V} \geq 10^{-2}$$

the values of the agents of each of the cycles can be selected by verifying that they satisfy the condition,

$$p_{f,\text{mode},1} > p_{B,1} + p_{M,1} + p_{A,1} \tag{3.443}$$

When $\Pr[M] = p_{m,1} < 10^{-4}$, the tsunami loading cycle will not be considered when calculating $p_{f,V}$.

3.12.2.6 Compatible values based on loading cycle regimes

The calculation of the probabilities should be carried out, according to one of the two following possibilities.

- Once the probability assigned to each failure mode is distributed among whatever cycles can be predominant in producing the failure, it is necessary to obtain the value of the peak or maximum value of predominant agent in the failure mode.
- The simultaneous and compatible values in relation to other agents of the cycle (if any such agents exist), and with the same origin as the predominant agent, are determined by applying the parametric functions of adimensional mononomials.
- The compatible values of the agents of other cycles are calculated based on medium regimes (calm cycles) or extreme regimes (loading cycles), depending on whether they are simultaneous without being predominant, or whether they are simultaneous and predominant, respectively. If one of the cycles is not simultaneous with the predominant agent, it should not be considered in this calculation.

Note. *The calculation of joint probability with Level I methods has two important constraints. The first is related to the compatibility of the values since on Spanish coasts, the occurrence of the loading cycle associated with the passage of extratropical storms produces peaks (maximum values) in all climate agents, wind, sea, and sea level, which are not necessarily simultaneous. There is no reason for the peak value of the maximum sea state to coincide with the peak value of the sea level. The second constraint resides in the approximation of the failure probability by means of the probability of exceedance of the maximum value of the predominant agent, namely, of the maximum*

state in the cycle. This calculation ignores the contribution of other loading cycle states to the failure probability, which, without being maximum states, can have waves or other oscillations capable of causing the failure.
This constraint is relevant in the case of rigid structures, such as vertical breakwaters, that can fail because of the arrival of a wave that is equal to or greater than the design wave.

3.12.2.7 Probability of exceedance and return period

For those subsets of the structure whose joint probability of failure for all failure modes is,

$$p_{f,V} \geq 10^{-2}$$

and when values for the agents in each cycle have been selected in accordance with verification context (a), it is possible to obtain an equivalent return period,

$$T_{R,e} \approx \frac{1}{p_{B,1}} + \frac{1}{p_{M,1}} + \frac{1}{p_{A,1}} \tag{3.444}$$

which is a rough confidence indicator of the structure's resistance to climate agents, though it does not provide any information regarding the reliability of the structure against other agents.

Probability values can be obtained from the extreme regime p_e. The return period, T_R, is the mean number of years (or tests), which should pass before that value is exceeded for the first time, and consequently,

$$T_R = \frac{1}{p_e} \tag{3.445}$$

If p_p is the probability of exceedance of a peak value in a year, and v. Is the mean number of annual peaks, that gives,

$$p_e = 1 - e^{-vp_p} \approx vp_p$$
$$T_R \approx \frac{1}{vp_p} \tag{3.446}$$

This equation can be used to calculate the return period of an annual maximum value based on the distribution function of the loading cycle peaks $F_{X_{peak}}(x)$ and of the mean number of peaks per year v.

3.13 GEOMETRIC FACTORS

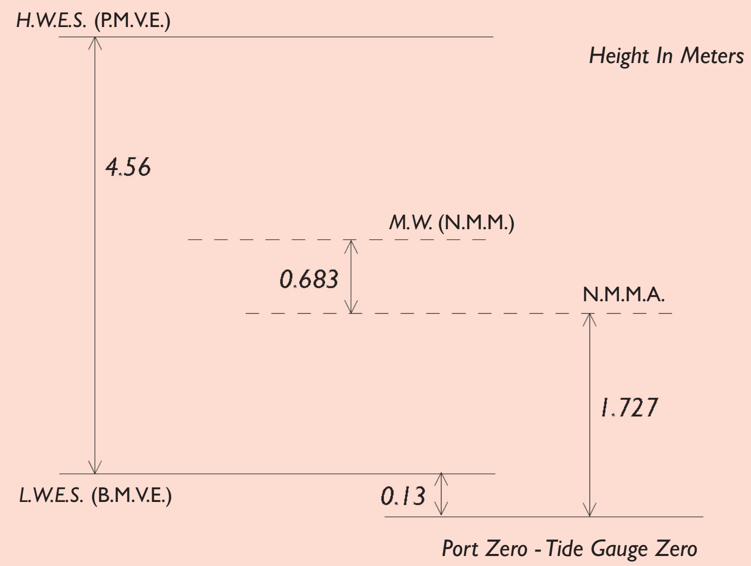
Geometric factors define the shape and dimensions of the structural elements of the construction and of the terrain, as well as its location. Accordingly, it is necessary to specify the land and maritime reference levels and their evolution over time.

3.13.1 Reference levels

The water depth or elevation of a point on land is described with respect to a level of reference. Port and maritime projects usually have two levels of reference: (i) terrestrial level of reference (TLR or TRL); (ii) a maritime level of reference (MLR or SRL). The terrestrial level refers to land elevations, whereas the maritime level of reference refers to water depth. In Spain the topographic level of reference is the mean sea level in Alicante, *NMMA* (see Figure 3.3.36).

Note. The Instituto Geografico Nacional *(IGN) [National Geographic Institute of Spain] uses the origin of altitudes, which in Spain corresponds to the Mean Sea Level in Alicante* (NMMA) *in 1870-1880. The geometric altitudes of the geodesic signals distributed throughout Spain all use it as a reference, and it is thus the national terrestrial reference. In the island territories, the IGN generally uses the mean local sea level as zero.*

Figure 3.3.36. **Maritime reference level (Port Zero) and land (NMMA) In the Port of Bilbao**



3.13.1.1 Temporal and spatial variability of reference levels

The reference levels can be assumed not to suffer modification during the project phases, particularly during the useful life of the projects, not for natural, climate, or geomorphologic causes. This also includes human-induced actions. It is advisable to periodically verify the validity of this work hypothesis. Otherwise, it is necessary to analyze the influence which the observed variations have on project requirements and proceed accordingly.

3.13.1.1.1 Long-duration variability of the NMMA

Also to be taken into account is the fact that the *NMMA* slowly varies for global causes related to climate variability and for specific causes associated with the behavior of the Mediterranean Sea. The mean sea level, *MW* (or NMM), at each point of the coastline is different and differs from the *MW* (or NMM), in Alicante.

Note. *The most recent data available indicates that in this century, the increase in the sea level due to climate variability may be 0.50-0.80 m higher than the current mean sea level (NMMA). This increase may affect the project designs of certain maritime structures whose minimum useful life is 50 years.*

3.13.1.1.2 NRM, NRT and spatial variability

Generally speaking, the *NRM* is the BMVE [29] though certain ports have their own port zero or local zero. The water depth beneath the LWES *(or BMMVE)* is referred to in these Recommendations as h_0, and the difference between local zero and the LWES *(or BMMVE)* as Δh_0.

According to its definition, the SRL or reference sea level, *NRM,* is a local attribute since it depends on the local conditions of the astronomical tide, and if $\Delta h_0 \neq 0$, it depends on the Port Authority. The *NRM* at one site should not be applied to another without previous verification.

(29) The Spanish acronym *BMMVE* refers to the water surface at the extreme low water of spring tides and generally corresponds to the mean value of the lowest low tides, measured at a certain location (LWES).

3.13.1.2 Project information

All maritime structure project designs should have a TRL or reference topography level, *NRT*, another SRL or reference sea level, *NRM*, and the deviation between them. It is advisable to determine the MW or mean local sea level (at the site), *NMML,* in regards to the *NRM* and the mean sea level in Alicante *NMMA*, which generally coincides with the reference topography level, *NRT*, at the site (see Figure 3.3.35).

In order to avoid confusion, each construction project plan should show the level at which the *NRT* or *NRM* elevations are referenced, the relation between both levels, the mean local sea level and the mean sea level at Alicante. It is advisable for maritime project plans to have the *NRM* as their reference.

Note. Reference levels should be perfectly defined in all the documents of the project, plans, memory, use and exploitation guides, etc. The possible errors in the establishment of those levels can have serious consequences for the construction of the structure and the use and exploitation of the harbor area. They can thus have serious economic repercussions.

3.13.1.3 Water depth in the state

Water depth $h(\vec{x}; t)$ is the vertical distance from the surface of the water to the plane tangent to the seabottom at the point. $h(\vec{x}; t)$ evolves over time, forced by the movements of the free sea surface, associated with the three period bands: short, medium, and long. The water depth in the state can be categorized as follows:

3.13.1.3.1 Water depth without sea oscillations

The water depth is the vertical distance, h, between the *NRM* and the plane tangent to the seabottom. This depth does not depend on the time (the effect of climate variability is negligible),

$$h(\vec{x}) = h_0(\vec{x}) + \Delta h_0 \qquad (3.447)$$

Except for when the contrary is indicated, $h(\vec{x})$ is the sounding shown on the nautical charts of the IHM [30].

3.13.1.3.2 Sea depth with long-period oscillations

Sea depth is composed of the depth $h(\vec{x}; t)$, measured with respect to the reference sea level, *NRM*, as well as the contribution of long waves. Depth evolves slowly in time. In a generic sea level state (identified by i), the contributions of the astronomical and meteorological tides are supposed to be constant and equal to their mean value in the state,

$$\begin{aligned} h_i(\vec{x}) &= h_{0,i}(\vec{x}) + \Delta h + \tilde{\eta}_{PL,i}(\vec{x}) \\ \tilde{\eta}_{PL,i}(\vec{x}) &= \eta_{MA,i}(\vec{x}) + \eta_{MM,i}(\vec{x}) \end{aligned} \qquad (3.448)$$

where,

$h_i(\vec{x})$ = water depth in the state i
$\tilde{\eta}_{PL,i}(\vec{x})$ = mean vertical displacement due to long-period oscillations.
$\eta_{MA,i}(\vec{x})$ = mean vertical displacement in the state due to the meteorological tide.
$\eta_{MM,i}(\vec{x})$ = mean vertical displacement in the state due to the astronomical tide.
$\Delta t_d = t_{di+1} - t_{d_i}$ = duration of the state.

(30) IHM stands for the *Instituto Hidrográfico de la Marina* in Spain.

This is the water depth that should be considered to study wave transformation, groups of waves, and when relevant, medium oscillations.

3.13.1.3.3 Water depth in the presence of climate agents

The water depth, $h_i(\vec{x})$, in the presence of climate agents, measured with respect to NRM can be estimated by the linear superposition of the mean quantities associated with short, medium, and long-period sea oscillations.

$$h_i(\vec{x}) = h_{0,i}(\vec{x}) + \Delta h_0 + \tilde{\eta}_{NMm,i}(\vec{x}) \tag{3.449}$$

where,

$$\tilde{\eta}_{NMm,i}(\vec{x}) = \tilde{\eta}_{BLW,i}(\vec{x}) + \tilde{\eta}_{PI,i}(\vec{x}) + \eta_{MA,i}(\vec{x}) + \tilde{\eta}_{MM,i}(\vec{x}) \tag{3.450}$$

h_{0i} = still water depth in reference to the LWES or port zero in state i.
Δh_0 = deviation (with its sign) from port zero with respect to the LWES.
$\tilde{\eta}_{BLW,i}(\vec{x})$ = mean sea level displacement because of the oscillation linked to the group.
$\tilde{\eta}_{PI,i}(\vec{x})$ = mean sea level displacement because of medium-period oscillations.

The water depth h_{0i} can vary from state to state because of erosion processes as well as sediment transport and accumulation. In each project phases, the values of the contributions to , are determined, depending on work conditions, loading cycles, combination type, and compatibility of factors and terms, as described in the ROM 0.0, section 4.9 and following sections.

3.13.2 Topography and bathymetry

In order to implement any of the project phases, it is essential to have information pertaining to the bathymetric and topographic surveys of the land and sea areas affected by the construction project. Cartographic land data is published by the *Instituto Geológico y Minero de España (www.igm.es)*. The cartographic database pertaining to the Spanish coastline is published by the *Instituto Hidrográfico de la Marina (www.ihm.es)*. This information can be useful for pre-project planning and design, as well as for initial dimensioning trials.

3.13.2.1 Topographic and bathymetric survey

For the project draft and the actual project design, it is advisable to carry out specific topographic and bathymetric surveys, depending on the representative wave periods and directions. This information should be associated with the characterization of the soil and the nature of the seabottom, and an estimate of the sources of error and their magnitude should also be performed. These include the expected deviations due to the curvature method of the bathymetric contour lines.

3.13.2.2 Extension of the bathymetric survey

The extension of the survey on the ground plan should be defined according to the objectives, extension to the port area and sub-areas (e.g. navigation channel, breakwater alignment, etc.) and the possible interaction of the construction work with the coastal morphodynamics and the quality of the water.

Generally speaking, the bathymetric survey is performed in two stages. The first stage is based on previous bathymetric data, such as that provided by IHM nautical charts. Depending on the nature of the construction project, the following five tasks must be performed, (though when sufficient information is not available, a preliminary survey should be conducted with sufficient quality guarantees):

1. Identification of the shape and nature of the seabed by applying the methods described in section 3.2.2.5 and the following sections.
2. Specification of the water depth in each direction at which the waves begin to experience bottom friction. This occurs when $h/L_0 < 1/2$, where $L_0 = gT^2/2\pi \simeq 1{,}56T^2$ is the wavelength in *(m/s)* in deep water, and *T(s)* is the largest representative period of the sea states in the direction considered.
3. Identification of the seabottom changes that can affect the propagation of water waves as well as other oscillations. This means applying the criterion that any change in water depth $\Delta h/L$ which has horizontal dimensions (width and relative length), $[b_x/L \geq 0.1]$ ó $[b_y/L \geq 0.1]$ or , and which modifies the length of local wave L by at least 5% affects the characteristics of the waves (wave height and direction). For this calculation, b_x, b_y are the dimensions of the obstacle in the wave propagation direction and the dimensions along the line of the wave crest, respectively. The bathymetric information should be detailed enough to quantify the effects produced on the wave action.
4. Specification of the water depth when waves begin to break. This depth is assumed to be approximately equal to the height of the largest waves expected to occur during the useful life of the structure. The bathymetry should be sufficiently detailed in the area where the waves undergo their final evolution before breaking. This zone covers 2-3 wavelengths towards the open sea from the breaking point.
5. Identification of the lateral extension of the bathymetry so that it is possible to quantify the transformation processes of waves coming from any of the representative directions at the project site.

When the available bathymetric data are not sufficiently detailed, the second step will begin with a new survey, whose specifications will be drafted, based on the previous step.

3.13.2.2.1 Recommended reductions

In the absence of more precise information or as long as the variation of the seabed is gradual with bathymetric contour lines that are parallel (e.g. sandy or muddy bottoms, see following section), the bathymetric survey should be extended at least in the following directions:

1. towards the sea, approximately five times the maximum depth of the structure;
2. laterally, between one to two wavelengths on each side of the coastal defense structures. (In this case, the wavelength corresponds to that of the maximum depth of the structure.)

3.13.2.2.2 Bathymetric survey scales

The survey scale should permit the study of wave propagation (or other sea oscillations) with grids large enough for a wavelength to be described by least eight points.

Note. *Currently, high-resolution digital models (obtained, for example by multibeam sounding) are available, which represent and allow the user to perceive the slopes and morphology of the seabed.*

3.13.3 Nature and variability of the seabed

Before beginning the project, it is a good idea to compile and analyze the available information pertaining to the nature of the seabed at the site location, the geomorphology, and the geological structure.

3.13.3.1 Information in the nautical charts of the IHM

The nautical charts represent the nature of the seabed surface when the survey was conducted for the chart. It is advisable to observe the distribution of the indications of the nature of the seabed. This means examining whether it is uniform or heterogeneous with alternating zones of sand, mud, rocks, etc. It also entails the identification of possible source of loose, unconsolidated material, cliffs, rivers, estuaries, etc.

Generally speaking, heterogeneity is indicative of minimal thicknesses of loose material. Even if the chart does not provide information regarding its thickness, jointly analyzed with the geological structure and the slope of the terrain, it is possible to have an initial idea of its carrying capacity, deformability, and erosionability. Moreover, with the help of a geological map and of the dip of geological structures, the thickness of the sediment mantle can be approximately dimensioned.

Accordingly, the continental shelf is exposed to the wave action as well as other oscillations, and the seabed slope can provide initial information regarding its nature (see Table 3.3.27).

Table 3.3.27. ***Most probable nature of the seabottom, depending on its slope***

Soil type	Seabed slope
Rock	Any
Gravel	$1/30 < tan\beta < 1/10$
Coarse sand	$1/60 < tan\beta < 1/30$
Fine sand	$1/120 < tan\beta < 1/60$
Sand - Mud	$1/600 < tan\beta < 1/120$
Mud	$tan\beta > 1/600$

For gravel or fine sand soils, the larger the magnitude or intensity of marine dynamics (mainly wave action), the more gentle the slope of the seabottom will be. When it is less than $1/120$, the mud will probably begin to appear in its composition, or there will be no loose material since the seabottom is the result of the erosion of the rocky bed.

3.13.3.2 Bathymetry and nature of the seabed

The shape of the bathymetric lines also provides an indication of the nature of the sea bed. If contour lines are very irregular and show brusque changes in the seabed, it is probable that the bottom is rocky. In contrast, when contour lines are regular, there is a greater probability that the seabed is made of sand, sandy mud or mud. The muddiest beds are thus those that have the most regular bathymetry. In any case, when this analysis is performed, the curvature method as well as the number and density of the points used in the design of the bathymetric chart are important factors. In areas near river mouths where there is a deltaic fan, the seabottom slope is defined by its morphological structure, dimensions, and composition.

Note. Currently, high-resolution digital models (obtained, for example by multibeam sounding) are available, which represent and allow the user to perceive the slopes and morphology of the seabed.

3.13.3.3 Processes and spatiotemporal evolution of the seabed

Sea oscillations and currents generate surface forces, pressures and tangential stresses, as well as mass forces on the seabed and soil particles. These forces can cause erosion, transport, and sedimentation of the cohesive and non-cohesive soil particles. Generally speaking, this action becomes more important as the relative depth and water velocity becomes less important. For this reason, it should be underlined that bathymetric surveys are only representative of the instant in which they were carried out, and that the bathymetry and composition of the seabed can change spatially and temporally with the climate agents.

3.13.3.3.1 EVOLUTION OF THE SEABED IN THE PRESENCE OF CONSTRUCTION WORKS

The construction of breakwaters substantially modifies the field of oscillations and currents downdrift and updrift from them. It also modifies the mass and surface forces, thus changing the erosion-transport-deposit pattern in the environment of the construction. These effects are especially relevant because of their negative impact on the stability at the toe of the structure, around the changes in the alignment and head of the breakwater as well as in the navigation channel.

Apart from the environmental effects, special attention should be paid to those environments in which vegetation is an important factor in the magnitude of incident wave action, and where the construction project can modify its extension an development. In such cases, in addition to the environmental studies, it is also necessary to evaluate alterations in the vegetation and their implications for marine dynamics and the stability of the seabed.

3.13.3.4 Processes and spatiotemporal evolution of coastal morphology

Furthermore, the construction of breakwaters substantially modifies the oscillation field in its immediate surroundings since it alters slopes, and as a result, the coastal circulation system. Consequently, it should be remembered that construction work also affects coastal circulation and the surf zone. It can also significantly alter coastal morphology, and for this reason, bathymetric surveys are only representative of the instant when they were performed.

All project designs of maritime structures should take these circumstances into account. The specific sections recommend that studies be carried out to analyze and verify seabed behavior and coastal morphology in the presence of breakwaters.

Chapter IV

Annex I.
Foundations and justification

Table of contents

CHAPTER IV. ANNEX I. FOUNDATIONS AND JUSTIFICATION

4.1 PROCESSES AND SCALES

The spatial and temporal variability of agents and their actions, and generally speaking, their randomness determine the spatial and temporal scales of studies and verifications as well as their uncertainty. For this reason, in Chapter 3 of the ROM 1.1, project factors are quantified at three temporal levels: short-term, medium-term, and long-term. On the short-term scale, instantaneous and basic manifestations are considered, as well as state manifestations of agents and actions. The medium-term scale includes manifestations that take place in the loading cycle and operationality cycle. Finally, the long-term temporal scale considers the annual and pluri-annual variability as well as the useful life of the structure.

4.1.1 Basic and instantaneous variables

Once a project site location is selected, the temporal description of the project factors can be based either on the continuous evolution of their values or on their cyclical evolution if they reflect this type of behavior. In the first case, such variables are known as instantaneous variables, and in the second, basic variables.

4.1.1.1 Instantaneous variables

Instantaneous variables describe instantaneous movements of the fluid, sailing vessel, particles, etc. They can be cinematic (e.g. velocity and acceleration of the fluid particle); dynamic (pressures and tangential stresses on the surface of the particle and forces per unit of volume).

The term, *instantaneous,* should be understood in its mathematical sense as the continuous sequence of the values at a point, which can be taken by a project factor, and which define a continuous curve (surface, volume, etc.). In the context of the physical environment, due to instrument limitations and data management, it is not possible to measure instantaneous values. For this reason, an instantaneous variable is characterized by discrete measurements in a time interval, which can be very small, but is still a time interval Δt. When the boundary conditions of the fluid medium determine the existence of a free surface, the instantaneous vertical displacement with respect to a fixed horizontal reference level can be regarded as an instantaneous variable, and thus, as the principal manifestation of fluid dynamics.

4.1.1.2 Basic variables

Marine and atmospheric climate agents are generally oscillatory and cyclical. The time period from when the instantaneous variable begins to oscillate until it stops oscillating and returns approximately to its initial situation defines the temporal variability rate and is known as the period of the cycle. Basic variables are those that characterize the agent or the action in a cycle, more specifically, the period (and the length) and the amplitude.

The astronomical tide can be quantified by the amplitudes of the high tide and low tide, measured with respect to a reference level, which is usually the low water equinoctial spring level. The tidal period is the duration of the tidal cycle and tidal range or amplitude (1), which is the vertical distance between the high tide and its subsequent low tide.

Note. *The mean sea level is generally used as the reference level for waves. The time that passes between two upwards crossings in respect to this reference level is the duration of the cycle or upwards zero-crossing period. Basic variables are the crest amplitude and the maximum value of the vertical displacement of the free surface with respect to the mean level, trough amplitude, or the maximum downwards movement in the vertical displacement of the free surface with respect to the mean sea level and the wave height, defined by the maximum vertical distance in the cycle. The wave height in itself is not sufficient since it has no associated*

(1) In reality, it is not an amplitude, but rather a tidal wave height: In the same way as waves, it had no location in regards to a general reference level.

reference level. One must assume that either the horizontal axis of the crest and trough is symmetrical, or adopt a theoretical wave shape to distribute that height with respect to a reference level.

4.1.2 State

Variables that define a random process evolve over time, and thus, are really non-stationary processes. In order to apply the theory of stationary processes, it is necessary to divide the manifestations of the process (e.g. the vertical displacement of the free sea surface in the wave band, or the wind speed measured at a height of ten meters) $\zeta(\vec{x},t)$, into smaller time segments of 30 minutes and one hour during which the energy of the system is conserved. In this supposition, it is advisable to decompose the signal into the sum of its mean value in the state and the deviation on this value, in other words,

$$\zeta_{state}(\vec{x},t) = \overline{\zeta_{state}(\vec{x},t)} + \eta_{desviation}(\vec{x},t) \quad (4.1)$$

$$\overline{\zeta_{state}(\vec{x})} = \frac{1}{\Delta t_d}\int_{t_{d_i}}^{t_{d_{i+1}}} \zeta_{state}(\vec{x},t)\,dt \quad (4.2)$$

where $\Delta t_d = t_{di+1} - t_{di}$ is the duration of the state (approximately one hour). The state is a manifestation of the agent during a bounded time interval, described by its basic and instantaneous variables, both fast and slow. In this state, it can be admitted that: (1) the agent is an ergodic, stationary, random process; (2) the basic variables that quantify the manifestation of the process are random variables; (3) a temporal sample of these variables represents the process at the point and in a time interval; (4) its temporal variability is described by probability models whose statistical parameters are state descriptors; (5) the values of these variables, within the hypotheses of statistical stationariness, can be said to be constant in the state; (6) the manifestations of other agents are different scales are bounded, and do not significantly influence the randomness of the agent. Similarly, a homogeneous random process can be defined in the spatial domain, in such a way that a spatial sample of random variables taken in a time interval represents the process in the domain and in that time instant.

4.2 LINEAR THEORY OF GRAVITY WAVES

From the perspective of its mathematical description, a monochromatic wave train is defined as any oscillation of the free sea surface with period T and height H. Generally speaking, the mean energy in a cycle per surface unit, associated with a wave train is proportional to H^2, and the mean energy flux in a section perpendicular to the propagation direction is proportional to the H^2C_g, where C_g is the energy propagation velocity. The waves have different origins, and once the movement has begun, mainly depending on their period, they have different restoring mechanisms, which are opposed to the persistence of the movement. Table 4.4.1 shows the main sea level oscillations, their principal generation causes, and restoration mechanisms.

Table 4.4.1. Sea oscillations, generation causes and restoration processes

Name	Period, T	Length, L	Height, H	Generating force	Restoring force
Capillary	0 – 0.1 s	2 – 7 cm	1 – 2 mm	Wind	Surface stress
Ultragravity	0.1 s – 1 s	Centimeters	Millimeters	Wind	Surface stress and gravity
Gravity	1 s – 30 s	Meters to hundred of meters	Centimeters – 15 m	Wind	Gravity
Infragravity	30 s – 5 min	100 – 200 m	Small	Wind	Gravity, Coriolis
Long-period	5 min – 24 h	–	1 – 5 m	Aerthquake, landslides, gravitational attraction between hevenly bodies	Gravity, Coriolis
Transtidal	more than 24 h	–	–	Attraction between hevenly bodies	Gravity, Coriolis

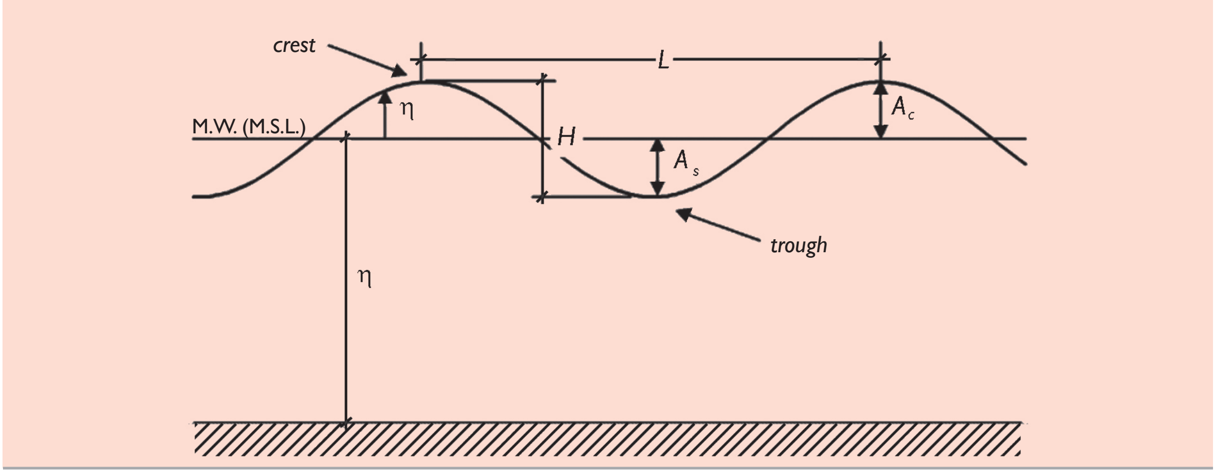
The observation of such oscillations points to the fact that the sea wave band has the greatest energy content. Furthermore, there are all those oscillations that significantly contribute to the local water depth, which oscillate with a larger period than that of the waves, and which can be assumed constant in a sea state. Such oscillations include the astronomical and meteorological tides.

This chapter is an introduction to the physical and mathematical foundations of the oscillatory motion of the free sea surface. It describes its principal cinematic and dynamic properties, and analyzes its range of mathematical validity and physical application.

4.2.1 Wave train description

Unlike sea waves, a monochromatic wave train is an oscillation that repeats itself in time and space. Basic characteristics of a wave are its length, L, height, H, and depth, h, in which it propagates (see Figure 4.4.1).

Figure 4.4.1. ***Wave magnitudes***



The wave profile is characterized by the following magnitudes:

η = vertical displacement of the free surface.
A_c = crest amplitude, maximum positive vertical displacement with respect to the mean sea level (MW).
A_t = trough amplitude, maximum negative vertical displacement with respect to the MW or mean sea level (M.S.L.).
H = wave height, maximum vertical displacement between two upwards crossings of the wave in respect to the M.S.L. If the wave is symmetrical, $A_c = A_t$ and $H = A_c + A_t = 2A$.
L = wavelength, horizontal distance between two contiguous crests or troughs.
T = period, time duration between the crossing of two contiguous crests or troughs through a fixed point.

These magnitudes can be used to define the following ones:

k = wave number, $k = 2\pi/L$.
σ = angular frequency, $\sigma = 2\pi/T$.

When the movement of the wave train is observed from a fixed point, the period T is the time that passes until the same position of the train (e.g. the crest) is observed on the free surface. In a photograph of the water surface, the wavelength L is the minimum horizontal distance between two consecutive points at the same position on the free sea surface.

When the profile propagates along a horizontal sea bed and travels a distance $L(m)$, wave length in time $T(s)$, period, maintaining its shape, the wave is said to be progressive and to travel with celerity, $c = L/T\ (m/s)$. When the profile oscillates in time, around fixed points (i.e. nodes) without propagating, the movement is said to be stationary.

When the heights of the wave trains are not equal or when their periods are slightly different, their movement is partially stationary with certain properties of stationary and progressive movements. More specifically, the nodes and antinodes are not well-defined, and their position in space does not have to be determined.

Note. *Waves in the open water, far from any boundary have the characteristics of a progressive wave train. In the face of a non-overtoppable, impermeable, vertical breakwater whose length is several times greater than the wavelength, the type of oscillatory movement is that of a stationary wave train. If it is a sloping breakwater that is permeable and non-overtoppable, the oscillatory movement is partially stationary. The astronomical tide, propagating in an estuary, whose width and length are less than, for example, a fourth of its wavelength and with the boundaries of a mild slope (e.g. 1/20) has stationary movement.*

4.2.1.1 Symmetry and reference sea level

η is the vertical displacement of the free surface with respect to a reference level and wave height H is the vertical distance between the amplitude of the crest A_c and the amplitude of the trough A_t,

$$H = A_t + A_c \tag{4.3}$$

Because of its spatial and temporal periodicity, the monochromatic train is symmetrical with respect to a vertical axis. However, the movement is not symmetrical around the reference level, since, in general, $A_c \neq A_t$. Only linear wave theory (first-order Stokes) represents a wave with both horizontal and vertical symmetry. In this case, $A_t = A_c = A$ and $H = 2A$, where A is the wave amplitude.

Vertical magnitudes are defined with respect to a reference level that can be fixed or mobile. The still water level, MSWL or SLW is what is measured in the absence of movement. Oscillatory motion (whether progressive or stationary) as well as the current cause the sea level to vary. This sea level is known as the mean level of the sea in movement, and it is different from the SLW. The MW or MSL significantly varies when the wave is non-linear and in the surf zone. When the wave train is in the form of a wave group, the MSL is a long wave, whose period is the period of the group.

In these Recommendations, the vertical displacement with respect to the SLW is represented by ζ. The vertical magnitudes η, A_t, A_c and A are defined with respect to a mobile reference, which generally is the still water level, MSL. Then,

$$\eta(x,y,t) = \zeta(x,y,t) + \Delta h_0(x,y,t) \tag{4.4}$$

where Δh_0 is the difference between the two levels, and is a local magnitude. In other words, it varies in time and space, generally slowly. The wave height H is a vertical distance without a reference level, which makes it difficult to objectively apply.

4.2.1.2 Wave number, angular frequency, and celerity

The wave number k is defined by

$$k = \frac{2\pi}{L}(1/m) \tag{4.5}$$

And the angular frequency σ as

$$\sigma = \frac{2\pi}{T}(1/s) \quad (4.6)$$

Sometimes, particularly in the frequency description of wave, the frequency f is used, which is defined as the inverse of the period,

$$f = \frac{1}{T}\,(cycles\ per\ second) = h_z\,(hertz) \quad (4.7)$$

$$\sigma = 2\pi f \quad (4.8)$$

4.2.1.2.1 PROGRESSIVE WAVE

In a system of Cartesian coordinates (x, y, z), where x is measured in the wave propagation direction; z is measured upwards from the MSWL or SLW; and y is orthogonal to x and z, the wave propagation velocity is

$$C = \frac{L}{T}(m/s) \quad (4.9)$$

$$C = \frac{\sigma}{k} \quad (4.10)$$

The first objective of a wave theory is to determine C, when H, L or T, and h are known, If the waves propagate at an angle α with respect to the x axis, the following magnitudes are obtained.

$$L_x = \frac{L}{\cos\alpha} \qquad L_y = \frac{L}{\sin\alpha} \quad (4.11)$$

And the following quantities are defined,

$$k_x = \frac{2\pi}{L_x} \quad (4.12)$$

$$k_y = \frac{2\pi}{L_y} \quad (4.13)$$

And the following relations are obtained between k, k_x and k_y,

$$k_x = k\cos\alpha \quad (4.14)$$

$$k_y = k\sin\alpha \quad (4.15)$$

Moreover, the wave number vector, $\vec{k}$, is defined, pointing in the wave propagation direction whose components are, $[k_x, k_y]$,

$$\vec{k} = k_x\vec{i} + k_y\vec{j} \quad (4.16)$$

whose vector module and argument are

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2} \quad (4.17)$$

$$\alpha = \tan^{-1}\left(\frac{k_y}{k_x}\right) \quad (4.18)$$

The mathematical description of a progressive wave train, traveling along the x axis is

$$\eta(x,t) = \eta\left(0, t - \frac{x}{C}\right) = A\cos\sigma\left(t - \frac{x}{C}\right) \tag{4.19}$$

$$\eta(x,t) = A\cos(\sigma t - kx) = A\cos[-(kx - \sigma t)] \tag{4.20}$$

The argument of the trigonometric function, $kx - \sigma t$, is the phase of oscillatory motion. Due to the circular nature of trigonometric functions, $\eta(x, t)$, it repeats itself in time and space, forming a monochromatic wave train.

4.2.1.2.2 Stationary wave

The mathematical description of a stationary wave train (x axis) is

$$\eta(x,t) = 2A\cos kx\cos\sigma t \tag{4.21}$$

The wave train does not propagate and oscillates around fixed points or nodal points that are found in $kx = (2n - 1)\pi/2$, $n = 1,2, \ldots$ In contrast, when $kx = n\pi$, $n = 0,1,2, \ldots$, the movement of the free surface is strictly vertical, reaching positive and negative maximum displacements, which are known as antinodes.

The previous mathematical expression can be obtained by superposing two equal waves, traveling in opposite directions,

$$\eta(x,t) = \mathrm{Re}\left[Ae^{i(kx-\sigma t)} + Ae^{-i(kx-\sigma t)}\right] \tag{4.22}$$

4.2.1.2.3 Partially stationary wave

When two waves of different amplitudes are superposed, and the same phase is traveling in opposite senses,

$$\eta(x,t) = Ae^{i(kx-\sigma t)} + Be^{-i(kx-\sigma t)} \tag{4.23}$$

a movement is obtained with two components. One component is strictly stationary and the other component is progressive, whose amplitude depends on the difference between the amplitudes,

$$\eta(x,t) = 2B\cos kx\cos\sigma t + (A - B)\cos(kx - \sigma t) \tag{4.24}$$

For this reason, strictly speaking, the movement does not have any nodes since the wave oscillates with an amplitude that depends on $A - B$.

4.2.1.2.4 Wave groups

The superposition of two wave trains of similar periods and wave numbers k_1, σ_1 and k_2

$$k_1 = k - \frac{\Delta k}{2} \tag{4.25}$$

$$\sigma_1 = \sigma - \frac{\Delta\sigma}{2} \tag{4.26}$$

$$k_2 = k + \frac{\Delta k}{2} \tag{4.27}$$

$$\sigma_2 = \sigma + \frac{\Delta\sigma}{2} \tag{4.28}$$

where Δk and $\Delta\sigma$ are small quantities that quantify the deviation of k and σ of each wave train. This gives the following:

$$\eta = \eta_1 + \eta_2 = Ae^{i(k_1 x - \sigma_1 t)} + Ae^{i(k_2 x - \sigma_2 t)} \tag{4.29}$$

$$\eta(x,t) = 2A\cos(kx - \sigma t)\cos\left[\frac{1}{2}\Delta k\left(x - \frac{\Delta\sigma}{\Delta k}t\right)\right] \tag{4.30}$$

The resulting profile are waves that propagate with celerity $C = \sigma/k$ and whose amplitude is modulated

$$2A\cos\left[\frac{1}{2}\Delta k\left(x - \frac{\Delta\sigma}{\Delta k}t\right)\right] \tag{4.31}$$

And its envelope propagates with velocity, $\Delta\sigma/\Delta k$.

4.2.1.2.5 *Short-crested and long-crested waves*

The waves that travel, and form an angle with the x axis,

$$\eta(x,y,t) = A\exp\left[i\left(k_x x + k_y y - \sigma t\right)\right] \tag{4.32}$$

whose phase is,

$$S = k_x x + k_y y - \sigma t \tag{4.33}$$

Once the vector position $\vec{r}(x, y)$ of a point $P(x, y)$ is defined, the phase can be written

$$S = \vec{k}\,.\,\vec{r} - \sigma t \tag{4.34}$$

where,

$$\vec{k} = \nabla S \; \nabla = \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \tag{4.35}$$

And

$$k.\nabla r - \sigma t = S \tag{4.36}$$

$$\sigma = -\frac{\partial S}{\partial t} \tag{4.37}$$

expressions that can be used to define $\vec{k}$ and σ, respectively.

The superposition of two wave trains with the same amplitude A and the same period, but traveling with senses $+\alpha$ and $-\alpha$, gives

$$\eta_1(x,y,t) = A\exp\left[i\left(k_{1x}x + k_{1y} - \sigma t\right)\right] \tag{4.38}$$

$$\eta_2(x,y,t) = A\exp\left[i\left(k_{2x}x + k_{2y} - \sigma t\right)\right] \tag{4.39}$$

$$\eta(x,y,t) = \eta_1 + \eta_2 = 2A\cos[yk\sin\alpha]\cos[xk\cos\alpha - \sigma t] \tag{4.40}$$

which represents a wave train, traveling in the direction, x, $2A\cos[yk\sin\alpha]$, which varies on the y axis. This pattern corresponds to short wave trains in contrast to long wave trains in which the crest is constant all along the y axis.

4.2.1.3 *Wave behavior according to adimensional monomials*

Since oscillatory motion satisfies the dispersion equation and since this equation contains five variables, these variables can be combined in adimensional monomials that allow oscillatory motion to be interpreted, according to their values,

$$\frac{A}{h}, \frac{h}{L} or \frac{h}{gT^2}, \frac{A}{L} or \frac{H}{gT^2}, \frac{C}{\sqrt{gh}}$$

4.2.1.3.1 *Relative depth, h/L*

The wave train parameters, celerity C, wavelength L, and height H are modified with the propagation depth h but the period T remains the same. The depth of the water column affected by oscillatory motion depends on the relation $\mu = h/L$ known as relative depth. Oscillatory motion with wavelength L is not perceived at depths such that $[h/L > 1/2]$. In that case, it is said that the wave is found in deep water. Furthermore, the movement of water particles associated with the wave train can be regarded as uniformly distributed in the water column when the relative depth is $[h/L < 1/20]$. This situation is known as shallow depths, and the wave is called a long wave. When the wave is in relative depths in the range $[1/20 < h/L < 1/2]$, it is in medium depths.

Note. On the shoreline and the inner continental shelf, meteorological and astronomical tides, as well as tsunamis, are long waves, and their movement affects the entire water column. In contrast, seas, because of their wide range of periods, can appear in the three ranges of relative depth.

4.2.1.3.2 *Surface profile: height and period*

The simplest mathematical representation of the surface profile is a trigonometric function, namely, sine or cosine. This profile has crest amplitude A_c, equal to trough amplitude A_t, and the wave height $H = 2A_c = 2A_s = 2A$. Furthermore, its permanence time over the mean level T_c is equal to the permanence time T_t, under that mean level. The crest period is said to be equal to the trough period, and equal to the half-period of the wave, $T/2$. The wave profile has a horizontal and vertical symmetry, and is the linear solution to the problem of a wave propagating over a horizontal seabed.

In deep water, $h/L > 1/2$, the sine wave is the closest approximation. However, as the wave propagates over medium and shallow depths, $h/L < 1/2$, the crests become progressively steeper. This reduces the permanence time over the mean level, and the troughs flatten, which increases the duration of the free surface under the mean level. In other words, $A_c > A_t$ and $T_c \neq T_t$. Nevertheless, it holds that $H = A_c + A_t$ and $T = T_c + T_t$. The wave is said to steepen because of depth-induced effects, and thus gradually or suddenly loses its horizontal symmetry. The wave can also steepen because of its interaction with other waves and with a current.

4.2.1.3.3 *Relative steepness, H/L*

The monomial that evaluates the horizontal asymmetry of the profile is the wave steepness, defined by H/L. When the steepness increases, the velocity and acceleration of the water particles under the crest also increase. The cinematic and dynamic characteristics of the oscillatory motion depend on the relative steepness of the wave, H/L. The relative steepness monomial can also be expressed as $\varepsilon = Ak = A(2\pi/L) = \pi(H/L)$. For its subsequent use, it is necessary to define the number, $\varepsilon_H = H/L$. When the steepness is very small, the wave is said to be linear because the cinematic and dynamic magnitudes (i.e. velocity, acceleration, and pressure) associated with oscillatory motion have a linear dependence on the steepness. They are said to be magnitudes of $O(Ak)$.

On the contrary, when the wave steepness grows and approaches its maximum limit, those magnitudes are a non-linear function of the steepness. The wave is then said to be non-linear. The cinematic and dynamic magnitudes (i.e. velocity, acceleration, and pressure) associated with oscillatory motion have a non-linear dependence on the steepness, and they are said to be higher-order magnitudes, $O((Ak)^n),\ n \geq 1$.

The maximum steepness of a gravity wave is $(H/L)_{lim} = 1/7$. This value can only be reached in deep water.

4.2.1.3.4 Relative wave amplitude, A/h

The relation between the amplitude of the wave (or wave height), and the depth at which it is found is known as the relative wave height, $\gamma = A/h$. It is a measurement of the magnitude of the maximum vertical movement with respect to the water depth. For its subsequent use, relative wave height, $\gamma_H = H/h$ must be defined.

The value of the monomial indicates indicate how close the wave is to the limit value of its relative amplitude, $\gamma_{lim} = (H/h)_{lim}$. When this value is expressed by the wave height, it is known as the breaking index, and its value depends on how and why it reaches that limit value.

4.2.1.3.5 Ursell parameter, U_r

The Ursell parameter, U_r, is a combination of the relative depth and steepness of the wave,

$$U_r = \frac{(H/L)}{(h/L)^3} = \frac{HL^2}{h^3} = \frac{\varepsilon_H}{\mu^3} \tag{4.41}$$

and classifies different solutions of the gravity wave problem. This monomial can also be seen as the quotient of the relative amplitude and the square of the relative depth,

$$U_r = \frac{H}{h}\left(\frac{L}{h}\right)^2 = \frac{\gamma_H}{\mu^2} \tag{4.42}$$

or as a relation of three adimensional monomials,

$$U_r = \left(\frac{H}{h}\right)^2 \frac{h}{H}\left(\frac{L}{h}\right)^4 = \frac{\varepsilon_H^2}{\gamma_H \mu^4} \tag{4.43}$$

4.2.2 Oscillatory motion equations and their solutions

Generally speaking, the propagation of a wave train on an impermeable sea bed are described by Navier-Stokes equations. If the fluid is also non-viscous, the system is reduced to Euler equations. If the motion (flow) is irrotational, the equation system is formed by the mass conservation equation (governing equation), the non-stationary Bernoulli equation and the boundary conditions on the surface, on the seabed, and on the sides of the integration domain (see Figure 4.4.2).

The only variable of the system is the power function, which is related to the cinematic variables (u, v, w) and the dynamic variables (pressure, p) of the fluid.

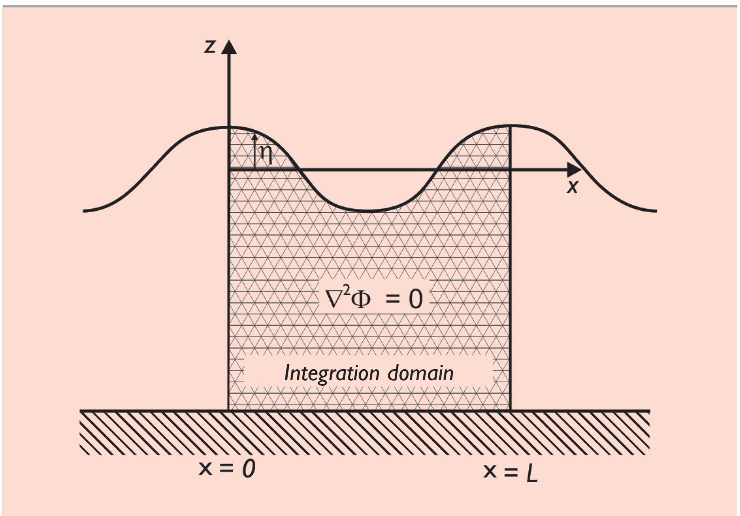
Since the fluid is non-viscous, there are no tangential stresses and the fluid slides along the boundaries, and only transmits forces per surface unit that are normal to it (pressure). This pressure has two components: (i) a hydrostatic component; (ii) a component associated with the oscillatory motion of the fluid (i.e. dynamic pressure).

$$p(x,y,z,t) = -\rho_w g z + \rho_d(x,y,z,t)$$

where the reference axes are placed on the still seawater surface , and the vertical axis z is positive in the direction away from the seabottom.

The integration of the previous equations with various orders of approximation, in a strictly mathematical sense, provides different solutions, which can be ordered in Stokes approximations and Boussinesq approximations. In the cases of simple topography, it is possible to obtain analytic solutions that can be useful in the calibration of numerical solutions or to perform pre-project work, such as preliminary studies and calculations.

Figure 4.4.2. ***Integration domain of governing equations***



Note. The solutions obtained, based on the approximation of non-viscous fluid are not valid near the boundaries. A boundary limit is developed there, and if the boundary is not in motion, then the water particle in contact with it also has zero velocity. Oscillatory motion is characterized by velocity inversion and by two velocity zero-crossings. It is thus difficult to completely develop the boundary layer. Its thickness δw is generally very small, and is measured in millimeters ($\delta_w \sim \sqrt{\nu/\sigma}$, $\nu \approx 10^{-6}\ m^2/s$, is the cinematic viscosity of the water, and σ is the angular frequency of the wave train). For this reason, the solution obtained with the non-viscous fluid hypothesis is practically valid at all depths.
Moreover, the previous equations consider that the fluid in which the wave train propagates is water without air bubbles. For this reason they should not be used to calculate the pressure on the wall of a vertical breakwater produced by a wave breaking against it and trapping air.

4.2.2.1 Equations integrated on the vertical axis

Euler equations can be integrated on the vertical axis. By applying the Leibniz rule and the boundary conditions, a system of three differential equations is obtained: one mass conservation equation and two horizontal momentum conservation equations. This system helps to solve problems of non-uniform, non-stationary, free-laminar flow, in other words, variable motion. The new variables of the system are the free surface η and the flow volume, $\vec{q}(q_x(x,y,t), q_y(x,y,t))$. These equations make up a hyperbolic system in η and q, known as the non-stationary mild slope equation, and are similar to equations obtained with the hyperbolic approximation of the elliptic mild slope equation.

The solution of the problem involves relating the dynamic variable to the two previous ones, $p_d = f(\eta,\vec{q})$. If it is also a question of working with finite-amplitude waves (non-negligible wave steepness, H/L), the vertical variation (profile) of the horizontal velocities should be specified. This means restoring the vertical structure that was lost in the integration. Based on this approach and depending on the functional relation between the dynamic pressure and the dependent variables of the system, the following solutions can be obtained.

4.2.2.2 Horizontal bed wave equation (2)

The law of dynamic pressures on the MW or MSL is hydrostatic, and varies with the depth underneath the MSL, in consonance with the hyperbolic cosine function, known as the depth function, *f(z)*,

$$p_d = \rho_w g \eta f(z) \tag{4.44}$$

$$f(z) = \frac{\cosh k(h+z)}{\cosh kh} \tag{4.45}$$

$$\sigma^2 = gk \tanh kh \tag{4.46}$$

Based on the hypothesis of small-amplitude motion (negligible H/L), the governing equation of oscillatory motion over a horizontal bottom is

$$\frac{\partial^2 \eta}{\partial t^2} = C^2 \left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right] \tag{4.47}$$

If the wave train is sinusoidal,

$$\eta(x,y,t) = a \exp(-i\sigma t) \tag{4.48}$$

this gives the Helmholtz equation, which can be applied in problems involving oscillations in wharfs and semi-enclosed water bodies,

$$\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + k^2 a = 0 \tag{4.49}$$

where $\alpha = H/2$ is the amplitude of the oscillatory motion.

4.2.2.3 Mild slope equation

When the sea bottom is variable, as long as the changes in depth related to the wavelength are gradual and the non-linear effects negligible, the Mild Slope Equation (MSPE) can be used. The slope is regarded as mild when the reflection produced by the depth change is negligible. The Mild Slope Equation is valid when

$$S_r = \frac{\nabla_h h}{kh} \ll 1 \tag{4.50}$$

This equation can be obtained by means of a weighted average in the water column of the governing equation, in other words, the Laplace equation,

$$\int_{-h}^{0} k_p(z) \left[\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \right] = 0 \tag{4.51}$$

with the weighting function,

$$k_p(z) = \frac{\cosh k(h+z)}{\cosh kh} \tag{4.52}$$

(2) Since this equation does not guarantee the conservation of the energy flux, it should not be applied to the study of refraction and shoaling processes.

By substituting the value of the potential, solving the integrals, and considering the boundary conditions in the seabed and the free surface, the following governing equation is finally obtained:

$$\frac{\partial}{\partial x}\left[\int_{-h}^{0}\left[k_p(z)\right]^2 dz \frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[\int_{-h}^{0}\left[k_p(z)\right]^2 dz \frac{\partial \phi}{\partial y}\right]+k^2\phi\int_{-h}^{0}\left[k_p(z)\right]^2 dz=0 \quad (4.53)$$

The integrals are directly obtained,

$$\int_{-h}^{0}\left[k_p(z)\right]^2 dz=\frac{\tanh kh}{k}\left[\frac{1}{2}\left(1+\frac{2kh}{\sinh 2kh}\right)\right] \quad (4.54)$$

When the dispersion equation is taken into account, the result can be written as follows:

$$\int_{-h}^{0}\left[k_p(z)\right]^2 dz=\frac{cC_g}{g} \quad (4.55)$$

The mild slope equation is thus expressed as

$$\frac{\partial}{\partial x}\left[cC_g\frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[cC_g\frac{\partial \phi}{\partial y}\right]+k^2 cC_g\phi=0 \quad (4.56)$$

When the operator is defined,

$$\nabla_h=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j} \quad (4.57)$$

the previous equation can be expressed as

$$\nabla_h\cdot\left[cC_g\nabla_h\phi\right]+k^2 cC_g\phi=0 \quad (4.58)$$

To solve this equation, it is necessary to specify the boundary conditions at the boundary of the integration domain. This is generally done numerically.

The governing equation of the wave can be obtained by

$$\phi(x,y)=-\frac{ig}{\sigma}A(x,y)e^{i\left(k_x x+k_y y\right)}=-\frac{ig}{\sigma}A(x,y)e^{iS(x,y)} \quad (4.59)$$

which makes explicit the dependence on x and y of the amplitude A and the phase $S(x,y) = k_x x + k_y y$. Since this is a complex field, two expressions equal to 0 are thus obtained. The first corresponds to the real part, and the second to the imaginary part.

Real part:

$$\nabla_h\cdot\left(cC_g\nabla_h A\right)-cC_g A\left|\nabla S\right|+k^2 cC_g A=0 \quad (4.60)$$

Imaginary part:

$$\nabla_h\cdot\left(cC_g A^2\nabla_h S\right)=0 \quad (4.61)$$

The real part is the Eikonal equation, in which the first term evaluates the diffraction. When this term is negligible, e.g. $\nabla_h A = 0$, the relation between the phase gradient and the wave number is recovered,

$$\left|\nabla_h S\right|=k;\quad \frac{\partial S}{\partial x}=k_x;\quad \frac{\partial S}{\partial y}=k_y \quad (4.62)$$

The imaginary part is an energy conservation equation. In fact, when the celerity c is replaced by the expression,

$$c = \frac{\sigma}{k} = \frac{\sigma}{|\nabla_h S|} \tag{4.63}$$

and when the vector of the energy propagation vector is defined as

$$\overrightarrow{C_g} = C_g \frac{\nabla_h S}{|\nabla_h S|} \tag{4.64}$$

the conservation equation is expressed as shown below,

$$\nabla_h \cdot \left(\sigma \overrightarrow{C_g} E \right) = 0 \tag{4.65}$$

where E is the mean spatial energy,

$$E = \frac{1}{2} \rho g A^2 \tag{4.66}$$

4.2.2.3.1 *Parabolic approximation of the mild slope equation*

The mild slope equation is elliptic and admits parabolic approximations, in the same way as those obtained for the Helmholtz equation. Accordingly, it is assumed that the wave train propagates, and forms a small angle with the x axis.

The potential function can thus be written,

$$\phi(x,y) = \varphi(x,y) e^{ikx} \tag{4.67}$$

and the function $\varphi(x,y)$ for a wave propagating over a horizontal bed is the following:

$$\varphi(x,y) = A e^{i(k_x - k)x} e^{ik_y y} \tag{4.68}$$

By substituting in the previous equation, and assuming that

$$\frac{\partial}{\partial x}\left[c C_g \frac{\partial \varphi}{\partial x} \right] \ll 1 \tag{4.69}$$

the parabolic approximation of the equation is obtained,

$$2ikcC_g \frac{\partial \varphi}{\partial x} + i\varphi \frac{\partial (kcC_g)}{\partial x} + \frac{\partial}{\partial y}\left(cC_g \frac{\partial \varphi}{\partial y} \right) = 0 \tag{4.70}$$

To resolve this equation, it is only necessary to impose the boundary equations at the initial boundary of the integration domain.

4.2.2.4 *Mild slope equation system*

If bottom effects are taken into account (i.e. mild changes in depth), the system becomes a set of mass conservation and horizontal momentum equations,

$$\frac{\partial \eta}{\partial t} + \nabla \vec{q} = 0 \tag{4.71}$$

$$\frac{\partial \vec{q}}{\partial t} + \nabla\left(C^2 \eta\right) - \frac{g\eta}{\cosh kh} \nabla h = 0 \qquad (4.72)$$

$\vec{q}$ [(3)] can be eliminated from this equation system to obtain a second-order differential equation in η,

$$\frac{\partial^2 \eta}{\partial t^2} - \nabla\left[\frac{C^2}{n} \nabla\left(n\eta\right)\right] = 0 \qquad (4.73)$$

This equation conserves mass flows, momentum, and energy, and is an essential tool in the project design of maritime structures, especially when a term is added that represents dissipation produced by bottom effects or wave breaking. This equation includes the temporal variation of the free surface. Often, it is enough to consider that the wave train is periodic with a constant angular frequency (i.e. the wave period is conserved). Thus, the free surface is

$$\eta\left(x,y,t\right) = \eta^*\left(x,y\right)\exp\left(-i\sigma t\right) \qquad (4.74)$$

which means that the equation can be written in terms of the variable $\eta^*(x,y)$, which is not time-dependent. This equation is the stationary MSP (see section 3.4.3.1), whereas the complete equation is non-stationary, and is usually known as the MSP-t (see section 3.4.4.1).

4.2.2.5 Shallow water non-linear wave equation

If the waves are very long, in other words, h/L is a very small magnitude, the vertical variations (accelerations) of the flow are negligible, and the wave motion is essentially horizontal. Thus, the application of the vertical momentum equation then indicates that the dynamic pressure is hydrostatic. In other words, the value of the depth function is equal to the unit since it is a long wave,

$$p_d\left(x,y,z,t\right) = \rho_w g \eta \qquad (4.75)$$

and the horizontal velocity does not vary with the depth. It can thus be represented by the averaged velocity in the vertical,

$$U\left(x,y,t\right) = \frac{1}{h+\eta}\int_{-h}^{\eta} u\left(x,y,z,t\right)dz \qquad (4.76)$$

$$V\left(x,y,t\right) = \frac{1}{h+\eta}\int_{-h}^{\eta} v\left(x,y,z,t\right)dz \qquad (4.77)$$

When these results are inserted in the Euler equations, this gives a hyperbolic quasi-linear system in $d = h + \eta$ and $\vec{U}(U,V) = (U_i, U_j)$, known as non-linear shallow water equations. The waves that satisfy these equations are non-dispersive. These waves can be formally derived from Navier-Stokes equations after vertically integrating them, and admitting that the vertical flows in the water column, particularly the local and convective vertical acceleration, are negligible when compared to the horizontal acceleration.

They are applied to the study of non-linear long waves, forced by the hydrostatic pressure gradient and by external forces. Such forces include the atmospheric action of the wind on the surface and the pressure gradient, gravitational attraction between heavenly bodies, and medium oscillatory movements caused by the radiation tensor gradients, as described in the section on long waves. There are a number of numerical codes that reliably resolve these equations, some of which are recommended in the section on long waves.

(3) The flow volume vector is expressed in Cartesian coordinates by $\vec{q} = q_x\vec{i} + q_y\vec{j}$. The operator $\nabla = (\partial/\partial x)\vec{i} + (\partial/\partial y)\vec{j}$ evaluates the variation of a vector magnitude in directions x and y. The scalar product $\nabla \cdot \vec{q} = (\partial/\partial x)\vec{q_x} + (\partial/\partial y)\vec{q_y}$ is a measurement of the net volume of water, flowing in and out of the infinitesimal control volume.

4.2.2.6 Boussinesq equations

When vertical accelerations in the water column and the vertical variation of the velocity field must be considered, it is preferable to work with Boussinesq equations. The same procedure can be used to derive these equations from vertically integrated Euler equations, considering the following laws of pressures and velocities:

$$p_d(x,y,z,t) = \rho_w g\eta + \rho_w z\frac{\partial}{\partial x_j}\left(h\frac{\partial u_j^{(r)}}{\partial t}\right) + \frac{1}{2}\rho_w z^2\frac{\partial^2 u_j^{(r)}}{\partial x_j \partial t} \tag{4.78}$$

$$u_i(x,y,z,t) = u_i^{(r)}(x,y,t) - \frac{\zeta h + 2z}{2}\frac{\partial^2\left(hu_j^{(r)}\right)}{\partial x_j \partial x_j} + \frac{\zeta^2 h^2 - 3z^2}{6}\frac{\partial^2\left(u_j^{(r)}\right)}{\partial x_j \partial x_j} \tag{4.79}$$

where $0 \le \zeta \le 1$ is a parameter that defines the water column level $z^{(r)} = -\zeta h$, which must be determined to optimize the dispersive nature of the wave, depending on the relative propagation depth. This mathematical "correction" of the standard Boussinesq equations allows the user to apply them to practically the entire range of relative depths.

$$\frac{\partial\eta}{\partial t} + \nabla\cdot\left[(h+\eta)\overrightarrow{u^{(r)}} - \nabla\cdot\left\{\frac{(1-3\zeta^2)h^3}{6}\nabla\left(\nabla\cdot\overrightarrow{u^{(r)}}\right) - \frac{(2\zeta-1)h^2}{2}\nabla\left(\nabla\cdot\overrightarrow{hu^{(r)}}\right)\right\}\right] = 0 \tag{4.80}$$

$$\frac{\partial\overrightarrow{u^{(r)}}}{\partial t} + \left(\overrightarrow{u^{(r)}}\cdot\nabla\right)\overrightarrow{u^{(r)}} + g\nabla\eta + \frac{1}{2}(\zeta h)^2\nabla\left(\nabla\cdot\overrightarrow{u^{(r)}}\right) - \zeta h\nabla\left[\nabla\cdot\left(\overrightarrow{hu^{(r)}}\right)\right] = 0 \tag{4.81}$$

The solution of these equations are waves whose amplitudes and frequencies are (weakly) dispersive. Their validity range extends to oscillatory movements in which Hk and $(kh)^2$ are small and of the same order.

These equations include radiation processes, such as diffraction and reflection, and wave-current interaction. Their resolution provides the free surface in time at any point in space.

Boussinesq equations can be used to study wave propagation in any direction, including both the incident and the reflected direction. When the waves are traveling in only one direction, either the velocity or the free surface displacement can be eliminated. The result is the Korteweg-deVries equation with slope,

$$\frac{\partial\zeta}{\partial t} + \frac{\partial(C\zeta)}{\partial x} + \frac{3}{2}\frac{C}{h_0}\zeta\frac{\partial\zeta}{\partial x} + \frac{1}{6}h_0^2 C\frac{\partial^3\zeta}{\partial x^3} - \frac{1}{2}\zeta\frac{\partial C}{\partial x} = 0 \tag{4.82}$$

where $C = \sqrt{gh_0}$ is the phase velocity of the wave, and h_0 is the still water depth.

Note. Cnoidal waves and solitons are solutions of the Kortweg-de Vries equation on a horizontal bed.

4.2.3 Transformation of the wave train in medium and shallow depths

When the wave train reaches medium and shallow depths, it is transformed because of the action of the seabed and boundaries. This transformation affects the cinematic properties, celerity, and wave height of the wave train. In such cases, the dispersion equation can be applied, in the absence of a current and maintaining the angular frequency [(4)], $\sigma = 2\pi/T$ (T is the period) of the waves. Accordingly, as the depth h becomes smaller, the wave number k becomes larger (i.e. wavelength decreases), and the celerity C diminishes. This causes a change in the wave propagation direction (refraction). Furthermore, if the mean energy of the wave train per unit of horizontal surface

(4) The angular frequency is usually represented by σ or ω. Generally speaking, σ is used when there is no currents, whereas ω is maintained to identify the angular frequency of the wave train when there is a current (see section 3.6.6.4).

is conserved, the variation of L and C causes a corresponding variation in wave height, which is induced by shoaling and refraction.

4.2.3.1 Wave conservation equation

According to the wave conservation equation, any time variation in the wave number should be compensated by spatial variations in the angular frequency, namely,

$$\frac{\partial k_x}{\partial t} + \frac{\partial \sigma}{\partial x} = 0 \tag{4.83}$$

$$\frac{\partial k_y}{\partial t} + \frac{\partial \sigma}{\partial y} = 0 \tag{4.84}$$

where (k_x, k_y) are the wave number components in the directions of the coordinate axes,

$$k = \sqrt{k_x^2 + k_y^2} \tag{4.85}$$

$$\theta = a\tan\frac{k_y}{k_x} \tag{4.86}$$

and θ is the angle formed by the perpendicular to the wave crest and the y axis,

$$k_x = k\sin\theta \tag{4.87}$$

$$k_y = k\cos\theta \tag{4.88}$$

4.2.3.1.1 Irrotationality of the wave number

When the wave number does not vary over time (e.g. a monochromatic wave train or the wave number corresponding to the mean zero-crossing period in a sea state), either the angular frequency is maintained or the period does not change as a result of the transformation.

However, the wavenumber vector changes during the propagation, though not arbitrarily. Rather, its module should satisfy the dispersion equation, and furthermore, the variation of its components should satisfy the following equation [5],

$$\frac{\partial k_x}{\partial x} + \frac{\partial k_y}{\partial y} = 0 \tag{4.89}$$

If the wave train variations along the y axis are negligible (e.g. when the seabed has straight and parallel bathymetric contours, or in the case of a long-crested wave train), then, $\partial k/\partial y = \partial H/\partial y = 0$,

$$\begin{aligned} \frac{\partial k_x}{\partial x} &= \frac{\partial (k\sin\theta)}{\partial x} = 0 \\ k\sin\theta &= \text{constant} \end{aligned} \tag{4.90}$$

and since σ is constant, Snell's law is obtained,

(5) Once the phase is defined by $\Omega = \vec{k} \cdot \vec{x} - \sigma t$, the wavenumber vector $\vec{k}$ is equal to the slope Ω which is scalar. Therefore, its rotational should be zero, $rot\,\vec{k} = 0$.

$$\frac{\sin\theta}{\frac{\sigma}{k}} = \frac{\sin\theta}{C} = constant \tag{4.91}$$

which provides the turning of the wavefront, depending on the depth change.

4.2.3.2 *Energy conservation equation*

For those cases in which the bathymetry is fairly regular and does not change because of erosion or sediment accumulation, and diffraction is negligible, the transformation of the wave train because of refraction and shoaling can be obtained by numerically resolving the wave energy conservation equation. This is regardless of whether a term is included that represents a dissipation induced by bottom effects and wave breaking in a system of Cartesian coordinates, along with the irrotationality of the wave number,

$$\frac{\partial\left(EC_g\cos\theta\right)}{\partial x} + \frac{\partial\left(EC_g\sin\theta\right)}{\partial y} + dissipation = 0 \tag{4.92}$$

$$\frac{\partial(k\sin\theta)}{\partial x} - \frac{\partial(k\cos\theta)}{\partial y} = 0 \tag{4.93}$$

This method (6) rapidly provides information regarding the refraction and shoaling of the wave train, but reflection and diffraction are not taken into account. The energy conservation equation in the control volume (7) formed by two crests and two separate orthogonal lines, b_0 and b_1, provide the relation between wave heights and control volume,

$$H_1 = H_0\sqrt{\frac{C_{g_0}}{C_{g_1}}}\sqrt{\frac{b_0}{b_1}} = H_0 K_S K_R \tag{4.94}$$

When the bathymetries are straight and parallel,

$$K_R^2 = \frac{b_0}{b_1} = \frac{\cos\theta_0}{\cos\theta_1} \tag{4.95}$$

where K_R is the refraction coefficient, and K_S is the shoaling coefficient.

The spatial variation of both coefficients only depends on the spatial variation of the relative depth, h/L. Since the equations are linear, the refraction not only depends on wave height, but also on the period (wave number). The height is a scale factor, and H_0 multiplies the value of the coefficient $K_S K_R$ to obtain the local height H_1.

4.2.3.3 *Numerical methods based on ray theory*

The ray method (based on Snell's method) can be numerically resolved with the following equations, written in a system of Cartesian coordinates. Parametric equations of rays are

$$\frac{ds}{dt} = C \tag{4.96}$$

$$\frac{dx}{dt} = C\cos\theta \tag{4.97}$$

(6) Refrac is a public domain numerical model that simultaneously resolves energy conservation and wavenumber equations.
(7) According to Iribarren, the control volume is the forward parallelepiped.

$$\frac{dy}{dt} = C \sin\theta \tag{4.98}$$

At the first order of approximation, the width between two rays b varies when the propagation angle has changed in $\Delta\theta$,

$$\frac{1}{b}\frac{\partial b}{\partial s} = \frac{\partial \theta}{\partial n} \tag{4.99}$$

and defining the new variable,

$$\beta = \frac{b}{b_0} = \frac{1}{K_R^2} \tag{4.100}$$

this gives,

$$\frac{1}{\beta}\frac{\partial \beta}{\partial s} = \frac{\partial \theta}{\partial n} \tag{4.101}$$

Considering that the variation rate of the propagation angle all along the ray can be expressed as

$$\frac{\partial \theta}{\partial s} = -\frac{1}{C}\frac{\partial C}{\partial n} = -\frac{1}{C}\left[\cos\theta \frac{\partial C}{\partial x} + \sin\theta \frac{\partial C}{\partial y}\right] \tag{4.102}$$

Finally, β satisfies the differential equation,

$$\frac{\partial^2 \beta}{\partial n^2} + \frac{1}{C}\frac{\partial^2 C}{\partial n^2}\beta = 0 \tag{4.103}$$

which can be transformed in

$$\frac{d^2 \beta}{ds^2} + p\frac{d\beta}{ds} + q\beta = 0 \tag{4.104}$$

where

$$p = -\frac{\cos\theta}{C}\frac{\partial C}{\partial x} - \frac{\sin\theta}{C}\frac{\partial C}{\partial y} \tag{4.105}$$

$$q = \frac{\sin^2\theta}{C}\frac{\partial^2 C}{\partial x^2} - 2\frac{\sin\theta\cos\theta}{C}\frac{\partial^2 C}{\partial x \partial y} + \frac{\cos^2\theta}{C}\frac{\partial^2 C}{\partial y^2} \tag{4.106}$$

This equation, along with the three parametric equations of rays form a system of ordinary differential equations which can be simultaneously resolved to obtain the position on the ray and the width between rays for a given bathymetry and celerity $C(x, y)$ at each point, based on the dispersion equation. The local height is obtained by

$$H_1 = H_0 K_S K_R = H_0 \sqrt{\frac{C_{g_0}}{C_{g_1}}}\sqrt{\frac{1}{\beta}} = H_0 \sqrt{\frac{C_0}{2C_{g_1}}}\sqrt{\frac{1}{\beta}} \tag{4.107}$$

4.2.3.4 Parabolic approximations to the diffraction problem

The equation that governs the diffraction on a horizontal bottom is obtained by substituting the expression of the potential function, which makes the dependence on z explicit in the Laplace equation. The resulting equation, known as the Helmholtz equation, is an elliptic equation in partial derivatives. Its solution for a flat wave of amplitude A, propagating over horizontal bottom, forming an angle θ, with the positive direction of the x axis, is

$$\phi(x,y) = -\frac{ig}{\sigma} A e^{i(k_x x + k_y y)} \tag{4.108}$$

where (k_x, k_y) are the components of the wave number vector, $\vec{k}$.

The solution of this equation for the case of a semi-infinite, thin, impermeable breakwater was obtained by Penny and Price (1952). There are a great variety of abacuses, based on these solutions and those of other entrance-induced diffraction problems, which help to obtain the turbulence at any point of the domain. Given the elliptic nature of the problem, in many cases numerical methods must be used to obtain solutions.

4.2.3.4.1 Parabolic approximation of the Helmholtz equation

Some of the difficulties derived from the elliptic nature of the Helmholtz equation can be solved by transforming it into a parabolic equation. For this reason, the potential $\phi(x, y)$ can be written as follows:

$$\phi(x,y) = \varphi(x,y) e^{ikx} \tag{4.109}$$

For the incident wave train, the function is $\varphi(x, y)$

$$\varphi(x,y) = a e^{i(k_x - k)x} e^{ik_y y} \tag{4.110}$$

where

$$a = \frac{igA}{\sigma} = -\frac{i\sigma A}{k \tanh kh} \tag{4.111}$$

If the propagation angle θ is small, then $k_x - k$ as well as k_y are wave numbers of small magnitude, and their corresponding wavelengths $l_x = 2\pi/(k_x - k)$ and $l_y = 2\pi/k_y$ are large. The function φ varies slowly along the x axis, the propagation axis, as well as along the y axis, the direction of the crest. A governing equation for the variable φ is

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + 2ik\frac{\partial \varphi}{\partial x} = 0 \tag{4.112}$$

Considering the following adimensional variables,

$$x' = \frac{x}{l_x}; \quad y' = \frac{y}{l_y}; \quad \varphi' = \frac{\varphi}{\alpha} \tag{4.113}$$

this gives

$$\left(\frac{k_x - k}{k_y}\right)^2 \frac{\partial^2 \varphi'}{\partial x'^2} + \frac{\partial^2 \varphi'}{\partial y'^2} + 2ik\frac{k_x - k}{k_y^2}\frac{\partial \varphi'}{\partial x'} = 0 \tag{4.114}$$

The first term is affected by the quantity $(k_x - k/k_y)^2$. When $\theta \approx 0$,

$$\lim_{\theta \to 0}\left(\frac{k_x - k}{k_y}\right)^2 = \lim_{\theta \to 0}\left(\frac{\cos\theta - 1}{\sin\theta}\right)^2 = 0 \tag{4.115}$$

It can thus be regarded as negligible in the face of $\partial^2\varphi'/\partial y'^2$, whereas for the term that affects $\partial\varphi'/\partial x'$, it holds that

$$\lim_{\theta \to 0} k\frac{k_x - k}{k_y^2} = \lim_{\theta \to 0}\frac{\cos\theta - 1}{\sin^2\theta} = -\frac{1}{2} \tag{4.116}$$

The third term is on the order $O(1)$ and is not negligible. Accordingly, at the order, $O(k_x - k/k_y)$, it holds that

$$\frac{\partial^2 \varphi}{\partial y^2} + 2ik\frac{\partial \varphi}{\partial x} = 0 \tag{4.117}$$

This equation is parabolic, and can be resolved numerically, advancing in the wave propagation direction as long as the incidence angle with respect to the local bathymetry is relatively small. The solution is valid for the small incidence angles (approx. $\theta < 40^{o}$).

One way to evaluate the error according to the incidence angle is to compare the wave number in the direction of the *x* axis, obtained from the Helmholtz equation *kx* and the wave number obtained from the parabolic problem, $k_x - k$. In the first case,

$$k_x = \sqrt{k^2 - k_y^2} = k\left[1 - \left(\frac{k_y}{k}\right)^2\right]^{\frac{1}{2}} \tag{4.118}$$

which for small values of $k_y/k = \sin\theta$ can be approximated by

$$k_x = k\left[1 - \frac{1}{2}\left(\frac{k_y}{k}\right)^2 + \left(O\left(\frac{k_y}{k}\right)^4\right)\right] \tag{4.119}$$

In the case of the parabolic problem, the wave number is

$$k_x - k = k\left[1 - \frac{1}{2}\left(\frac{k_y}{k}\right)^2 + O\left(\frac{k_y}{k}\right)^4\right] - k = \frac{1}{2}\left(\frac{k_y}{k}\right)^2 + O\left(\frac{k_y}{k}\right)^4 \tag{4.120}$$

As a result, $k_x - k \approx 0$, as long as $k_y/k = \sin\theta$ is small.

4.2.3.5 Refraction and diffraction contributions

To resolve the propagation problem of a wave train in a Eulerian equation system (8) which moves with the group celerity, it is necessary to evaluate the turning rate of the front, $d\theta/dt$, instead of evaluating it on the ray, $d\theta/ds$. A propagation velocity of the energy as $C_g = \Delta s/\Delta t$ gives the following:

$$\left(\frac{d\theta}{dt}\right)_{refraction} = -\frac{C_g}{C}\frac{\partial C}{\partial n} = \frac{1}{k}\frac{\partial k}{\partial n} \tag{4.121}$$

If the wave train, besides changing direction because of refraction, experiences spatial variations of the wave height, the turning rate should include both effects, as expressed below:

$$\left(\frac{d\theta}{dt}\right)_{ref+dif} = C_g\left(\frac{1}{k}\frac{\partial k}{\partial n} + \frac{1}{2(1+\delta_a)}\frac{\partial \delta_a}{\partial n}\right) \tag{4.122}$$

where δ_a is a modified diffraction parameter,

$$\delta_a = \frac{\nabla CC_g \nabla a}{k^2 CC_g a} \tag{4.123}$$

(8) The transformation processes of control volumes can be analyzed by moving with the control volume (i.e. Lagrangian description) or by leaving it fixed in space or propagating with a constant velocity and quantifying the flow on the surface of the quantities (such as mass, momentum, energy, heat, humidity, etc.), which can cause temporal alterations of their magnitude in the control volume. This is a Eulerian description.

and $a = H/2$ is the amplitude of the movement, such that the train can be expressed by

$$\eta(x,y,t) = a(x,y)\sin\left[\sigma t + \alpha(x,y)\right] = A(x,y)\cos(\sigma t) + B(x,y)\sin(\sigma t) \tag{4.124}$$

where $\alpha(x,y)$ is the phase. The amplitudes A, B satisfy the mild slope equation (see the following section),

$$\frac{\partial}{\partial x}\left(CC_g \frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(CC_g \frac{\partial A}{\partial y}\right) + k^2 CC_g A = 0 \tag{4.125}$$

$$\frac{\partial}{\partial x}\left(CC_g \frac{\partial B}{\partial x}\right) + \frac{\partial}{\partial y}\left(CC_g \frac{\partial B}{\partial y}\right) + k^2 CC_g B = 0 \tag{4.126}$$

The numerical resolution of this system of equations along with the irrotationality of the wave number is rather more complex than the energy flux conservation system described in the previous section.

4.2.3.6 Reflection with oblique incidence

When a wave train obliquely impinges on an obstacle or uniform depth change all along the *y* axis, part of the energy is reflected in the sense and direction of a vector whose angle with the negative sense of the *x* axis is identical to the angle of incidence (see Figure 4.4.3). The wavenumber vectors of the incident and reflected wave trains, k_I, k_R are, respectively,

$$\overrightarrow{k_I} = (k\cos\theta, k\sin\theta); \overrightarrow{k_R} = (-k\cos\theta, \sin\theta) \tag{4.127}$$

In these conditions, the potential in the region of the obstacle can be expressed as the superposition of the incident and reflected potentials of the amplitudes, A'_I and A'_R, respectively, as follows,

$$\begin{aligned}\Phi &= \Re\left\{-\frac{ig}{\sigma}\frac{\cosh k(h+z)}{\cosh(kh)}\left[A'_I e^{i\overrightarrow{k_I}\cdot\vec{x}} + A'_R e^{i\overrightarrow{k_R}\cdot\vec{x}}\right]e^{-i\sigma t}\right\} \\ &= \frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh(kh)}\left[A_I\sin\left(\overrightarrow{k_I}\cdot\vec{x} - \sigma t + \varphi_I\right) + A_R\sin\left(\overrightarrow{k_R}\cdot\vec{x} - \sigma t + \varphi_R\right)\right] \\ &= \frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh(kh)}A_I\sin\left(k\cos\theta x + k\sin\theta y - \sigma t + \varphi_I\right) \\ &- \frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh(kh)}A_R\sin\left(k\cos\theta x - k\sin\theta y - \sigma t + \varphi_R\right)\end{aligned} \tag{4.128}$$

4.2.3.7 Wave train that obliquely impinges on an impermeable vertical wall

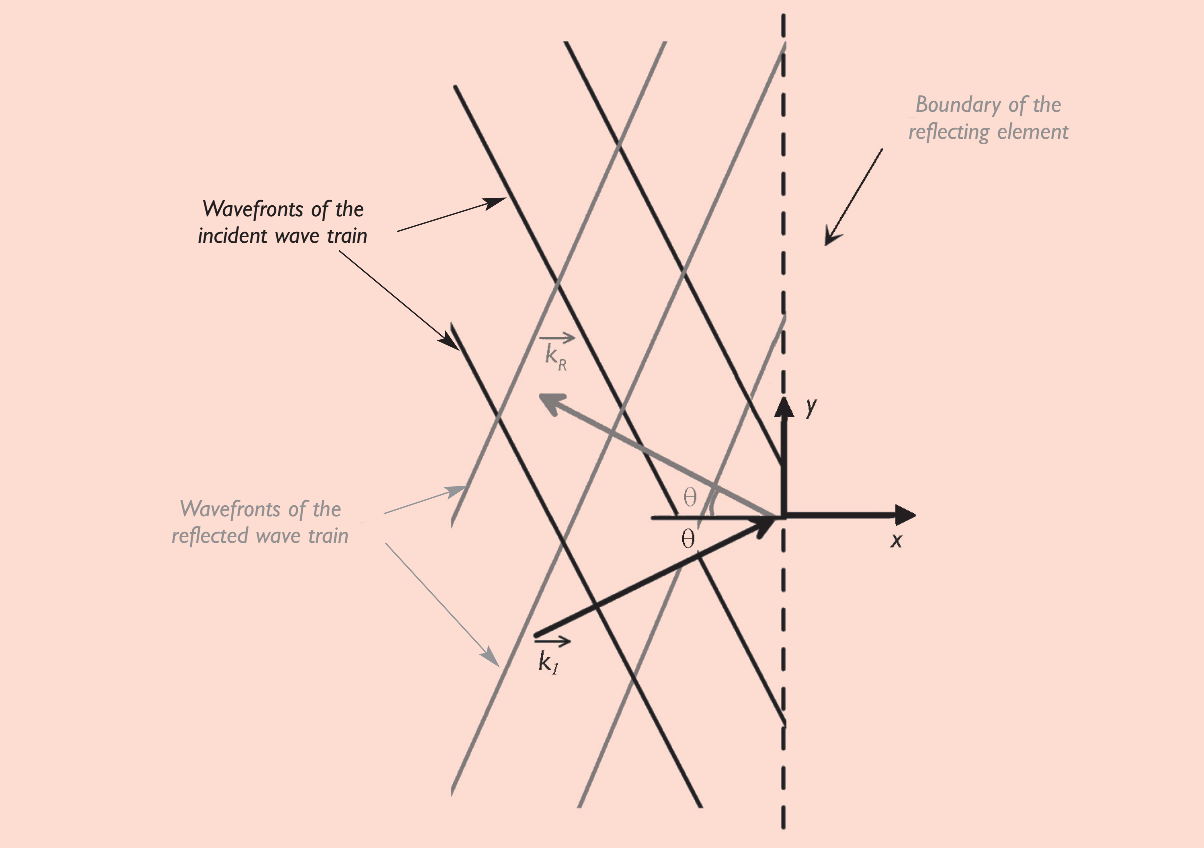
In the case of an impermeable wall, the zero flow condition is always fulfilled and when it is verified,

$$A'_I = A'_R \tag{4.129}$$

In these conditions, a stationary oscillation is produced, whose free surface is

$$\eta = \Re\left\{2A'_I\cos(k_x x)e^{i(k_y y - \sigma t)}\right\} = 2A'_I\cos(k_x x)\cos(k_y y - \sigma t) \tag{4.130}$$

Figure 4.4.3. Incident and reflected wave fronts by a uniform obstacle along the y, axis when the incidence is oblique



On the lines, whose coordinate x is worth $x_a^m = -mL/(2\cos\theta)$ with $m = 0,1,2, \ldots$ the free surface reaches, in an absolute value, the maximum values (antinodes), whereas for those defined by $x_n^m = -L/(4\cos\theta) - mL/(2\cos\theta)$, $m = 0,1,2, \ldots$ there is no movement of the free surface (nodes). The distance between antinodal lines is the following:

$$x_a^m - x_a^{m+1} = \frac{L}{2\cos\theta} \tag{4.131}$$

The first antinodal line is located on the wall, $x = 0$. The first nodal line can be found at $L/(4\cos\theta)$ of the wall, and successive lines are located at distances

$$x_n^m - x_n^{m+1} = \frac{L}{2\cos\theta} \tag{4.132}$$

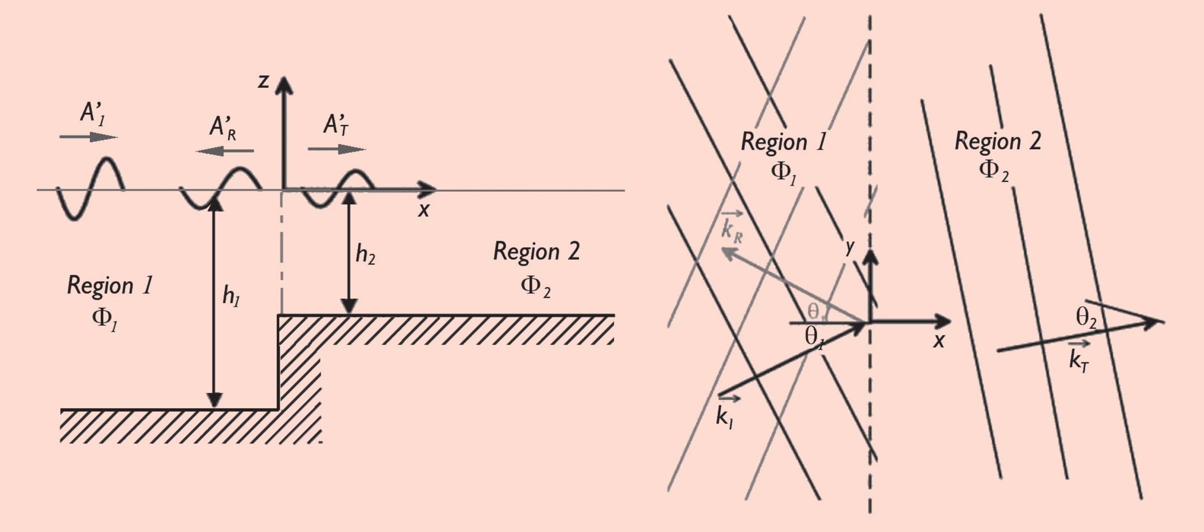
from the previous one.

If the wall is not totally impermeable, part of the energy is transmitted through the wall, and the resulting oscillation upstream from the obstacle is partially stationary.

4.2.3.8 Wave train propagation due to sudden oblique depth change

In this case, the difference in depth produces, apart from the total and partial reflection of the energy, a change in the direction of propagation. To resolve this problem, it is necessary to consider two regions located at both sides of the discontinuity (see Figure 4.4.4).

Figure 4.4.4. *Configuration of the problem of a progressive wave, propagating because of a sudden oblique depth change*



For a differential element of the water column centered in the limit between the two regions, $x = 0$, there is one pressure and one horizontal velocity. For this reason, their expressions, calculated on the basis of the Φ_1 and Φ_2 potentials should coincide. Therefore, it holds that,

$$\left.\begin{array}{l} \Phi_1 = \Phi_2 \\ \dfrac{\partial \Phi_1}{\partial x} = \dfrac{\partial \Phi_2}{\partial x} \end{array}\right\} in \begin{array}{l} x = 0 \\ \\ -h_2 < z < 0 \end{array} \tag{4.133}$$

Furthermore, there cannot be any flow through the vertical wall. Accordingly, it should hold that

$$\frac{\partial \Phi_1}{\partial x} = 0 \; in \; x = 0, \; -h_1 < z < -h_2 \tag{4.134}$$

Once known the complex amplitude and the propagation direction of the wave train A_I and θ_I, the unknowns are (apart from the complex amplitudes of the wave trains reflected and transmitted in $x = 0$) the propagation angle in region 2, θ_2.

For particle movement to be possible at the boundary between the two regions, the wave numbers, according to the direction y at $x = 0$, $k_{y,1}$ and $k_{y,2}$ must be equal.

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 \tag{4.135}$$

This equation gives the relation between the angles, depending on the wave period and different depths. To obtain the rest of the unknowns, the procedure is similar to that for normal incidence. Thus, the conditions are multiplied according to the functions $f_i(z)$ and integrating the result in the domain in which they are defined. When the change in depth is $h_2 > h_1$, it is possible for the values of k_1, k_2 and θ_1 to be such that

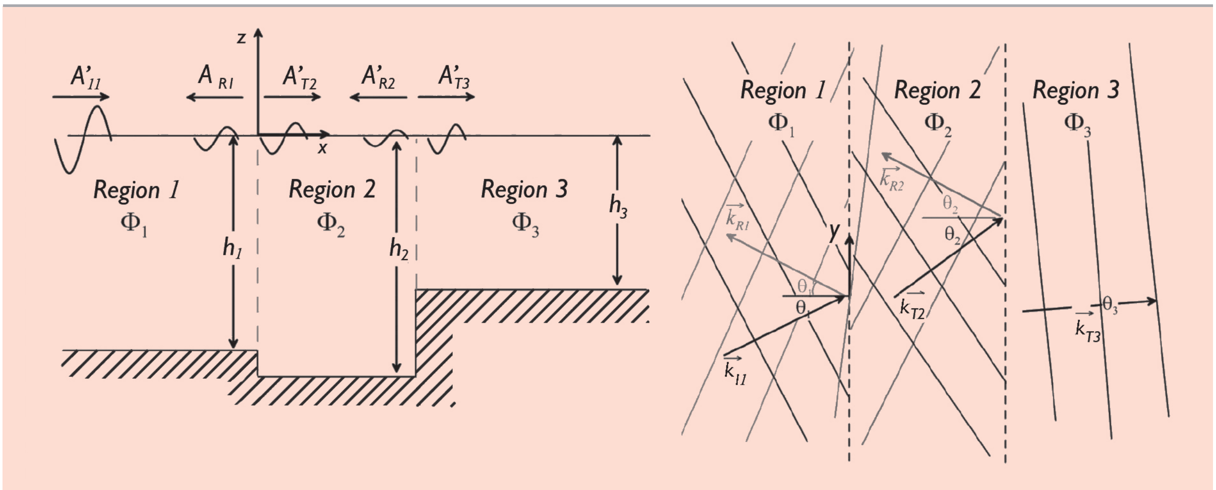
$$\left| \frac{k_1}{k_2} \sin\theta_1 \right| > 1 \tag{4.136}$$

In this case, the problem does not have a real solution in θ_2. In these conditions, the wave does not propagate to the region of greatest depth, and thus the oscillation is confined to region 1. In that case, the wave is said to be trapped in that region.

4.2.3.9 The case of the navigation channel

The presence of a navigation channel can greatly modify the characteristics of the incident wave train. According to linear theory, the calculation is simply an extension of the previous sections. The work domain should be divided in three regions (see Figure 4.4.5) and expressed in each the resulting potential of the superposition of incident/transmitted and reflected wave trains (when relevant), due to changes in depth. In this case, there is a double change in the propagation direction, given by

Figure 4.4.5. Configuration of the problem of a progressive wave, obliquely propagating in a channel



As previously mentioned in regards to one change in depth, a wave may be trapped in region 1 without propagating through the channel to the third region. In the case of a submerged breakwater, this entrapment may occur on the breakwater, which would then become an integral barrier for this oscillation.

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 \tag{4.137}$$

4.2.4 Numerical models based on the first-order Stokes regime

The vertically integrated models formulated in the first-order Stokes regime are widely used in maritime engineering because of the accuracy of their results as well as their relative simplicity. Given their linear nature, they can be applied to the study of a regular wave train represented by the basic variables H_0, T, θ_0, (where the subscript 0 refers to the initial point or reference point), to an irregular wave train represented by the state descriptors, $H_{rms,0}$, $\overline{T_z}$, $\overline{\theta_0}$, and to each component of the spectral density function, formulated in the frequency domain $S(f,\theta)$ or in the wavenumber domain $S(\vec{k},\theta)$.

4.2.4.1 Mild slope models: MSPE and approximations

If the seabed slope α is mild, it is possible to again separate the calculation of the vertical coordinate from the calculation of the horizontal coordinates. Depending on the relative values of the adimensional monomials, the following depth-integrated models hold.

At any depth, with slight variations, $\alpha \ll kh$ and with weak non-linearity, the equations can be expanded into a Stokes series. This results in the mild slope equation (MSPE). In order for the behavior of this equation in shallow water not to be overly divergent, it should satisfy the condition, $H/h \ll (kh)^2$.

This equation has a hyperbolic, non-stationary solution. Its stationary form assumes the harmonic nature of the waves. In its most general form, the equation to be resolved is elliptic. The stationary version of the MSPE allows additional approximations, known as the parabolic and hyperbolic approximations. The terms, *hyperbolic, elliptic*, and *parabolic,* are in consonance with the classification of second-order differential equations, expressed in canonical form.

Note. From a physical viewpoint, the integration of the MSPE equation in its non-stationary form requires the specification of the initial conditions of the velocity potential or the free surface as well as the boundary conditions at all of the boundaries of the integration domain. The resolution of the elliptic version of the stationary MSPE equation requires the specification of the problem at all of the boundaries of the integration domain. The parabolic equation provides solutions that advance through space in only one sense. For this reason, it is enough to specify the problem with upstream boundary conditions. Finally, the hyperbolic equation has solutions of waves that propagate, according to characteristic curves. Moreover, by using a Fourier decomposition, it is possible to decompose an arbitrary signal into its harmonic components and simulate turbulence conditions, induced by irregular waves, thanks to the stationary version of the MSPE equation. In any case, to obtain information concerning the transitory effects induced by irregular wave trains, it is necessary to use the non-stationary mild slope equation or another type of non-stationary, phase-resolving model (e.g. Boussinesq equations).

4.2.4.1.1 Elliptic model

The stationary linear MSPE equation, which includes the first-order contributions to the seabed variation, is the following:

$$\nabla \cdot \left(CC_g \nabla \phi \right) + k^2 CC_g \phi = 0 \tag{4.138}$$

where is the velocity potential, and C, C_g are the phase velocity and the group velocity, respectively.

4.2.4.1.2 Parabolic approximation

A parabolic approximation of the MSP is the following:

$$\frac{\partial \phi}{\partial x} - \left(ik - \frac{1}{2kCC_g} \frac{\partial \left(kCC_g \right)}{\partial x} \right) \phi - \frac{1}{2kCC_g} \frac{\partial \left(CC_g \frac{\partial \phi}{\partial y} \right)}{\partial y} = 0 \tag{4.139}$$

where x is the main propagation direction of the wave train.

4.2.4.1.3 Hyperbolic approximation

The hyperbolic approximation of elliptic MSPE equations differs from non-stationary, hyperbolic mild-slope equations.

$$\nabla \cdot \left(CC_g \nabla \eta \right) - \frac{C_g}{C} \frac{\partial^2 \eta}{\partial t^2} = 0 \tag{4.140}$$

This hyperbolic equation is expressed with a pair of first-order equations:

$$\nabla Q + \frac{C_g}{C} \frac{\partial \eta}{\partial t} = 0 \tag{4.141}$$

$$\frac{\partial Q}{\partial t} + CC_g \nabla \eta = 0 \tag{4.142}$$

where Q is an integration function on the vertical.

4.2.5 Numerical models based on the Boussinesq regime

This type of numerical model is based on Boussinesq equations, namely, a continuity equation and momentum equation, whose solution is transitory. These equations have two important parameters: (i) relative depth (dispersion), $\mu = kh$, where k is the wave number and h is the depth; (ii) non-linearity defined as $\varepsilon = H/L$, where H is the wave height.

The derivation of Boussinesq-type equations does not consist of only one procedure, but rather are obtained under different assumptions. For this reason, there are a wide variety of approximations. One of the main differences lies in the order of the linear and non-linear dispersive terms in the formulations. Consequently, conventional equations are known as weakly non-linear since they conserve the hypothesis of assuming the non-linearity and dispersion to the same order of approximation. Furthermore, if higher-order terms are included in the equations, the models are known as completely non-linear. This means that all the velocity expansion terms up to the second order are retained in the expansion of the equations. Boussinesq equations that assume that the frequency dispersion and non-linear effects are weak provide an accurate description of wave evolution in coastal zones, as long as the theory is applied within the validity limits of each individual approximation.

The classification of Boussinesq numerical models can be based on the resolution methods of governing equations. Generally speaking, they can be grouped in two large families: (i) models that use finite-difference schemes with structured grids; (ii) models that use the finite-element method on non-structured triangular grids. Funwave is a public domain Boussinesq model that resolves the continuity equation and the non-linear momentum equation. These equations are, respectively:

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \mathrm{M} = 0 \tag{4.143}$$

$$\frac{\partial \bar{u}_\alpha}{\partial t} + \varepsilon \left(\bar{u}_\alpha \cdot \nabla\right)\bar{u}_\alpha + \nabla \eta + \mu^2 \mathrm{V}_1 + \varepsilon\mu^2 \mathrm{V}_2 - O\left(\mu^4\right) = 0 \tag{4.144}$$

where

$$\mathrm{M} = \left(h + \varepsilon\eta\right)\left[\bar{u}_m + \mu^2 \left\{ \begin{array}{l} \left[\frac{1}{2}z_\alpha^2 - \frac{1}{6}\left(h^2 - h\varepsilon\eta + \left(\varepsilon\eta\right)^2\right)\right]\nabla\left(\nabla \cdot \bar{u}_\alpha\right) \\ + \left[z_\alpha + \frac{1}{2}\left(h + \varepsilon\eta\right)\right]\nabla\left(\nabla \cdot \left(h\bar{u}_\alpha\right)\right) \end{array} \right\}\right] + O\left(\mu^4\right) \tag{4.145}$$

$$\mathrm{V}_1 = \frac{1}{2}z_\alpha^2 \nabla\left(\nabla \cdot \frac{\partial \bar{u}_\alpha}{\partial t}\right) + z_\alpha \nabla\left(\nabla \cdot \left(h\frac{\partial \bar{u}_\alpha}{\partial t}\right)\right) - \nabla\left[\frac{1}{2}\left(\varepsilon\eta\right)^2 \nabla \cdot \frac{\partial \bar{u}_\alpha}{\partial t} + \varepsilon\eta\nabla \cdot \left(h\frac{\partial \bar{u}_\alpha}{\partial t}\right)\right] \tag{4.146}$$

$$\mathrm{V}_2 = \nabla\left[\begin{array}{l} \left(z_\alpha - \varepsilon\eta\right)\left(\bar{u}_\alpha \cdot \nabla\right)\left(\nabla \cdot \left(h\bar{u}_\alpha\right)\right) + \\ + \frac{1}{2}\left(z_\alpha^2 - \left(\varepsilon\eta\right)^2\right)\left(\bar{u}_\alpha \cdot \nabla\right)\left(\nabla \cdot \bar{u}_\alpha\right) \end{array}\right] + \frac{1}{2}\nabla\left[\left(\nabla \cdot \left(h\bar{u}_\alpha\right) + \varepsilon\eta\nabla \cdot \bar{u}_\alpha\right)^2\right] \tag{4.147}$$

Note. In these expressions, when the order of development is expanded, the range of validity is increased to waters of greater depth. This is mostly due to their capacity of more effectively resolving the problem of the interaction between components.

4.2.6 Wave train transformation in the surf zone

The moment when the wave train breaks, the breaker height, and its evolution in the surf zone (or on the slope) depends on the characteristics of the incident wave train. These include the water depth [$h_{min} \leq h \leq h_{max}$], bottom slope, tan β, and whether there are currents or other wave trains (e.g. reflected wave trains), apart from the incident wave train.

4.2.6.1 Breaking criteria of a wave train

The physical boundary of the wave height can be determined, according to wave steepness, $(H_I/L)_{\text{lim}}$, or relative wave height [9] $(\gamma_I)_{\text{lim}} = (H_I/L)_{\text{lim}}$.

The steepness criterion is applied in the Stokes wave regime, whereas the breaking index is more frequently applied in the Boussinesq wave regime. In both cases, the value of h is the total water depth, including medium-period and long-period oscillations.

Note. *When the wave breaks (or does not break) was analyzed by Stokes, who proposed two cinematic hypotheses or conjectures. The wave begins to break (1) when the orbital velocity of the water particle in crest u_c exceeds the propagation velocity of wave C; (2) when the vertical acceleration of the water particle in crest a_z exceeds the acceleration of gravity g.*

4.2.6.1.1 Horizontal bed or with a very mild slope

On a horizontal bed or on a very mild slope, when there is no current and wind on the surface, the steepness limit of a wave train depends on the relative depth and the maximum steepness in deep water.

Progressive wave train. In the Stokes regime, the steepness limit is

$$\left(\frac{H_I}{L}\right)_{lim} \leq 0.142 \tanh\frac{2\pi h}{L} \approx \frac{1}{7}\tanh\frac{2\pi h}{L} \tag{4.148}$$

In the Boussinesq regime (solitary wave), for the same conditions, the breaking index limit (relative wave height) is

$$(\gamma_I)_{lim} \equiv \left(\frac{H_I}{h}\right)_{lim} \lesssim 0.80 \tag{4.149}$$

where H_I is the incident wave height.

Stationary wave train. In the Stokes regime, the steepness limit of two wave trains with the same period and the same height H_I, but traveling in opposite directions, is the following:

$$\left(\frac{H_I}{L}\right)_{lim} \leq 0.11 \tanh\frac{2\pi h}{L} \tag{4.150}$$

Partially stationary wave train. In the Stokes regime, the steepness limit of the two wave trains of the same period, one of which as height, H_I, and the other has height, RH_I, but traveling in opposite directions, can be approximated by,

$$\left(\frac{H_I}{L}\right)_{lim} \leq \left(0.11 + 0.03\frac{1-|R|}{1+|R|}\right)\tanh\frac{2\pi h}{L} \tag{4.151}$$

where $|R|$ is the module of the reflection coefficient.

Note. *Both criteria are complementary, and it is enough to approximate the hyperbolic tangent by means of its argument for the steepness limit to be transformed into the breaker index, though with a slightly higher value (approximately 0.88). Moreover, the steepness limit converges with the steepness limit in deep water (see section 3.4.6) (when $h/L > 1/2$, $\tanh(2\pi h/L) \Rightarrow 1$).*

(9) This is also known as the *breaking index*.

The breaking criterion for a partially stationary wave train can be applied when the waves are reflected against a breakwater or a cliff formation, and $|R| < 1$ *is the reflection coefficient of the incident wave train.*
When applying these breaking criteria to the individual waves of an irregular wave train, it should be taken into account that these waves generally break with values somewhat lower than those of the wave train.

4.2.6.1.2 *Steepness limit on a sloping seabed*

Generally speaking, when the seabed has a mild slope, $m = \tan\beta \lesssim 1/15$, the wave steepness limit differs very slightly in respect to that that of the horizontal bottom.

However, when necessary, the following equations should be applied.

If the seabed slope is mild, $m \leq 1/40$, and the relative depth is $h/L > 1/20$, the limit value of the wave height can be evaluated, based on the possible steepness limit, by using with the following equation,

$$\left(\frac{H_I}{L_0}\right)_{lim} = a_1\left\{1-\exp\left[-a_2 k_{z0} h\left(1+a_3 (m)^{\frac{4}{3}}\right)\right]\right\} \tag{4.152}$$

where the experimental coefficients are $a_1 = 0.17$, $a_2 = 0.75$, and $a_3 = 15$, and $k_{z0} = 2\pi/L_{z0}$ and L_{z0} are the wave number and wavelength in deep water, respectively.

If the seabed slope is in the interval, $1/15 \lesssim m \leq 1/40$, the following equation should be applied,

$$\begin{aligned} \left(\frac{H}{gT^2}\right)_{lim} &= 2\pi\alpha_{lim}\tanh\left[\frac{1}{2\pi}\beta_{lim}\left(\frac{h}{gT^2}\right)^{\gamma_{lim}}\right] \\ \alpha_{lim} &= 0.0277 \\ \beta_{lim} &= 17.7 \qquad (m < 1/15) \\ \gamma_{lim} &= 0.90 \qquad (m < 1/10) \end{aligned} \tag{4.153}$$

When the seabed slope is $1/15 < m = \tan\beta < 1/7$, the breaker steepness and the breaker type, and consequently, the coefficients of the limit steepness equation depend on it,

$$\begin{aligned} \left(\frac{H}{L_0}\right)_{lim} &= 2\pi\alpha_{lim}\tanh\left[\frac{1}{2\pi}\beta_{lim}\left(\frac{h}{L_0}\right)^{\gamma_{lim}}\right] \\ \alpha_{lim} &= 0.0277 \\ \beta_{lim} &= 152m + 6.6 \qquad (m \geq 1/15) \\ \gamma_{lim} &= 1.92m + 0.72 \qquad (m \geq 1/10) \end{aligned} \tag{4.154}$$

Note. *When the seabed slope is very mild, the equation asymptotically approximates the limit steepness equation on the horizontal bed (section 3.6.4.1).*

4.2.6.2 *Limit relative wave height*

The relative wave height limit is defined by the quotient of the wave height and its water depth (H/h). When it is expressed in the breaking point, it is usually known as the breaking index γ_b of $(\gamma_I)_{\mathrm{lim}}$.

The subscript I indicates that the wave height is that of the incident wave train.

4.2.6.2.1 *Spilling and plunging breakers*

For flat beds and beds with a mild slope, $tan\ \beta < 1/60$, waves whose Iribarren number satisfies $I_{r,I} < 2$, the limit value of the wave height can also be calculated based on the breaking index, according to the following equation [(10)].

$$(\gamma_I)_{lim} = \left(\frac{H_I}{h}\right)_{lim} \le b_1 I_{r,I}^{b_2} + b_3 \tag{4.155}$$

where $[b_1 \approx 1,\ b_2 \approx 0.17,\ b_3 \approx 0.08]$ are experimental coefficients that depend on the slope and its permeability. Generally speaking, the increase in the permeability of the slope reduces the heat of the breaker.

4.2.6.2.2 *Collapsing and surging breakers*

On flat, impermeable beds and relative depths, $h/L < 1/20$, waves whose Iribarren number is $I_{r,I} > 2$, the limit value of the wave height can be calculated with the following equation,

$$(\gamma_I)_{lim} = \left(\frac{H_I}{h}\right)_{lim} \le 0.84 \exp\left(6.42 S_{r0}\right) \tag{4.156}$$

where,

$$S_{r0} = 1.52 \frac{\tan\beta}{\sqrt{\dfrac{H_I}{h}}} \tag{4.157}$$

In these cases, the wave reflection on the slope plays an important role in the breaking process.

Note. *The equation of the breaking index, depending on S_{r0}, is implicit, and it should be numerically resolved. The breaking index increases as the wave height decreases, which means that waves of a small height can reach values close to nine, and thus break very near the shoreline. Moreover, when the slope tan$\beta > 1/5$, long waves whose slope parameter is $S_{r0} > 0.37$ do not break, and as a result, are totally reflected.*

4.2.6.2.3 *Spilling breaker on a flat bed*

For a spilling breaker with a constant breaking index all along the slope, the surf zone is said to be saturated. In these conditions, there are various waves breaking, the dissipation of each wave is gradual, and the variation in wave height can be described by an exponential law (following the variation in depth of the beach profile),

$$H_{zr}(x) = H_b \exp\left[\alpha\left(x - x_b\right)\right] \tag{4.158}$$

where H_b is the height of the breaker; α is a dissipation parameter; and x_b is the distance from the coastline to the breaking point measured in the direction normal to the wave crest. The dissipation parameter depends on the breaker type. In the case of spilling or plunging breakers, the parameter value can be obtained by modeling the dynamics and kinematics of the water volume involved in the breaking [(11)].

On beach profiles with a slope $tan\ \beta < 1/60$, it is best to assume that the surf zone is saturated. In other words, at each depth, the following relation is satisfied:

(10) This equation was obtained for impermeable beds, but it cal also be applied to muddy bottoms and those with fine and coarse sand. In those cases, it is expected that the real maximum wave height will be lower than calculated wave height.
(11) This volume is known as a *roller*.

$$H_{zr}(x) \approx (\gamma_1)_{lim} h_{zr}(x) \qquad (4.159)$$

where $h = h_{zr}$ is the total depth in the surf zone, including the contribution of short-period, medium-period, and long-period oscillations.

4.2.6.3 Evolution of the energy of wave trains in the irregular surf zone

In these conditions, it is advisable to represent the broken wave by the superposition of the oscillatory part upon which the body of water or roller rides. The roller extracts the energy of the wave by rotating on it. The propagation of the wave train in the surf zone can be obtained by solving the equation system formed by the conservation of mass, momentum (including the roller contribution), and the middle equation.

4.2.6.3.1 Stable energy flux of the broken wave

On occasions, the bathymetry shows a gradual change in depth and the subsequent reduction of the slope. In these cases, when it is a spilling or non-violent plunging breaker, the broken wave first evolves rapidly, and then stabilizes and reaches an equilibrium height. During the breaking process, the ordered energy of the wave is transformed in turbulence and heat.

The analogy between the broken wave and the hydraulic jump suggests that the energy loss can be expressed in the same way as the dissipation in a bore. In these conditions, it is possible to describe the attenuation of the energy in the surf zone, admitting that at a certain depth for an individual wave, the energy dissipation produced by the breaking per unit of area D_* is proportional to the excess energy flux with respect to the presumably stable energy flux F_S. This quantity corresponds to the maximum wave height H_S at which the wave does not break, and whose value depends on the local depth $H_S = \Gamma d$, where Γ is a constant value. This is

$$D_* = \frac{\kappa}{d}(F - F_S) = \frac{\kappa}{d}\frac{1}{8}\rho g C_g \left(H^2 - H_s^2\right) = \frac{\kappa}{d}\frac{1}{8}\rho g C_g \left(H^2 - (\Gamma d)^2\right) \qquad (4.160)$$

where κ is an empirical parameter. Consequently, the energy conservation equation that is resolved is the following:

$$\frac{dF}{dx} = \begin{cases} -\frac{\kappa}{d}\frac{1}{8}\rho g C_g \left(H^2 - (\Gamma d)^2\right) & si\ H > \Gamma d \\ 0 & si\ H \le \Gamma d \end{cases} \qquad (4.161)$$

The values of κ and Γ depend on the breaker type, thus, on the Iribarren number. For spilling and plunging breakers, the values considered acceptable are $\Gamma = 0.4$ and $0.15 \lesssim \kappa \lesssim 0.20$.

4.2.6.3.2 Beach profile with a constant slope

When the beach profile has a constant slope, $h(x) = mx$, where m is the beach slope, the following analytical solution can be obtained,

$$\left(\frac{H(x)}{H(x_b)}\right)^2 = (1+\alpha_r)\left[\frac{h(x)}{H(x_b)}\right]^{\left(\frac{\kappa}{m}-\frac{1}{2}\right)} - \alpha_r \left(\frac{h(x)}{H(x_b)}\right)^2 \qquad (4.162)$$

$$\alpha_r = \frac{\kappa \xi_e}{m\left(\frac{5}{2} - \frac{K}{m}\right)} \left(\frac{h(x_b)}{H(x_b)}\right)^2 \qquad (4.163)$$

When $\kappa/m = 5/2$, the solution is

$$\left(\frac{H(x)}{H(x_b)}\right)^2 = \left(\frac{h(x)}{H(x_b)}\right)^2 \left[1-\beta \ln\left(\frac{h(x)}{H(x_b)}\right)\right] \tag{4.164}$$

$$\beta = \frac{5}{2}\xi_e \left(\frac{h(x_b)}{H(x_b)}\right)^2 \tag{4.165}$$

The energy dissipated in the breaking process is
La energía disipada en el proceso de la rotura es,

$$\frac{\Delta E}{\rho_w g} \approx \frac{1}{8}\left[H_b^2 - H_e^2\right] \tag{4.166}$$

This equation should be simultaneously resolved with the equation of the sea level variation in the surf zone.

Note. This approximation can be applied to obtain the energy reduction of each frequency spectrum component because of wave breaking. This is done by uniformly distributing the dissipation among all frequency components. In addition, it can be used to obtain the probability function of the wave height at a point in the surf zone. With the required conditions, this problem can be resolved with models in the public domain SMC (Sistema de Modelado Costero) [System of Coastal Modeling] at www.smc.unican.es.

4.3 FOUNDATIONS OF WAVE DESCRIPTION

4.3.1 State, basic, and instantaneous variables

The instantaneous sea wave variable, in the same way as for other sea oscillations, is the vertical displacement of the free sea surface, $\eta(t, \vec{x})$ in a given time interval with respect to a reference level. It is a continuous variable that is registered by means of pressure sensors, level sensors, accelerometers, etc. Its analysis is performed by taking samples at half-second time intervals.

The sea description is based on the assumption that at deep water, in a specific time interval and spatial domain, $\eta(t, \vec{x})$ is a random Gaussian variable, whose variance, σ_η^2, is proportional to the energy of the process,

$$\sigma_\eta^2(\vec{x}) = \frac{1}{D_t}\int_t^{t+D_t}\left[\eta(t,\vec{x}) - \bar{\eta}\right]^2 dt \tag{4.167}$$

where $\bar{\eta}$ is the mean vertical displacement of the record during the time interval or duration, D_t. This hypothesis loses validity as the waves are transformed by boundaries and the seabed, as well as by the passing of time.

In shallow water, $\eta(t, \vec{x})$ is no longer a Gaussian variable. However, in most cases, its deviation is not significant in maritime engineering.

Note. A practical way of getting around this limitation is, on the one hand, to limit the duration of the interval by defining the sea state, and on the other, to limit the space, by defining the subset. In these conditions, it can be admitted that in the subset and the state: (1) $\eta(t, \vec{x})$ is an approximately random Gaussian variable; (2) the record measured at a point of the subset contains necessary and sufficient information to obtain the statistical descriptors and the spectral function that are representative of the expected stochastic variability in the state.

4.3.1.1 Statistic-mathematical model

If the vertical displacement on the free surface is expressed at a point in the sea $P(x, y)$ with respect to a reference level at $z = 0$.

$$\eta(t) = \sum_n a_n \sin(\omega_n t + \varphi_n) \tag{4.168}$$

and

$$\eta^2(t) = \sum_n \sum_m \frac{1}{2} a_n a_m \left\{ \cos\left[(\omega_n - \omega_m)t + (\varphi_n - \varphi_m)\right] - \cos\left[(\omega_n - \omega_m)t + (\varphi_n - \varphi_m)\right]\right\} \tag{4.169}$$

By averaging data over a long period of time, the mean total energy per unit of horizontal surface obtained is

$$E_T = \frac{1}{D_t} \int_t^{t+D_t} \eta^2(t) dt = \sum_n \frac{1}{2} a_n^2 \tag{4.170}$$

and the variance of the sea state is equal to the sum of the variances of the spectral components. If one considers the variance ΔE in an interval of frequencies *(f, f, Δf)*, then a function $S(f)$ can be defined, called the wave variance spectrum, such that,

$$S(f) = \frac{\Delta E}{\Delta f} \tag{4.171}$$

$S(f)$ is finite when $\Delta f \Rightarrow 0$. This can be used to obtain the following:

$$E_T = \int_0^\infty S(f) df \tag{4.172}$$

The spectral density function of the waves or wave energy spectrum is obtained by multiplying $S(f)$ by $\rho_w g$,

$$E(f) = \rho_w g S(f) \tag{4.173}$$

Generally speaking, the free surface displacement as a function of time and position can be written as

$$\eta(x,y,t) = \sum_n \sum_m a_{nm} \cos\left[k_n (x \cos\theta_m + y \sin\theta_m) - 2\pi f_n t + \varphi_{nm}\right] \tag{4.174}$$

assuming that the sea state is a superposition of N sinusoidal waves (spectral components) (see Figure 4.4.6). Each of the waves is traveling with its own amplitude a_n, angular frequency f_n, wave number k_n, in different directions θ_m and with different phases φ_{nm}. Consequently, each component satisfies the dispersion equation, given by

$$(2\pi f_n)^2 = g k_n \tanh k_n h \tag{4.175}$$

where h, represents the depth. When the phases φ_n are assumed to be uniformly distributed in a random way in interval $[0, 2\pi]$, this gives

$$\sum_f^{f+\Delta f} \sum_\theta^{\theta+\Delta\theta} \frac{1}{2} a_n^2 = S(f,\theta) df d\theta \tag{4.176}$$

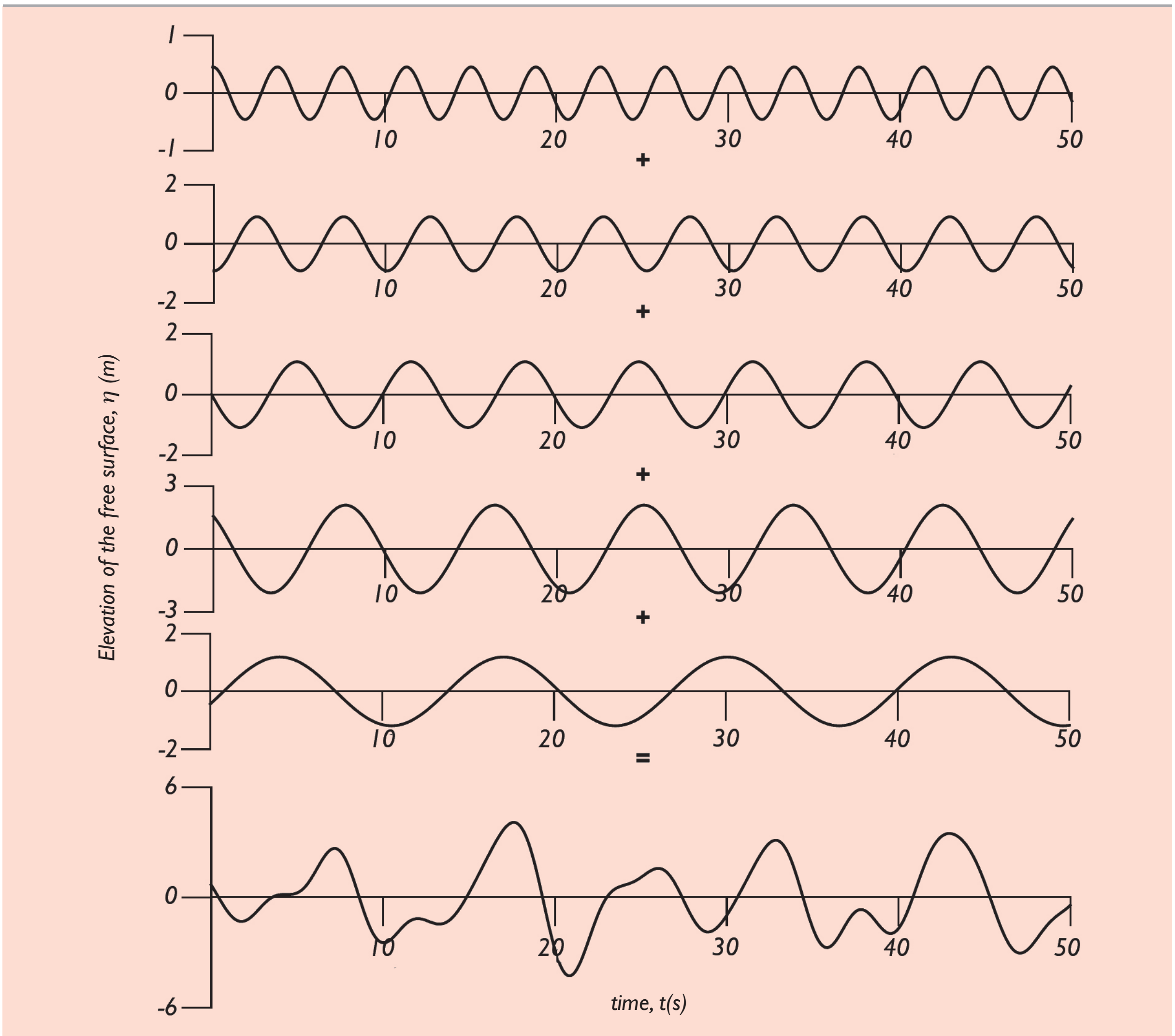
$$E_T = \int_0^\infty \int_{-\pi}^{\pi} S(f,\theta) df d\theta \tag{4.177}$$

4.3.1.2 Basic wave variables: description parameters

Once the reference level is defined in the record, the time between two upwards crossings of the reference level is known as upwards zero-crossing period, T_z. The inverse of the period is known as frequency f (with units $H_z = 1/s$), and if it is expressed in radians/second, it is known as angular frequency, ω [12],

(12) The angular frequency is usually defined by σ. It is generally when there is a current, and known as the *intrinsic frecuency*. In this case, the dispersion equation of the linear wave train is the following $\sigma^2 = (\omega - kU)^2 = gk \tanh kh$

Figure 4.4.6. Simulation of the sea as a linear superposition of sinusiodal waves of different amplitude and period and random phase



$$f = \frac{1}{T_z} \quad (4.178)$$

$$\omega = \frac{2\pi}{T_z} \quad (4.179)$$

Between two upwards zero-crossings, the following magnitudes can be defined (see Figure 4.4.7).

H_z, wave height: maximum vertical distance;
η_c, crest height: vertical distance from the reference level to a relative (concave) maximum;
η_s, trough height: vertical distance from the reference level to a relative (convex) minimum;
a_c, crest amplitude: maximum crest height;
a_t, trough amplitude: maximum trough height.

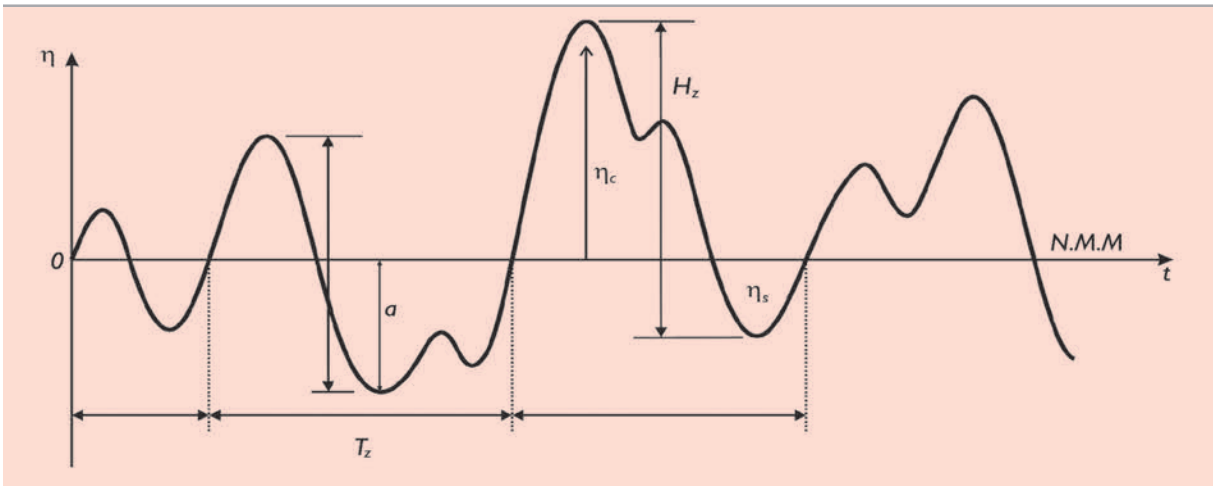
Apart from the upwards zero-crossing, the following periods are usually identified:

T_c, crest period: time interval between consecutive crests;
T_s, trough period: time interval between consecutive troughs.

In consonance with each of the period definitions, the following horizontal distances or lengths are defined:

L_z, wave length: horizontal distance between two consecutive upwards crossings of the reference level;
L_c, crest length: horizontal distance between two consecutive crests;
L_s, trough length: horizontal distance between two consecutive troughs.

Figure 4.4.7. Spatial and temporal parameters that define irregular oscillations on the sea surface in the sea wave band



The wave number k is defined (in units of radians/meter):

$$|k| = \frac{2\pi}{L_z} \tag{4.180}$$

The wave number is a vector magnitude of components, $\vec{k} = (k_x, k_y)$,

$$k_x = |k| \sin\theta_w \tag{4.181}$$

$$k_y = |k| \cos\theta_w \tag{4.182}$$

where θ_w is the wave propagation direction.

The wavelength L_z is related to the period, T_z and the water depth h by the dispersion equation, which in linear theory and in the absence of the current is

$$\omega^2 = gk \tanh kh \tag{4.183}$$

The wave steepness is the quotient of the wave height and length, $s_s = H_z/L_z$ and the celerity C is the velocity at which oscillatory movement propagates. It is a vector magnitude, $\vec{C} = (C_x, C_y)$. When the wave propagates and the form is conserved, this satisfies

$$|C| = \frac{L_z}{T_z} \tag{4.184}$$

$$C_x = \frac{\omega}{k_x} \qquad C_y = \frac{\omega}{k_y} \tag{4.185}$$

In a wave state, the values of the wave height and period (H_z, T_z) are random variables with a joint probability distribution, whose parameters depend on state descriptors. In addition, these state descriptors depend on the age (13) of the waves, their relative depth, and thus, on the sea level. Consequently, it is the same for all the magnitudes derived from the wave height and period (e.g. celerity, wavelength, and steepness). Similarly, the maximum values of the record, ($H_{z,max}$, $a_{c,max}$, $a_{s,max}$, $\eta_{c,max}$, $\eta_{t,max}$), are random variables, whose distribution function depends on the wave number in the record, in other words, its duration.

Note. *Although in maritime engineering, the definition by upwards crossings is the most usual one, there are other ways of defining the basic wave variables. One of them is to select the downwards crossing period as a time interval of the statistic trial, and at each time interval, define the basic wave variables. It is also possible to select crests and troughs, and define the crest period and trough period. Since it is the standard definition, the wave height defined by upwards crossings is not usually identified by the subscript z. However, it is advisable to maintain it in the period.*

4.3.1.3 Probability density functions

By definition, the basic variables are random. In a sea state, a record of N waves can be used to create a combined sample or individual samples of basic variables, heights, periods, crest and trough amplitudes, etc. and to infer their joint and individual probability density functions. The density function parameters are state descriptors. For example, in certain conditions, the wave heights in a sea state follow a Rayleigh distribution with the mean square wave height as the parameter in the record.

4.3.1.4 Statistical state descriptors

In a sea state, there are oscillations of the free sea surface in the sea wave period band, which locally satisfies the hypothesis of stationarity and statistical uniformity in the widest sense. Once a representative sample of sea state wave heights, defined by the upwards zero-crossing, the statistical values generally used as state descriptors are the following:

$\overline{H}$, mean height or sample mean value;
H_{rms}, mean square wave height or sample mean square wave height;
H_s, significant wave height or mean value of the top third of the wave heights;
$H_{1/n}$, mean value of the 1/n of the highest wave heights of the sample
H_{max}, maximum wave height in the wave height sample.

The most usual periods used as a state descriptor are

$\overline{T}_z$, mean period of the upwards zero crossing or the mean value of the sample;
T_s, significant period, mean value of the top third of the largest wave periods;
T_{H_s}, mean period of the top third of the waves of the record;
$T_{H_{max,N}}$, period of the maximum wave height in the state;
$\overline{\theta}_w$, mean propagation direction in respect to the geographic North;
$\overline{L}_z$, wavelength associated with the mean period.

The state descriptor of the wavelength can be determined by applying the dispersion equation,

$$\overline{L_z} = \frac{g\left(\overline{T_z}\right)^2}{2\pi}\tanh\left(\frac{2\pi h}{\overline{L_z}}\right) \tag{4.186}$$

(13) The age of the sea waves is defined by the quotient of their celerity and generating wind speed (C/U_{10}) which depends, among other things, on the time of generation and fetch. During the generation phase, the sea is said to be partially developed and $(C/U_{10}) < 1$, whereas at the end of this phase, the sea is said to be totally developed, and $(C/U_{10}) \gtrsim 1$.

In deep water, $h/Ls > 1/2$, the wave length is approximately $\overline{L}_{z,0} \approx 1.56(\overline{T}_z)^2$, and in shallow water, $h/\overline{L}_s < 1/20$, $\overline{L}_z \approx \sqrt{gh}(\overline{T}_z)$, in SI units $(g = 9{,}81\ m/s^2)$, $\overline{T}_z$ in seconds, and $\overline{L}_{z,0}$, $\overline{L}_z$, and h in meters. The significant celerity (m/s) is defined by

$$C_s = \frac{H_s}{T_{H_s}} \tag{4.187}$$

It is advisable to define the mean propagation direction $\overline{\theta}_w$ in respect to the directional spectrum, or when relevant, in terms of the propagation direction of the wavefronts with the greatest wave height.

Generally speaking, the sea state should be characterized by the statistical descriptors of the significant incident wave height at the project site $H_{s,I}$ or the mean square wave height $H_{rms,I}$ and the mean period of the upwards zero-crossing $\overline{T}_z$, the mean propagation direction , in reference to the geographic North, and when relevant, a wave group descriptor.

The significant steepness does not an exact physical meaning, but in the same way as for the regular wave train, it is a measurement of the steepness of the irregular wave train. In deep water, it is the following:

$$s_s = \frac{H_{m_0}}{\overline{L}_{z,0}} = \frac{2\pi H_{m_0}}{g\overline{T}_z^2} \tag{4.188}$$

Note. *The significant steepness for the storms in the North Atlantic (fetch on the order of $1000\ Km$) should be in the interval $0.045 \lesssim S_s \lesssim 0.055$, whereas in the Alboran Sea (fetch on the order of $200\ Km$), the steepness values should be in the interval $0.055 \lesssim S_s \lesssim 0.065$.*

4.3.1.5 Visual state descriptors

For many years, the most trustworthy source of information regarding sea states was the visual database containing the data as observed by ships on their sea voyages. This information was transmitted, organized, and managed by the WMO, and implemented by different international meteorology institutes. The visual wave description parameters are the following:

H_v, visual wave height;
T_v, visual wave period;
θ_v, visual direction of the waves;
$H_{v,\max}$, height of the maximum visual wave of the storm;
$T_{v,\max}$, period of the maximum visual wave associated with $H_{v,\max}$.

The visual descriptors are equivalent to the statistical state descriptors. They usually have a statistical correlation, for which the best fit curve is usually a polynomial function (generally of two parabolic terms), although it is more frequent to opt for a linear solution.

Visual descriptors are individual values of the wave. The relation between visual descriptors and statistical or spectral descriptors are specific of the work zone. For this reason, they should not be extrapolated from one site to another without verification.

Note. *To correlate visual wave height and significant wave height, the ROM 0.3-91 recommended one of the empirical validity relations, such as the following:*

$$H_s = 1.23 + 0.88H_v$$

This linear relation can be used in preliminary studies and project calculations. In other cases, and as recommended in the ROM 0.3-91, its validity should be verified with instrument measurements taken at the project site.

4.3.1.6 Description of the sea state in the frequency domain

A time series can be decomposed into a number of individual periodic components, each of which has its own frequency and propagation direction. Figure 4.4.8 shows the spectral representation in the frequency domain of the free surface, constructed from the superposition of five regular waves of Figure 4.4.6. The basic variables of irregular wave action in the frequency domain are the mean total energy per unit of horizontal surface, $E(\vec{x})$ and per spectral component, $S(f_i, \vec{x})$, the frequency f_i (or the angular frequency, ω_i), the amplitude a_i, and the propagation velocity of the energy $C_g(f_i)$,

$$S(f_i, \vec{x}) = \frac{1}{2}\rho_w g a_i^2 \tag{4.190}$$

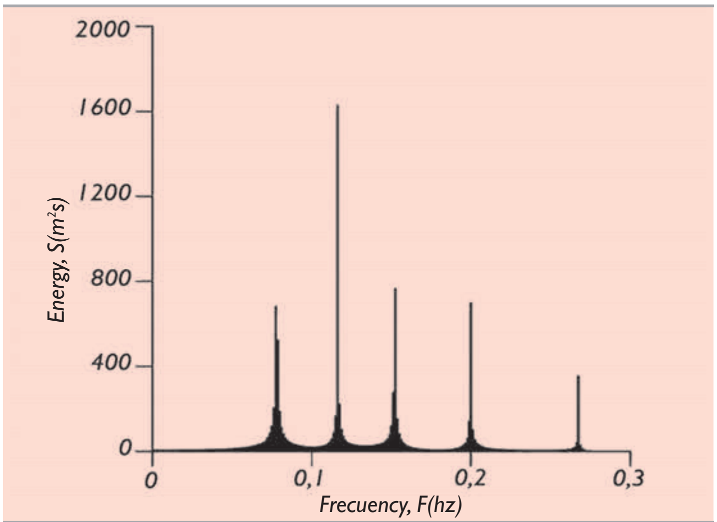
$$C_{g,i} = \frac{1}{2}C\left(1 + \frac{2k_i h}{\sinh 2k_i h}\right) \tag{4.191}$$

$$\omega_i^2 = g k_i \tanh k_i h \tag{4.192}$$

$$E(\vec{x}) = \alpha_E \int_{f_{\inf}}^{f_{\sup}} S(f, \vec{x})\, df \tag{4.193}$$

where, h is the water depth; k_i is the wave number of the frequency component $f_i = 2\pi\omega_i$, g is the gravity acceleration; ρ_w is the sea water density; and i, $[1 \leq i \leq n]$ identifies the spectral component; α_E is a constant that depends on the formulation of the spectral density function; $[f_{inf}, f_{sup}]$ are the lower and upper limits, respectively, of the sea wave band $(3 < T(s) < 30)$. Generally speaking, they are defined in terms of the period of maximum energy in the spectrum or the peak period, $T_p = 1/f_p$, $[f_{inf} \approx 0.6 f_p, f_{sup} \approx 2.5 f_p]$.

Figure 4.4.8. Representation of spectral density function (frecuency domain) from the superoposition of regular waves



4.3.1.7 Directional and frequency spectrum of the sea waves

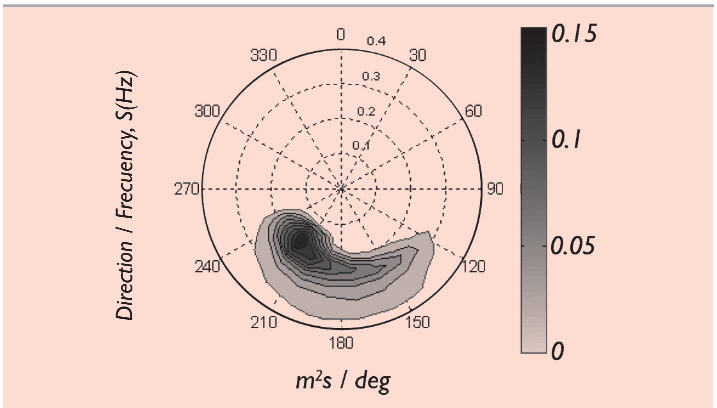
The distribution of the wave energy due to each of the frequency and directional components is known as the energy spectrum or spectral density function. The frequency and directional spectrum $S_w(f, \theta)$ (units $L^2 * T$, for example, $m^2 * s$) represents the directional and frequency distribution of the wave energy density in the sea state, where θ is the angle measured (positive in the clockwise sense) with respect to the principal wave propagation direction θ_0 (see Figure 4.4.9).

If there is no better information regarding the directional and frequency spectrum of the waves, $S_w(f, \theta)$ can be described by the product of two functions: (i) the frequency spectrum $S_w(f)$; (ii) the directional spectrum $D_w(\theta, f)$

$$S_w(f, \theta) = S_w(f) D_w(\theta, f)$$

The units of the frequency spectrum are the same as those of the total spectrum $(L^2 * T)$. The directional spectrum is adimensional.

Figure 4.4.9. **Spectral representation of the frecuency and direction of the waves**



The frequency function $S_w(f)$ has two branches, one going upwards, which is proportional to $S_1(f) \sim f^{-5}$ and the other going downwards, $S_2(f) \sim f^{-4}$. The crossing of both branches defines the peak or maximum value of the spectral function $S_1(f_p)$, which measures the energy supersaturation of the components around the frequency of peak, f_p. This shape depends on the age and type of sea.

The directional function $D_w(\theta, f)$ provides the energy density in the directional domain in each frequency f, relative to the energy density in that frequency. $D_w(\theta, f)$ usually fits a $\cos^{2s}$ –type function, where s is the angular dispersion parameter, which depends on the fetch or area (or time) and the generation depth and mean speed of the generating wind.

4.3.1.8 Spectral moments

The spectral n-order moments defined by (in SI units, $L^2(m^2)$),

$$m_n = \int_{f_{\inf}}^{f_{\sup}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f^n S_w(f,\theta)\, df d\theta \qquad (4.194)$$

where $f_{\inf}$, $f_{\sup}$ define the frequency band in which the integration takes place. Evidently, it should hold that

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} D_w(\theta, f)\, d\theta = 1 \qquad (4.195)$$

The zero-order moment (m^2) represents the total energy of the irregular wave train,

$$m_0 = \int_{f_{\inf}}^{f_{\sup}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f^n S_w(f,\theta)\, df d\theta = E \qquad (4.196)$$

Note. *Though from a strictly theoretical viewpoint, $f_{inf} \Rightarrow 0$ y $f_{sup} \Rightarrow \infty$, in practice, these two boundaries should fit the frequency band in which there is wave energy. In one-peak spectra, it these values are usually bounded, depending of the frequency of the peak f_p, for example, $(0.3f_p, 3f_p < 0.04)$.*
In the sea, the waves generated by the wind do not propagate in only one direction. On the contrary, their energy is distributed in various directions. The energy associated with frequencies with a value near the modal frequency is mainly propagated in the direction of the wind. A directional analysis of the waves consists in determining the shape in which the energy is distributed over frequencies (or wave numbers) and propagation directions.

4.3.1.9 Wave number spectrum

In the same way as for the frequency, it is possible to express the spectral density function in terms of the wave number $S(k,\theta)$. Given the fact that the energy of the process should be the same, it should be verified that

$$S(k,\theta)\, dk = \frac{1}{2\pi} S(f,\theta)\, df = S(\omega,\theta)\, d\omega \qquad (4.197)$$

The units of the wavenumber spectrum are L^3, for example, m^3. For a specific propagation direction, it holds that

$$S(k) = \frac{1}{2\pi}\left[S(f)\right]\frac{df}{dk} = \frac{C_g}{2\pi}S(f) \qquad (4.198)$$

$$C_g = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)C = nC \qquad (4.199)$$

4.3.1.10 Spectral state descriptors

The spectral descriptors define wave height, period, and direction in the sea state, calculated on the basis of the spectral density function.

4.3.1.10.1 Period and height descriptors

For the definition of the *n*–order spectral moments in accordance with section 3.6.2.3, the most usual descriptors in maritime engineering are the following:

m_0, zero-order: $n = 0$ is the area under the frequency spectrum of the sea wave, and it is equal to the variance of the system (m^2).
m_2, second-order: $n = 2$ is the moment of inertia of the spectrum around the axis $f = 0$.

For Gaussian processes, it holds that

H_{m_0}, significant spectral wave height (m) that is defined depending on m_0 by

$$H_{m_0} = 4\sqrt{m_0} \qquad (4.200)$$

$T_{0,1}$, mean spectral period $(0, 1)$, *(s)*

$$T_{0,1} = \frac{m_0}{m_1} \qquad (4.201)$$

T_m, mean spectral period $(0, 2)$, *(s)*

$$T_m = T_{0,2} = \sqrt{\frac{m_0}{m_2}} \qquad (4.202)$$

$\tilde{T}_c$, mean spectral period of the crest, *(s)*

$$\tilde{T}_c = \sqrt{\frac{m_2}{m_4}} \qquad (4.203)$$

T_p, spectral peak period or period in which the spectrum has its maximum energy content;
L_p, peak wavelength associated with the peak period;
f_p, peak frequency or frequency with the maximum energy content = $1/Tp$, $(1/s)$.
$f_{0,1}$, mean spectral frequency $(0,1) = 1/T_{0,1}$
$f_{0,2}$, mean spectral frequency $(0,2) = 1/T_{0,2}$
Qp, parameter that quantifies the spectrum shape and is defined by (for wind sea states, its value is near 2).

$$Q_p = \frac{2}{m_0^2}\int_0^\infty f\left[S(f)\right]^2 df \qquad (4.204)$$

υ spectral width that is the normalized turning radius of the spectrum near its mean frequency, $f_{0,1}$,

$$v^2 = \frac{m_0 m_2}{m_1^2} - 1 \qquad (4.205)$$

The spectral width varies in the interval $[0 \leq \nu < 0{,}5]$, depending on the spectral shape, according to Table 4.4.2.

Table 4.4.2. Spectral widths for different spectral density functions

Spectral shape	ν
Narrow band	≈ 0
Pierson-Moskowitz	0.425
Jonswap $(\gamma = 3.3,\ \sigma_a = 0.7,\ \sigma_b = 0.9)$	0.390

Note. *The parameter ε is often also used.*

$$\varepsilon^2 = 1 - \frac{m_2^2}{m_0 m_4} = 1 - \left(\frac{\tilde{T}_c}{T_m}\right)^2 \qquad (4.206)$$

The calculation of the moment involves an integration of the upper and lower limit. For this reason, the truncation error has a greater effect on higher-order moments. The preceding definitions are associated with the frequency definition of the energy spectrum. If the energy spectrum frequency is defined by the angular frequency, some of the expressions will be affected of 2π.

4.3.1.10.2 Relation between different spectral periods

For one-peak frequency spectra, the relation between the peak and mean spectral periods depends on the spectral shape and the estimation method. Table 4.4.3 gives this relation for the Jonswap and PM theoretical spectral density functions (see section 3.6.3.5),

Table 4.4.3. Quotient of the peak and mean values for different spectral density functions

Spectral shape	T_p/T_m
Narrow band	1.10
Pierson-Moskowitz	
Jonswap $(\gamma = 3.3,\ \sigma_a = 0.7,\ \sigma_b = 0.9)$	1.25

The *Puertos del Estado* database should be used to evaluate this relation at the project site. When no other information is available, it can be said that,

$$T_p \approx 1.35 T_m \qquad (4.207)$$

4.3.1.10.3 Directional descriptors

The mean frequency direction is defined by the following:

$$\bar{\theta}(f) = \arctan \frac{\int \sin\theta \cdot D(f,\theta)\, d\theta}{\int \cos\theta \cdot D(f,\theta)\, d\theta} \qquad (4.208)$$

and the mean wave direction by

$$Mdir = \arctan \frac{\int_{f_{\inf}}^{f_{\sup}} \sin\bar{\theta} \cdot S(f)\, df}{\int_{f_{\inf}}^{f_{\sup}} \cos\bar{\theta} \cdot S(f)\, df} \qquad (4.209)$$

These descriptors can be defined by different frequency bands. In this way, it is possible to identify, for example, the mean propagation direction of high and low frequency bands. The wave direction in the spectral peak is $\overline{\theta}(f_p)$.

The angular dispersion is defined by

$$\sigma(f) = \int_0^{2\pi} (\theta - \overline{\theta})^2 \cdot D(\theta, f)\, d\theta \tag{4.210}$$

θ_0, the principal propagation direction of the sea waves is that in which the maximum energy flux occurs, in other words,

$$S_w(f, \theta_0) C_g(f, \theta_0) \Delta f \Delta\theta = max\left[S_w(f, \theta) C_g(f, \theta) \Delta f \Delta\theta\right] \tag{4.211}$$

where C_g is the propagation velocity of the energy in the frequency band f. In the θ_0 and Δf, $\Delta\theta$ are the width of the directional and frequency intervals in which the spectrum is divided. The relation between the group and wave celerities of each spectral component is:

$$C_g(f) = n(kh) C(f) \tag{4.212}$$

$$n(kh) = \frac{1}{2}\left[1 + \frac{2kh}{\sinh(2kh)}\right] \tag{4.213}$$

$$(2\pi f)^2 = gk \tanh kh \tag{4.214}$$

4.3.1.11 Functional relation between frequency and statistic descriptors

The description of the sea waves is based on the hypothesis that in a state or interval limited in time, and in a domain bounded in space, $\eta(t, \vec{x})$ is a Gaussian random variable, whose variance σ_η^2 is proportional to the total energy of the process, $E(\vec{x})$. Between the spectral and statistical descriptors, there are functional relations (see section 3.5.2.11). Especially relevant are the relations between the variance of the time signal, σ_{η}^2, the area under the spectral density function or zero-order moment m_0, and the mean square wave height H_{rms},

$$m_0 \approx \sigma_\eta^2 \tag{4.215}$$

$$H_{m_0} \approx 4\sqrt{m_0} \tag{4.216}$$

$$H_{m_0} = \alpha_s H_s \tag{4.217}$$

where α_s depends on the type of sea. Generally speaking, it can be said that $\alpha_s \approx 1$. It is advisable to use the relations between state descriptors to verify the coherence and validity of calculations and data.

4.3.1.12 Description of the kinematics and dynamics of the sea waves

The description of fluid movement can include other instantaneous variables, such as velocity and acceleration, tangential pressures and stresses, whose temporal variability contains the same information as the oscillations that cause them. In particular, if their origin is due to the presence of the sea waves, they can be described by following the same schema as for the sea waves, namely by first defining the fast basic and instantaneous variable, the period of the basic cycle, and afterwards, the value as a slow variable or state descriptor. This expansion can be performed on the basis of measurement data.

However, since in maritime engineering, linear wave theory is generally sufficient, the probability functions and frequency spectra of any of the cinematic or dynamic variables can be directly obtained from the results obtained for the instantaneous variable, free surface displacement, and the associated basic variables. In the time domain, statistical methods are sufficient to obtain the derived probability functions, whereas in the frequency domain, it is enough to evaluate the transfer frequency function. Multiplied by the sea wave spectrum, it provides the frequency spectra of the derived random variable, as show in the following two sections.

4.3.2 Generation processes: foundations

The principle energy transfer mechanisms that intervene in wave generation processes are

1. Energy sources: wind action;
2. Conservative wave-wave interaction between four waves in deep water;
3. Conservative wave-wave interaction between three waves in medium and shallow depths;
4. Energy sinks: wave breaking because of maximum steepening in deep water, bottom-induced breaking, and bottom friction.

4.3.2.1 Source mechanism

The generation mainly begins because of turbulent wind action and the resonant coupling between vertical pressure fields and waves freely propagating in the ocean. This mechanism seems to provide an increase in the linear wave amplitude over time. This incipient development modifies the airflow pattern on the sea surface, such that the air pressure on it pushes the wave on the side of the wind and sucks the water from behind, thus causing the wave to steepen. This mechanism depends on the wave amplitude, and becomes more and more effective as the sea waves increase. However, the increase in surface roughness also begins to be a crucial factor in the structure of the wind fields and surface pressures. These two mechanisms are identified by $S_{in}(f, \theta)$, where S is the wave energy density at frequency f and propagation direction θ. They represent energy input from the atmosphere to generate waves, in other words, the source term.

4.3.2.2 Energy redistribution mechanism

The second mechanism that intervenes in the development of the sea is known as the non-linear four-mode interaction of components, and which consists of an energy exchange between two groups of spectral components $(f_1, \overline{k}_1; f_2, \overline{k}_2)$ and $(f_3, \overline{k}_3; f_4, \overline{k}_4)$. For the mechanism to be resonant, the sum of the frequencies of the two groups of two waves should be equal $(f_1 + f_2 = f_3 + f_4)$ as well as the result of the vector sum of their respective wave numbers (14) $(\overline{k}_1 + \overline{k}_2 = \overline{k}_3 + \overline{k}_4)$. The final result is that the total energy in the spectrum does not vary, but rather is only redistributed among components. The redistribution affects all spectral components. The highest and lowest frequencies receive energy from centered frequencies. In fact, this mechanism guarantees that the spectral shape in the domain of frequencies higher than that of the peak $f \geq f_p$ has the shape f^{-4}. That mechanism is $S_{nl4}(\overline{k}_4)$.

4.3.2.3 Dissipating mechanism

Waves in deep water have a steepness limit, which when waves ride on each other, cross each other, etc. is lower than the generally accepted value. This value is derived from the condition that the horizontal velocity of particles in the crest exceeds wave celerity (i.e. Stokes conjectures),

$$\left(\frac{H}{L}\right)_{lim} \lesssim \frac{1}{7} \tag{4.218}$$

(14) See definition in section 3.6.2.1.

The breaking of waves over others, generally on the lee side of the wave, is a pressure pulse which works against the growth of that part of the wave by the suction mechanism. In this sense, the breaking acts as a dissipating mechanism expressed by $S_{wc}(f,\theta)$, and is thus linear. However, it finally depends on the amount of energy in the spectrum.

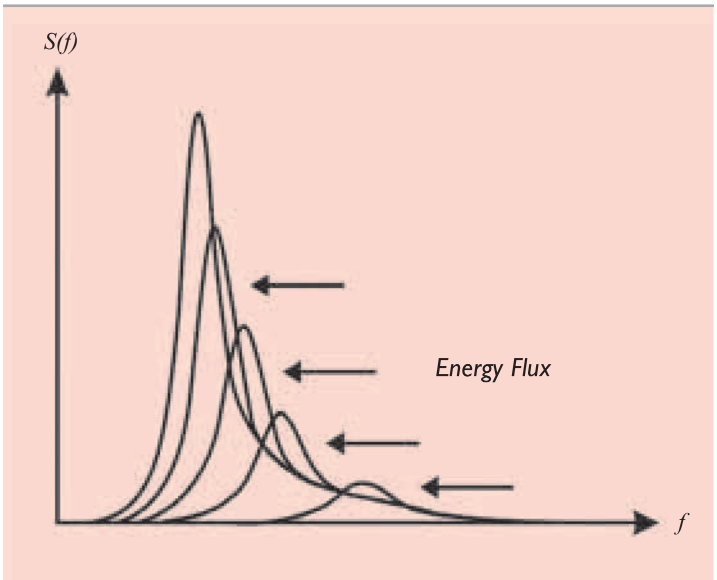
4.3.2.4 Redistribution mechanism in shallow waters

In shallow waters, the interaction mechanism between components can be fulfilled with only three spectral components, such that $(\overline{k}_1 + \overline{k}_2 = \overline{k}_3)$. As a result, components exchange energy, either gaining it or losing it. Nevertheless, the total energy (i.e. the sum of the three components) remains the same.

4.3.2.5 Energy increase per component and energy flux

The temporal evolution of the energy spectrum shape is the result of the equilibrium between the input of the wind, the four-mode interaction of components, and the wave breaking. In this process, each of the components of the upper tail of the spectrum, near the peak, receive the wind energy and redistribute it by means of non-linear interaction in higher and lower frequencies, whereas part of the energy is dissipated by breaking processes. In the high frequency zone, the energy flux tends to be stable, and the non-linear mechanism competes with the breaking mechanism. In contrast, the low frequency zone $f \leq f_p$ continuously receives energy by means of the interaction process and the wind. In this way, the spectrum peak advances towards its components, and the spectral shape becomes progressively filled in the low frequency domain (see Figure 4.4.10).

Figure 4.4.10. Evolution of the shape of the frequency spectrum



The temporal evolution of a frequency component initially without energy (for example, less than the peak) begins to receive energy and exponentially increases until reaching a maximum. From that moment on, the component begins to transfer energy through the interaction between components and the breaking of other components, but it also begins to capture wind energy.

4.3.2.5.1 Totally developed sea, TDS

The totally developed sea is the sea state with the maximum possible energy content for the generation conditions due to wind speed $\overline{u_{10}}$, without limit in fetch or time (see Figure 4.4.11). The spectral function is formed by the frequency band, each of which has the maximum energy that can be transported. In such conditions, the spectrum has two branches, one in which the energy increases as the frequency decreases, and the other, after reaching a maximum energy peak, in which the energy decreases as the frequency increases.

The upwards branch is mainly related with the frequencies, which have wind energy transfer. In TDS conditions, there is an equilibrium between energy that is received, dissipated, and transferred to other frequencies. This range of frequencies in which this frequency occurs is known as the saturation range. Based on the dimensional analysis, the shape of the spectral density function in the equilibrium range is the following:

$$S(f) = \alpha g^2 (2\pi f)^{-5} \tag{4.219}$$

where α is a constant with an approximate value of $\alpha = 8{,}1 * 10^{-3}$, $g = 9{,}81 m/s^2$ and f is the frequency of the oscillation in $1/s$. The spectral units of $S(f)$ are m^2s (in SI).

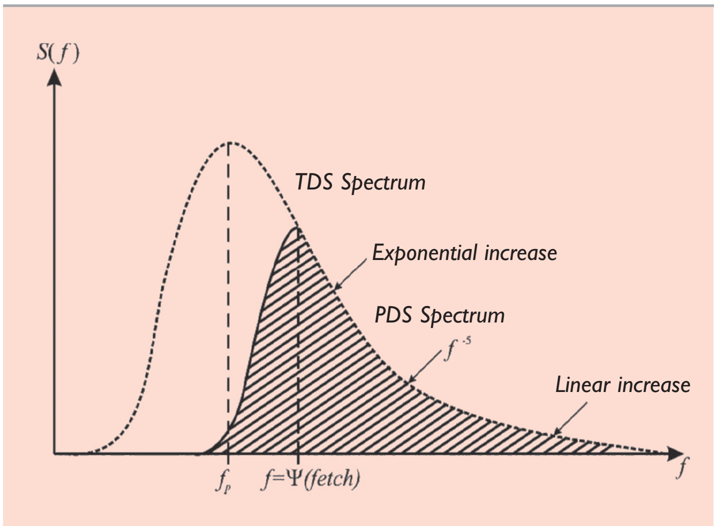
4.3.2.5.2 Partially developed sea, PDS

PDS represents the sea states in which, either because of time limitations or limitations of the generation surface, some of the spectral components of the frequency band related to the generating wind do not contain the energy that they would contain in the case of a PDS (see Figure 4.4.11). Certain frequencies in the upper part of the spectrum can have more energy, in other words, they are supersaturated. Other frequencies (generally those in the lower branch) can have less energy or none at all. In such situations and in deep water, the upper branch maintains the shape of f^{-5}, but the constant α depends on $\tilde{F}$ and $\tilde{t}_g$,

$$S(f) = g^2 f^{-5} \varphi(\tilde{F}, \tilde{t}_g) \tag{4.220}$$

$$\tilde{F} = \frac{gX}{u_{10}^2} \quad \tilde{t}_g = \frac{gt}{u_{10}^2} \tag{4.221}$$

Figure 4.4.11. Spectra for the totally and partially developed sea by applying the spectral density function, PM, and omitting the energy supersaturation around the spectral peak



4.3.2.6 Propagation outside the generation zone

When the waves leave the generation zone, they begin to travel on their own energy, and thus, they do it, depending on their direction and frequency. Consequently, their steepness gradually decreases. The system is said to be dispersive in respect to direction and frequency. In these conditions, the shape of the energy spectrum is considerably removed from the function f^{-4}.

The arrival at the coastline of waves that have abandoned the generation area, occurs out of phase, and depends on the energy propagation velocity. For this reason, the spectral components of the largest period are the first to appear.

4.3.2.7 Depth dependency

When waves are generated in small water bodies, the energy increase of some of the spectral components is less than in the deep water of the open sea. This circumstance should be taken into account when selecting the generation method, as recommended in the following sections.

4.3.2.8 Increase in the energy density function

Knowledge of the spatial and temporal evolution of the waves should be gained by quantifying the evolution of each spectral component from its beginning. This means that

$$S(f, \theta) = S(f, \theta, x, y, t) \tag{4.222}$$

In order to know the spectral shape at any point in the sea and at any instant, it is necessary to calculate the evolution of the energy of each component. This is done by following the ray with its group celerity, in other words, the Lagrangian energy conservation formula,

$$\frac{dS(f,\theta,x,y,t)}{dt} = S^*(f,\theta,x,y,t) \tag{4.223}$$

$$S^*(f,\theta,x,y,t) = f\left(\overline{u_{10}}(x,y,t),\theta_u(x,y,t),F,t_g\right) \tag{4.224}$$

where S^* quantifies the main physical processes in wave generation; and $\overline{u_{10}}(x,y,t)$, $\theta_u(x,y,t)$ are the speed and direction of the wind acting on the water surface F during time T_g. When the wind field slowly evolves, $\overline{u_{10}}$, θ_u the rise and development of the waves can be studied by applying simple graphical methods (15), which can calculate the influence of the width and obstacles in the generation area.

When the wind field is uniform and stationary and the configuration of the fetch does not affect the generation, the Jonswap spectrum formula can be applied, according to the principal generation parameters $(\overline{u_{10}}, X, t)$ or their corresponding adimensional monomials,

$$\tilde{F} = \frac{gX}{\overline{u_{10}}^2} \quad \tilde{t}_g = \frac{gt}{\overline{u_{10}}^2} \tag{4.225}$$

$$S(f,\theta_u) = f\left(\frac{gX}{\overline{u_{10}}^2},\frac{gt}{\overline{u_{10}}^2}\right) \tag{4.226}$$

where X, t represents the distance to the source from the generation field, and t is the time of wind action .

4.3.2.9 *Increase of statistical descriptors*

In conditions of windfield stationarity and uniformity, the growth of wave action can be quantified by using parametric functions of dimensional monomials,

$$\tilde{H}_{m_0} = f\left(\frac{gX}{\overline{u_{10}}^2},\frac{gt}{\overline{u_{10}}^2}\right) \tag{4.227}$$

$$\tilde{T}_p = f\left(\frac{gX}{\overline{u_{10}}^2},\frac{gt}{\overline{u_{10}}^2}\right) \tag{4.228}$$

$$\tilde{H}_{m_0} = \frac{gH_{m_0}}{\overline{u_{10}}^2} \quad \tilde{T}_p = \frac{gT_p}{\overline{u_{10}}} \tag{4.229}$$

Note. The first study of parametric functions was carried out in the SMB project during the World War II. The fit functions continue to be very useful in deep water,

$$\tilde{H}_s = 0.238\tanh\left(0.0125\tilde{F}^{0.42}\right) \tag{4.230}$$

$$\tilde{T}_z = 7.54\tanh\left(0.077\tilde{F}^{0.25}\right) \tag{4.231}$$

(15) E.g. the Integrated Method of Suarez Bores.

4.3.3 Models of wave generation, transformation and breaking

4.3.3.1 Phase-averaged models: Formulation

In phase-averaged models, the vertical coordinate is first calculated by vertically integrating the equations, whereas the horizontal coordinates define the propagating plane with the phase. This results in transport equations, which describe the horizontal propagation of the wave train, and which in most cases are numerically integrated. Generally speaking, these models are formulated by establishing the energy (or wave action) balance in each of the grid cells for each spectral component. Consequently, it is an Eulerian formulation. Once the wave action, $\tilde{A}$, is defined by the quotient of the energy density E and intrinsic frequency σ, these models use a balance in a control volume to resolve the spectral distribution function of the wave action, assuming that it slowly evolves in time and space. This equation can be expressed in a variety of different ways (e.g. in Cartesian coordinates, polar coordinates, spherical coordinates, spectral domain, wave number or frequency and direction, and the stationary or non-stationary temporal evolution).

An equation system, valid for slight changes in the source terms and current is the following:

- Dispersion equation

$$\sigma^2 = g \tanh kh = \left(\omega - k_i U_i\right)^2, \; i = 1,2 \tag{4.232}$$

where σ is the intrinsic or relative frequency, and ω is the absolute frequency; $\vec{k} = (k_x, k_y)$ is the wavenumber vector, and k is its module, and $U_1 = U, U_2 = V$ are the components.

- Propagation equations:

$$T_i = C_{g,i} + U_i = \frac{\partial \sigma}{\partial k_i} + U_i \tag{4.233}$$

$$K_i = \frac{\partial k_i}{\partial t} + T_j \frac{\partial k_i}{\partial x_j} = -\frac{\partial \sigma}{\partial h}\frac{\partial h}{\partial x_i} - k_j \frac{\partial U_j}{\partial x_i} \tag{4.234}$$

$$\Omega = \frac{\partial \omega}{\partial t} + T_i \frac{\partial \omega}{\partial x_i} = \frac{\partial \sigma}{\partial h}\frac{\partial h}{\partial t} + k_i \frac{\partial U_i}{\partial t} \tag{4.235}$$

This equation system considers the temporal and spatial variations of the current and depth.

- Wave action balance or transport equation

$$\frac{\partial \tilde{A}}{\partial t} + T_i \frac{\partial \tilde{A}}{\partial x_i} + K_i \frac{\partial \tilde{A}}{\partial k_i} = \sum_i S_i^* \tag{4.236}$$

$$\tilde{A}\left(\vec{k},\vec{x},t\right) = \frac{E\left(\vec{k},\vec{x},t\right)}{\sigma} \tag{4.237}$$

where E is the energy density function at a point in the sea at an instant t, expressed according to the wavenumber vector $\vec{k}$, and S_i^* are the energy source, energy consumer, or energy supplier terms to the system. These equation systems should be resolved in a rectangular grid.

Note. *The preceding equation is the Eulerian formulation of the sea wave conservation equation in deep water. When it is assumed that there is no current, it can be expressed in the following way:*

$$\frac{\partial E\left(f,\theta\right)}{\partial t} + C_{g,x}\frac{\partial E\left(f,\theta\right)}{\partial x} + C_{g,y}\frac{\partial E\left(f,\theta\right)}{\partial y} = S^*\left(f,\theta\right) \tag{4.238}$$

4.3.3.2 Phase-averaged numerical models: open sea and continental shelf

When meteorological data are more accurate than the information in meteorological charts, it is advisable to apply numerical methods to integrate the equations of wave generation and propagation in the vicinity of the project site.

Note. Since1992, the European Center for Medium Range Weather Forecasting (ECMWF) has been working with Puertos del Estado to apply the WAM numerical model, ocean version, to the Atlantic Ocean and to apply the sea and continental shelf version to the Mediterranean Sea.

4.3.3.2.1 Open sea and ocean: WAM

WAM is a wave generation model, which in version 4 integrates the preceding equation system. The basic transport equation describes the evolution of a bidimensional spectrum of the wave action with respect to the frequency and direction with making any initial assumptions regarding the shape of the spectrum.

With no currents, the spectral evolution mainly depends on the following source terms: wind action, resonant four-mode non-linear interaction, and steepness-induced wave breaking in deep water with.

Using a two-way nesting scheme, *Puertos del Estado* has developed a WAM application for oceans (and thus, no seabed-induced phenomenon is taken into account). It is for the Spanish Atlantic Coast with a resolution of a one-fourth degree, and is for deep water.

4.3.4 Generation and transformation models of waves or irregular wave trains

Near the coastline, there is a rapid transformation of the waves, due to changes in seabed, obstacles and boundaries, as well as the intensification of currents that cause rapid changes in the propagation vector of wave energy. Thus, the wave transformation processes are usually dominant. This transformation occurs at a short distance, and as a result, the energy gain processes are not important. Since the integration of transformation models requires small computational grids, models specifically conceived for these processes are the most efficient.

4.3.4.1 Formulation of the problem

In consonance with previous sections, the Eulerian formulation of the spectrum propagation problem is the most effective. It includes shoaling, refraction, and diffraction processes, and considers source terms of three-wave interaction, bottom friction, and bottom-induced breaking. The numerical model that gives the closest approximation is the SWAN model.

The energy conservation equation (absence of currents) in each of the grid cells for each spectral component with its frequency and direction is now

$$\frac{\partial E(f,\theta,x,y,t)}{\partial t}+\frac{\partial C_{g,x}E(f,\theta,x,y,t)}{\partial x}+\frac{\partial C_{g,y}E(f,\theta,x,y,t)}{\partial y}+\frac{\partial C_{\theta}E(f,\theta,x,y,t)}{\partial \theta}=\sum_i S_i^*(f,\theta,x,y,t) \quad (4.239)$$

where the fourth term on the left side of the equation represents the refraction and diffraction effect (change in propagation direction) because of the spatial variation of the bottom and wave height.

If the waves propagate when there is a current, the problem is formulated in terms of the wave action, as explained in the section on SWAN models. Finally, in the source term on the right, apart from the terms associated with wave generation in deep water, it is necessary to include three-wave interaction, bottom and friction-induced dissipation, and wave breaking.

4.3.4.2 Modeling spectral transformation

These Recommendations propose different degrees of solution to evaluate the transformation of the wave energy spectrum, based on phase-averaged models and phase-resolving models, and which are not formulated in terms of harmonic functions.

Phase-averaged models are generally formulated in terms of the harmonic potential function. Taking into account that the energy density function is an averaged quantity, these methods are applicable to port areas, particularly when wave breaking is not a determining factor. These models can be applied to the following:

a. state descriptors, stationary solution.
b. spectral components, stationary solution.
c. non-stationary formulation, wave groups.

Furthermore, it is possible to formulate and resolve integral numerical models of the spatio-temporal evolution of the SWAN spectrum, as described in the previous section. This is an Eulerian formulation of the problem of the spectrum propagation, including the processes of shoaling, refraction and diffraction, and considering the source terms of three-wave interaction, bottom-induced friction, and bottom-induced breaking.

4.3.4.3 SWAN

SWAN is a model that studies wave propagation in shallow water, and conserves the open-water source terms, besides adding dissipation terms because of friction-induced dissipation and bottom-induced breaking. This model is an extension of the WAM model, and includes shallow water phenomena. The transport equation can be formulated in terms of σ, the intrinsic or relative frequency and the propagation direction θ, direction normal to the crest of each spectral component,

$$\frac{\partial N_*}{\partial t}+\frac{\partial\left(C_{gx}N_*\right)}{\partial x}+\frac{\partial\left(C_{gy}N_*\right)}{\partial y}+\frac{\partial\left(C_{g\sigma}N_*\right)}{\partial \sigma}+\frac{\partial\left(C_{g\theta}N_*\right)}{\partial \theta}=\sum_i \frac{S_i^*}{\sigma} \qquad (4.240)$$

where N_* is the wave action as defined by

$$N_*\left(\sigma,\theta,\vec{x},t\right)=\frac{S\left(\sigma,\theta,\vec{x},t\right)}{\sigma} \qquad (4.241)$$

which is conserved in the presence of the current, something that does not occur with the energy S.

The first term is the local variation of the wave action. The second and third term represent the propagation of the wave action in physical space in directions *x* and *y* with their respective propagation speeds C_{gx}, C_{gy}. Consequently, they evaluate shoaling. The fourth term evaluates the modification of the relative frequency due to variations in depth and current, whereas the last term on the left side of the equation represents the refraction induced by changes in the depth and current. The terms on the right are the source terms, which represent the effects of generation, dissipation because of steepness-induced breaking in deep water, nonlinear wave-wave interaction, bottom friction, bottom-induced breaking, and three-mode interaction.

Generally speaking, the term, $\partial(C_{g\theta}N_*)/\partial\theta$, does not include the complete diffraction effect. Consequently, this model should not be used in those situations in which diffraction is expected to be important. In reality, SWAN incorporates an evaluation of the diffraction parameter in terms of the energy of each component, ignoring wave phase, and decoupling the two effects produced by direction changes, namely refraction and diffraction.

In the models of coastal areas, it is important to accurately model the source term (sink) of energy dissipation due to bottom-induced wave breaking. Given the relevance of the wave energy dissipation in the radiation tensor gradients (the motor of the circulation in the surf zone), special attention should be paid to the modeling of this term.

4.3.5 Wave transformation models

4.3.5.1 State descriptor transformation

In certain cases in which there is only slight variations in the bathymetry, and the wave action is a narrow-band process, any of the methods described for a regular wave train can be applied to the sea state descriptors in the open sea $(H_{rms}, T_p, \bar{\theta})_0$, as though they behaved like a regular wave train. At any point of the sea, the values obtained $(H_{rms}, T_p, \bar{\theta})_x$ represent the transformed sea state from the open sea. Consequently, the local wave height can be described by a Rayleigh model of parameter $H_{rms,x}$. In principle, with their intrinsic limitations, it is possible to apply all the methods recommended: ray theory, as well as the parabolic, hyperbolic, and elliptic approximations of the MSP.

4.3.5.2 Spectrum transformation models: Foundations

The theoretical foundations of the evolution of the wave spectrum $E(f, \theta, x, y, t)$ are based on the fact that the spectral density function, expressed in wave number terms $E(k_x, k_y, x, y, t)$ remains constant when the waves advance with the group celerity along a ray, namely,

$$\frac{\partial E}{\partial t} + \frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial k_x}\frac{dk_x}{dt} + \frac{\partial E}{\partial k_y}\frac{dk_y}{dt} = 0 \tag{4.242}$$

The first term represents the local variation of the spectrum, which is annulled for a sea state. The two following terms give the convective variation or the energy variation because of spatial change. Finally, the two last terms provide shoaling and refraction,

$$\frac{dx}{dt} = \frac{d\sigma}{dk_x} = C_g \cos\theta \tag{4.243}$$

$$\frac{dy}{dt} = \frac{d\sigma}{dk_y} = C_g \sin\theta \tag{4.244}$$

where the angular frequency $\sigma(k, x, y)$ is related to the depth $h(x, y)$ and the wave number, k,

$$\sigma^2 = gk \tanh kh \tag{4.245}$$

The following relation exists between the wavenumber spectrum and the angular frequency spectrum:

$$E(f,\theta,x,y,t) = \frac{2\pi k}{C_g} E(k_x,k_y,x,y,t) \tag{4.246}$$

The condition of the energy conservation equation in space of the wave number is expressed in function of the frequency spectrum by

$$\frac{C_g}{2\pi\sigma}\left\{\cos\theta\frac{\partial\left[CC_gE(f,\theta)\right]}{\partial x} + \sin\theta\frac{\partial\left[CC_gE(f,\theta)\right]}{\partial y} + \frac{1}{C}\left(\sin\theta\frac{\partial C}{\partial x} - \cos\theta\frac{\partial C}{\partial y}\right)\frac{\partial\left[CC_gE(f,\theta)\right]}{\partial\theta}\right\} = 0 \tag{4.247}$$

Note. A unidirectional spectrum, propagating with crests parallel to a straight, parallel bathymetry, simply experiences shoaling. The frequency spectrum at any depth $h(x)$ is obtained by,

$$S(f,h) = S_0(f) * \frac{C_{g,0}(f)}{C_g(f,h)} \tag{4.248}$$

$$K_s^2(f,h) = \frac{C_{g,0}(f)}{C_g(f,h)} \tag{4.249}$$

When the irregular wave train obliquely impinges on the straight, parallel bathymetry, apart from shoaling, it is refracted, and the refraction coefficient should include the Jacobian of the transformation of the directions, such that,

$$S(f,\theta,h) = S_0(f,\theta_0) * K_S^2(f,h) * K_R^2(f,\theta,h) * \frac{\partial\theta_0}{\partial\theta} \tag{4.250}$$

$$K_R^2(f,\theta,h) = \frac{\cos\theta_0}{\cos\theta} \tag{4.251}$$

$$\theta_0 = \Psi(f,\theta,h); \quad \frac{\partial\theta_0}{\partial\theta} = \frac{\partial\Psi}{\partial\theta} \tag{4.252}$$

where Ψ *is the inverse function of the direction which, for a ray, is*

$$\theta_0 = \Psi(f,\theta,h) = arc\sin\left(\frac{k}{k_0}\sin\theta\right) \tag{4.253}$$

$$\frac{\partial\theta}{\partial\theta_0} = \frac{C}{C_0}\frac{\cos\theta_0}{\cos\theta} = \frac{k_0}{k}\cos\theta_0 \tag{4.254}$$

and finally,

$$k_R^2(f,\theta,h) * \frac{\partial\theta_0}{\partial\theta} = \frac{k}{k_0} = \frac{1}{\tanh kh} \tag{4.255}$$

$$S(f,\theta,h) = S_0(f,\theta_0) * \frac{C_{g,0}(f)}{C_g(f,h)}\frac{k}{k_0} = \frac{1}{\left[\tanh kh\left(1+\frac{2kh}{\sinh 2kh}\right)^{\frac{1}{2}}\right]^2} \tag{4.256}$$

universal form of the independent transformation of the spectral shape for straight, parallel bathymetry. Furthermore, it is necessary to bear in mind that the following should hold:

$$\sin\theta_0 = \frac{k}{k_0}\sin\theta = \frac{\sin\theta}{\tanh kh} \le 1 \tag{4.257}$$

$$|\sin\theta| \le \tanh kh \tag{4.258}$$

Consequently, the spectral components that do not locally satisfy this condition do not appear in the transformed spectrum unless they have been generated in the propagation zone. In other words, the angles of the spectral component in deep water θ_0 *and in shallow water* θ *should belong to the same quadrant.*

4.3.5.3 Propagation of the spectral components

Except in situations in which non-linear phenomena and the interaction between components are important, the propagation of the spectrum can be carried out by applying to each of the frequency and directional component of the spectrum linear transformation models, the elliptic MSP or its parabolic approximation.

All of them are formulated, assuming that the energy associated with a (narrow) frequency band stays on that band during the transformation, and that the processes admit linear superposition. For each frequency band, the quantity of energy contained and quantified by the square of the displacement on the free surface is an invariant during the transformation.

Consequently, the laws and methods to quantify the transformation of monochromatic wave trains can be directly applied to each one of the spectral components, taking into account that its energy travels on its corresponding ray with its group velocity.

4.3.5.4 Transformation of the spectrum by the integration of the energy conservation equation

Similarly to a wave train, the transformation of the energy spectrum, taking into account refraction and diffraction, can be obtained by numerically integrating the energy conservation equation in a rectangular grid with the x axis parallel to the coast. A stationary regime is assumed, and source terms are incorporated, as described in previous sections,

$$\frac{\partial C_{g,x}E(f,\theta,x,y,t)}{\partial x}+\frac{\partial C_{g,y}E(f,\theta,x,y,t)}{\partial y}+\frac{\partial C_{\theta}E(f,\theta,x,y,t)}{\partial \theta}=S^{*}(f,\theta,x,y,t) \tag{4.259}$$

$$C_{g,x}=C_g\cos\theta \tag{4.260}$$

$$C_{g,y}=C_g\sin\theta \tag{4.261}$$

$$C_{\theta}=\frac{C_g}{C}\left[\frac{\partial C}{\partial x}\sin\theta-\frac{\partial C}{\partial y}\cos\theta\right] \tag{4.262}$$

4.3.6 Medium-term and long-term description of the sea waves

4.3.6.1 Loading cycles

A loading cycle is the sequence of meteorological states in which there is a significant probability of the occurrence of a failure mode in the maritime structure. More specifically, it refers to one of the failure modes assigned to the ultimate limit or serviceability states used to calculate the joint failure probability in a structure's useful life. In the case of breakwaters, wave action is usually the predominant agent of such modes.

Once the threshold limit state is defined, a loading cycle consists of all of the sea states, which are consecutively above a critical threshold value $H_{s,cr}$, or in the case that it moves downward and again crosses the threshold value, all of the states that have the same cause, in other words, are caused by the same storm [16].

The cycle is made up of a (random) time sequence of crests and troughs, and corresponding relative maximum and minimum values (see Figure 4.4.12), until the first time it moves downwards below the threshold value. These maximums and minimums are found in the instants in which

$$\frac{dH_s(t)}{dt}=0 \tag{4.263}$$

Among the maximums (or minimums), one of them is the largest (or smallest), or the absolute maximum of the cycle that corresponds to the maximum state or peak of the loading cycle.

The variables that describe the cycle are, among others, the following:

$H_{s,cr}$, value of the critical threshold value or the value of the exceedance threshold;
$H_{s,\max}$, relative maximum value of the state descriptor;
$H_{s,\max,cycle}$, absolute maximum or peak of the state descriptor in the cycle;
τ_{cs}, duration of the cycle that is generally equal to the exceedance time over the threshold;
$\tau_{g,cs}$, $\tau_{peak,cs}$, $\tau_{d,cs}$, durations of the growth phases, maximum state and dissipation of the cycle.

(16) This usually accurs in the Alboran Sea. The storm travels from west to east, causing a change in the mean direction of the wind, which goes from W-WSW to WSW-SW. This signifies a change in period, and an initial reduction of the H_s, which afterwards increases again.

4.3.6.1.1 *Threshold value of the loading cycle*

In the case of breakwaters, the threshold limit state is usually defined by the significant wave height $H_{s,cs}$, accompanied by an interval of mean periods and mean propagation directions.

There is no general criterion to determine the threshold value of a loading cycle since it depends on the failure mode being considered. Nevertheless, practically speaking, and when no other information is available, the following can be used:

$$H_{s,cs} \sim (1.5 \; a \; 2)\,\overline{H_s} \tag{4.264}$$

where $\overline{H_s}$ is the mean significant wave height in the time interval adopted for the statistical description. In middle latitudes, this interval is usually the meteorological year.

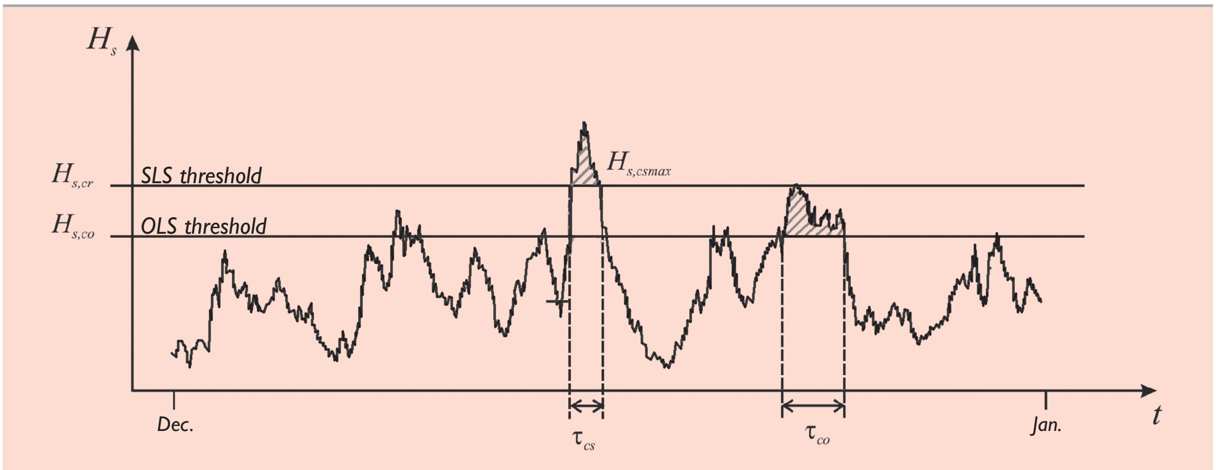
Generally speaking, the threshold values of the serviceability failure modes are lower than the threshold values of the ultimate failure modes. However, the difference is usually small, and thus, the same threshold value can be used for the analysis of the subset and the structure in regards to its safety as well as its serviceability.

4.3.6.2 *Cycle of use and exploitation or operationality*

From the perspective of the operationality of the maritime structure, the cycle of use and exploitation is the sequence of meteorological states in which there is a significant probability of the occurrence of one of the stoppage modes assigned to the operational limit states, used to calculate the joint probability of operational stoppage in the structure's useful life, the number of operational stoppages, or the duration of the operational stoppage.

The cycle is made up of a (random) time sequence of crests and troughs, and the corresponding relative maximum and minimum values until it goes for the first time over the threshold value (see Figure 4.4.12).

Figura 4.4.12. Loading cycles, use and exploitation cycles, and threshold values



The variables that describe the cycle are, among others:

$H_{s,co}$, critical threshold value or the non-exceedance threshold value;
τ_{co}, duration of the cycle that, generally speaking, is equal to the non-exceedance threshold time.

4.3.6.2.1 THRESHOLD VALUE OF THE CYCLE OF USE AND EXPLOITATION

Usually, for the breakwaters and port and shoreline areas, the threshold limit state was defined by the significant wave height $H_{s,co}$, accompanied by an interval of mean periods and mean propagation directions.

There is no general criterion to determine the threshold value of a cycle of use and exploitation since it depends on the stoppage mode considered. Generally speaking, this threshold value is lower than the threshold value that defines the beginning of the loading cycle. For all practical effects, and if no better information is available, the following can be adopted:

$$H_{s,co} \sim \overline{H_s} \tag{4.265}$$

where $\overline{H_s}$ is the mean significant wave height in the time interval adopted in the statistical description, which in middle latitudes is usually the meteorological year.

4.3.6.3 Meteorological year, useful life

In the meteorological year, a sequence of loading cycles can occur, whose number, peak magnitude, duration, and time between cycles are random variables. In other words, in the meteorological year, the following random variables can be defined:

$n_{c,year}$, number of loading cycles in the meteorological year;
$H_{s,\max,year}$, values of the maximum state in the meteorological year;
τ_o, time interval two consecutive cycles.

Similarly, the useful life is formed by a given sequence of meteorological years. In the useful life, it is possible to define the random variables of the loading cycle without identifying their variability in the meteorological year. Examples of these variables include the following:

$n_{c,V}$, number of loading cycles in the useful life of the structure;
$H_{s,\max,V}$, value of the maximum state of the useful life;
$\tau_{c,V}$, duration of the cycle in the maximum state in the useful life.

4.3.6.4 Cascade of probability models

Each of these random variables has a probability model that is valid in the time interval (state, cycle, year, and useful life) in which the variable has been defined. In each time interval, each of these random variables can have other associated random variables with different degrees of dependence (e.g. mean period, mean direction, sea level, etc.). In these cases, it is advisable to estimate the corresponding joint distribution functions.

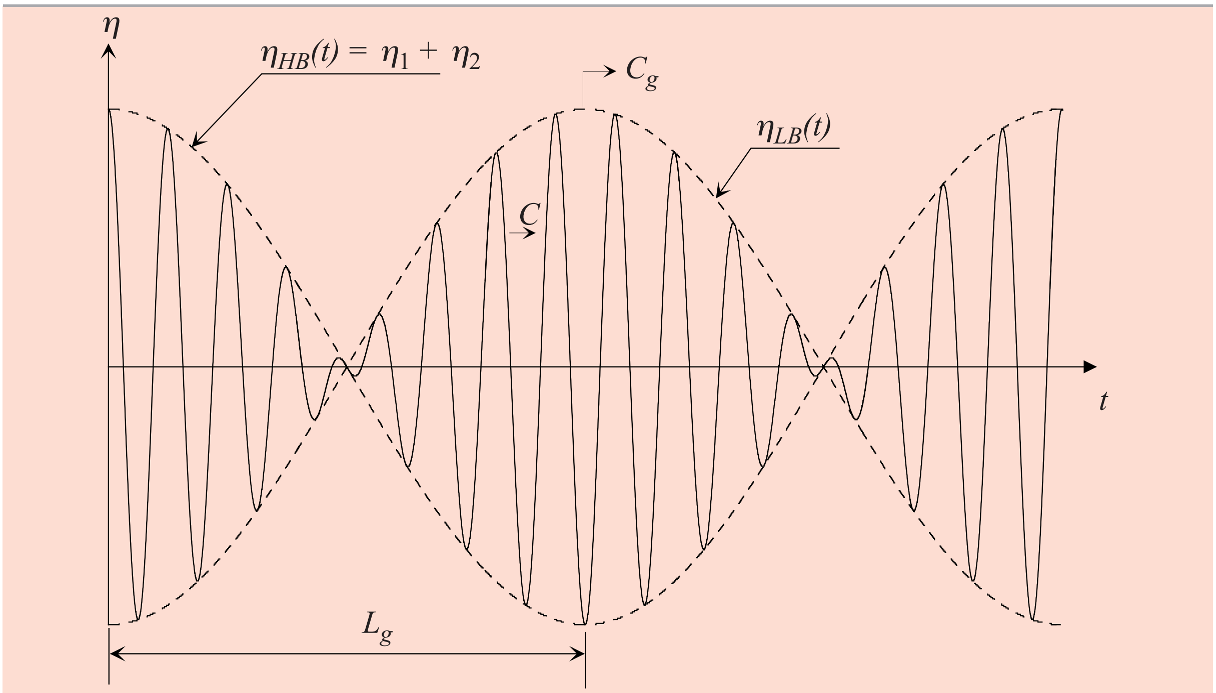
Moreover, in many cases between the probability models and their parameters corresponding to different time intervals, there are functional or statistical relations. This circumstance is used to obtain probability models of certain random variables in the useful life since available information is generally scarce.

For example, under certain assumptions, based on probability models of the exceedance of $H_s > H_{s,cr}$, it is possible to derive probability models of $H_{s,\max,cycle}$, $H_{s,\max,year}$ and $H_{s,\max,V}$. This use probability models in cascade is useful to optimize the available information and to verify the models obtained.

4.4 WAVE GROUPS

The linear superposition of two frequency components (f_1, f_2), relatively near to each other, is another wave that has wave group structure (see Figure 4.4.13). By working in the Stokes regime and expanding the analysis to the second order of approximation, it can be verified that two new oscillations appear when the wave train propagates in medium and shallow water. One of these oscillations has a frequency equal to the difference between frequencies $(f_1 - f_2)$ and the other has a frequency equal to the sum of these frequencies $(f_1 + f_2)$. These two waves are linked or bound to the linear waves. However, even though they together travel with the linear waves, their movements are clearly different from them.

Figure 4.4.13. Characteristics of a wave group formed by the sum of two sinusoidal waves with periods close to each other



The low-frequency wave $(f_1 - f_2)$ is a long wave that travels with the celerity of the wave group (i.e. it does not satisfy the linear dispersion equation). Its vertical displacement with respect to a specific reference level $\eta_{LB}(t)$ describes the oscillations of the mean sea level. It is also known as a subharmonic oscillation. Furthermore, the high-frequency oscillation $(f_1 + f_2)$ is a short-period wave that travels with the celerity that satisfies the dispersion equation of the linear wave train. Its vertical displacement $\eta_{HB}(t)$ makes the wave train asymmetric (i.e. crests higher than troughs), as can be observed in the case in which both components have the same frequency $f_1 = f_2$. Then, $f_1 + f_2 = 2f_1$ is a double frequency harmonic (period that is half of the linear wave train), which travels with the celerity of the primary wave.

This causes the steepening of the crest and the flattening of the trough. This wave is known as a second-order Stokes wave.

4.4.1 Wave group and envelope function

In the presence of wave groups, the vertical displacement on the free surface, $\eta(\vec{x};\, t)$, in the sea domain can be approximated by the linear superposition of Fourier components, whose amplitude a_n is defined by the spectral function, and its phase, ε_m is a random variable, uniformly distributed in the interval $[0,2\pi]$. Consequently, $\eta(\vec{x};\, t)$ at a point in the sea can be approximated by

$$\eta(t) = \sum_{n=1}^{N} c_n \exp\left[i\left(2\pi f_n t + \varepsilon_n\right)\right] \tag{4.266}$$

$$c_n = \sqrt{2S_\eta(f)\Delta f} \tag{4.267}$$

By selecting a representative mean frequency of the short-period oscillation $\bar{f}$, it is possible to write

$$\eta(t) = \sum_{n=1}^{N} c_n \exp\left[i\left(2\pi f_n t + \varepsilon_m\right)\right] \tag{4.268}$$

$$\eta(t) = \exp\left(i2\pi\bar{f}t\right) \sum_{n=1}^{N} c_n \exp\left[i\left\{2\pi\left(f_n - \bar{f}\right)t + \varepsilon_n\right\}\right] = \eta_E(t)\exp\left(i2\pi\bar{f}t\right) \tag{4.269}$$

which permits the definition of the envelope function,

$$\eta_E(t) = \sum_{n=1}^{N} c_n \exp\left[i\left\{2\pi\left(f_n - \bar{f}\right)t + \varepsilon_n\right\}\right] = A(t)\exp\left[i\phi(t)\right] \tag{4.270}$$

where $A(t)$ is the amplitude of the envelope function, and $\phi(t)$ is its phase. The energy flux of the irregular wave train is approximately proportional to $H^2(t) \approx 4A^2(t)$.

4.4.1.1 Envelope function

One of the methods most often used to identify the structure of the group is by means of the *SIWEH* function $E_g(t)$, which calculates the time series of the energy record.

$$\begin{aligned} E_g(t) &= \frac{1}{T_p}\int_{-\infty}^{\infty} \eta^2(t+\tau)Q(\tau)d\tau \\ Q(\tau) &= 1 - \frac{|\tau|}{T_p} \qquad |\tau| < T_p \\ Q(\tau) &= 0 \qquad |\tau| \geq T_p \end{aligned} \tag{4.271}$$

where $Q(\tau)$ is Bartlett's smoothing function.

However, to avoid signal contamination with undesirable oscillations, the envelope function is best calculated, based on a signal $z(t)$, formed by the record $\eta(t)$ and its corresponding Hilbert transform, $\widehat{\eta(t)}$,

$$z(t) = \eta(t) + i\widehat{\eta(t)} \tag{4.272}$$

either as amplitude,

$$A(t) = |z(t)| \tag{4.273}$$

as energy,

$$E(t) = |z(t)|^2 \tag{4.274}$$

4.4.1.2 Basic variables

Basic variables that define the group structure are the following:

H_*, threshold wave height in reference to which the large wave group is started;
j_1, number of waves that consecutively exceed the threshold height, namely, in the time interval between the exceedance and the following downwards crossing of the threshold height;

j_2, total number of waves in the group contained in time interval between the instant at which the threshold height is exceeded until it is exceeded again. Therefore, the time interval is defined in the same way as for the zero-crossing period;
T_{g1}, period of the larger wave group;
T_{g2}, period of the wave group or time interval between two upwards crossings of the threshold height;
$\rho_{H_1H_2}$, correlation coefficient between two consecutive waves.

4.4.1.3 State variables

The state statistics that define the wave group structure are the following:

$\bar{j}_1$, mean number of waves that consecutively exceed the threshold height;
$\bar{j}_2$, mean total number of waves in the group contained in the time interval between the instant in which threshold height is exceeded until it is exceeded again;
$\overline{T_{g1,2}}$, wave group period;
$\bar{L}_g$ mean wavelength of the group;
r_{HH}, correlation coefficient in the time domain.

The spectral state variables are the following:

$\bar{\kappa}$, spectral correlation coefficient;
$\prod$, mean wave number in a group.

4.4.1.4 High and low-frequency waves bound to irregular wave trains

The instantaneous variable is the vertical displacement $\eta_{LB}(t)$, as described in section 3.10.1.2. The basic variable is the wave amplitude ξ_{LB}, which is the maximum positive or negative value of $\eta_{LB}(t)$ with respect to the mean value of $\overline{\eta_{LB}}$. From a record with sufficient time length to obtain a statistically representative sample of ξ_{LB}, it is possible to calculate $\xi_{LB,rms}$, namely, the mean square displacement of the amplitudes or any other statistic.

When the wave train is irregular, and is formed by a large number of components, its transformation in medium and shallow water depths involves the generation of a great number of harmonic and subharmonic oscillations with values pairs $(f_n - f_m)$ and $(f_n + f_m)$, where *n, m* are the frequencies in the wave energy spectrum, If two arbitrary spectral components are expressed in the following way,

$$\eta_{nm}(x,t) = \eta_n(x,t) + \eta_m(x,t) \tag{4.275}$$

$$\eta_n(x,t) = a_n \cos(2\pi f_n - k_n x) + b_n \sin(2\pi f_n - k_n x) \tag{4.276}$$

$$\eta_m(x,t) = a_m \cos(2\pi f_m - k_m x) + b_m \sin(2\pi f_m - k_m x) \tag{4.277}$$

the short-period wave is expressed as shown below:

$$\eta_{HB}(t) = \sum_{n=f_{min}}^{N} \sum_{m=f_{min}}^{M} \begin{Bmatrix} (a_n a_m - b_n b_m) G^{+}_{nm} \cos[2\pi(f_n + f_m)t - (k_n + k_m)x] + \\ (a_n b_m - a_m b_n) G^{+}_{nm} \sin[2\pi(f_n + f_m) - (k_n + k_m)x] \end{Bmatrix} \tag{4.278}$$

and the long-period wave is expressed as shown below

$$\eta_{LB}(t) = \sum_{n=f_{min}}^{N} \sum_{m=f_{min}}^{M} \begin{Bmatrix} (a_n a_m - b_n b_m) G^{-}_{nm} \cos[2\pi(f_n - f_m)t - (k_n - k_m)x] + \\ (a_n b_m - a_m b_n) G^{-}_{nm} \sin[2\pi(f_n - f_m) - (k_n - k_m)x] \end{Bmatrix} \tag{4.279}$$

where $[G^+_{nm}, G^-_{nm}]$ are the transfer functions for the waves of frequencies $(f_n + f_m)$ and $(f_n - f_m)$, respectively. Figure 4.4.14 represents G^+_{nm} in respect to $(\sqrt{h/g})f$ and the quotient, $\Delta f/f_n (m<n)$. Figure 4.4.15 represents G^-_{nm} in respect to $(\sqrt{h/g})f$ and the quotient, $\Delta f/f_n$, $\Delta f = f_n - f_m$.

Note. *Figure 4.4.15 makes it possible to estimate the order of magnitude of long and short bound waves. In the case of a sea state with descriptors $H_s = 4$ meters, $T_p = 14$ seconds, and water depth, $h = 15$ meters, the amplitude of the long wave can be obtained in the following way.*
It is assumed that the two spectral components are equal, each of which has an amplitude equal to $H_s/4$.
It is calculated that $(\sqrt{h/g})f_p \approx 0.085$. Supposing that the number of waves in the group lies in the range of 4 to 10, then the quotient is $0,1 \le \Delta f \le 0,25$ since Figure 4.4.15 gives a value of the function $G^-_{nm}h \approx 5$, the estimated value of the amplitude of the long wave is:

$$\widehat{\eta_{LB}} = G^-_{nm} \frac{H_s}{4} \frac{H_s}{4} = 0.30m \qquad (4.280)$$

Figure 4.4.15 gives the amplitude of the second Stokes component. If one considers a wave of $T_z = 8$ seconds, a depth of $h = 25$ metros, $h/L_o = 0.25$. In this case, $f_n = f_m$, the transfer function $(f_n/f_m = 1)$ is $G^+_{nm} = 0.044$. If the linear wave has an amplitude $a_{(1)} = 2$ m, the second-order Stokes component of double frequency has an amplitude of $a_{(2)} = 2^2 * 0.044 = 0.018$ m. The wave crest then reaches a height of 2,18 and the trough, –1,82.

4.4.1.5 Energy spectrum of high and low-frequency bound waves

The energy spectrum of high-frequency oscillations $(f_n + f_m)$ is obtained by applying the transfer function, G^+_{nm}, to the product of the components of the spectral density function,

Figure 4.4.14. G^+_{nm} transfer functions

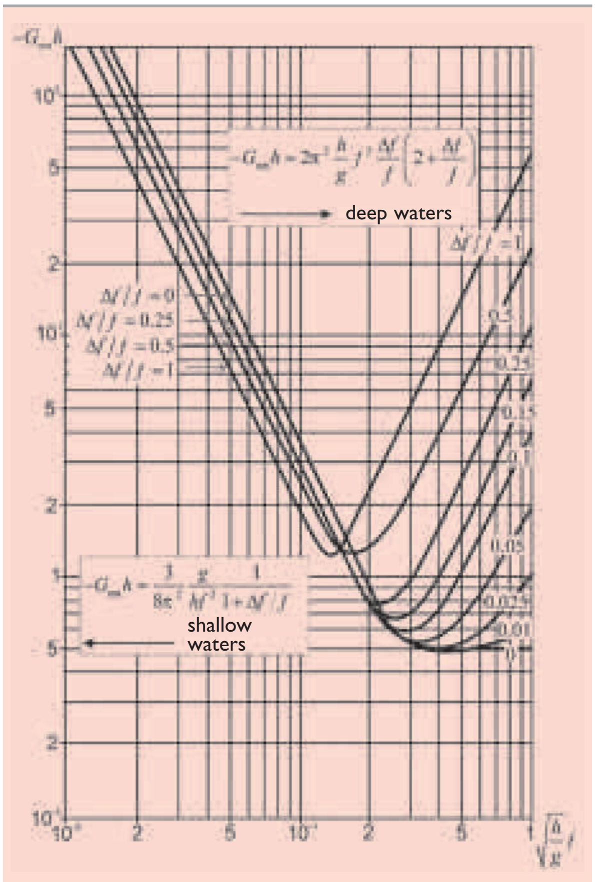
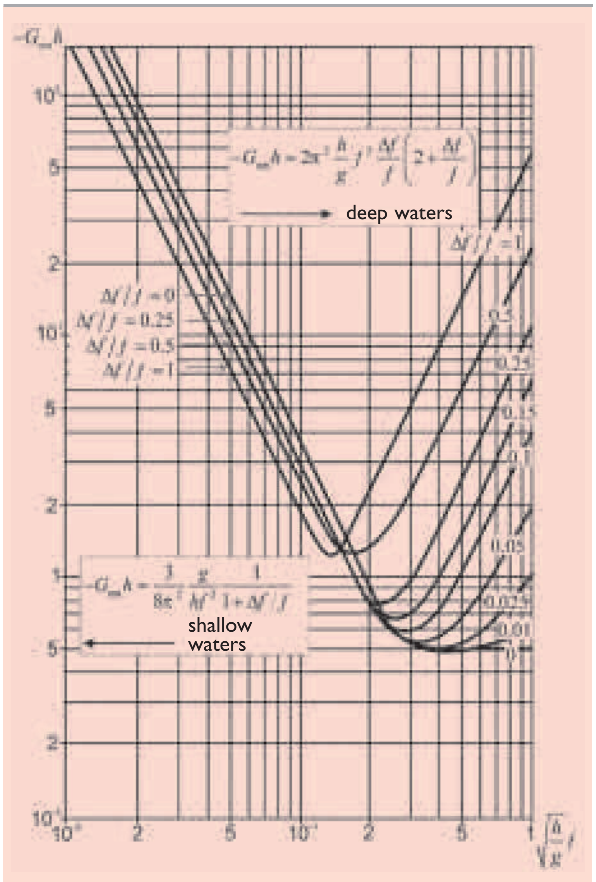


Figure 4.4.15. G^-_{nm} transfer functions



$$S_{\eta_{HB}}(f) = 2\int_0^{f/2} S_\eta(f) S_\eta(f - f^*) \left[G^+(f, f^*)\right]^2 df^* \tag{4.281}$$

where $S_\eta(f)$ is the local wave spectrum, as described in section 3.5.2.8. Because of the relative phases between the linear components and the high-frequency components, the total spectrum of the two movements is not equal to the sum of the two spectra. Similarly, the energy spectrum of the low-frequency oscillations $(f_n - f_m)$ is obtained by applying the transfer function to the wave spectrum $G^-(f, f^*)$,

$$S_{\eta_{LB}}(f) = 2\int_0^{f/2} S_\eta(f) S_\eta(f + f^*) \left[G^-(f, f^*)\right]^2 df^* \tag{4.282}$$

These are long waves that make up the structure of the wave group. The amplitude of the spectral component is proportional to the square of the envelope wave.

4.4.1.5.1 Information in the signal

When wave action is recorded at a given point in the sea, where high-frequency and low-frequency components jointly propagate with the irregular wave train, the record contains information concerning all three of these elements, namely:

$$\eta_{register}(t) = \eta_{wave}(t) + \eta_{LB}(t) + \eta_{HB}(t) \tag{4.283}$$

Consequently, when analyzing the record, it is necessary to filter the low-frequency signal, for example, by discarding the components in the frequency domain, $f < 0.04\ hz$, and subsequently, calculating the second-order components, beginning with the lowest frequencies on the spectrum, whose contribution to the harmonic motion is negligible, and progressively calculating the second-order contribution and subtracting it from the spectrum. In such cases in which one wishes to eliminate the high and low frequencies, the cut-off point should be in the range, $0.5 < f/f_p < 1.5$.

4.5 METEO-TSUNAMIS

Changes in atmospheric pressure, in cyclonic as well as anti-cyclonic conditions, can generate sea level oscillations in a period band that ranges from a few minutes to several hours. Such oscillations occur in the form of atmospheric gravity wave trains or sudden isolated pressure jumps.

4.5.1 Generation

These perturbations can have various origins. They can be due to dynamic instability, orography, warm fronts, northwest winds, tornados, etc. Generally speaking, the atmospheric pressure jump, and consequently, the energy content of these perturbations is small, even in very severe conditions. For this reason, in the absence of any other mechanism, their presence only causes the sea level to vary by a few centimeters. However, sometimes, these perturbations can cause sea level oscillations of a larger magnitude. For this reason, their expansion should be associated with other resonant mechanisms between the ocean and the atmosphere.

Thus, for sea level oscillations or meteo-tsunamis to occur, it is necessary for a complex chain of processes to ensue, i.e. the existence of an atmospheric perturbation, the coupling of sea and atmospheric waves, and finally the forcing of the port or coastal area.

4.5.1.1 Barometric pulses in the open sea

These mechanisms intervene in the propagation phase and acquire their importance in one of the three regions of topographic transformation of the tsunami (which are the same as those for seismic tsunamis): (i) open sea; (ii) regional or continental shelf; (iii) local (e.g. bay, inlet, estuary or port area). The possible resonant mechanisms are the following:

1. **Oceanic resonance** occurs when on the open sea, the propagation velocity of the barometric pulse is equal to the celerity of the wave $(U_{pa} \approx C)$ where $C = \sqrt{gh_0}$ and h_0 is the ocean depth in the generation zone.
2. **Edge wave resonance** occurs when the length component of the barometric pulse is equal to the celerity of one of the edge wave modes.
3. **Continental shelf resonance** occurs when the periods of the barometric pulse and the corresponding ocean wave coincide with periods of the continental shelf.
4. **Local resonance** occurs when the oscillation that reaches the port or shoreline area has periods close to those of the overall area or to those of the wharfs or coastal zones.

In this case, the occurrence of the local oscillation at the port entrance is a necessary condition for the occurrence of the resonant port oscillation. Nevertheless, its magnitude depends on the transfer function or the amplification function,

$$H^2(f) = \frac{Q^2}{\left[Q\left(1-\frac{f}{f_0}\right)\right]^2 + \left(\frac{f}{f_0}\right)^2} \qquad (4.284)$$

where f, f_0 are the frequencies of the signal of the tsunami and of the port area; and Q is a parameter that measures the energy damping capacity of the area. Consequently, it permits the evaluation of the amplification of the system, and the time required to reduce the oscillation to a certain value. Q basically depends on the geometry of the area and on its communication with the sea in which it is forced to oscillate.

4.5.1.2 Local barometric pulses

In the shoreline areas, due to the orographic influence, buildings, large installations, and moored or anchored vessels, there are local barometric pulses related to the emission of vortexes and the re-adaptation of the airflow on the leeside of the obstacle or because of local thermal gradients associated with spatial gradients of the albedo.

These pulses can generate edge waves on nearby beaches that propagate at the entrance of the port area or generate oscillations in the wharf. The oscillation can be resonant either because the barometric pulse is stationary (attached to the obstacle), or because the component of its propagation velocity in the direction parallel to the coast is equal to the celerity of one of the edge wave modes or to the same (longitudinal or transversal) oscillation mode of the wharf itself.

In both cases, it can be identified and quantified in the same way as the atmospheric pulses in the open sea.

4.5.1.3 Simplified equations and models

When the pressure signal in the zone is known because it has been measured with microbarographs, such that its propagation direction (if it is propagating) can be identified, the sea level oscillations can be calculated by resolving the general system of equations with boundary conditions of the work domain, which appear in the section on long waves.

The scale of the pressure gradients is related to the generating agent by the emission of vortexes or the re-adaptation of the flow. Its mathematical expression (F_x, F_y) also depends on its origin, and thus, it should be modeled on the basis of measurements taken directly at the site. Sometimes, it is necessary to include the propagation velocity, C_p. If this velocity is assumed to be approximately constant, it is easy to suppose that the atmospheric pressure is a function of $(C_p t - x)$, where t is the time, and x is the propagation direction of the barometric wave, $p_a = f(C_p t - x)$.

4.6 TSUNAMIS

A tsunami can have many different causes, as described in the following section, and propagates from the generation zone. As it travels, it is transformed by changes in the seabed. It can force oscillations in the port or coastal area, which can be resonant, generate currents of considerable magnitude, and flood large areas of the coastline.

4.6.1 Generation

A tsunami can occur for different reasons, such as sudden movements of the seabottom, landslides of submerged or emerged slopes, volcanic eruptions, and the impact of asteroids. All of these events can cause vertical movements in the water column. Once the vertical movement occurs, the action of gravity tries to restore the equilibrium of the water mass. The result is a wave train that radially propagates from the generation zone.

The most active and important vertical movements in the Earth's crust occur at the edges of the tectonic plates, mainly at the meeting places of the thickest oceanic plates that slide beneath the continental plates in a process, known as *subduction*.

The magnitude of the tsunami, particularly the initial water volume, depends on a variety of factors. These mainly include the centroid location of the earthquake or the forcing mechanism (location of the energy liberation zone, slope of the subduction plane, epicenter, propagation of the crack and its orientation), water depth in the generation zone, acceleration and maximum velocity reached by the generating movement. This generating movement can be a submarine earthquake, landslide, etc., and the efficiency and coupling between these movements and the water displacements, namely, the efficiency of the mechanisms transmitting energy to the water body, which are still being analyzed and modeled. This important number of factors that control the initial magnitude of the tsunami makes their formulation and modeling more difficult.

Note. Generally speaking, most tsunamis are caused by seismic action. Large tsunamis must be generated by submarine earthquakes of a magnitude greater than 7 *on the Richter scale, and whose epicenter is at less than* 30 *km in depth inside the Earth.*
However, tsunamis can have other causes, such as landslides or volcanic eruptions. This term is also used to refer to meteo-tsunamis.

4.6.2 Transformation models

The mathematical description of a seismic tsunami should contain the following phases: generation, propagation, and flooding (see Figure 4.4.16). In the generation phase, it is necessary to simulate the behavior of the Earth's crust during the earthquake, its velocity fields and accelerations, which is a fairly complicated problem. It is thus common practice to reduce this phase to the definition of the oscillation form of the free surface.

As a result, the generation of a tsunami by a submarine earthquake can be regarded as a process involving pulses, and consequently can be treated like a Cauchy-Poisson problem. Although the seabed movement can be accurately represented by using Heaviside functions, there is little information regarding the dimensions and kinematics of the movement.

Accordingly, the initial elevation of the free surface can be said to be equal to the displacement of the bottom (without taking into account the dynamic nature of the fracturing, and considering the initial velocity field to be zero), such that the problem of the propagation and transformation of the tsunami becomes a wave propagation problem with a known initial value.

The equations that govern the tsunami movement are Navier-Stokes equation, integrated into the water column, as described in the section on long waves. Moreover, the pressure p is hydrostatic (i.e. the vertical accelerations in the fluid are negligible) and the equation system is written, depending on the instantaneous variable η, which represents the vertical displacement of the free surface with respect to a reference level,

$$p = \rho g \left(h_0 + \eta \right) \tag{4.285}$$

If the bottom is horizontal, the movement equations are reduced to the wave equation,

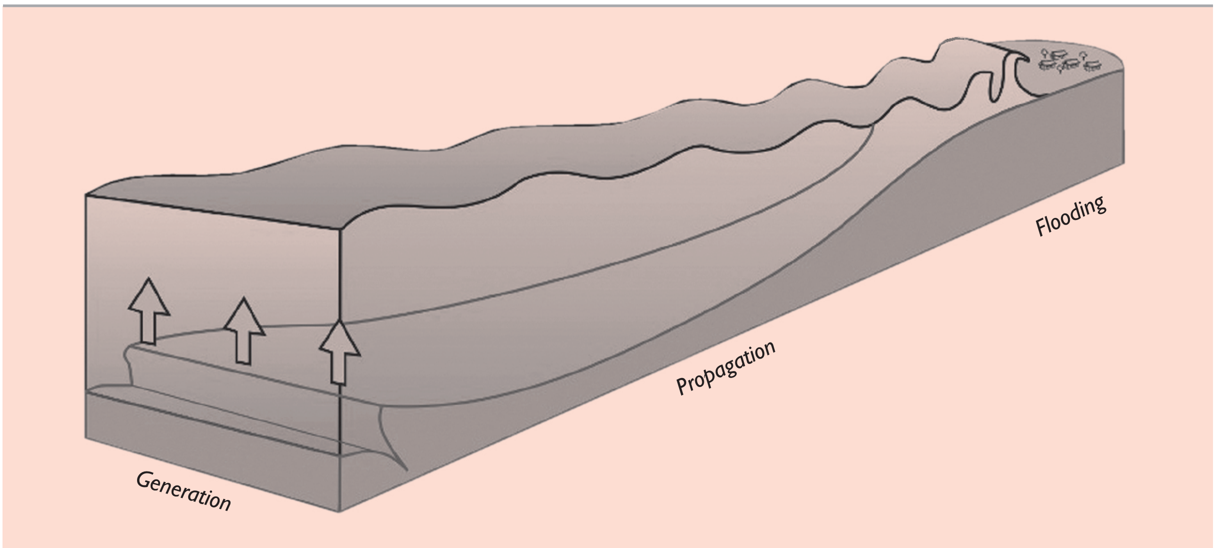
$$\nabla^2 \eta = \frac{1}{C^2} \frac{\partial^2 \eta}{\partial t^2} \tag{4.286}$$

where $C = \sqrt{gh_0}$ is the wave celerity. This equation can have simple analytical solutions.

If the seabed varies and the hypothesis of hydrostatic pressure is maintained, this results in shallow water linear equations. If convective acceleration is included, nonlinear shallow water equations are obtained. And if the wave celerity is not constant, but rather frequency and amplitude-dispersive, Boussinesq equations are obtained.

In recent years, significant advances have been made in the three-dimensional integration of the complete problem. The resolution of any of these equation systems require numerical techniques similar to those recommended in the sections regarding wave propagation and wave action.

Figure 4.4.16. Scheme of the phase of a seismic tsunami



4.6.2.1 Modeling the area around the port and coastal zone

There is sometimes no need to completely evaluate the generation and propagation of the tsunami until the site location. Rather, it is sufficient to evaluate, for example, the worst overtopping conditions or stability conditions of a maritime structure. In these cases, the problem is best solved wave by wave. This means defining as an initial condition of the problem each of the individual waves of the tsunami in terms of their amplitude, and representing their shape by means of positive or negative Gaussian functions, or solitary waves.

The propagation of this signal towards the coast can be carried out by integrating complete nonlinear shallow water equations, transformed in a hyperbolic equation, and obtaining the maximum up slope and down slope movement, propagation in the port area, etc. This method of analysis is not very difficult when it is a question of analyzing the wave propagation in two dimensions.

When it is a question of analyzing the transformation of the signal, taking into account the bathymetry at the site, the problem can be resolved by integrating the nonlinear shallow water equation system or by formulating and resolving the propagation of the tsunami as a complete gravity wave problem, in other words, with nonlinear boundary conditions.

In the latter case, the solution can follow the same steps as those recommended for the problem of regular wave trains. These steps include decomposing the initial signal (solitary wave or Gaussian curve) in Fourier components, propagating them with MSP elliptic models, and reuniting at the site the partial results of the different components propagated to reconstruct the transformed signal. This method has been found to be effective even for the propagation of solitary waves through porous mediums, where the diffraction at the port entrance is taken into account. Furthermore, each component of the signal can be used to analyze its capacity to force resonant oscillations in port and shoreline areas.

When the temporal evolution of the surface profile needs to be studied, the solution of the problem can be obtained by applying the non-stationary mild slope equation or by using one of the Boussinesq models.

It should be underlined that the initial form of the signal considerably influences the results. Generally, the positive Gaussian signal and negative Gaussian signal produce the same up-slope and down-slope values, but a negative signal followed by positive one of greater or lesser volume causes a differentiated laminar flow water sheet behavior on the differentiated slope.

4.6.2.2 Oscillatory forcing of port and coastal areas

Apart from the devastating capacity of large tsunamis to flood extensive areas of the coast, less intense tsunamis can also have serious consequences if they can force resonant oscillations in port areas, coves, inlets, and estuaries. The extensive forcing capacity of a tsunami is due to the wide range of periods of its temporal signal, which goes from a few minutes to four hours. The lower band of 2-4 minutes is the typical oscillation band of wharfs and port areas, whereas the band of 10-50 minutes is the typical oscillatory band of small estuaries, inlets, and very long wharfs. In those zones in which tsunamis can occur, it is advisable to analyze the forcing of the port, shoreline area, estuary, or inlet for the period band contained in the tsunami signal. When relevant, the result should be taken into account in the development of the port area or in the modifications carried out over time.

4.7 LONG WAVES AND SEA LEVEL

This section describes the kinematics and dynamics related to long waves that can affect port and shoreline areas, more specifically, meteorological and astronomic tides, tsunamis, and barometric pulses or meteo-tsunamis. Forced oscillations in port installations, along with edge and shear waves, because of their importance in port use and exploitation, are analyzed in the ROM 1.1. The river flood wave can be important in ports located in estuaries and inlets. Their hydrodynamic analysis can be performed in the same way as described for the tidal bore in a river or estuary.

4.7.1 Introduction

A wave is regarded as a long wave when its length L is much greater than the depth h where it propagates. The following criterion is usually adopted:

$$\frac{h}{L} \lesssim \frac{1}{20}$$

When the semi-diurnal astronomic tide propagates in a depth of $200\ m$, it has a wavelength of approximately $150\ km$, thus satisfying the previously mentioned requirement. The astronomical tide refers to the regular oscillations of the sea or ocean surface, due to variations in the force of gravity caused by the relative movements of heavenly bodies, mainly those of the moon revolving around the Earth, and those of the Earth, revolving around the Sun.

There are other long waves that are generated by movements in the lateral, bottom, and atmospheric surface boundaries. The meteorological tide is the set of oscillations of the free sea surface, generated during the passage of tropical or extratropical cyclones and by the associated wind action. The short-period pressure waves (band of periods ranging from minutes to a few hours) are generated by atmospheric dynamics, topographically induced or caused by thermal processes, and can force sea oscillations in that period band. Since they approximately coincide with the period band of seismic tsunamis, they are known as meteo-tsunamis. Tidal waves or tsunamis are long waves, generated by movements of marine boundaries, such as the seabottom because of a submarine earthquake, or of a sea slope because of a sediment slump or landslide above or underneath the water. Types of tsunamis are often differentiated according to their cause. Accordingly, there are earthquake-induced tsunamis, generated by submarine seismic movements and landslide-induced tsunamis, generated by submarine or coastal landslides.

When these waves propagate along lateral boundaries and variable beds, they are transformed by refraction, shoaling, reflection, and bottom friction. The longest waves, such as the astronomical tide, only break in very special cases. However, tsunamis usually reach the coast as they break. Generally speaking, linear theory can be used to describe long waves, though for this purpose, its relative amplitude, A/h, should be small (around 0.1). In those cases in which the linearity hypothesis (a negligible wave amplitude in relation to depth) does not hold, wave transformation is performed by applying complete equations previous to the analysis of the importance of each equation term with a view to maximally simplifying the system to be resolved.

Note. When a long wave (e.g. astronomical tide) reaches the continental shelf, there is a sudden change of depth since the depth varies, approximately, $h_{pc}/h_o \approx 1/10$ *at a distance,* $l_t \ll L$. *It is as though the wave collides against a step. It is thus reflected on the slope, and part of the energy returns to the sea. Moreover, when the wave propagates on the continental slope, given that the celerity depends on the depth, this produces the shoaling and refraction of the wave with the subsequent changes in amplitude and propagation direction in the same way as for other waves. Finally, when the wave reaches the coastline, it is reflected by this boundary. Since the depth is shallow, friction effects during the propagation on the platform are not negligible, and consequently, energy is dissipated. This effect is even more accentuated in estuaries and inlets. The wave amplitude at each point is an equilibrium between the shoaling and refraction effects, which generally cause an increase in wave amplitude and the friction effects that reduce it.*

Because a long wave is generally not steep, even though its amplitude increases because of shoaling, unlike other waves, it rarely breaks when it propagates in the coastal zone. However when a long wave propagates in confined areas, such as inlets, coves and estuaries, or when the slope of the coastal zone is so mild that the wave floods areas of dry land, the long wave can break. This is a frequent occurrence in the case of tsunamis and the meteorological tide.

4.7.2 Longwave (shallow water) equations

The equations governing the movement of longwave or shallow water equations are Navier-Stokes turbulent flow equations, which are integrated in the water column. They are based on the fact that in a long wave, there is considerably less vertical motion than horizontal motion, such that the flow is assumed to be flat and the pressure, hydrostatic. These equations can be applied to the study of the propagation, transformation, and dissipation of astronomical and meteorological tides, as well as to tsunamis, or any long wave whose wave length is much greater than the propagation depth. Shallow water equations with hydrostatic pressure are

- the mass conservation equation,

$$\frac{\partial(h_0+\eta)}{\partial t}+\frac{\partial\left[U(h_0+\eta)\right]}{\partial x}+\frac{\partial\left[V(h_0+\eta)\right]}{\partial y}=0 \qquad (4.287)$$

- the horizontal momentum conservation equations,

$$\frac{\partial\left[U(h_0+\eta)\right]}{\partial t}+\frac{\partial\left[U^2(h_0+\eta)\right]}{\partial x}+\frac{\partial\left[UV(h_0+\eta)\right]}{\partial y}=\frac{(h_0+\eta)}{\rho}\left(\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}\right)+Q_x^* \qquad (4.288)$$

$$\frac{\partial\left[V\left(h_0+\eta\right)\right]}{\partial t}+\frac{\partial\left[VU\left(h_0+\eta\right)\right]}{\partial x}+\frac{\partial\left[V^2\left(h_0+\eta\right)\right]}{\partial y}=\frac{\left(h_0+\eta\right)}{\rho}\left(\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}\right)+Q_y^* \qquad (4.289)$$

where h_0 is the still water depth, measured on a horizontal plane of reference (generally, the still water level); η is the superelevation of the sea level, based on a reference plane; and U and V are the horizontal velocity-field components, according to the x and y axes, respectively, averaged in the total depth of the water column, $h = h_0 + \eta$,

$$U=\frac{1}{h}\int_{-h_0}^{\eta} u\,dz \qquad V=\frac{1}{h}\int_{-h_0}^{\eta} v\,dz \qquad (4.290)$$

Consequently, $U(h_0 + \eta)$ and $V(h_0 + \eta)$ express the total volumes per unit of width or water flow through the planes perpendicular to the x and y axes, respectively. The first term at the right of the moment conservation equations, $(\partial\tau_{xx}/\partial x + \partial\tau_{yx}/\partial y)$, are the horizontal gradients of the tangential stresses in the fluid, which represent the horizontal diffusion generally related to turbulent processes. Finally, there are the terms, Q_x^*, Q_y^*, which include the effects of the following:

1. The spatial gradients of the free surface $(\partial\eta/\partial x, \partial\eta/\partial y)$,
2. The spatial gradients of the atmospheric pressure on the water surface, $[\partial p_a/\partial x, \partial p_a/\partial y]_\eta$
3. The tangential stresses on the water surface η, due to wind action $[\tau_x, \tau_y]_\eta$
4. The friction caused by the bottom $-h_0$, on the fluid $[\tau_x, \tau_y]_{-h_0}$
5. The Coriolis force, $[r_x = -f_C V, r_y = f_C U]$, where f_C is the Coriolis coefficient, depending on the latitude.

A mathematical structure of the source term is generally the following:

$$Q_x^*=\underset{(1)}{-g\frac{\partial\eta}{\partial x}}+\left[\underset{(2)}{\frac{\partial\left(-\frac{p_a}{\rho}\right)}{\partial x}}+\underset{(3)}{\frac{\tau_x}{\rho\left(h_0+\eta\right)}}\right]_\eta\underset{(4)}{-\frac{\left[\tau_x\right]-h_0}{\rho\left(h_0+\eta\right)}}\underset{(5)}{-f_C\frac{V}{\left(h_0+\eta\right)}} \qquad (4.291)$$

$$Q_y^*=\underset{(1)}{-g\frac{\partial\eta}{\partial y}}+\left[\underset{(2)}{\frac{\partial\left(-\frac{p_a}{\rho}\right)}{\partial y}}+\underset{(3)}{\frac{\tau_y}{\rho\left(h_0+\eta\right)}}\right]_\eta\underset{(4)}{-\frac{\left[\tau_y\right]-h_0}{\rho\left(h_0+\eta\right)}}\underset{(5)}{+f_C\frac{U}{\left(h_0+\eta\right)}} \qquad (4.292)$$

The unknowns in this equation system are U, V and η, and their solution requires the specification of initial conditions and suitable boundary conditions. Moreover, it is necessary to specify the tangential wind action on the free surface, namely, $[\tau_x, \tau_y]_\eta$, and the tangential stresses on the bottom or bottom friction $[\tau_x, \tau_y]_{-h_0}$.

Note. It should be mentioned that in the books and articles on geophysical fluids, the water depth is usually represented with the uppercase letter, H. Given the fact that in maritime engineering, this nomenclature is generally used to represent wave height, for convenience sake, h_0 is used to refer to still water depth, and h is used to refer to the total water column. Furthermore, the curvature of the free surface of long waves is very large. For this reason, in practically all cases, there is no need to consider the effects of the surface stress in the pressure on the interphase.

4.7.2.1 Current velocity profile

Generally speaking, for the dimensioning of port areas, knowledge of the velocity profile of the current is not an important factor. It is sufficient to know the mean velocity, (U,V), integrated at depth h, and the friction coefficient

(e. g. Chezy). When knowledge of the velocity profile is necessary, it is advisable to measure the mean current velocity *u* at a distance z near to the water surface and assume a potential velocity profile [(17)],

$$u(z) = u(z_1)\left(\frac{x+h}{z_1+h}\right)^{\frac{1}{7}} \qquad (4.293)$$

where h is the depth at the measurement point.

4.7.2.2 Importance and specification of the terms

Given the complexity of equations and taking into account the different situations that can occur, this section and the following ones analyze the terms in general equations, and how they can be simplified.

The momentum conservation equations represent the equilibrium between the forces of inertia, pressure gradients and density, the internal consumption of momentum due to viscosity and turbulence, and external forces. Some are applied in the water body because of celestial action (generation of the astronomical tide), and the rotation of the Earth (Coriolis effect), whereas others are applied to the lower boundary, due to bottom friction and their accelerations (meteorological tide).

The following sections analyze the importance of these forces, beginning with the Coriolis effect and the stratification of the water column. Afterwards, the rest of the terms are adimensionalized.

4.7.2.2.1 Importance of the Earth's rotation

The rotation of the Earth or the Coriolis effect can be an important factor in the movement of sea oscillations. The period of the Coriolis effect is $T_\Omega = 1/\Omega\ (s)$, where $\Omega = 7{,}29 * 10^{-5}\ (1/s)$ is the rotation of the Earth. If T, L and U are, respectively, the temporal scale, the spatial scale, and the scale of the fluid velocity motion, where $T = O(L/U)$, then a water particle that travels at velocity U, covers a distance L in a time period T. If $T > T_\Omega$, the path of the particle will be influenced by the rotation of the Earth. Consequently, if the quotient of both periods is $\xi = T_\Omega/T = 2\pi U/\Omega L \lesssim 1$, the rotation is important in the fluid motion and the Coriolis effect should be considered in the motion equations. Table 4.4.4 summarizes the length and velocity scales of the motion which fulfill the condition, $\xi \lesssim 1$. Therefore, it is necessary to include the Coriolis term in the analysis of the motion, where the last distance is the radius of the Earth, $R = 6371\ km$.

Table 4.4.4. Length and velocity scales ffor which Coriolis force must be considered

L (Km)	U (m/s)
10^{-3}	$\leq 1.2 * 10^{-5}$
10^{-1}	$\leq 1.2 * 10^{-3}$
1	$\leq 1.2 * 10^{-2}$
10^{2}	≤ 1.2
10^{3}	$\leq 1.2 * 10$
6371	$\leq 7.2 * 10$

These velocities and scales are found in different longwave propagation situations, meteorological and astronomical tides, mainly, in the ocean and on the continental shelf.

(17) When the velocity of the current is represented in uppercase letters, it refers to its nature of velocity integrated in the water column. The use of lowecase means that the velocity is measured at a certain depth.

4.7.2.2.2 IMPORTANCE OF STRATIFICATION

Generally, geophysical fluids consist of fluid volumes with different densities which, because of the action of gravity, tend to become organized in vertical columns corresponding to a minimum potential energy state. The movement of the oceans continuously alters this state of equilibrium, producing the upward movement of dense fluid and the downward movement of lighter-weight fluid. This increase in potential energy should be carried out at the expense of cinematic energy. Consequently, the importance of the stratification in the dynamics of oscillations can be evaluated by comparing the potential and cinematic energy.

If $\Delta\rho$ is the density variation scale and h is the water column scale, the upwards movement Δh, of dense water with density, $\rho_0 + \Delta\rho$, should be compensated by the downward movement Δh of lightweight water of density ρ_0. Accordingly, the variation in the potential energy per unit of volume is $(\rho_0 + \Delta\rho)g\Delta h - \rho_0 g\Delta h = \Delta\rho g\Delta h$. For a representative velocity of the fluid U, the available cinematic energy per unit of volume is $1/2(\rho_0 U^2)$. The quotient of both energies σ_s is the following:

$$\sigma_s = \frac{\frac{1}{2}\rho_0 U^2}{\Delta\rho g \Delta h} \tag{4.294}$$

If $\sigma_s \sim 1$, the potential energy available is consumed in cinematic energy. As a result, the stratification substantially modifies the movement of the fluid. If $\sigma_s \ll 1$, there is not sufficient cinematic energy to significantly modify the flow. Finally, if $\sigma_s \gg 1$, very little cinematic information is consumed in the process, and the stratification does not usually affect the flow. Consequently, stratification should be considered in all cases in which $\sigma_s \lesssim 1$.

For the stratification and rotation of the Earth to be important, and consequently, for both to be considered in the study of oscillations, it should hold that $\xi \lesssim 1$, $\sigma_s \lesssim 1$. This occurs if, $L \sim U/\Omega$ and $U \sim \sqrt{(\Delta\rho/\rho_0)g\Delta h}$. When the velocity is eliminated, this gives the following spatial scale,

$$L \sim \frac{1}{\Omega}\sqrt{\frac{\Delta\rho}{\rho_0} g\Delta h} \tag{4.295}$$

Note. *For values, $\rho_0 = 1028$ kg/m^3, $\Delta\rho = 2$ kg/m^3, $\Delta h = 10$ m the critical scale of movement is $L \sim 6000$ m, and $U \sim 0,4$ m/s. These scales are not frequent in propagation studies of long waves in the port area. However, in those cases where freshwater and saltwater mix together, the effects of the stratification are not negligible.*

4.7.2.3 Evaluation of the bottom-friction term

The effect of bottom friction in the propagation of long waves is evident in the gradual reduction of their amplitude. In many occasions, such as the propagation in inlets and estuaries, this damping should not be ignored.

Generally speaking, the friction term $[\tau_{xz}, \tau_{yz}]_{z=-h_0}$ is expressed by a square function of the horizontal velocity field, affected by a friction coefficient, which depends on the roughness of the seabed and the hydraulic conditions of the fluid. These conditions can be evaluated by means of two adimensional numbers, namely, the relative roughness, k_s/h, and the Reynolds number, Uh/ν.

This produces the conventional organization of the hydraulic flow as laminar or turbulent, smooth or rough. Thus, *ks* is known as Nikuradse roughness; $h = h_0 + \eta$ is the total height of the water column; and ν is the cinematic viscosity of the water $[\sim 10^{-6}(m^2/s)]$.

This functional relation is found in the calculations of pressure pipes, and has been imported to free laminar regimes. The equations of Darcy-Weisbach, Manning-Strickler, and Chezy follow this scheme. The three can be expressed as follows:

$$[\tau_x]_{-h_0} = \nu_f U|U| \quad [\tau_y]_{-h_0} = \nu_f V|V| \tag{4.296}$$

where v_f is a friction coefficient that depends on the bedforms and the forms of the hydraulic regime. The relations between this global friction coefficient and the Chezy, Manning-Stickler, and Darcy-Weisbach coefficients are the following:

	Darcy – Weisbach	Manning	Chezy
v_f	$\frac{1}{8}\frac{f}{h^2}$	$\frac{g n_f}{h^{\frac{7}{3}}}$	$\frac{g}{C_f h^2}$

where f is the friction coefficient of Darcy-Weisbach; C_f is the coefficient of Chezy; and n_f is the coefficient of Manning. Both f and n_f are directly proportional to the friction. Accordingly, f takes values in the range of 10^{-1} - 10^{-4}. Since C_f is in the denominator, the larger its value, the less important is the bottom friction. In estuaries and the continental shelf, it has a range of 30-80.

4.7.2.4 Importance of the inertia and friction terms

In order to facilitate its presentation, the momentum conservation equation is only adimensionalized in the direction of the *x* axis, without considering the external forces, $F_{c,x}$, F_x, (since their order of magnitude has already been analyzed),

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial(h+\eta)}{\partial x} + g\frac{U|U|}{C_f^2(h+\eta)} = 0 \tag{4.297}$$

The first term (1) is the local acceleration; the second (2) represents the convective terms; the third (3) is the force due to the gradient of the water column; and the fourth term (4) represents the bottom friction in the Chezy format. If T, L and D are a time interval, a horizontal length, and a vertical length characteristic of the movement, and υ is the flow velocity scale, $\upsilon = L/T$, the previous equation can be adimensionalized by considering the following adimensional variables, $x^* = x/L$, $U^* = U/\upsilon$, $t^* = t/T$, $\eta^* = \eta/D$, $h^* = h/D$.

$$\frac{\upsilon}{T}\frac{\partial U^*}{\partial t^*} + \frac{\upsilon^2}{L}U^*\frac{\partial U^*}{\partial x^*} + \frac{gD}{L}\frac{\partial(h^*+\eta^*)}{\partial x^*} + \frac{g\upsilon^2}{C_f^2 D}\frac{U^*|U^*|}{(h^*+\eta^*)} = 0 \tag{4.298}$$

Each equation term is multiplied by a monomial with the dimensions of the term. Since the object of analysis is longwave movement, the friction term should always appear.

For this reason, its monomial is given the value of a unit and the other terms are multiplied by their inverse, the following equation is obtained:

$$\frac{C_f^2 D}{g\upsilon T}\frac{\partial U^*}{\partial t^*} + \frac{C_f^2 D}{gL}U^*\frac{\partial U^*}{\partial x^*} + \frac{C_f^2 D^2}{\upsilon^2 L}\frac{\partial(h^*+\eta^*)}{\partial x^*} + \frac{U^*|U^*|}{(h^*+\eta^*)} = 0 \tag{4.299}$$

The following representative values of the movement of a long wave $C_f = 50\ m^{1/2}/s$ and a Froude number $F_r = u^2/gD \approx 0.10$ give the following orders of magnitude of the monomials, (see Table 4.4.5).

Table 4.4.5. Orders of magnitude of the adimensional monomials of the longwave propagation equation

Term	
(1)	$1000\sqrt{D}/T$
(2)	$250D/L$
(3)	$25000D/L$
(4)	1

When these results are applied to different port contexts in relation to long waves, this gives the information found in Tables 4.4.6, 4.4.7, and 4.4.8.

Table 4.4.6. Orders of magnitude of the monomials for the astronomical tide

Astronomical tide	*D(m)*	*T(s)*	*L(m)*	(1)	(2)	(3)	(4)
Continental shelf	10^2	10^4	10^6	1	10^{-2}	1	—
Tidal inlets and estuaries	1	10^4	10^4	10^{-1}	10^{-1}	10	1
Wharfs	10	10^4	10^5	1	10^{-1}	10	1

In each case, it is necessary to add the forcing or generating term F, which is of $O(1)$ to higher-order terms, In the case of propagation on the continental shelf, the Coriolis effect F_c is crucial in the study of the astronomical tide. However, the friction effect is negligible because, generally speaking, the velocity is very small. The tables show that in the majority of cases, the convective terms need not be considered. The convective terms play a predominant role in the governing equation only in those cases in which the velocities, $U \sim 1\ m/s$ and their gradients are important. In such conditions, an analytical solution is usually found. This means that numerical methods must be used.

4.7.3 Linear problem and Coriolis effect: Kelvin waves

The influence of the Coriolis effect on the propagation of long waves, whose scale satisfies the previous table (e.g. the astronomical tide propagating in the Bay of Biscay, $l = O(1000Km)$, $U < 1m/s$) can be evaluated by considering the motion equations without friction and a horizontal bed,

$$\frac{\partial \eta}{\partial t} + h\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = 0 \qquad (4.300)$$

$$\frac{\partial U}{\partial t} + f_c V + g\frac{\partial \eta}{\partial x} = 0 \qquad (4.301)$$

$$\frac{\partial V}{\partial t} + f_c U + g\frac{\partial \eta}{\partial y} = 0 \qquad (4.302)$$

Table 4.4.7. Orders of magnitude of the monomials for the barometric pulses

Barometric pulses	*D(m)*	*T(s)*	*L(m)*	(1)	(2)	(3)	(4)
Continental shelf	10^2	10^3	10^4	1	1	10^2	—
Tidal inlets and estuaries	1	10^3	10^3	1	10^{-1}	10	1
Wharfs	10	10^3	10^3	1	10^{-1}	10	1

Table 4.4.8. Orders of magnitude of the monomials for tsunamis

Tsunamis	*D(m)*	*T(s)*	*L(m)*	(1)	(2)	(3)	(4)
Continental shelf	10^2	10^3	10^4	1	10^{-1}	10	—
Tidal inlets and estuaries	1	10^3	10^4	1	10^{-1}	10	1
Wharfs	10	10^3	10^3	1	10^{-1}	10	1

By orienting the coordinate axes such that the y axis coincides with the cliff line, it can be said that there is no flow in the direction of the y axis. Thus the momentum equation in the y direction is reduced to the equilibrium between the longitudinal slope of the free surface and the transversal flow,

$$f_c U + g\frac{\partial \eta}{\partial y} = 0 \tag{4.303}$$

The progressive long wave of only one frequency component σ to this problem is

$$\eta(x,y,t) = \alpha_\eta \exp\left(-\frac{f_c y}{c}\right)\exp\left[i(kx - \sigma t)\right] \tag{4.304}$$

$$U(x,y,t) = \frac{\alpha_\eta c}{d}\exp\left(-\frac{f_c y}{c}\right)\exp\left[i(kx - \sigma t)\right] \tag{4.305}$$

where c is the celerity of the wave, $c = \sqrt{gh} = \sigma/k$; k is the wave number; and σ_η is the amplitude of the wave at $y = 0$.

The Coriolis effect appears in the exponential term, $f_c y/c$. When the wave travels a distance, $y = c/f_c$, the amplitude of the wave decreases by approximately 60%. The slope of the free surface in the direction y is $\partial\eta/\partial y = (-fc/c)\eta$. This gradient is balanced by a current perpendicular to it. When the axis is oriented in the wave propagation direction, the solution indicates that in the wave crest, the movement is in the direction $y > 0$, and that the amplitude and velocity decrease. In contrast, in the trough, the movement is in the direction $y < 0$, and thus the amplitude and velocity become larger.

Note. This solution is known as Kelvin waves in honor of Sir William Thomson, Lord Kelvin. The superposition of two equal Kelvin waves, traveling in opposite directions, permits the calculation of the spatial distribution of the contour lines of the high tide in the tide cycle and the definition of the amphidromic points or points at which the tide does not oscillate, and is always at high tide.

4.7.4 Linear problem and astronomical forcing

The astronomical tide is generated in the ocean where the forces of gravity, due to the action of the sun and the moon are more effective. There, the amplitude is small, but the reflection on the continental platform and the quasi-resonant conditions of the oceanic basins for the semi-diurnal tide are important factors. Until the wave stops propagating on the platform and nears the coastline, friction terms are not important.

For this reason, when it is necessary to work with a generation and transformation model of the astronomical tide, the following equation system should be resolved:

$$\frac{\partial h}{\partial t} + \frac{\partial (Uh)}{\partial x} + \frac{\partial (Vh)}{\partial y} = 0 \tag{4.306}$$

$$\frac{\partial (Uh)}{\partial t} = -gh\frac{\partial h}{\partial x} + \frac{1}{\rho}F_x \tag{4.307}$$

$$\frac{\partial (Vh)}{\partial t} = -gh\frac{\partial \eta}{\partial y} + \frac{1}{\rho}F_y \tag{4.308}$$

when F_x, F_y includes the force of gravity and the force of Coriolis, as explained in the following. In the open sea, the amplitude of the sea is on the order of a meter,

$$h = h_0 + \eta \approx h_0 \tag{4.309}$$

where h_0 is the still water depth at the point considered, and η is the vertical displacement of the free surface, due to the astronomical tide. As a result, the previous equations can thus be simplified,

$$\frac{\partial h}{\partial t}+h_0\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}\right)=0 \tag{4.310}$$

$$\frac{\partial U}{\partial t}=-g\frac{\partial \eta}{\partial x}+\frac{1}{\rho h_0}F_x \tag{4.311}$$

$$\frac{\partial V}{\partial t}=-g\frac{\partial \eta}{\partial y}+\frac{1}{\rho h_0}F_y \tag{4.312}$$

$$F_x=\rho h_0\left(fV+F_{MM,x}\right) \tag{4.313}$$

$$F_y=\rho h_0\left(-fU+F_{MM,y}\right) \tag{4.314}$$

where $(F_{MM,x}, F_{MM,y})$ are the gravitational components per unit of mass or generating forces in the directions of the coordinate axes. Of all these components, the most important is the lunar component, due to the M_2. In this case, the generating force is the result of the equilibrium between the force of attraction of the Earth-Moon system and the centrifugal force due to the rotation of this system around a center of gravity. The solution of the equation system can be obtained by any of the usual methods to resolve shallow water equation systems.

4.7.5 Linear problem without forcing or friction

Linear wave theory is resolved for a non-compressible non-viscous fluid in irrotational flow with the boundary condition at the smooth bed, in other words, a bed without friction.

This solution can also be obtained, based on general longwave equations, only using linear terms, assuming that $\eta \ll h_0$, and not considering the terms representing friction and external forces. In this case, supposing that the wave propagates in the direction of the x axis, and that the bottom is horizontal, $h = h_0 + \eta \approx h_0$, the general equations can be simplified, such that finally:

Mass conservation equation,

$$\frac{\partial \eta}{\partial t}+h_0\frac{\partial U}{\partial x}=0 \tag{4.315}$$

Horizontal momentum equation,

$$\frac{\partial U}{\partial t}+g\frac{\partial \eta}{\partial x}=0 \tag{4.316}$$

By deriving the first equation in respect to t, and the second in respect to x, and subtracting both equations, the longwave linear equation is obtained,

$$\frac{\partial^2 \eta}{\partial t^2}+gh_0\frac{\partial^2 \eta}{\partial x^2}=0 \tag{4.317}$$

whose solution is a wave that constantly advances in the direction of the x, axis with celerity $c=\sqrt{gh_0}$, and whose surface profile and horizontal velocity are

$$\eta=a\cos(kx-\omega t);\qquad U=c\frac{\eta}{h_0} \tag{4.318}$$

where k and ω are the wave number and angular frequency, respectively. The previous equation indicates that the velocity integrated in the water column is proportional to the vertical displacement of the free surface. This wave only exists when the celerity of wave c and the angular frequency, $\omega = 2\pi/T$, satisfies the relation or dispersion equation,

$$\omega^2 \eta \left(1 - \frac{gh_0}{c^2}\right) = 0 \tag{4.319}$$

This occurs when $c^2 = gh_0$, in other words, the same celerity that was obtained from the potential theory. The wavenumber, $k = 2\pi/L$, is obtained directly from the relation, $c = L/T = \omega/k$. L is the wavelength.

Note. *The semi-diurnal astronomical tide has a period of approximately* 12 *hours. In the Atlantic Ocean with a mean depth of* $\overline{h} \approx 4000$, *the tide travels with a celerity on the order of* $c \approx \sqrt{g\overline{h}} = 200$ *m/s, and its wavelength is on the order of* $L \approx cT = 8500$ *km, such that* $\overline{h}/L < 1/20$. *The amplitude of the tide wave in the open sea is on the order of centimeters, and thus, its steepness is very small. This is the necessary condition for the application of linear theory.*

4.7.5.1 Energy flux in a long wave

According to linear wave theory, the energy of a long wave per unit of horizontal surface is

$$E = \frac{1}{2}\rho g a^2 \tag{4.320}$$

where A is the amplitude of the wave, and the energy flux in the water column in the propagation direction is

$$F_x = Ec_g = Ec = \frac{1}{2}\rho g A^2 c \tag{4.321}$$

Since this is a long wave, the energy propagation celerity c_g, and the wave celerity c coincide.

4.7.5.2 Conservation of the energy flux in a control volume

With a unit-width control volume in the wave propagation direction, the wave entering on the ocean side has an amplitude A_1 and the depth is h_1. The wave from the landside has an amplitude, A_2 in relation to the depth, h_2. When friction and reflection effects are negligible, the energy conservation equation implies that the variation of the energy flux in the control volume is zero. The energy flux is thus conserved, and consequently,

$$\left(F_x\right)_1 = \left(F_x\right)_2 \tag{4.322}$$

$$\frac{1}{2}\rho g A_1^2 c_1 = \frac{1}{2}\rho g A_2^2 c_2 \tag{4.323}$$

$$\frac{A_2}{A_1} = \left(\frac{h_1}{h_2}\right)^{\frac{1}{4}} \tag{4.324}$$

This solution is known as Green's Law, and establishes that wave amplitude increases with $h^{-1/4}$.

Note. *If, for example,* $h_1 = 200m$, $h_2 = 10m$ *is considered as the characteristic magnitude of the change in depth from the continental slope to the inner continental shelf, according to Green's Law:*

$$c_1 \approx 30m/s, \quad c_2 \approx 10m/s; \quad A_2 \approx 2{,}1A_1$$

If $A_1 = 0.5m$, $A_2 = 1.05m$, *these values can be regarded as orders of magnitude since reflections have not been considered, and friction-induced energy losses are not relevant.*

4.7.5.3 Transformation on the continental shelf

Within the context of the previous restrictions, linear longwave theory provides information regarding certain processes that further on will be used in this chapter to analyze the astronomical and meteorological tides. In all cases, it is assumed that the long wave is formed by a single harmonic component. Firstly, longwave transformation is analyzed when the wave meets the continental edge, which is a sudden change of depth from the abyssal zones to the outer edge of the continental platform. Then, the transformation of the wave as it travels along the continental shelf is analyzed. On the shelf, the wave shoals, is refracted, slides over the seabed, reflects off the edge of the coastline, and eventually breaks.

The first case studied is that of normal wave incidence on the continental edge, and the second case studied is oblique case incidence.

4.7.5.3.1 Reflection and transmission on the continental shelf

In the case of an incident long wave η_I, propagating in the positive direction of the x axis perpendicular to a sudden change in slope or a ridge of infinite width, the incident energy flux is transformed or partitioned. Part of it is reflected, and the other part is transmitted (see Figure 4.4.17). The process is assumed to occur without any energy loss or dissipation. Since the analysis is linear, the partition of the energy flux takes place without any interaction between waves. This means that the period and form of the waves are conserved. The oscillation on the free surface in the deep-water zone η_1 is formed by the superposition of the incident wave and the reflected wave, whereas in the shallow-water zone, the only wave is the transmitted wave η_2. This is expressed as follows:

$$\eta_1 = \eta_I + \eta_R \tag{4.325}$$

$$\eta_2 = \eta_T \tag{4.326}$$

If the origin of the coordinates is located in the transition, and the positive x axis in the propagation direction of the incident wave train, and if A_I, A_R, A_T are the incident, reflected, and transmitted wave heights, respectively, these oscillations can be expressed as follows:

$$\eta_I + \eta_R = \frac{A_I}{2}\cos\left(k_1 x - \omega t\right) + \frac{A_R}{2}\cos\left(k_1 x + \omega t + \varphi_R\right) \tag{4.327}$$

$$\eta_T = \frac{A_T}{2}\cos\left(k_2 x + \omega t + \varphi_T\right) \tag{4.328}$$

and φ_R, φ_T are the phase lags that are possibly produced in the transformation.

On the ridge, $x = 0$, the free surface on one side and the other should be continuous. In other words, there is no break in the laminar flow, and the inflowing water volume or mass should be equal to the outflowing ones. These two conditions are expressed as follows:

$$\eta_1 = \eta_2 \qquad x = 0 \tag{4.329}$$

$$(Uh)_1 = (Uh)_2 \qquad x = 0 \tag{4.330}$$

When the free surface expressions are substituted in these conditions, and given the fact that the horizontal velocity in a long wave is expressed according to its celerity, $U = c(\eta/h)$, which only depends on the depth, $c = \sqrt{gh}$, this gives the following equation system,

$$1 \pm R = \pm T \tag{4.331}$$

$$1 \mp R = \pm T \left(\frac{h_2}{h_1} \right)^{\frac{1}{2}} \tag{4.332}$$

where

$$R = \frac{A_R}{A_I} \tag{4.333}$$

$$T_r = \frac{A_T}{A_I} \tag{4.334}$$

are the reflection coefficient and transmission coefficient, respectively.The possible values of the phases determine the corresponding signs, namely:

$$\alpha = \frac{C_2}{C_1} = \sqrt{\frac{h_2}{h_1}} \tag{4.335}$$

The expression of the reflection and transmission coefficients in a sudden transition for the linear approximation of the equations is the following:

$$R = \left| \frac{1-\alpha}{1+\alpha} \right| \tag{4.336}$$

$$T_r = \left| \frac{2}{1+\alpha} \right| \tag{4.337}$$

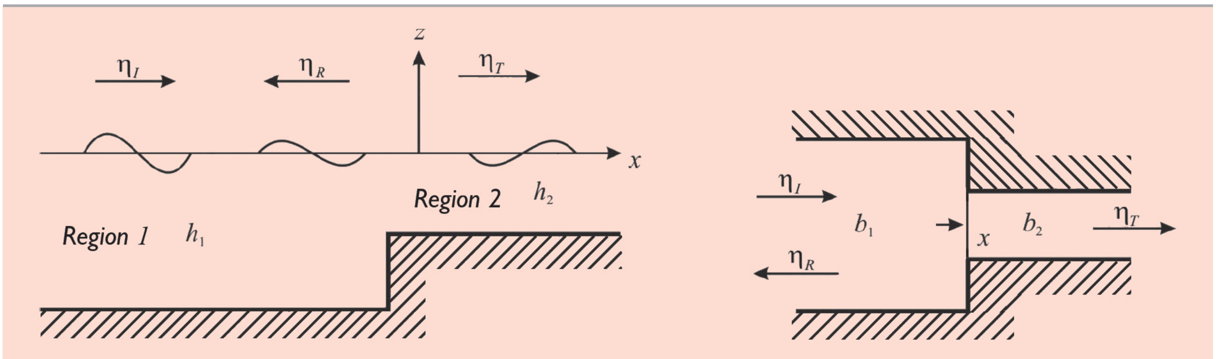
Various cases can be considered, defined according to the values adopted by α,

- $\alpha = 0$. Cases in which $h_1 \gg h_2$. In this case, the following values are obtained:

$$|R| \simeq 1, \quad |T_r| \simeq 2 \tag{4.338}$$

which indicates that due to the great difference in depth between the ocean (1) and the continental shelf (2), an almost perfect reflection is produced in the phase lag, thus causing a double-amplitude wave in region 1.

Figure 4.4.17. Side view and ground view of a channel with a ridge



- $0 < \alpha < 1$. The values of the reflection and transmission coefficients are in the regions,

$$0 < |R| < \square\square < |T_r| < 2 \tag{4.339}$$

The reflection is partial, and the wave propagates over the continental shelf.

- $\alpha < 1$. When $h_2 > h_1$, the values of the reflection and transmission coefficients are in the following regions,

$$0 < |R| < 1, \quad 0 < |T_r| < 1 \tag{4.340}$$

The latter case represents the propagation of the long wave at greater depths.

Note. *The fact that transmission coefficient is greater than the unit is because of the form in which it has been defined. When the wave propagates through waters of shallower depths, its amplitude should increase, simply because of the conservation of the energy flux, $Ec_g = cte$. Since c (c_g) decreases on the shelf, E should increase, and consequently, A_T should also increase. As a result, $T_r > 1$.*

Assuming that the continental shelf is horizontal, the transmitted wave propagates until it reaches the edge of the coastline, whether this border happens to be a beach, a cliff formation or the continental boundary of the estuary in which it is generally reflected. The reflected wave returns to the sea, and its encounter without any interaction with the incident wave produces, in the case of a horizontal bottom, a stationary wave with nodes and antinodes, as obtained for the case of the sea waves.

$$\eta_T + \eta_{TR} = \frac{A_T}{2}\cos\left(k_2 x - \omega t\right) + \frac{A_{TR}}{2}\cos\left(k_2 x + \omega t + \varphi_{TR}\right) \tag{4.341}$$

where A_{TR} is the amplitude of the reflected wave. The other variables have been previously defined. The nodes appear in $k_2 x = \pi/2$.

4.7.5.3.2 Refraction and shoaling on the continental shelf

When the bathymetry of the continental edge is assumed to be straight and parallel, and the long wave obliquely impinges on it, a control volume can be defined with its faces parallel to the wavecrest lines. This expresses the energy conservation of the control volume, defined by two orthogonals or lines perpendicular to the crest lines. Once separated a width b_1 on the side of the ocean, and b_1 on the side of the land, without considering dissipation, the conservation of the energy flux in the control volume provides the generalization of Green's Law,

$$\frac{A_2}{A_1} = \left(\frac{h_1}{h_2}\right)^{\frac{1}{4}} \left(\frac{b_1}{b_2}\right)^{\frac{1}{2}} \tag{4.342}$$

Furthermore, in the case of a straight and parallel bathymetry, the wave enters the control volume at an angle α_1 to the sea bathymetry, and leaves the control volume at an angle α_2 with respect to the land. The variation of the propagation angle can be evaluated by establishing the conservation of the longitudinal wave number or Snell's Law

$$k_1 \sin\alpha_1 = k_2 \sin\alpha_2 \tag{4.343}$$

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{k_1}{k_2} = \frac{c_2}{c_1} = \left(\frac{h_2}{h_1}\right)^{\frac{1}{2}} \tag{4.344}$$

The variation in the amplitude because of refraction is obtained by the ratios in the following expression:

$$K_r = \frac{b_1}{b_2} = \left(\frac{\cos\alpha_1}{\cos\alpha_2}\right)^{\frac{1}{2}} = \left(\frac{1-\sin^2\alpha_1}{1-\sin^2\alpha_2}\right)^{\frac{1}{4}} \tag{4.345}$$

These two equations provide a simple evaluation of the refraction and shoaling of the long wave when it propagates along the continental slope and the continental shelf with a mild slope. In this case, no reflection is produced on the coastline, either because the wave breaks or because it dissipates during propagation.

If the long wave is assumed to be a monochromatic wave, once the wave amplitude and propagation direction are known, thanks to linear theory, it is possible to determine the wave profile and its cinematic and dynamic properties. This model does not consider the fact that the continental shelf has a fairly mild slope, which is about $1/80$ that of the Cantabrian shelf, and that generally, the long wave is reflected on the coastal edge. For this reason, the following section analyzes the case of the propagation of a long wave by a shelf with a constant slope and a reflecting boundary.

Note. In the case of the Cantabrian continental shelf, the abyssal zone is located at a depth of $d_1 \approx 2000m$. The continental shelf begins at a depth of $d_2 \approx 200m$, and a long wave impinging at an angle of $\alpha_1 = 60^o$ has a propagation angle at the edge of the continental shelf of $\alpha_2 \approx 60^o$, and a refraction coefficient, $K_R \approx 0.72$.

4.7.5.3.3 TRANSFORMATION BECAUSE OF A MILD SLOPE WITH A REFLECTING BOUNDARY

When a monochromatic long wave propagates along a continental shelf, whose slope linearly varies, such that $h = h_0 + \beta(x_R - x)$, where β is a small angle (on the order of 10^{-2}), and which ends in a reflecting boundary in $x = x_R$ at depth h_0, the equations governing the movement are:

$$\frac{\partial(Uh)}{\partial x} = -\frac{\partial\eta}{\partial t} \tag{4.346}$$

$$\frac{\partial U}{\partial t} = -g\frac{\partial\eta}{\partial x} \tag{4.347}$$

These two equations can be combined to reduce the problem to a problem with only one unknown,

$$g\frac{\partial}{\partial x}\left(h\frac{\partial\eta}{\partial x}\right) - \frac{\partial^2\eta}{\partial t^2} = 0 \tag{4.348}$$

which has simple analytical solutions for a monochromatic wave of angular frequency σ. In other words,

$$\eta(x,t) = \xi(x)\exp(-i\sigma t) \tag{4.349}$$

The substitution of this form in the previous equation gives an ordinary differential equation in x,

$$\frac{d}{dx}\left(h\frac{d\xi(x)}{dx}\right) + \frac{\sigma^2 h}{g}\xi = 0 \tag{4.350}$$

and by specifying the variation of the sea bottom, $h(x) = h_0 + \beta(x_R - x)$ and changing the variable, $X = h(x)$, one obtains Bessel's equation,

$$\frac{d^{2\xi}}{dX^2} + \frac{1}{X}\frac{d\xi}{dX} + k^2\xi = 0 \tag{4.351}$$

where $k^2 = \sigma^2/g\beta^2$ is the wave number, and the free surface is

$$\eta(x,t) = \xi(x)\cos\sigma t = \left\{\alpha_1 J_0(kX) + \alpha_2 Y_0(kX)\right\}\cos\sigma t \tag{4.352}$$

If the wave, depending on the boundary type, is partially reflected at $x = x_R$ with a reflection coefficient of $R = |R|\,exp(i\varphi_R)$, where $|R|$ is the module of the coefficient and φ_R is its phase, the free surface solution is

$$\eta(x,t) = A_T\left[H_0^{(1)}\left(\frac{\sqrt{1+\beta(x_R - x)}}{\beta}\right) + RH_0^{(2)}\left(\frac{\sqrt{1+\beta(x_R - x)}}{\beta}\right)\right]\}\sigma t \tag{4.353}$$

where $H_0^{(1)}, H_0^{(2)}$ are Hankel functions, and A_T is the wave amplitude at the edge of the coastal shelf, or the wave transmitted along the continental slope.

4.7.5.3.4 *Transformation in estuaries and inlets*

The linear long-wave equations, including the width, are derived from general equations by integrating in the width previously to integrating in depth. This gives

$$\frac{\partial (Uhb)}{\partial x} = -b\frac{\partial \eta}{\partial t} \tag{4.354}$$

$$b\frac{\partial U}{\partial t} = -g\frac{\partial \eta}{\partial x} \tag{4.355}$$

where U represents the horizontal velocity in width b. These two equations can be combined to reduce the problem to only one unknown,

$$\frac{g}{b}\frac{\partial}{\partial x}\left(bh\frac{\partial \eta}{\partial x}\right) - \frac{\partial^2 \eta}{\partial t^2} = 0 \tag{4.356}$$

To resolve this equation, it is necessary to specify two spatial boundary conditions and two initial conditions. This equation has simple analytical solutions for a monochromatic wave of angular frequency σ, propagating on a horizontal bottom and with a linear variation of the width, as follows:

$$\eta(x,t) = \xi(x)\cos\sigma t \tag{4.357}$$

horizontal bed and linear variation of the width. The substitution of this form in the previous equation gives an ordinary differential equation in x,

$$\frac{g}{b}\frac{d}{dx}\left(bh\frac{d\xi(x)}{dx}\right) + \sigma^2\xi = 0 \tag{4.358}$$

By specifying a linear law of channel width, $b(x) = (B_0/l)x$, where l is the channel length, it is possible to obtain the Bessel equation,

$$\frac{d^{2\xi}}{dx^2} + \frac{1}{x}\frac{d\xi}{dx} + k^2\xi = 0 \tag{4.359}$$

where $k = \sigma^2/\sqrt{gh}$, is the wave number and the free surface is

$$\eta(x,t) = \xi(x)\cos\sigma t = \{\alpha_1 J_0(kx) + \alpha_2 Y_0(kx)\}\cos\sigma t \tag{4.360}$$

where α_1, α_2 are two constants that should be evaluated by forcing the solution to satisfy the two boundary conditions. If it is specified that the wave at the entry of the inlet is bounded, and that finally it has a certain height. e. g. $\xi(x = l) = H_{sup}$, the constant α_2 should be zero to eliminate the function Y_0, which takes the infinite value at $x = 0$, and $\alpha_2 = H_{sup}/2J_0(kl)$. The final solution is thus:

$$\eta(x,t) = \xi(x)\cos\sigma t = \frac{H_{\sup}}{2}\frac{J_0(kx)}{J_0(kl)}\cos\sigma t \tag{4.361}$$

The longwave amplitude increases inside the estuary with a wavelength of approximately $L_a = L/4$, where $L_a = 2\pi/k$. In estuaries, whose length l is on the order of that distance, the oscillation in the inlet can resonate. In those cases, bottom friction plays an important role. The following section analyzes the effect of the friction.

4.7.6 Linear problem without forcing and with friction

When nonlinear terms are negligible, the conservation equation of the horizontal movement can be expressed by including a friction term, which is also linear with the velocity, and has the following form,

$$\{\tau_x\}_{-h_0} = \lambda_f \hat{U} \qquad \{\tau_x\}_{-h_0} = \lambda_f \hat{V}$$

where λ_f is a friction coefficient, which in the linearization process should be constant. And $\hat{U}, \hat{V}$ are the representative velocities of the velocity fields in the temporal domain as well as the spatial domain under consideration, e.g. the mean value, the mean square value, or by applying the equivalent work criterion of Lorentz. λ_f takes values on the order of $10^{-4}(1/s)$. When this term is substituted in the linear equation of horizontal momentum, this gives,

$$\frac{\partial U}{\partial t} = -g\frac{\partial \eta}{\partial x} - \lambda_f U \tag{4.362}$$

where the mass conservation equation does not change,

$$\frac{\partial (U h_0)}{\partial x} = -\frac{\partial \eta}{\partial t} \tag{4.363}$$

When both equations are combined, this gives,

$$\frac{\partial^2 \eta}{\partial t^2} + \lambda_f \frac{\partial \eta}{\partial t} - g h_0 \frac{\partial^2 \eta}{\partial x^2} = 0 \tag{4.364}$$

Note. In the case of simple harmonic oscillatory motion in which the velocity fields can be expressed by $U = U_0 \cos(\sigma t)$, *where* $T = 2\pi/\sigma$ *is the period of oscillatory motion, the linearization process consists of a Fourier series expansion of the square term,* $U|U|$, *in the following form,*

$$U|U| = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\sigma t)$$

where

$$a_o = \frac{U_0^2}{T}\int_0^T \cos(\sigma t)|\cos(\sigma t)|dt \tag{4.365}$$

$$a_n = \frac{2U_0^2}{T}\int_0^T \cos(\sigma t)|\cos(\sigma t)|\cos(n\sigma t)\,dt \tag{4.366}$$

When the integrals are evaluated, this gives:

$$a_o = 0 \qquad a_1 = \frac{8U_0^2}{3\pi} \qquad a_2 = 0 \qquad a_3 = \frac{8U_0^2}{15\pi} \; \ldots.$$

the even coefficients are all zero. If only the first term a_1 *is taken, the linearized expression of the friction in the case of the Chezy formula is the following:*

$$[\tau_x]_{-d} = \frac{8U_0^2 \cos(\sigma t)}{3\pi}\frac{g}{C_f h_0^2} = \lambda_f \hat{U} \cos(\sigma t) \tag{4.367}$$

$$\lambda_f = \frac{8g}{3\pi}\frac{U_0}{C_f h_0^2} \qquad \hat{U} = U_0 \tag{4.368}$$

The other formulas give

$$Darcy - Weisbach \qquad \lambda_f = \frac{1}{3\pi}\frac{fU_0}{h_0^2} \tag{4.369}$$

$$Manning - Strickler \qquad \lambda_f = \frac{8g}{3\pi}\frac{n_f^2 U_0}{h_0^{\frac{7}{3}}} \tag{4.370}$$

4.7.6.1 Analytical solutions

The solution to this equation contains the effect of friction on the oscillatory motion, which consists of a reduction in its amplitude and a modification in the phase. If the wave is stationary, the evolution occurs over time, and thus, the effect of the friction is analyzed in the time domain by expressing the angular frequency of the wave as a complex number,

$$\sigma = \sigma_o - i\sigma_f \tag{4.371}$$

where σ_0 or the real part of the angular frequency provides the period of the stationary oscillation, $T_0 = 2\pi/\sigma_0$, whereas σ_f is the frequency or temporal damping rate of the amplitude. This behavior is observed when the previous expression is substituted in the temporal part of the wave phase,

$$\eta(x,t) = \xi(kx)\exp(-i\sigma t) = \xi(kx)\exp\left(-\sigma_f t\right)\exp\left(-i\sigma_o t\right) \tag{4.372}$$

when the wave is progressive, dissipation occurs in the spatial domain. In this case, the effect of the friction can be analyzed by expressing the wave number in a complex form,

$$k = k_p + ik_f \tag{4.373}$$

where k_p is the propagating wave number, which provides the distance or length of wave L_p between two equal wave phase points, $L_p = 2\pi/k_p$ and k_f is the spatial damping rate of the waves. This behavior is observed when the previous expression is substituted in the spatial part of the wave phase,

$$\eta(x,t) = \Upsilon(\sigma t)\exp(ikx) = \Upsilon(\sigma t)\exp\left(-k_f x\right)\exp\left(ik_p x\right) \tag{4.374}$$

Both solutions conserve the relations between the cinematic variables of the longwave propagation,

$$c = R_e\left(\frac{\sigma}{k}\right) \tag{4.375}$$

where R_e stands for the real part of the expression in the parenthesis.

4.7.6.1.1 STATIONARY LONG WAVE WITH FRICTION

The solutions to this equation for a stationary wave and a progressive wave are immediately obtained by means of the variable separation method, $\eta(x, t) = \xi(x)\Upsilon(t)$. In the case of the stationary wave, the decomposition can be written in the following way:

$$\xi = \xi(kx) = A_e \exp(ikx) \qquad \Upsilon = \Upsilon(\sigma t) \tag{4.376}$$

where $\sigma = \sigma_0 - i\sigma_f$, k are the angular frequency and the wave number of the incident long wave without damping, and A_e is the amplitude of the wave in the initial instant, $t = 0$. This gives an ordinary differential equation in t:

$$\frac{d^2\Upsilon}{dt^2} + \lambda_f \frac{d\Upsilon}{dt} + gdk^2\Upsilon = 0 \tag{4.377}$$

whose solution is

$$\Upsilon(t) = \exp\left(-\sigma_f t\right)\exp\left(-i\sigma_p t\right) \tag{4.378}$$

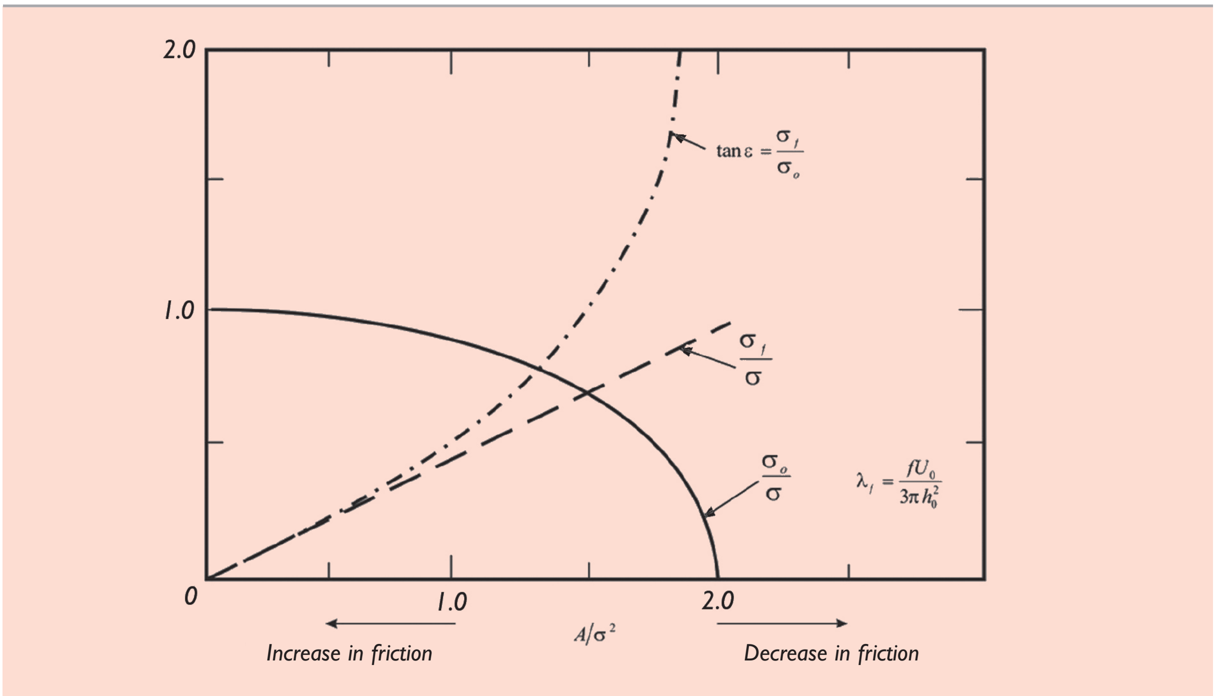
where

$$\sigma_f = \frac{\lambda_f}{2} \qquad \sigma_o = \sigma\sqrt{1 - \frac{1}{4}\left(\frac{\lambda_f}{\sigma}\right)^2} \tag{4.379}$$

$$\sigma^2 = \sigma_f^2 + \sigma_o^2 \tag{4.380}$$

σ_f is the dampening frequency of the wave, and σ_0 is the angular frequency of the damped oscillation; σ is the angular frequency of the incident motion without dampening, such that $c^2 = \sigma/k = \sqrt{gh}$. For the problem to have an oscillatory solution, it is necessary for σ_0 to be a real number. For this purpose, it should hold that $\lambda_f/\sigma < 2$. In another case, the solution is not oscillatory in time.

Figure 4.4.18. ***Wave number and phase for the stationary wave with friction***



The solution of the complete problem, in other words, in space and time is

$$\eta(x,t) = \xi(kx)\Upsilon(\sigma t) = \left\{A_e \exp(-\sigma_f t)\right\}\cos(kx)\exp(-i\sigma_o t) \tag{4.381}$$

The part of the equation in brackets is the decreasing amplitude in time, whereas the part outside the brackets has the functional structure of a stationary wave with nodes and antinodes.

Note. This solution provides information concerning the behavior of a stationary long wave inside a wharf or port, or concerning the propagation of the tidal wave in a tidal inlet or small estuary compared with the wavelength. It is interesting to observe that as the friction λ_f increases, σ_f also increases and σ_0 decreases. Consequently, the oscillation period grows (see Figure 4.4.18).

4.7.6.1.2 Progressive long wave with friction

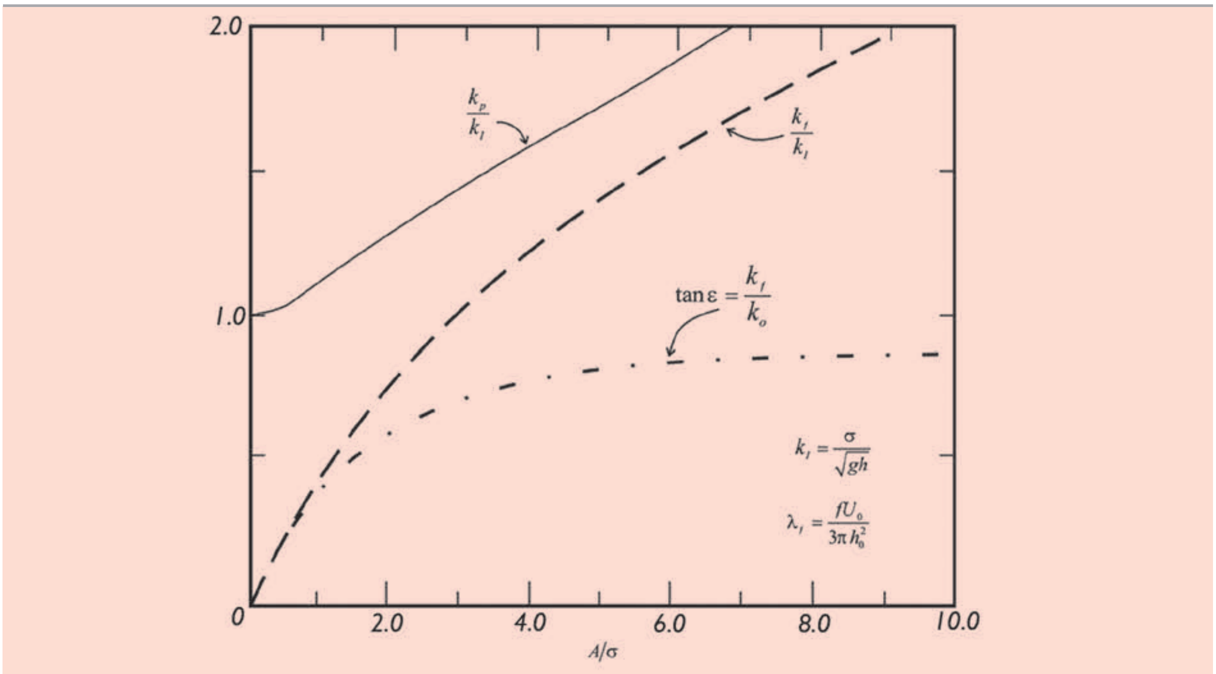
In this case, similarly to the stationary problem, the separation of the functions $\eta(x, t) = \xi(kx)\Upsilon(t)$ is $k = k_p + ik_f$,

$$\xi(x) = A_p \exp(-k_f x)\exp(ik_p x) \quad \Upsilon(\sigma t) = \exp(-i\sigma t) \tag{4.382}$$

which substituting in the differential equation in η provides an ordinary differential equation in x,

$$\frac{d^2\xi(x)}{dx^2} + \frac{\sigma^2}{gh}\left(1 + i\frac{\lambda_f}{\sigma}\right)\xi(x) = 0 \tag{4.383}$$

Figure 4.4.19. **Wave number and phase for a progressive long wave with friction**



$$\frac{d^2\xi(x)}{dx^2}+\frac{\sigma^2}{gh}\left(1+i\frac{\lambda_f}{\sigma}\right)\xi(x)=0 \qquad (4.383)$$

and when the expression of $\xi(x)$ is substituted, this gives the dispersion equation of the progressive wave with friction, which can be decomposed in two equations, corresponding to its real part and imaginary part at zero, respectively. This results in the following:

$$k_p=\frac{\sigma}{\sqrt{gh}}\frac{\left[1+\sqrt{1+\left(\frac{\lambda_f}{\sigma}\right)^2}\right]^{\frac{1}{2}}}{\sqrt{2}} \quad k_f=\frac{\sigma}{\sqrt{gh}}\frac{\left[-1+\sqrt{1+\left(\frac{\lambda_f}{\sigma}\right)^2}\right]^{\frac{1}{2}}}{\sqrt{2}} \qquad (4.384)$$

Consequently, the solution is

$$\eta(x,t)=\xi(kx)\Upsilon(\sigma t)=\left\{A_p\exp(-k_f x)\right\}\cos(k_p x-\sigma t) \qquad (4.385)$$

The part of the equation in brackets is the decreasing amplitude in space, whereas the part outside the brackets has the functional structure of a progressive wave, traveling with celerity, $c_p = \sigma/k_p$. It is interesting to observe that as the friction λ_f increases, k_f also increases, as well as the damping of the wave. As k_p increases, the wavelength decreases (see Figure 4.4.19).

Note. *Where $c=\sqrt{gh}$ is the celerity of the undamped wave; $c_p = \sigma/k_p$ is the celerity of the damped progressive wave; and $c_e = \sigma_0/k$ is the relation of the damped stationary wave, the following inequalities are satisfied,*

$$k<k_p, \quad c_p<\sqrt{gh} \qquad (4.386)$$

$$\sigma<\sigma_o, \quad c_e<\sqrt{gh} \qquad (4.387)$$

In other words, for both waves, the friction has the effect of slowing the oscillatory motion. When the progressive wave is slowed down, there is a reduction in propagation celerity, and when the stationary wave is slowed down, there is an increase in its oscillation period.

4.7.7 Linear problem with atmospheric forcing and friction

Summarizing the hypotheses concerning linear equations and the linear term of bottom friction, the duration of atmospheric forcing (meteorological tide) is much less than the temporal scale of the geostrophic forcings, due to the acceleration of Coriolis (the inertial period is $2\pi/f_C$). The variation of the free surface η due to the meteorological tide is small in relation to the local depth h_0, $(\eta \ll h_0)$, and does not vary with time. Diffusive terms are not important.

With these hypotheses, the general system of equations is reduced to the following,

$$\frac{\partial \eta}{\partial t}+\frac{\partial (Uh_0)}{\partial x}+\frac{\partial ((Vh_0)}{\partial y}=0 \tag{4.388}$$

$$\frac{\partial (Uh_0)}{\partial t}=-gh_0\frac{\partial \eta}{\partial x}+g(h_0+\eta)\left[-\left\{\frac{\partial\left(\frac{p_a}{\rho g}\right)}{\partial x}\right\}_\eta+\frac{\{\tau_x\}_\eta}{\rho g(h+\eta)}\right]-\lambda_f(Uh) \tag{4.389}$$

$$\frac{\partial (Vh_0)}{\partial t}=-gh_0\frac{\partial \eta}{\partial y}+g(h_0+\eta)\left[-\left\{\frac{\partial\left(\frac{p_a}{\rho g}\right)}{\partial y}\right\}_\eta+\frac{\{\tau_y\}_\eta}{\rho g(h+\eta)}\right]-\lambda_f(Vh) \tag{4.390}$$

$$Uh=q_x=\int_{-h_0}^{0}u\,dz \qquad Vh=q_y=\int_{-h_0}^{0}\upsilon\,dz \tag{4.391}$$

$$F_x=g(h_0+\eta)\left[-\frac{\partial\left(\frac{p_a}{\rho g}\right)}{\partial x}+\frac{\tau_x}{\rho g(h_0+\eta)}\right]_\eta \tag{4.392}$$

$$F_y=g(h_0+\eta)\left[-\frac{\partial\left(\frac{p_a}{\rho g}\right)}{\partial y}+\frac{\tau_y}{\rho g(h_0+\eta)}\right]_\eta \tag{4.393}$$

where (q_x, q_y) are the linearized horizontal components, integrated in the water column, of the water transported by the fluid movement; λ_f is a constant friction coefficient, which for Chezy's model, can be estimated by $\lambda_f = g/C_f^2 h^2 \langle u \rangle$; and $\langle u \rangle$ is a representative discharge volume of the spatial and temporal variability of the real volume of water in the work domain. The forcing terms are (F_x, F_y). Moreover, if the depth of the water column is not time-dependent, the local acceleration terms can be written $[(\partial U/\partial t)h, (\partial V/\partial t)h]$.

The scale of the pressure gradients is related to the generating agent, either by open sea pulses, or by the emission of vortices or flow re-adaptation. Its mathematical expression (F_x, F_y) also depends on its origin. For this reason, it should be modeled on the basis of on-site measurements. Sometimes, it is necessary to include the propagation velocity C_p. If this velocity is said to be approximately constant, it is easy to assume that the atmospheric pressure is a function of $(C_p t - x)$, where t is the time, and x is the propagation direction of the barometric wave $p_a = f(C_p t - x)$.

If the depth of the water column is not time-dependent, the local acceleration terms can be written $[(\partial U/\partial t)h, (\partial V/\partial t)h]$. This system can be solved numerically although in cases in which the boundary conditions are simple, there are analytical solutions that permit the importance of the problem to be rapidly analyzed.

Note. These equations have three unknowns: the free surface η, and the water volumes in each of the horizontal directions (U, V), which depend on the horizontal spatial coordinates, x and y, and time coordinate t. The action of the atmospheric perturbation in each of the coordinated directions is represented by two terms, One designates the atmospheric pressure gradient $(\partial p_a/\partial x, \partial p_a/\partial y)_\eta$, and the other quantifies the tangential stress *on the water surface due to the wind* $(\tau_x, \tau_y)_\eta$. *These two terms are known as forcing terms since they cause the variation of the free sea surface and the movement of the particles of water. For this reason, they are said to give the system momentum. The units of the forcing terms are the same as those of the other terms in the momentum conservation equation, in other words* $(L^2/T^2$, *e.g.* $m^2/s^2)$.

The term $\lambda_f Uh$, $\lambda_f Vh$ *evaluates the consumption of the momentum by bottom friction when the fluid moves. The quantity of positive or negative residual movement, defined by the difference between that contributed by atmospheric action and that consumed by the friction accelerates the fluid (positively or negatively), thus modifying the water volumes transported. This produces temporal and spatial variations of the free surface. Generally speaking, this representative volume is the mean volume, the mean square volume, or the volume obtained when the work performed by the linear term is obliged to be equal to the work carried out by the square term in a given time interval and distance.*

The state of equilibrium of the system consists of a spatial and temporal variation of the free surface and the spatial and temporal transport of water. From the time that the forcing begins until the state of equilibrium is reached, there is a transitory one, which in certain cases, is dominant, as shall be seen further on.

Note. *To find a solution for the equation, it is advisable to define a system of coordinates that travel with the wave, and which converts the problem into a stationary one* $x' = C_p t - x$.

4.7.7.1 Unidimensional problem

This section describes different case studies that illustrate how the previously mentioned equations are used. Firstly, an analysis is given of what happens in a narrow channel, in which the forcings and transversal motion are assumed to be zero, in other words, $(\tau_y = \partial p_a/\partial y = 0)$ and $(\partial\eta/\partial y = V = 0)$ and in which the bed is horizontal, $h = h_0$. The problem is thus reduced to one dimension, in this case, to the *x* axis. The equations can thus be simplified as follows:

$$\frac{\partial \eta}{\partial t} + \frac{\partial U h_0}{\partial x} = 0 \tag{4.394}$$

$$\frac{\partial U h_0}{\partial t} = -gh\frac{\partial \eta}{\partial x} + \lambda U h_0 + F_x \tag{4.395}$$

$$F_x = g\left(h_0 + \eta\right)\left[-\frac{\partial\left(\dfrac{p_a}{\rho g}\right)}{\partial x} + \frac{\tau_x}{\rho g\left(h_0 + \eta\right)}\right]_\eta \tag{4.396}$$

4.7.7.1.1 Forcing by the spatial gradient of the stationary pressure

Initially, the two possible forcing cases should be analyzed separately. In the first, there is a spatial variation of the atmospheric pressures, but no associated wind, $\{\tau_x\}_\eta = 0$. The next step is to study the case in which there is only wind, $\{\partial(p_a/\rho_g)/\partial_x\}_\eta = 0$.

The first case involves a channel whose transversal dimension is smaller than its length, and whose margins are parallel to the x axis. Let us suppose that the atmospheric perturbation that produces the pressure gradient appears instantaneously, remains indefinitely over the channel without changing, and does not propagate. In other words, it is fixed in space, and does not generate wind. As a result, the pressure gradient is not time-dependent, and thus the forcing term is

$$F_x = -g\left(h_0 + \eta\right)\left[\frac{\partial\left(\dfrac{p_a}{\rho g}\right)}{\partial x}\right]_\eta \tag{4.397}$$

Furthermore, if the water body in the channel is assumed to instantaneously respond to the forcing action of the pressure variation, equilibrium is instantaneously attained. That means that the sea level variation is not time-dependent, $\partial\eta/\partial t = 0$, and according to the mass conservation equation, the spatial gradient of the water volume is zero, $\partial q_x/\partial x = 0$. This equation indicates that the volume, $q_x = Uh_0$, is constant throughout the domain. Similarly, it can be concluded that q_x is not time-dependent. As a result, if the channel has a fixed, impermeable boundary in which the water velocity is zero, it can be concluded that the volume is zero, $q_x = 0$, in the entire channel.

Within the context of these hypotheses, the momentum conservation equation is

$$0 = -gh_0 \frac{\partial \eta}{\partial x} + F_x \tag{4.398}$$

$$\frac{d\eta}{\partial x} + \left[1 + \left(\frac{\eta}{h_0}\right)\right] \frac{d\left(\dfrac{p_a}{\rho g}\right)}{dx} = 0 \tag{4.399}$$

which in the hypothesis of $\eta/h_0 \ll 1$ for the linearization process, results in the spatial variations of the atmospheric pressure and of the mean sea level or superelevations being proportional, $\eta + p_a/\rho g = c^{te}$.

Note. The simplified unidimensional expression obtained permits a rough estimate of the magnitude of the pressure area. A variation of 1 millibar of atmospheric pressure produces a variation of 0.01 m. in the sea level. Furthermore, the storm is assumed to be approximately circular in shape, and the atmospheric pressure exponentially decreases as the distance increases from the center of the storm,

$$p_a(r) = p_e - (p_e - p_0)\left(1 - \exp\left(-\frac{R-r}{r}\right)\right) \tag{4.400}$$

where:

p_r, pressure of a point at distance r from the center of the storm.

p_c, pressure outside the storm.

p_0, pressure at the center of the storm.

R, radius of the storm.

r; distance of the point considered, as measured from the center of the storm

When the x coordinate is replaced by the radial coordinate r it is obtained that that variation rate of the mean level of the free sea surface, moving along a vector radius of the storm is constant,

$$\eta(r) + \frac{p_a(r)}{\rho g} = c^{te} \tag{4.401}$$

4.7.7.1.2 Spatial gradient forcing of the stationary wind speed

Generally speaking, the tangential stresses on the surface, due to the wind, $[\tau_x, \tau_y]_\eta$, are expressed, according to the square of the mean wind speed in the direction considered, e. g. $[\tau_x]_\eta/\rho = k_f \overline{u_{10}}\,|\overline{u_{10}}|$, where the proportionality constant k_f is on the order of 10^{-6}, when the wind speed is expressed in m/s. Regarding the direction and $[\tau_y]_\eta/\rho$, the expression is similar, and generally, the same proportionality constant is applied.

For the channel considered in the previous case, with a horizontal bottom of constant depth, the forcing term is $F_x = \{\tau_x\}_\eta/\rho$. Because of this term, the water mass will begin to move, though the bottom will go against this movement. When equilibrium is reached, the superelevation η and the current velocity U will not be time-dependent. For this reason, the simplified linear equation of the superelevation of the sea level, according to the shear stress produced by the wind, should also include the linearized bottom-friction term,

$$-g(h_0 + \eta)\frac{d\eta}{\partial x} + \frac{\{\tau_x\}_\eta - \{\tau_x\}_{-d_0}}{\rho} = 0 \tag{4.402}$$

In order to resolve this problem, the bottom friction needs to be specified. To avoid having to resolve the equation in velocities, the friction term can be expressed as proportional to the tangential stress due to the wind, $\{\tau_x\}_{-d_0} = (1 - n)\{\tau_x\}_\eta$, where n is a proportionality factor that evidently depends on the nature of the bed, but is usually in the range, $1.15 < n < 1.30$. When $\{\tau_x\}_\eta - \{\tau_x\}_{-d_0} = n\{\tau_x\}_\eta$ is substituted in the equation, this gives the following:

$$\frac{d\eta}{\partial x} = \frac{n\{\tau_x\}_\eta}{\rho g(h_0 + \eta)} \tag{4.403}$$

This approximation is not consistent with the linear analysis performed in the preceding sections since all the terms are not of the same order. If the channel has a length *l*, in the conditions defined for the stationary case, the resulting equation can be written as

$$\frac{1}{2}\frac{d(h_0 + \eta)^2}{dx} = \frac{n\{\tau_x\}_\eta}{\rho g} \tag{4.404}$$

whose solution is

$$\eta(x) = \sqrt{h_0^2 + \frac{2n\{\tau_x\}_\eta x}{\rho g}} - h_0 \tag{4.405}$$

which, when written in adimensional variables, is

$$\frac{\eta(x)}{h_0} = \sqrt{1 + \frac{2Ax}{l}} - 1 \tag{4.406}$$

where the adimensional parameter, $A = n\{\tau_x\}_\eta l/(\rho g h_0^2)$, defines the relation between the resulting force of the stresses produced by the wind on the surface as well as those produced by bottom friction on the entire length of the channel and the hydrostatic pressure.

Note. *This particular solution was obtained by considering that $\eta = 0$ at the beginning of the channel. As the depth becomes greater, (theoretically, when $h_0 \to \infty$) the free surface tends to zero, $\eta \to 0$. A solution of the preceding equation is immediately obtained for the case in which the bottom slope is constant, $h(x) = h_0[1 - (x/l)]$, where h_0 is the depth at the channel entrance. The implicitly expressed solution is*

$$\frac{x}{l} = \left(1 - \frac{h + \eta}{h_0}\right) - A \ln\left(\frac{\frac{h+\eta}{h_0} - A}{1 - A}\right)$$

for this reason, the equation should be resolved for η, (e.g. Newton-Raphson). The free surface slope increases as the water depth decreases.
In order to perceive the magnitude of the phenomenon, consider a wind speed of 20 m/s. The resulting tangential stress is on the order of 1 N/m^2. At a depth of 50 m, the slope of the free sea surface is approximately $2{,}10^{-6}$. If the coastal shelf has an extension of 500 km, the superelevation at a point near the coast is on the order of 1 m.

4.7.7.1.3 Wind speed and pressure

Finally, there is the case in which the forcing includes the wind action as well as the pressure gradient. Based on the previous linear system of equations,

$$\frac{\partial \eta}{\partial t} + \frac{\partial q_x}{\partial x} = 0 \tag{4.407}$$

$$\frac{\partial q_x}{\partial t} = -gh\frac{\partial \eta}{\partial x} + \lambda_f q_x + F_x \tag{4.408}$$

$$F_x = g(h+\eta)\left[-\frac{\partial\left(\frac{p_a}{\rho g}\right)}{\partial x} + \frac{\tau_x}{\rho g(h+\eta)}\right]_\eta \tag{4.409}$$

where $q_x = Uh_0$ represents the total volume in the water column, due to the passing of the wave. Deriving the mass conservation equation with respect to time, and the momentum conservation equation with respect to x, and eliminating the terms in the volume, the following equation in partial derivatives is obtained,

$$\frac{\partial^2\eta}{\partial t^2} - C^2\frac{\partial^2\eta}{\partial x^2} + \lambda_f\frac{\partial\eta}{\partial t} = -\frac{\partial F_x}{\partial x} \tag{4.410}$$

Operating in a similar way, but eliminating the terms in η, the following governing equation for water volume is obtained,

$$\frac{\partial^2 q_x}{\partial t^2} - C^2\frac{\partial^2 q_x}{\partial x^2} + \lambda_f\frac{\partial q_x}{\partial t} = -\frac{\partial F_x}{\partial t} \tag{4.411}$$

where $C = \sqrt{gh}$ is the wave propagation velocity. If this velocity and the friction coefficient λ_f are constant, the two previous equations have analytical solutions that provide complete information concerning the behavior of the meteorological tide on the coastline. The differential equation is hyperbolic and non-homogeneous since it contains a forcing term, $\partial F_x/\partial_x$ (o $\partial F_x/\partial t$). Since it is a second-order term in time and space, to obtain a solution, it is necessary to specify two initial conditions (in the initial instant, $t = 0$), and two spatial or boundary conditions (for a finite domain, at its limit points, $x = 0$ and $x = 1$).

Very generally, the initial conditions for the problem described in η can be

$$\eta(x,0) = f(x) \quad y \quad \frac{\partial\eta(x,0)}{\partial t} = g(x) \tag{4.412}$$

The boundary conditions can be of the Dirichlet or Neuman-type, such that,

$$\eta(0,t) = \varphi_0(t) \quad o \quad \frac{\partial\eta(o,t)}{\partial x} = \varphi_0(t) \tag{4.413}$$

$$\eta(l,t) = \varphi_l(t) \quad o \quad \frac{\partial\eta(l,t)}{\partial x} = \varphi_l(t) \tag{4.414}$$

Since, generally speaking, $\varphi_0(t) \neq 0$, $\varphi_l(t) \neq 0$, the boundary conditions are not homogeneous.

Note. *When λ and $\partial F_x/\partial_x$ are zero, the free surface equation is the standard long-wave equation or the Helmholz equation, whose solution can be obtained by variable separation. This equation can be used to obtain the oscillations in closed water bodies, and thus provide an approximation to the periods in wharfs, ports, or semi-closed water bodies. The solution thus obtained is only approximated because in the Helmholtz equation there is no term that explicitly evaluates the radiation of the oscillatory energy at the entrance of the port.*

4.8 ASTRONOMICAL TIDE

4.8.1 Astronomical data

The characteristics of the system composed of the Earth, Moon, and Sun is defined by the combination of four basic movements: (i) the rotation of the Earth on its own axis; (ii) the rotation of the Moon on its own axis; (iii) the rotation of the Earth around the Sun; (iv) the rotation of the Moon around the Earth. A complete rotation of the Earth on its axis is known as a mean solar day and its duration is 24 *hours*. The Earth makes a complete revolution around the Sun in a tropical year, whose duration is 365 *days*, 5 *hours*, 48 *minutes*, and 45.68 *seconds*. These time intervals are measured in reference to the Sun. If fixed stars are taken as a reference, the duration of the Earth's rotation on its axis (sidereal day) is 23 *hours*, 56 *minutes*, and 4.09 *seconds* in solar time. In this context, the duration of the Earth's orbit around the Sun is 366,256 *sidereal days* (sidereal year).

In its elliptic orbit around the Sun, the Earth is located nearest to the Sun on January 3rd (perihelion), whereas it is farthest from the Sun on July 4th (aphelion).

The Moon revolves on its own axis, which is approximately parallel to the Earth's axis, and traveling in the same direction, it revolves completely around the Earth in 27.3216 *sidereal days*. Its elliptical translation forms an angle of 5.15º with the plane that contains the (elliptical) orbit of the Earth. In its orbit around the Earth, its closest point to the Earth is known as the perigee, whereas the farthest point is known as the apogee. The time between two successive crossings by the Moon of the same meridian of the Earth is 24 *hours*, 50 *minutes*, and 28.33 *seconds*.

The declination is the angle that forms the line joining the center of a celestial body and the center of the Earth, in respect to the plane of the Equator. The declination of the Sun varies from 23.45º in the summer solstice to –23.45º in the winter solstice, whereas the Moon has a total range of 57.20º, reaching its maximum 28.60º every 28.5 *years*. The duration of the Moon's complete declination cycle from the maximum south declination to the maximum north declination, and back again to the south is 27.2 *days*, and is known as the tropical month.

4.8.2 Spring tides and neap tides

The amplitude of the tidal movement depends on the relative position of the stars. When they are aligned, they cause maximum tidal ranges, given that the effects are in phase. These states are known as spring tides, and coincide approximately with the full moon and the new moon. The difference between both of these events is generally from one to three days. This phase lag is known as the age of the tide. In contrast, when the stars are at a right angle, this causes the smallest tidal ranges, known as neap tides. This situation approximately coincides with the lunar phases of first quarter and the last quarter. The time interval between the spring tide and the neap tide is 14.8 *days*.

The equinoctial spring tide occurs when the alignment of the three stars coincides when the Sun and of the Moon are closest to the Earth. This is the time of the highest tides. In contrast, the lowest tides or neap tides occur when Sun and the Moon are farthest away from the Earth.

4.8.3 Effects of the Moon-Earth system on the tide

During a lunar day, a given point on the Earth's surface will experience two maximum levels and two minimum levels. This movement is known as the semi-diurnal lunar tide. Its period can be calculated, and is 12.42 *hours*. Given the lunar declination in respect to the Equator, the maximums and minimums will not have the same amplitudes. Furthermore, the declination of the Moon affects the nature of the tide. It remains semi-diurnal near the Equator and becomes diurnal in higher latitudes. If the declination were zero, in other words, if the Moon always remained at the Equator, the tides generated by the Moon would be semi-diurnal in the entire planet.

The time period between the perigee and the following perigee is 27.55 *days*. The lunar tide intensifies during the perigee and lessens during the apogee. The position of the perigee in the orbit varies with a period of 8.85 *years*.

The lunar nodal tide occurs because of the Moon's backward movement along the ellipsis of the ascending node, defined as the point where the Moon crosses the ellipsis from south to north. This backwards movement completes a revolution in 18.61 *years*, and is responsible for the fact that the declination of the Moon varies between 18.5º and 28.5º in that period. This variation in the declination produces what is known as the nodal tide. If the (lunar) equilibrium conditions of the maximum tidal range were calculated, this would give an approximate value of 0.53 *m*. This value is close to the values measured in deep water. However, its magnitude is smaller than the magnitude of values observed in coastal regions.

4.8.4 Effects of the Sun-Earth system on the tide

The solar tide can be evaluated in the same way as the lunar tide. The relation between the maximum ranges of the solar and lunar tide is $0.46\ m$. Evidently, the larger mass of the Sun is compensated by the fact that it is at a greater distance from the Earth in the universal law of gravity.

Note. *The intensification of the tide when the declination is zero logically occurs in the case of the Sun as well as the Moon. The semi-diurnal lunar tides decrease 23% when the Moon reaches its maximum declination. In contrast, the semi-diurnal solar tides decrease 16% in June and December when the maximum solar declinations occur. The extreme tidal forces occur when the Sun and the Moon are aligned with the Earth and their positions are closest. Moreover, in the case of semi-diurnal tides, the two stars should also be over the Equator. However, for this to occur (i.e. for the perihelion and the zero solar declination or equinox to coincide), it will be necessary to wait until the year 6581.*
The lunar perigee and the zero lunar declination coincide ever six years. The coincidence of the lunar perigee and either the summer or winter equinox has a period of $8.85\ years$.

4.8.5 Equilibrium tide

The equilibrium tide is understood as the instantaneous response of the ocean to the force of gravity, assuming that it covers the entire surface of the Earth. The equilibrium hypothesis is used to define the distribution of elevations of the sea surface, according to the generating force and their relative position. The orbits of the Moon around the Earth and the orbits of the Earth around the Son are elliptic. This causes maximums and minimums in the forces of gravity. Furthermore, the axes of the Earth and the Sun are both tilted in reference to the planes of their respective orbits. This means that the analysis of the forces causing the tide on Earth is a complicated though deterministic exercise, not exempt of periodicity.

Note. *The instantaneous response of water masses is a hypothesis that facilitates the resolution of motion equations. Consequently, there are deviations from the values obtained with the equilibrium tide model as well as from observational values. For example, at a given point, one would expect a high tide when the Moon crosses the meridian corresponding to the point. Nevertheless, a phase lag occurs because of the inertia of the water masses. The same is true for spring tides, which occur in phases of one to three days after the full or new moon. The phase lag depends on the geographic location of the point.*

4.8.6 Tide components

By making use of the periodicity of the movements of the Moon, Earth, and Sun system, it is possible to express the potential function of the generating force of the tide by a linear combination of sinusoidal terms with its own amplitude, phase, and period. In other words, it is possible to approximate the function by means of a finite sine series expansion. Each term in the expansion is a harmonic component. If it is assumed that the tide has the same periodicity as the generating force, the tide can be described by a finite series of sines with the same frequencies, in other words, with the same components known as astronomical tide components. Although the frequency of each component remains the same, the amplitude as well as the phase of each component can be significantly modified in coastal regions.

4.8.6.1 Bottom and boundary effects

In shallow waters, the non-linear interaction between different components can create new components. The force of friction causes variations in phase and the geometric and bathymetric configuration. Besides creating new components, it can produce important amplifications in the amplitudes of the tide components. Therefore, the equilibrium tide is not sufficient to characterize tide components at a point in shallow water. In such cases, systematic measurements of the tide are needed over a sufficiently long period to define the main components.

4.8.6.2 Compound tides

Compound tides are the components calculated as the sum or subtraction of two or more astronomical components. Compound tides, whose origin is the M_2 and S_2 components, are the most salient. For example, the $2MS_6$ components multiply by two the angular velocity of the semi-diurnal lunar tide and the semi-diurnal solar tide. Table 4.4.9 lists the most relevant overtides and compound tides.The coefficients of each one depend on the location.

4.8.7. Importance of the tidal circulation

The currents generated by the tide wave can affect the maritime structure even more greatly than the superelevations on the sea surface. These currents are more pronounced in semi-enclosed coastal regions, such as bays, estuaries and river areas.

The activities in port and shoreline areas, besides being affected by the astronomical tidal range, can also be affected by some of the phenomena associated with the tidal currents, such as the following:

Table 4.4.9. Astronomical tide components of the astronomical tide. Most relevant overtides and compound tides an associated period

Symbol	Origin	Period (h)	Symbol	Origin	Period (h)
MNS_2	$M_2 + N_2 - S_2$	13.1273	M_6	$3M_2$	4.1402
$2MS_2$	$2M_2 - S_2$	12.8718	$2MS_6$	$2M_2 + S_2$	4.0924
$2MS_2$	$2S_2 - M_2$	11.6069	$2MN_6$	$2M_2 + N_2$	4.1663
MK_3	$M_2 + K_1$	8.1771	$2SM_6$	$2S_2 + M_2$	4.0457
$2MK_3$	$2M_2 - K_1$	8.3863	MSN_6	$M_2 + S_2 + N_2$	4.1179
SK_3	$S_2 + K_1$	7.9927	S_6	$3S_2$	4.00
SO_3	$S_2 + O_1$	8.1924	M_8	$4M_2$	3.1051
M_4	$2M_2$	6.2103	$3MS_8$	$3M_2 + S_2$	3.0782
MS_4	$M_2 + S_2$	6.1033	$2(MS)_8$	$2M_2 + 2S_2$	3.0517
MN_4	$M_2 + N_2$	6.2692	$2MSN_8$	$2M_2 + S_2 + N_2$	3.0926
MK_4	$M_2 + K_2$	6.0949	S_8	$4S_2$	3.00
S_4	$2S_2$	6.00			

- The magnitude of the velocities
- The transport of natural substances as well as pollutants in coastal, estuary, and river areas
- Phenomena associated with saline transport:
  - Formation of salt wedges or salt fronts with non-uniform depth distributions
  - Changes in the salt distribution, effecting the flora and fauna in estuaries and rivers

Note. *The tidal currents are an important factor in pollutant transport. The location of an sewage pipe or other maritime structure that discharges a polluting substance should evidently depend on the tidal currents in the area. The same is true for solid transport and saline transport. In the latter case, as well as for pollutants, it is crucial to consider environmental factors related to marine and river life.*

4.9 THE LAND IN RELATION TO MARINE AGENTS

According to the ROM 0.0, the land is regarded as an agent when it can cause actions such as pressures, thrusts, and other forces, movements, and strain deformations of the various elements of a load-bearing structure.

These actions can be ordered, according to whether they are due to direct action $Q_{t,1}$, (for example, thrusts against the walls) or indirect action because of earth movements $Q_{t,2}$ (for example, parasite effects in piles).

Because of its importance in the safety and serviceability of breakwaters, it is advisable to analyze the behavior of the land by identifying: (1) its surface and resistance to erosion and liquefaction in relation to the action of the sea oscillations on the bottom; (2) the soil as a solid load-bearing structure and its geotechnical stability in relation to the sliding and sinking because of the actions of the breakwater and the sea oscillations propagated through it.

Since the sea oscillations vary in amplitude and period over time, and propagate over the land, they can force very different behaviors from those of the permanent hydraulic regime. These oscillations are random and are manifested by meteorological states that evolve with the loading cycle. These states occur randomly in the meteorological years of the useful life of the structure. Consequently, the geotechnical behavior of the soil also evolves locally with the passage of time during the useful life of the structure because of the temporal evolution of the forcing agent (sea oscillations) as well as because of the dynamic response of the soil. Since the forcing agent is random, the dynamic response of the soil in the useful life is also random.

Note. The project design of a breakwater usually involves the need to lay the foundations for a rigid structure, generally of large dimensions, on a soil with a certain load-carrying capacity under strain. This is the case of vertical breakwaters, mixed breakwaters and even breakwater crown walls. For this purpose, it is necessary to lay the foundations for these rigid structures on rockfill or gravel banks in order to reduce differential foundations and the excentric loads. The bank transmits the movements of the structure to the soils or to the nucleus in the case of a sloping breakwater. The soil responds to these movements and all breakwater project designs should evaluate this response.
In those cases in which the conditions of the soil or materials make it possible, the study of their behavior in relation to the sea waves and other sea oscillations can be performed by using the Biot's physical-mathematical model, based on complete poro-elastic soil equations.

4.9.1 Interaction between the land and sea waves

Because of their nature, sea oscillations are a dynamic load on the ground. The behavior of the soil, which is composed of a mineral skeleton and an interstitial phase, depends on the interaction between phases. Generally speaking, the strain and strength of a soil mass are controlled by the interactions between individual particles, mainly because of the sliding between them.

The interstitial phase affects the chemical nature of the mineral surfaces. Therefore, it affects the transmission of forces at the contact points between particles. Moreover, the interstitial phase forced by the sea oscillations can move through the soil, affecting the mineral skeleton that modifies the magnitude of forces on the contact points between particles. In this way, it influences the strength and the strain of the soil.

When a load is suddenly applied to the soil, the action is jointly transmitted to the mineral skeleton and the interstitial fluid. The load distribution depends on the compressibility of each of the phases. Since the fluid is practically incompressible in comparison to the solid skeleton, it is the fluid that absorbs the load applied. The load applied to the fluid increases the interstitial pressure, which, in certain cases, forces the fluid to move through the soil.

As the water drains, the solid skeleton must support an increasingly larger part of the load. The movement of the water through the soil causes a variation in the properties of the soil with respect to time. If the load applied to the soil is constant because of the structure or because of a permanent, uniform flow, it is probable that the drainage will attain a permanent regime with a well established filtration network. Moreover, if the medium is not very permeable, the hydraulic gradient will be proportional to the discharge velocity. It is indicative of a laminar regime, known in geotechnical circles as Darcy's filtration. In such cases, the stress-strain relation of the soil is usually known.

Finally, if the load conditions do not change significantly, it may be possible to reach a state in which the solid skeleton is able to bear the totality of the load, and in which the water pressure returns to its initial value.

4.9.1.1 Hydraulic oscillatory regime

When there are sea oscillations, the geotechnical behavior of the soil, its load-bearing capacity and deformability locally varies over time and space. This means that it is possible to force very different behaviors from those obtained in the permanent hydraulic regime. When the soil is only slightly permeable, the hydraulic regime may be laminar, but oscillatory, in other words, significantly influenced by the local flow acceleration. For more permeable soils, fillers and layers, the hydraulic regime of the interstitial flow is usually Forchheimer's. In this regime during the semi-cycles, the flow is turbulent, whereas near the crest and the trough, the regime is laminar. This makes the resolution of the problem more difficult since the consumption of momentum depends on the hydraulic regime, which is not known until the problem is solved.

Depending on the principal period of the oscillatory energy (magnitude proportional to the square of the oscillation amplitude) the sea oscillations can be classified in four oscillatory bands. Of these oscillations, only those whose period is of the same order of magnitude or smaller than the time necessary for the drainage to reach the permanent regime, significantly influence its behavior. Except in the case of long waves (astronomical tide), it is improbable that during drainage, the permanent regime will be reached.

The stress-strain relation of the soil in presence of sea oscillations is complex and can be influenced by previous load conditions. Depending on what this relation is, a soil can be either rigid or deformable. Both rigid and deformable soils can be porous. In other words, they can have the capacity to permit water flow through them.

4.9.1.2 Soil classification in relation to sea oscillations

A rigid soil suffers strain deformation when a stress is applied to it. Its behavior varies, depending on whether it is porous. In non-porous rigid soils, a constitutive relation of the behavior of the material is not necessary since the strain is zero. However, it should be verified that the stress applied to the material is less than its load-bearing capacity. Porous rigid soils allow water to pass through them, and their strain deformation by the action of the sea is negligible.

A deformable soil experiences a certain strain deformation under the action of a stress. The idea behavior of deformable soils can be categorized, depending on whether they are time-independent. The behavior of time-independent soils can be described by elastic and plastic models, whereas the behavior of time-dependent soils requires viscoelastic and viscoplastic models.

In these Recommendations, the behavior of the soil in relation to the variations of the oscillatory pressure in the interstitial phase is organized in three large groups: (1) porous rigid soils; (2) porous elastic soils; (3) viscoplastic soils. The first group includes the soils whose surface does not follow a forcing periodic motion. Rather the water particles flow between the pores and drain the excess interstitial pressure. The wave propagates or oscillates in phase in the outer fluid or in the soil. The water flow between the pores causes a loss in momentum, and dissipates the mechanical energy into turbulence, and finally, into heat. The cinematic characteristics of the wave are due to a bi-phasic medium, water and soil. The dispersion equation combines the depth of the outer fluid, the depth of the soil and the properties of the porous medium, porosity and friction. This behavior is found in sand and gravel beds, and in classified fillers, cores and layers of natural and artificial stones.

The surface of a non-drained porous elastic soil follows periodic movement. In other words, it oscillates, forced by the outer pressure, generating tangential pressures, tangential strains, and the movement of the soil particles, apart from the circulation of the interstitial water. Once the forcing oscillatory movement has stopped, the soil recovers its initial situation. The horizontal velocity of the soil is also out of phase π with respect to the velocity of the outer water. The percolation to and from the soil, induced by oscillatory movement is less than the percolation predicted of a rigid porous soil.

Note. – *Elastic soil. A soil is considered to be elastic if the strain deformation caused by the applied stress, disappears when stress disappears. Furthermore, if the relation between the stress and strain is linear, it is considered to be linear elastic soil.*

– *Plastic soil. If the strain deformation does not disappear when the stress disappears, the soil is considered to be plastic. A particular case is that of elasto-plastic soil, characterized by an elastic behavior of the material when the stresses applied are small, and by a plastic behavior when the stress exceeds a certain value. Elasto-plastic behavior can occur when there is a growth in strength because of an increase in strain deformations.*
– *Viscoelastic soil. A soil is considered viscoelastic when it is subjected to stress, and the strain deformation experienced by the soil is time-dependent and totally disappears.*
– *Viscoplastic soil. A soil is considered viscoplastic when it is subjected to stress, and the strain deformation experienced by the soil is time-dependent, and does not totally disappear when the stress disappears.*

4.9.1.3 Interstitial pressures in the soil

The presence of sea oscillations can substantially modify the interstitial pressures in the soil, whose magnitude varies in space and time, depending on the oscillatory regime and the mechanical and hydrodynamic properties of the soil. The interstitial pressure at a point in the soil is an instantaneous variable that varies over time. In a time record, it is possible to identify cycles and in each cycle, maximum and minimum pressure amplitudes. Generally speaking, the period of the cycle is related to the period of the forcing oscillation.

The frequency analysis of the signal provides the interstitial pressure spectrum and the area under the spectrum is a state descriptor that can be related to the mean square amplitude of the pressure. These are state descriptors.

The variations in interstitial pressure, besides being associated with external forcing, depend on the soil drainage regime. They can substantially modify the effective stresses on the soil matrix, and this can ultimately affect the drainage regime. This stress scheme can have a state treatment similar to that of other oscillations.

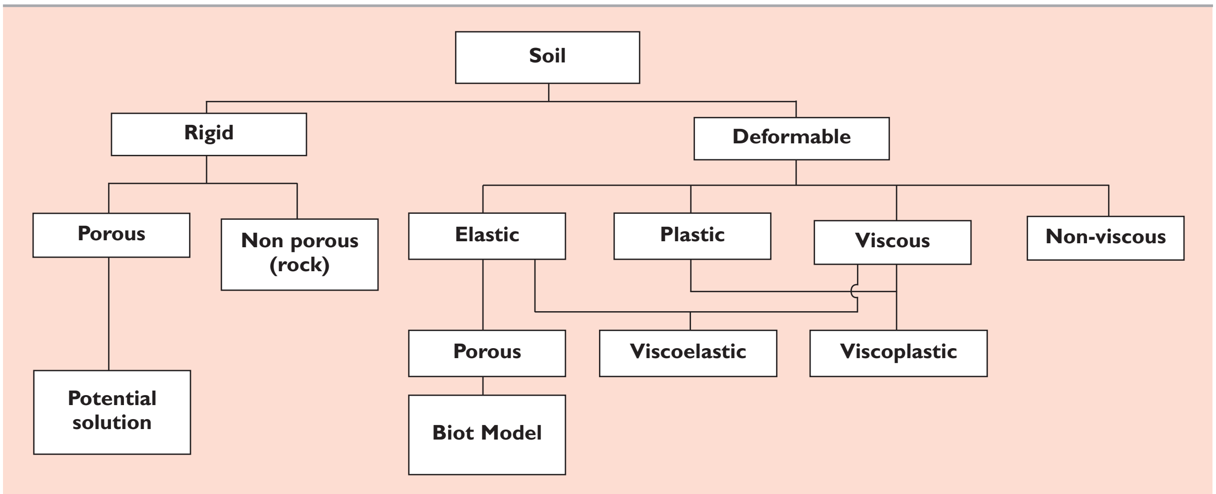
4.9.2 Description in the state

4.9.2.1 Models of soil behavior with oscillatory flow

According to the previous classification, soil behavior affected by sea oscillations should be analyzed in the context of the following models (see Figure 4.4.20):

- Porous and non-porous rigid soil
- Viscous, elastic, and viscoelastic deformable soil
- Poroelastic deformable soil

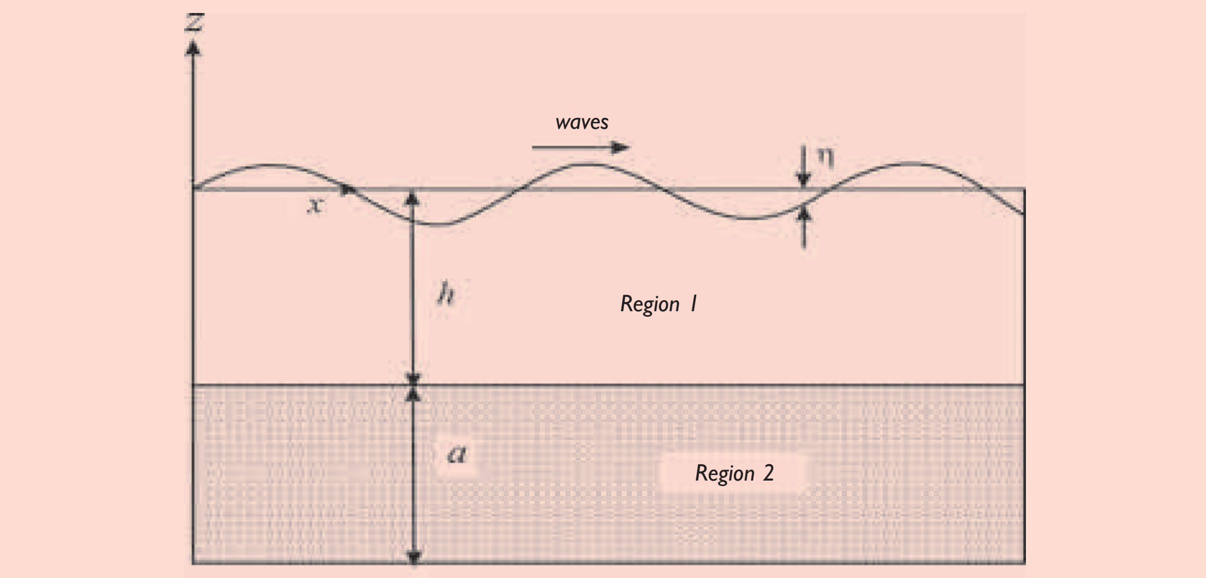
Figure 4.4.20. ***Soil behavior models***



4.9.2.2 Progressive sea waves over a porous rigid soil

To formulate the propagation of an oscillatory train over a porous rigid bed, the fluid is assumed to be incompressible. The flow over the porous rigid soil, expressed in terms of filtration velocity, is irrotational in the same way as the outer flow, and the movement is linear (see Figure 4.4.21).

Figure 4.4.21. Progressive sea waves over a porous rigid soil



The solution is a linear oscillatory motion that propagates without a phase lag in the outer fluid and the porous medium that satisfies the equation,

$$\sigma^2 - gK \tanh Kh_w = F\left(\omega^2 \tanh Kh_\omega - gK\right) \tag{4.415}$$

$$F = \left(1 - \frac{n}{S_p - if}\right) \frac{\tanh Kh_p}{1 - \dfrac{n}{S_p - if} \tanh^2 Kh_p} \tag{4.416}$$

which gives the wavenumber K for an angular frequency, $\sigma = 2\pi/T$, where n represents the porosity of the soil; S_p, the added mass coefficient (usually, $S_p = 1$); f is the friction coefficient; h_p is the thickness of the porous medium, and h_w is the depth of the water column. The wave number thus obtained is complex, $K = K_R + iK_I$, $L = 2\pi/K_R$, and the free surface oscillation is

$$\eta = \frac{H_I}{2} \exp\left[-K_I\left(x - x_0\right)\right] \cos\left[K_R\left(x - x_0\right) - \sigma t\right] \tag{4.417}$$

where $(x - x_0)$ is the distance from the point at which the wave is calculated and the reference point (x_0). The term $\exp[-K_I(x - x_0)]$ quantifies the reduction in height because of the porous medium.

4.9.2.2.1 Penetration depth of the oscillatory movement in the porous rigid soil

The soil thickness h_p measured from the bottom, under which the oscillatory motion in the porous medium is negligible, can be calculated in the following way,

$$h_p > \frac{\pi}{K_R} - h_w, \; for \; \frac{h_w}{L_w} < \frac{1}{2} \qquad (4.418)$$

$$L_w = \frac{2\pi}{k_w}, \; \sigma^2 = g k_w \tanh k_w h_w \qquad (4.419)$$

$$\sigma = \frac{2\pi}{T} \qquad (4.420)$$

where h_w, L_w are the water depth on the bottom and the wavelength when it propagates over an impermeable bed, respectively.

4.9.2.3 Hydraulic regimes in porous rigid soils

In non-permanent, oscillatory regimes, the horizontal pressure gradient, $\partial p/\partial x$ can be related to the horizontal filtration velocity u_f by means of the following equation,

$$I = \frac{1}{\gamma_w}\frac{\partial p}{\partial x} = a u_f + b u_f \left|u_f\right| + c\frac{\partial u_f}{\partial t} \qquad (4.421)$$

$$u_f = nu \qquad (4.422)$$

where n is the porosity and u is the instantaneous velocity of the water. The coefficients, a, b, have units (T/L) and $(T/L)^2$, respectively, and the coefficient, c, which evaluates the added mass, usually has a value close to the unit. These coefficients should be estimated with the following expressions:

$$a = \alpha_n \frac{(1-n)^2}{n^3}\frac{v}{g D_{50}^2} \qquad (4.423)$$

$$b = \beta_n \frac{(1-n)}{n^3}\frac{1}{g D_{50}} \qquad (4.424)$$

where v is the cinematic viscosity of the water $[\sim 10^{-6}(m^2/s)]$, and α_n and β_n depend, among other things, on the size and shape of the particles. If no specific information is available, it can be assumed that $\alpha_n = 1000$ and $\beta_n = 1.1$. The square term evaluates the consumption of turbulent momentum, whereas the linear term evaluates the consumption of momentum by the viscous friction. In the laminar regime, $\beta_n = 0$, and the equation is simplified as

$$bI = \frac{1}{\gamma_w}\frac{\partial p}{\partial x} = a u_f \qquad (4.425)$$

and the permeability is defined in the laminar or Darcy regime by $K_f = 1/a$. The permeability values for the permanent regime are well-known, but these values are usually not representative of the oscillatory regime. The solution of the problem is obtained by linearizing the momentum consumption term by means of the Lorentz's hypothesis of equivalent work,

$$a u_f + b u_f \left|u_f\right| \approx f u_f \qquad (4.426)$$

whereas the inertial term explicitly appears in the boundary problem.

4.9.2.4 Friction coefficient and porosity of rigid porous soils

If no other information is available, it is advisable to use the following values of f, according to the median diameter of the sediment (see Table 4.4.10).

Table 4.4.10. **Friction coefficient and porosity of porous rigid soils**

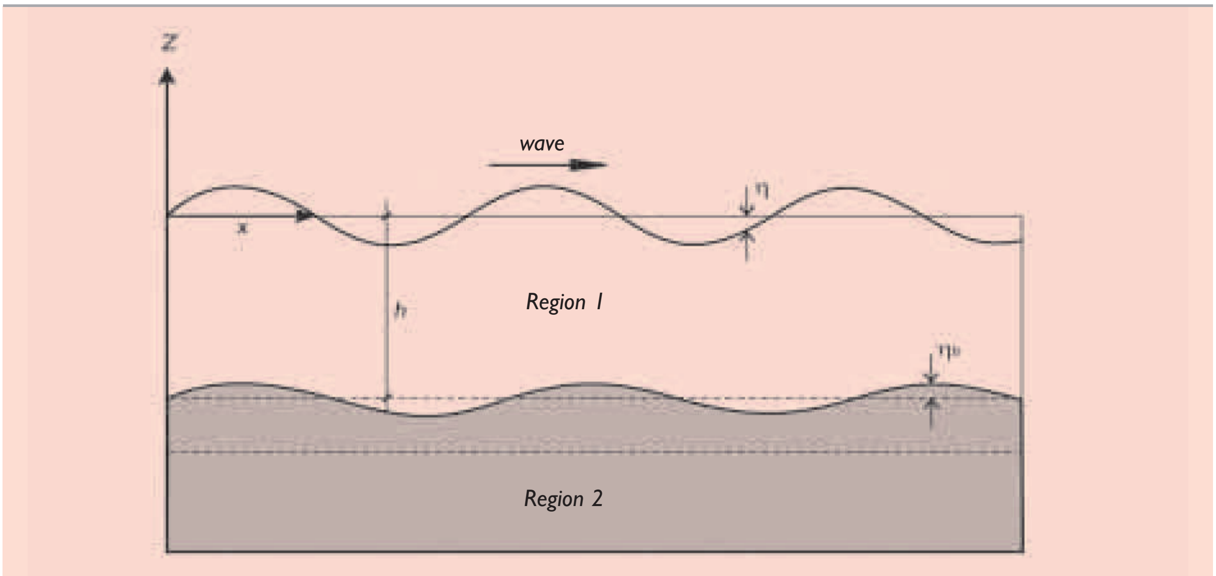
Sediment size	Porosity, n	Friction, f
Gravel, rockfill	0.40	0.5
Coarse sand and core	0.38	1.0
Medium sand	0.35	1.5
Fine sand	0.32	2.0
Silty soil	0.30	10.0

In the case of heterogeneous soils, water flow is conditioned by the pore size, defined by smaller diameters. In such cases, the values of f to be applied are those closest to the values proposed for the smallest size of the sample.

4.9.2.5 Progressive sea waves on elastic soils

A soil is regarded as elastic and incompressible when its movements are slight, and when its stress-strain relation satisfies Hooke's Law (see Figure 4.4.22).

Figura 4.4.22. **Progressive sea waves on elastic soils**



Solid displacements are expressed in terms of a potential function ϕ and a shear function ψ. Furthermore, it is assumed that the fluid is non-viscous; the flow is irrotational; and the flow governing equations are the Laplace and the linearized Bernoulli equation. The dispersion relation of the hydroelastic gravity wave is

$$\frac{\frac{\rho_w}{\rho_s}\left(1-\left(\frac{Kg}{\sigma^2}\right)^2\right)\tanh Kh}{\left(\frac{Kg}{\sigma^2}\right)\tanh Kh-1}+\frac{Kg}{\sigma^2}-\left(\frac{2K^2}{s^2}-1\right)^2+\left(\frac{2K^2}{s^2}\right)^2\left(1-\frac{s^2}{K^2}\right)^{\frac{1}{2}}=0 \qquad (4.427)$$

which relates the angular frequency σ and K, the wave number. In this expression, s represents the wave number of the elastic shear wave in the soil,

$$s^2 = \frac{\rho_s \sigma^2}{G} \tag{4.428}$$

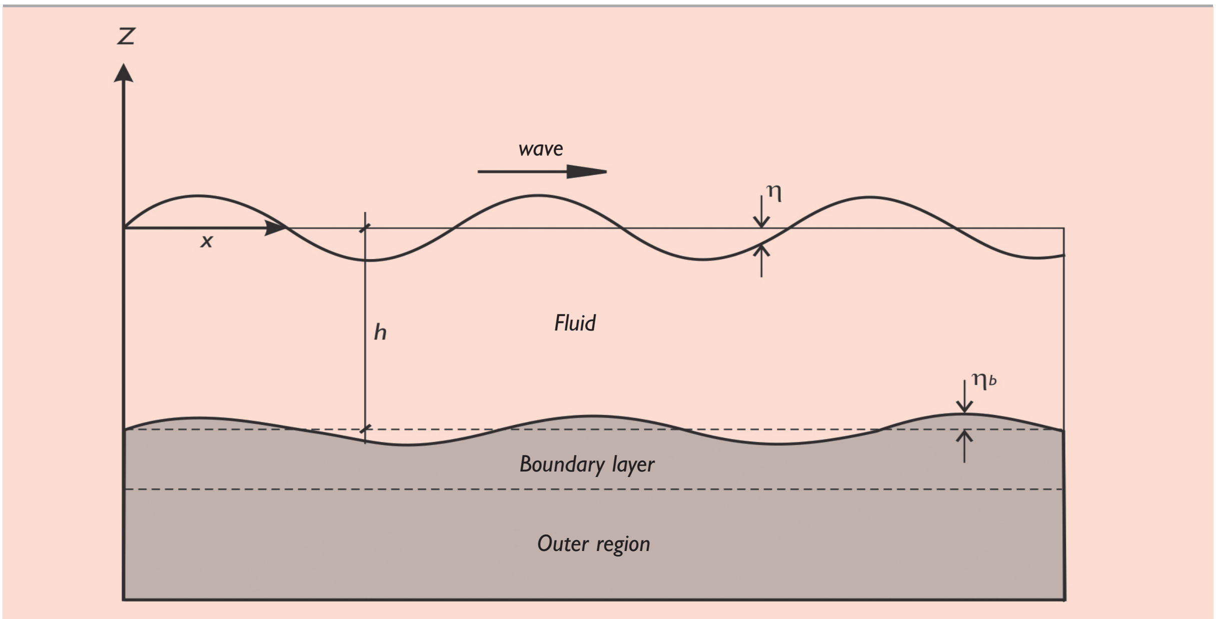
where G is the shear modulus of the soil, and ρ_s is its density. The wave height in the water H and the amplitude of the soil movement around the bottom level at depth h are related by the following expression,

$$\frac{A_b}{H} = \frac{1}{2}\left[\cosh Kh - \frac{gK}{\sigma^2 \sinh Kh}\right] \tag{4.429}$$

4.9.2.5.1 GEOTECHNICAL PARAMETERS OF ELASTIC POROUS SOILS

Wave propagation over a porous bed can produce significant modifications in wave properties and in the interstitial pressures in the bed (see Figure 4.4.23).

Figura 4.4.23. Progressive sea waves over a poro-elastic soil



Especially significant are the reduction of the wavelength and wave energy. These modifications depend on the soil thickness and porosity. In the case of cohesive soils, it also depends on the composition of the sediment, as well as on the cinematic characteristics of the wave, period, and height. The seabed thickness h_p measured from the bottom, below which the oscillatory motion in the porous medium is negligible, can be calculated as follows,

$$h_p > \frac{\pi}{K} - h_w, \quad for \quad \frac{h_w}{L_w} < \frac{1}{2} \tag{4.430}$$

$$L_w = \frac{2\pi}{k_w}, \quad \sigma^2 = g k_w \tanh k_w h_w \tag{4.431}$$

$$\sigma = \frac{2\pi}{T} \tag{4.432}$$

where K_R is the real part of the wavenumber obtained from the dispersion equation; and h_w, L_w and T are the water depth over the seabed, the wavelength, and wave period, propagating over an impermeable bed.

4.9.2.6 Progressive sea waves over viscous-elastic soils

The dispersion equation in this case is given by:

$$\frac{\frac{\rho}{\rho_s}\left(1-\left(\frac{kg}{\omega^2}\right)^2\right)\tanh kh}{\left(\frac{kg}{\omega^2}\right)\tanh Kh-1}+\frac{kg}{\omega^2}-\left(\frac{2k^2}{s^2}-1\right)^2+\left(\frac{2k^2}{s^2}\right)^2\left(1-\frac{s^2}{k^2}\right)^{\frac{1}{2}}=0 \qquad (4.433)$$

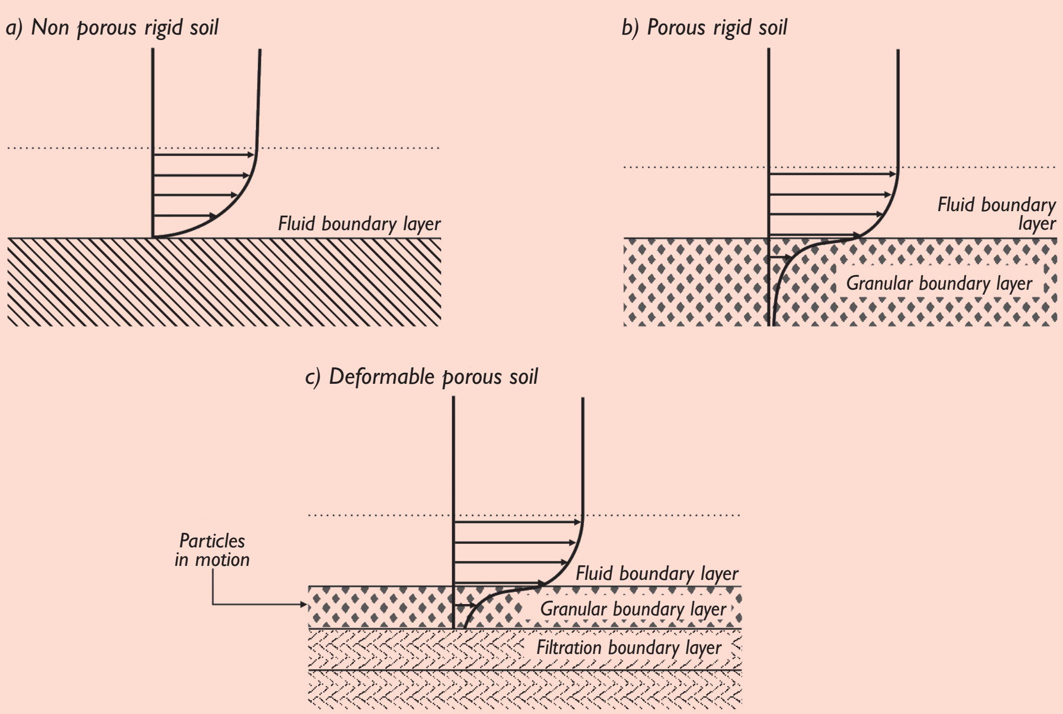
with

$$s^2=\frac{\rho_s\omega^2}{G_v} \qquad (4.434)$$

4.9.2.7 Stratum-stratum and soil-water interphases: boundary layer

The soil-water interphase is characterized by the fact that the response to the problem of the interaction between the fluid and the soil has a sudden transition. It is generally assumed that water is a compressible fluid (constant density), and can be either viscous or no viscous. If it is regarded as non-viscous, it cannot have any tangential stresses and thus, it cannot have a boundary layer. If it is regarded as a viscous fluid, the existence of tangential stresses produces a boundary layer. The various types of boundary layer are described in the following sections (see Figure 4.4.24).

Figura 4.4.24. **Types of boundary layer**



4.9.2.7.1 Boundary layer in a non-porous rigid soil

The tangential stresses of the fluid on the soil develop this viscous boundary layer or the (generally turbulent) fluid boundary layer. Because of the finite length of sea waves, the non-linear convective terms for inertia are not zero. The product of the harmonic terms of the sea waves generates new harmonics. Of special importance is the zero-order harmonic, which represents a weak water flow, induced by the waves, and which is capable of transporting mass over the soil.

4.9.2.7.2 Boundary layer in a porous rigid soil

When the soil is porous, the boundary layer extends underneath the soil-water interphase. The upper part of the boundary layer, composed of the previously described fluid boundary layer, is connected to the granular boundary layer, which is made up of a thin layer on the soil surface in which the velocity profile is adapted to the profile within the soil.

4.9.2.7.3 Boundary layer in a deformable porous soil

In the case of a deformable porous soil, underneath the fluid boundary layer, there is a thin layer composed of a flux of granular particles extracted from the soil. The granular flux is maintained by the effect of the tangential force exerted by the fluid over it. This particle flux comes in two types: bed transport and suspension transport. Bed transport involves those particles whose weight is supported by solid stresses, due to particle collisions and abrasion, which are transmitted to the bed as effective stresses. Suspension transport involves the particles whose weight is supported by the pressure of the fluid, which is transmitted to the bed as an excess of interstitial pressure.

Along with this granular flux in which the vertical distribution of the tangential stress is almost uniform, there is a sliding transport mode, confined to the granular base layer, which would be a second-order boundary layer. In this case, the movement, due to the shear stress, occurs in a thin layer, whereas the rest of the particles move at an almost constant velocity, practically without mixing.

Besides the previously described boundary layers, there is a boundary layer inside the non-fluid soil. In this layer, the fluid moves between the soil particles in what is known as filtration flux.

4.9.3 Behavior of the ground surface

The ground surface is the upper part of the soil on which the fluid boundary layer develops, and which is subject to the direct action of velocity fields and accelerations due to the oscillatory motion of the water. These actions can result in the erosion, transport, and accumulation of soil particles.

The thickness of the surface soil depends on the erosion capacity of the oscillations and its resistance to this action. In non-cohesive ground surfaces, the thickness can be on the order of meters, depending on the size of the material and the hydrodynamic properties of the action, whereas in cohesive ground surfaces, the thickness is on the order of decimeters.

When the ground surface is rocky, its mineralogical composition and other parameters should be determined, as described in the section on the load-bearing ground surface. In the case that the seabed is a loose cohesive or non-cohesive surface, the power of the sediments and their properties should be determined, according to the following section.

The principal failure modes that occur in this zone are liquefaction, erosion and undercutting, and the penetration between strata.

4.9.3.1 Instantaneous, basic, and state variables

According to the ROM 0.0, the ground surface should be characterized at least in respect to the following parameters.

4.9.3.1.1 *Parameters of identification*

The different layers of the ground surface can be identified by determining the grain-size and composition of non-cohesive soils, and the grain-size and plasticity of fine particles and the composition of cohesive soils. The soil samples for the analysis of soil composition and grain-size should be taken approximately 10-30 *cms* underneath the ground surface in order to avoid the contamination of the sample. In regards to the other layers, soil samples should be extracted without contaminating the upper layers.

Consequently, the granulometric curve is expressed by a dry weight percentage of the total sample, retained in the corresponding screen. Moreover, the pore index of the different layers should be calculated.

4.9.3.1.2 *State parameters*

It is also necessary to calculate the dry density of the ground surface. If this information is not available, the following nominal values can be considered in the predimensioning.

4.9.3.1.3 *Mechanical parameters*

Parameters to be determined are the Mohr-Coulomb strength parameters, internal friction angle in cohesive and non-cohesive soils, and the non-drained shear strength in cohesive soils.

4.9.3.1.4 *Oscillatory flow resistance*

Soil permeability against oscillatory motion should be determined on the basis of porosity and water flow resistance. For this purpose, it is necessary to evaluate the hydraulic regime (i.e. laminar, transitory or turbulent) to which the soil is subjected because of sea oscillations. This should be done in terms of the Reynolds number, Re.

Note. The flow resistance parameters are strictly mechanical parameters. However, because of their importance in the behavior of breakwaters, they are included in another type of parameter, related to their behavior in regards to the action of the water flow on the surface and through the porous medium.

4.9.3.2 *Erosion, undercutting, and deposit*

Under the action of sea oscillations, the loose ground surface particles can begin the movement, produce short-length rhythmic forms (i.e. ripples and dunes) that can be eroded, transported, or deposited. They can generate large-scale rhythmic forms, longitudinal or oblique bars, sand lens, etc. In the project design of a port area, it is important to know the variability of the thickness of the ground surface in order to take corrective measures, when necessary.

The depth of the wave action on the seabed depends on the wave period and the magnitude of the waveheight. The response of the seabed depends on the sediment diameter and density as well as its thickness. In the case of cohesive soils, it also depends on their composition.

In any case, the presence of a maritime structure can significantly modify the sea oscillations, in magnitude as well as direction. For this reason, once the plant layout of a breakwater is dimensioned, the surface behavior of the ground surface should be analyzed again.

4.9.3.3 *Flow action on the bed particles in the state*

The water flow due to the oscillatory motion, independently of its period, produces surface forces, tangential stress, and pressure as it moves over the ground surface. These forces depend on the structure of the boundary layer. In the case of cohesive and non-cohesive soils, it is best to evaluate the condition of initial bed movement according to the safety margin equation,

$$\tau_t - \tau_c \geq 0 \tag{4.435}$$

where τ_c is the critical tangential stress of initial movement; and τ_t is the tangential stress that the fluid generates on the seabed.

4.9.3.3.1 Progressive oscillatory motion

In the case that the beginning of the movement is only related by the presence of waves propagating over a mildly sloping bed, with an angular frequency $\omega = 2\pi/T_z$, the amplitude of the velocity on the bed u_{bw}, can be obtained by applying a wave theory, selected on the basis of the relative depth, h/L, and the relative steepness, H_I/L, of the sea waves at the site location. In these conditions, the tangential stress on the bed, due to the presence of the sea waves, in the propagation direction of the wave train, can be calculated by

$$\tau_w = \frac{1}{2}\rho_w f_w u_{bw} \tag{4.436}$$

$$u(t) = u_{bw} \sin \omega t \tag{4.437}$$

$$f_w = \exp\left[-6.0 + 5.2\left(\frac{A_{bw}}{k_r}\right)^{-0.19}\right] \tag{4.438}$$

$$f_{w,max} = 0.3 \tag{4.439}$$

where A_{bw} is the amplitude of the movement on the bed due to the sea waves, and which in linear theory is related to the amplitude of the velocity on the bed,

$$u_{bw} = \omega A_{bw} = \frac{\omega H_I}{2 \sinh kh} \tag{4.440}$$

where the incident wave height, $H_I \approx 1.8 H_{s,I}$, and the angular frequency, $\omega = 2\pi/\overline{T_z}$, and k_r is the bed roughness, which in the case of the beginning of moment and non-cohesive soil in the sand and gravel phase can be approximated by

$$k_r = 2 D_{90} \tag{4.441}$$

where D_{90} is the non-exceeded diameter by 90% (in weight) of the particles in the sample.

4.9.3.3.2 Stationary and partially stationary motion

To determine the velocity on the bed in the case of stationary and partially stationary regimes, it is necessary to know the reflection coefficient and the reflected and irradiated wave phase. The velocity module can be determined by

$$\left|u_{b,e}\right| = \omega A_b \left(1 + \left|C_R\right|\right) = \frac{\omega H_I \left(1 + \left|C_R\right|\right)}{2 \sinh kh} \tag{4.442}$$

In these cases, it should be remembered that the oscillation has nodal and antinodal points. Depending on the characteristics of the incident waves, including the direction of approach, level of reflection, and the grain-size of the soil particles can form different ondulatory structures on the bed and at the toe of the breakwater with localized erosive and accumulative zones.

In the presence of wave and current

In the case that the sea waves are accompanied by an oblique current (or an oscillatory motion with a period much larger than that of the sea waves, such as astronomical and meteorological tides, river, current, etc.), the structure of the boundary layer is very complex and the tangential stress and its resulting direction.

In order to determine the stress on the soil because of the sea wave action in the presence of a current, the following equation can be applied,

$$\tau_{w,c} = \rho_w \kappa u_{w,c}^2 \tag{4.443}$$

$$u_{w,c}^2 = \frac{g}{\kappa^2 C_f} u_c^2 + \frac{f_w}{2\kappa^2} u_b^2 (\sin\omega t)^2 + s\frac{\sqrt{g}}{\kappa C_f} u_c \frac{1}{\kappa}\sqrt{\frac{f_w}{2}} u_b (\sin\omega t)\sin\phi \tag{4.444}$$

where C_f is the Chezy coefficient; $\kappa \approx 0.4$; u_c is the velocity of the current; and ϕ is the angle between the current and the waves ($\phi = 0$, waves perpendicular to the current). The Chezy coefficient generally has a range of range $[35 < C_f < 85]$. The smallest value corresponds to the maximum level of friction, and the largest value corresponds to the minimum level of friction.

4.9.3.4 Small-scale rhythmic forms on a flat bed

Once the tangential stress of the beginning of moment has passed, non-cohesive soils begin to develop small-scale rhythmic forms, such as ripples and dunes. This continues until a value is reached, beyond which the entire bed undergoes a laminar flow. This causes the soil particles to move until a certain depth, and consequently, the resulting bed thicknesses can be on the order of $10D_{50}$.

The presence of ripples and dunes can be important because in most cases, their evolution is accompanied by reinforced sediment suspension. In these conditions the presence of a weak current helps to transport this sediment in preferential directions.

4.9.3.5 Large-scale rhythmic bedforms

When the wave action is partially stationary (e.g. at a breakwater), longitudinal and radial bars are formed, whose amplitude wavelength, and location depend on the characteristics of the oscillatory movement, mainly the reflection coefficient, phase lag, and the diameter and density of the soil particles in the bed. Depending on them, sediment can accumulate at the toe of the breakwater or undermine the structure, causing it to sink.

Chapter 2 of the ROM 1.0-09, on the predimensioning of breakwaters, describes the local structure of the flow produced by the interaction between the movement of the fluid and the presence of the breakwater, alignment changes and the head, particularly the vortex generation and emission at different scales. This aspect is relevant, and should be taken into account in the project design of breakwaters on erosionable beds.

4.9.3.6 Liquefaction and "tubing"

Liquefaction occurs when the granular material loses its shear strength and behaves like a fluid mass. The effective strength can be negated by the increase of interstitial pressure from the application of static or dynamic loads (earthquakes, storm waves, landslides, etc.). The sea bottoms that are most sensitive to liquefaction are those that are made of fine sand, saturated sand without any drainage, and loose sand (of low relative density). The presence of gas mixed with water increases the probability of liquefaction. Wave-induced liquefaction can be momentary (induced siphoning) or residual.

Note. *When the free sea surface oscillates, the pressure on the bed varies periodically. In the case of a progressive movement, the pressure increases with the passage of the wave crest and is reduced with the passage of the trough. If the motion is stationary, the pressure does not change at the nodes and goes from maximum to minimum in a semi-period at the antinodes. Consequently, the soil is compressed during the crests and in the positive antinodes. It expands with the troughs and in the negative antinodes. The free sea surface and the soil surface are out of phase on the order of π. This generates normal and tangential stresses in the soil that periodically vary with time. The tangential strain deformations in the soil cause a re-accommodation of soil particles at the expense of the volume of porous soil, pressing the water out of the particles, and possibly increasing the interstitial pressure if the drainage or flow is not immediately established, and takes a certain amount of time to begin.*

Since the external, forcing motion is periodic, the process in the soil is repeated with period of the motion. Consequently, if the period is smaller than the time that the soil takes to recover (i.e. drainage of the excess interstitial pressure), the pressure accumulates and becomes excessive. This excess of interstitial pressures can modify the magnitude of the forces at the contact points between particles, thus influencing the parameters of strength and strain of the soil. In this process, the confining pressure of the soil particles can be exceeded. The particles are thus liberated and the soil begins to act like a liquid. This state is known as the residual liquefaction of the soil.

Furthermore, during the passing of the trough in the progressive wave, and in the antinode of a stationary wave, the excess of the interstitial pressure on the hydrostatics is negative. If the soil is saturated, the pressure gradient is not very high, but if the soil is not saturated because it contains air or gas pockets, the pressure gradient is high, particularly near the bed surface. The vertical water flow, simultaneous with the pressure gradient, produces a drag force on the soil particles, which, along with the action of the pressure gradient, can exceed the submerged weight of the soil particles of the bed. This causes the soil to reach a state of liquefaction. This process is known as momentary liquefaction.

The role of the wave groups in both liquefaction processes can be relevant. In the case of residual liquefaction, the accumulation of the interstitial pressure depends on the time, passing from maximum to minimum pressures in the semi-period of the group. Moreover, the mean sea level follows the bottom surface movement. It falls with the passage of the large waves in the group, and rises with the passage of the small ways. Both processes increase the possibility that soil liquefaction will occur. This situation is even more relevant when the wave group is reflected from the breakwater because the processes described are reinforced even more in the antinodes, in other words, at the toe of the breakwater face. Furthermore, the maximum increasing velocities are produced $T/4$ before the maximum pressure. This causes the soil particles to become even looser.

The residual liquefaction of the soil can also occur because of earthquakes. Instantaneous liquefaction can occur when there are tsunamis, oscillations in wharfs, and other long waves that cause vertical pressure gradients and important velocity.

4.9.3.6.1 RESIDUAL LIQUEFACTION

Residual liquefaction is generated by the cyclical action of the sea waves and tangential stresses. The main factors that affect this process are the relative density of the sand and the drainage conditions. It can occur, not only because of the action of sea waves, but also because of earthquakes, submarine explosions, landslides, and the pitching motion induced in vertical breakwaters (e.g. breakwaters built with prefabricated caissons) because of the sea oscillations. At the site location, in the absence of reflecting boundaries, residual liquefaction occurs during the passage of progressive waves.

4.9.3.6.2 MOMENTARY LIQUEFACTION

In layers of sand particles or smaller, it is best to verify its sensitivity to liquefaction due to excess interstitial pressure, depending on its flow drainage capacity induced by sea oscillations and their temporal and spatial scales.

This includes waves, wave groups, and tides, as well as different oscillation types (e.g. progressive, stationary, and partially stationary).

Once the critical velocity is reached, the particle floats and the soil becomes fluid. It is advisable to consider the beginning of the liquefaction when the upwards force due to the outgoing flow equals the specific weight of the saturated soil,

$$I_{lim} = (1-n)(S-1) \tag{4.445}$$

$$S = \frac{\rho_s - \rho_w}{\rho_w} \tag{4.446}$$

where n is the porosity of the soil, and ρ_s aand ρ_w are the densities of the sediment and of the water, respectively. If the thickness of the soil layer is d_s the condition of liquefaction can be written similarly by expressing the hydraulic gradient, I

$$I = \left(\frac{\Delta h}{d_s}\right)_{lim} = (S-1) \tag{4.447}$$

The problem is intensified if the erosion progresses, and a continuous channel or tube is formed, which propagates in the propagation direction of the waves. This reduces the path or length of the filtration flow, L_f. In these conditions, the maximum hydraulic gradient to avoid the formation of such a tube should satisfy the following inequality,

$$I_{lim} = \left(\frac{\Delta h}{L_f}\right)_{lim} \leq 0.87\alpha_f c_f (S-1)\left(0.68 - 0.10\ln c_f\right) \tag{4.448}$$

$$\alpha_f = \left(\frac{d_s}{L_f}\right)^{\frac{0,28}{\left(\frac{d}{L_f}\right)^{2,8}-1}} \tag{4.449}$$

$$c_f = 0.25 D_{70}\left(\frac{g}{\nu K_f L_f}\right)^{\frac{1}{3}} \tag{4.450}$$

where $g = 9.8$ (m^2/s). In absence of more specific information, it can be supposed that $D_{70} \approx 1.25 D_{50}$.

4.9.4 Behavior of layers, core, and fillers

This section provides recommendations for the description and characterization of the ground and soil in terms of their geotechnical properties. This includes their mechanical properties, deformability, and their hydraulic behavior in the face of sea oscillations. These properties can be applied to the ground as well as its layers, fillers, and cores, and are widely used in maritime and port structures.

4.9.4.1 Description in the state

The description and characterization of the ground, layers, core, and fillers in relation to the sea oscillations is carried out in relation to the values of parameters of identification, state parameters, and mechanical parameters as well as others, in accordance with the ROM 0.0 (18). In any case, the predimensioning as well as the verification phase should follow the recommendations in the ROM 0.5-05.

(18) The R.O.M. 0.5-05 provides a detailed description of the parameters of ground, soil and rocks

4.9.4.1.1 Sands and gravel

In the case of layers of sand and gravel, the internal friction angle, ϕ is a property of the material and its state, as shown in Table 4.4.11.

Table 4.4.11. Internal friction angle of granular system

Type of soil	ϕ
Loose gravel	35
Very dense gravel	44
Loose sand	30
Very dense sand	39

The material discharged from the tube is loose, and consequently, the active and passive friction angles are the same.

4.9.4.1.2 Rocks

In the case of rocks, the internal friction angle depends on the state of the contact points or surfaces between the rocks of the layer since these are possible areas of rock fracture. In such cases, the angle ϕ depends on the angle at rest ϕ_0 of the rock without fractures, of a parameter R^* $[0 \leq R^* < 15]$ representing the shape of the rock, its degree of packing (porosity) and of the quotient of the equivalent effective stress of the quotient of the (particulated) rock s^*, and of the effective normal stress σ_c obtained from the simple compressive strength texts. Generally, this quotient depends on the diameter of the rock, $[0.2 \leq s^*/\sigma_c \leq 1.0]$ and $s^*/\sigma_c = f(l_p)$, where l_p is the equivalent cube face or median diameter of the rock.

$$\phi = \phi_0 + R^* \log\left(\frac{s^*}{\sigma_c}\right) \quad (4.451)$$

4.9.4.1.3 Friction between mantles, layers, and fillers

The mean angle of internal friction between mantles and layers depends on the relation of the diameters of the materials belonging to each mantle or layer, on the shape of the particles, and on their uniform or non-uniform grain size. For relatively uniform rocks and soils, a general equation to determine this mean angle is the following:

$$\phi = a\left(\frac{l_{s,i}}{l_{s,i+1}}\right)^{-0.3} \quad (4.452)$$

where $l_{s,i}$, $l_{s,i+1}$ are the equivalent cube faces of the upper and lower mantle, respectively; and a is a coefficient that varies with the shape of the rocks (see Table 4.4.12),

Table 4.4.12. Coefficient to calculate the mean friction angle between rock layers

	Irregular	Rounded	Spherical
a	70.0	61.5	50.0

4.9.4.2 Geotechnical properties

The geotechnical properties of loose ground, rubble-mound layers, core, fill material, and soils are best described by the Mohr-Coulomb model in terms of effective pressures and by strain deformability parameters, Poisson elasticity modules, and compression and stiffness indexes in edometric models.

The geotechnical properties of a rock layer depend on the grain-size, shape, and weight of the rocks as well as their spatial distribution that defines their apparent density d_{ap} and their permeability.

The geotechnical behavior of the rocky massifs can be analyzed by means of the following state parameters: geological strength index (*GSI*) and disturbance factor (*D*). Other parameters of analysis are mechanical parameters such as the simple compressive strength of the rock matrix σ_{ci} and the lithology of the rock massif m_i.

Except when more specific information is available, the geotechnical properties for predimensioning can generally be described with the nominal values listed in the ROM 0.5-05.

4.9.4.3 Hydraulic properties

The two main parameters that define the hydraulic properties of rock layers, core, fill material, and soil are porosity and permeability. In the same way as the geotechnical properties, they depend on the grain-size, shape, and weight of the rocks, as well as the spatial distribution that defines their apparent density d_{ap}.

4.9.4.3.1 NDEX OF OPENING SPACES AND POROSITY

The real porosity of the core, the main layer, and the secondary layer also depends on the degree of packing, and thus, of the construction method. The real porosity should fit the intervals shown on Table 4.4.13.

Table 4.4.13. Real porosity of rock layers

Core and layers	Porosity
All one	[0.32 – 0.38]
Rocks and rockfill	[0.35 – 0.42]

In the case of rock layers, the porosity value is associated with the compaction value and the number of pieces per surface unit, which should fit the following criteria.

4.9.4.3.2 PERMEABILITY

The permeability of the rock layers depends on the hydraulic regime and the porosity. In the sea and breakwaters, the regime is usually turbulent. In the laboratory, the regime can be laminar or turbulent, depending on the scale of the experiment [(19)]. Table 4.4.14 gives the representative values of the permeability of rock fill and soils.

Table 4.4.14. Permeability of rocks and seabeds

	Size, D_{50}	Hydraulic regime	Permeability *(m/s)*
Rockfill 1	[0.85 – 2.5] *(m)*	Turbulent	1.00
Rockfill 3	[0.1 – 0.3] *(m)*	Turbulent	0.30
Gravel	[1 – 8] *(cm)*	Turbulent	0.10
Coarse sand	[0.7 – 3] *(mm)*	Transitory	10^{-3}
Fine sand	[0.12 – 0.75] *(mm)*	Laminar	10^{-5}
Silt and clay	[0.001 – 0.05] *(mm)*	Laminar	10^{-7}

(19) See the section in the Annex, "Dimensional analysis and scale effects in hydraulic experiments".

Generally speaking, permeability increases with the square of the rock size D_{50} in a laminar regime and with the square root of its size in a turbulent regime. D_{50} is the mean size of the granulometry of the rocks or soil.

4.9.4.4 *Hydraulic regimes*

The cyclical nature of sea oscillations and the temporal variability of the amplitude can significantly influence the behavior of the ground. Drainage velocity depends on permeability, which depends on the hydraulic regime. This depends on the period and amplitude of the oscillation, and of the porosity of the granular bed.

4.10 RANDOM VARIABLE AND PROBABILITY

A random variable, X, is a measurable function that assigns real values to the elements of the sample space on which the experiment is performed $X : S_p \to (-\infty,\infty)$, with $\omega \to x = X(\omega)$. Depending on the nature of the set of values that the variable can take, this variable is discrete or continuous. A random variable that can take values in a countable set $\{x_1, x_2, x_3, ..., x_n, ...\}$ is a discrete variable. In contrast, a variable that can take any real value between two limits (bounded or not) is a continuous variable.

4.10.1 Density and distribution functions

The purpose of these functions is to describe as accurately as possible the uncertainty of the random variable. The probability density function of a continuous random variable X, namely $f(x)$, measures the intensity or probability rate of the value x. Consequently, the probability of occurrence of an event A is determined as

$$\Pr[A] = \int_A f(x)dx \qquad (4.453)$$

where $f(x)$ is a non-negative, integrable function, and the domain of the variable X is $(-\infty,\infty)$ and is verified by

$$\lim_{x\to\infty} f(x) = 0 \quad \lim_{x\to-\infty} f(x) = 0 \quad \int_{-\infty}^{\infty} f(x)dx = 1 \qquad (4.454)$$

Note. *If the domain of a random variable is discretized in tiny increases* Δx, *the probability density function can be defined by the probability that the value,* x_0, *the random variable will fall within the interval* $[x - \Delta x/2, x + \Delta x/2]$, *such that*

$$\Pr\left[x - \frac{\Delta x}{2} \le x_0 \le x + \frac{\Delta x}{2}\right] = f(x_0)\Delta x \qquad (4.455)$$

When this definition is compared to that in the section, it is evident that event A is effectively the event in which the value of the random variable lies in the interval $[x - \Delta x/2, x + \Delta x/2]$.

The probability distribution function of X or the accumulated distribution function is a function $F(x)$ which assigns its probability of occurrence to each event $X(\omega) \le x$,

$$F(x) = \Pr[X(\omega) \le x] \qquad (4.456)$$

where $F(x)$ is an increasing function that takes values in the interval $[0,1]$, which is calculated, based on the density function as

$$F(x) = \int_{-\infty}^{x} f(x)dx, \quad F(x = \infty) = 1 \qquad (4.457)$$

Moreover, the density function $f(x)$ can be calculated as the derivative of the distribution function,

$$f(x) = \frac{dF}{dx} \qquad (4.458)$$

If X is a discrete random variable, the probability mass function is defined as the function that assigns to each event, $X(\omega) = x_i$, its probability of occurrence, $p = \Pr(X = x_k)$, where x_k is a value that can take a random variable, and k indicates the order number of the value. $F(x)$ is calculated as the sum of the probabilities of all possible values of X, less than or equal to x,

$$F(x) = \sum_{x_k \le x} \Pr(X = x_k) \tag{4.459}$$

Note. The discrete random variables can only take values at concrete points. Consequently, the value of the density function at point x_0 is the value of the probability mass concentrated there, $p(x_0)$. When the random variable is continuous, the value of the probability in the interval is equal to the product of the unitary value of the probability or the probability density, $f(x)$ at the width of the interval $f(x_0)\Delta x$. In each of these cases, the name of the probability function refers to either mass or density. For the sake of simplicity, the term, probability density function is used here to designate both functions since in the context of application (discrete and continuous random variables), its meaning is sufficiently clear.
In both cases, the distribution function is accumulated, which in one case, explains how it is calculated as the sum of discrete values, $p(x_0)$. In the other case, it is calculated by the sum or integral of the area under the density function in the interval, $f(x_0)\Delta x$.

4.10.2 Population and Sample

The population of a certain project factor is composed of all the possible values that it can take. Generally speaking, it is not possible to know all the population, but rather only a small part of it. This small part or sample can only provide incomplete information regarding the total population.

The sample is the available information, and it can be used to obtain information regarding the population by means of the elaboration of a probability model.

The process of constructing this model is known as *statistical inference*. This process is inductive since it uses a specific example (the sample) to arrive at more general information regarding the entire population. The most frequently used procedures for such inferences are the point estimator, the estimate of the confidence interval, and the hypothesis test.

Once a probability model is available, this can be used to obtain information regarding a sample. The procedure is governed by probability rules, and is basically a deductive process that applies a general model to a particular sample.

4.10.2.1 Sample space S_p and sample S_m

When it is not possible to anticipate the values of a project factor from repeated observations, it is best to describe the factor as a random variable U which represents the population. The set of all the possible values of the population is known as the sample space S_p. The realizations (observations) of U are represented by u. A set of realizations is S_m, which is a subset of S_p.

4.10.2.2 Sample measurements and sample values

An experiment in which a project factor U is randomly measured or observed n times in a time interval provides a sample of *n* values, u_i, $i = 1, \ldots, n$, o $\{u_i\}_n$. This sample can be used to obtain sample values or descriptive statistics by following the following procedure:

1. Ordering of the data according to in order of increasing magnitude. This results in an ordered sequence of values $\{u_{(i)}\}_n$, where $u_{(1)}$ is the observation of the smallest value, and $u_{(n)}$ is the observation of the largest value. The range of data is defined by $[u_{(n)} - u_{(1)}]$.

2. Sample values include the following:

- sample mean: $\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$
- sample mean square: $u_{rms} = \frac{1}{n}\sqrt{\sum_{i=1}^{n} u_i^2}$
- sample mode: $\breve{u}$: the most frequent value in the sample
- sample median: $\tilde{u}$: the centered value of the measurements
- sample variance: $s_u^2 = \frac{1}{n-1}\sum_{i=1}^{n} u_i \left(u_i - \bar{u}\right)^2$
- standard deviation of the sample: s_u
- sample variation coefficient: $c_\upsilon = \frac{s_u}{\bar{u}}$

For the case of two or more variables, it is possible to define, among others, the sample covariance and the sample correlation coefficient:

- sample covariance: $C_{u,v} = \frac{1}{n}\sum_{i=1}^{n}\left(u_i - \bar{u}\right)\left(\upsilon_i - \upsilon\right)$
- sample correlation coefficient, $r_{u,v} = \frac{C_{u,v}}{s_u s_v}$, $-1 \le r_{u,v} \le 1$

Note. During the experiment, more than one random variable can be observed simultaneously, such that the sample would be composed of n vectors, $\hat{u}_i$, $i = 1, \ldots, n$, which can be jointly or separately analyzed.

For the case of two or more variables, it is possible to define, among others, the sample covariance and the sample correlation coefficient:

- sample covariance: $C_{u,v} = \frac{1}{n}\sum_{i=1}^{n}\left(u_i - \bar{u}\right)\left(\upsilon_i - \upsilon\right)$
- sample correlation coefficient, $r_{u,v} = \frac{C_{u,v}}{s_u s_v}$, $-1 \le r_{u,v} \le 1$

4.10.2.3 Frequency histogram

If the size of the sample is sufficiently large, it is useful to group the data in k intervals of width Δu_j of frequencies q_j, equal to the number of x, included in Δu_j. By defining the relative frequency by $f_j = q_j/n$, it is possible to draw a histogram or frequency distribution of the sample, $[j * \Delta u_j, f_j]$ and the histogram of the accumulated relative frequencies, $[j * \Delta u_j, F_j]$, where $F_j = \sum_{i=1}^{j} f_i$.

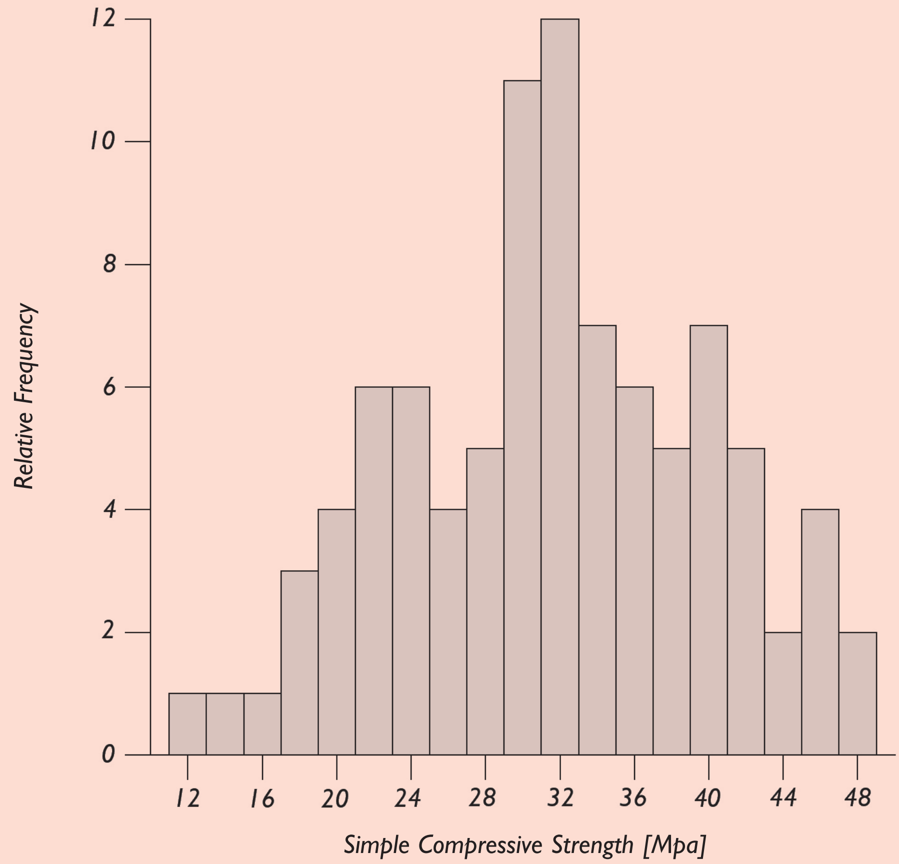
Example. Figure 4.4.25 simultaneously shows two properties of concrete, namely, density and simple compressive strength. Density is a state parameter, and simple compressive strength is a mechanical parameter.

4.10.3 Discrete and continuous random variables

A random variable, which takes values in a countable set $\{x_1, x_2, x_3, \ldots, x_n, \ldots\}$ is a discrete random variable. A random variable that can take any real value between two limit values (whether bounded or not) is a continuous random variable.

Example. At a fixed point in the sea, the vertical displacement of the free sea surface, $\eta(t)$, measured during a time interval $t_o \le t \le t_o + D_t$ is a random variable. In principle, $\eta(t)$ can be any real value. This variable can be used to define the variable wave height as the maximum vertical distance between two consecutive upwards crossings of the mean level. The wave height is a discrete random variable that takes values in the set of natural numbers.

Figure 4.4.25. **Example of a frecuency histogram for the simple compressive strength of concrete**



Generally speaking, the values that take a random variable are uncertain, unpredictable, and non-deterministic. For this reason, the random variable is described by the probability of occurrence of events or possible sets of values that the variable can take.

The uncertainty of the random variable U is modeled by the distribution function, $F(u;\ \theta)$, where F is a mathematical function of the values of u, which the variable U can take in the sample space S_p. The theoretical model has one or various parameters represented by θ. Thus, $F(u;\ \theta)$ represents a family of functions, depending on the value of the parameter or parameters, $\theta \subset \Omega_p$, where Ω_p is the parametric space.

4.10.3.1 Density and distribution functions of a discrete variable

The density function $p(u;\ \theta)$, is defined such that $Pr[U = u] = p(u;\ \theta)$. Since this function is only defined for concrete values of u, it is also known as the mass probability function. The distribution function F is calculated by $F(u;\ \theta) = \sum_{t \leq u} p(t;\ \theta)$, and thus, is also known as the accumulated probability function.

4.10.3.2 Density and distribution functions of a continuous variable

The probability density function, $f(u;\ \theta)$, is defined by

$$\Pr\left[u - \frac{du}{2} \leq U \leq u + \frac{du}{2}\right] = f\left(u;\theta\right) du \qquad (4.460)$$

The function, $F(u;\ \theta)$, is the accumulated distribution function, and is defined by $Pr[U \le u] = F(u;\ \theta)$. The probability of the random variable U, taking a value less than or equal to u is given by F. If the variable U can take values in the domain $[a,\ b]$, these function are interrelated by

$$F(u;\theta) = \int_a^u f(t,\theta)\,dt \qquad (4.461)$$

$$\Pr[a \le u \le b] = F(b,\theta) = \int_a^b f(u,\theta)\,du = 1 \qquad (4.462)$$

Example. *The height of successive waves that occur in a sea state is random. In certain general conditions, the random variable wave height $H = U$ follows a Rayleigh distribution,*

$$F(H;H_{rms}) = 1 - \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] \qquad (4.463)$$

where $H_{rms} = \theta$ is the mean square wave height, and is the distribution parameter. The density function f is obtained by deriving the function of distribution F,

$$f(H;H_{rms}) = \frac{2H}{H_{rms}}\exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] \qquad (4.464)$$

The distribution parameter θ is not the only one. In the case of the Rayleigh distribution, any other statistical parameter can be used, such as the significant wave height H_s, mean wave height H_m, etc.

4.10.3.3 State descriptors: expectation and moments

The expectation of a function h of a discrete random variable is defined as

$$E[h(U)] = \sum_{\forall t} h(t)\,p(t;\theta) \qquad (4.465)$$

If the variable is continuous in a bounded domain $[a \le u \le b]$, the expectation of the function h is defined by

$$E[h(U)] = \int_a^b h(u)\,f(u;\theta)\,du \quad a \le u \le b \qquad (4.466)$$

thus proving that the integral is finite [20].

4.10.3.3.1 Expected value and variance

The r-order moment around the source is calculated by means of the previous operation with $h(U) = U^r$. The expected value of U, $E[U] = \mu_1$ is obtained for $r = 1$. The mean square value of U is defined by $r = 2$, $E[U^2] = \mu_2$.

The r-order moment around the mean is defined by means of the previous operation by

$$h(U) = [U - E[U]]^r \qquad (4.467)$$

The variance of U is obtained with

$$Var[U] = E\left[\{U - E[U]\}^2\right] = E[U^2] - \{E[U]\}^2 = \mu_2 \qquad (4.468)$$

(20) $p(u;\theta)$ y $f(u;\theta)du$ are weighting factors that permit the evaluation of the weighted average of $h(U)$.

The square root of the variance is the standard deviation $s[U]$ of U. Furthermore, apart from the expected value $E[U]$ and the variance, these Recommendations also use other values related to the function F.

4.10.3.3.2 QUANTILE

The q-order quantile represented by u_q is defined by the equation $F(u_q;\theta) = q$ (21).

4.10.3.3.3 MODE

For a continuous variable, the mode $\breve{u}$, is the most probable value of the sample, and is obtained as the root of the equation $d/du\{f(\breve{u};\theta)\} = 0$, which is the maximum value of the density function f.

4.10.3.3.4 VARIATION COEFFICIENT

The variation coefficient is the quotient of the standard deviation and the mean,

$$c_v(U) = \frac{S(U)}{E[U]} = \frac{\sqrt{\mu_2}}{\mu_1} \tag{4.469}$$

4.10.3.3.5 KURTOSIS AND BIAS

Kurtosis and bias are factors that define the shape of the function F. The bias is defined by $\gamma_1 = \mu_3/(\mu_2)^{3/2}$ (22), and kurtosis is defined by $\gamma_2 = \mu_4/(\mu_2)^2$ (23).

4.10.3.3.6 CHARACTERISTIC VALUES AND CONFIDENCE INTERVAL

As already shown, given the random variable, U, and a required probability level, $p(0 \le p \le 1)$, the worst quantile value u_p is defined as the lowest value of u that satisfies the inequality, $F(u) > p$.

For any random variable, it is possible to define a range of values, with and upper boundary and a lower boundary, known as a confidence interval. In this interval, when a statistical trial is performed and repeated in identical conditions, it can be expected that successive sample values will take values in a certain percentage of the cases.

The values that define the confidence interval are known as the characteristic upper value and lower value, respectively. By definition, for continuous variables, the probability that a given value of the parameter or project variable will fall within the confidence interval is

$$\Pr\left[U_{k,\inf} < U < U_{k,\sup}\right] = \Pr\left[U_{k,\sup} < U\right] - \Pr\left[U_{k,\inf} > U\right] = (1-\alpha) \text{ [illegible] } < \alpha < 1 \tag{4.470}$$

The probability value $(1 - \alpha)$ is known as the confidence level or coefficient.

(21) For example, the 0.05-order quealilty, also known as the 5%, percentile is exceeded by 95% of all the measurements. The 0.5 order-quantile corresponds to the median value of the sample.
(22) Positive bias indicate that the right tail of the density function is greater than the left tail. When the bias is zero, the density function is symmetrical.
(23) The normal density function has a kurtosis equal to 3. When the density function has $\gamma_2 > 3$, the function is more peaked than normal.

4.10.4 Joint distribution and density functions

The joint distribution and density functions with various random variables can be defined in the same way as when there is only one random variable. In the case of two continuous random variables, $(u - v)$, the joint density function is defined in the following way:

$$\Pr\left[\left(u-\frac{du}{2}\le U\le u+\frac{du}{2}\right)\cap\left(\upsilon-\frac{d\upsilon}{2}\le V\le \upsilon+\frac{d\upsilon}{2}\right)\right]=f\left(u,\upsilon;\theta\right)dud\upsilon \quad (4.471)$$

and the joint distribution function can be defined as

$$F\left(u,\upsilon;\theta\right)=\int_a^u\int_c^\upsilon f\left(u,\upsilon;\theta\right)dud\upsilon \quad (4.472)$$

where θ represents the function parameters, and the domains of the variables are $[a \le u \le b]$ and $[c \le \upsilon \le e]$. Consequently,

$$f\left(u,\upsilon;\theta\right)=\frac{\partial^2 F\left(u,\upsilon;\theta\right)}{\partial u\partial \upsilon} \quad (4.473)$$

4.10.4.1 Conditioned functions

The density function and the distribution function of one of the variables conditioned to the occurrence of a value or range of values of another variable are known as conditioned functions. The density function of the variable u, conditioned to the presence of the variable v in the interval $[\upsilon - d\upsilon \le \upsilon \le \upsilon+d\upsilon]$ is

$$f\left(u|\upsilon;\theta\right)=\frac{f\left(u,\upsilon;\theta\right)}{f\left(\upsilon;\theta\right)} \quad (4.474)$$

$$f\left(u,\upsilon;\theta\right)=f\left(u|\upsilon;\theta\right)f\left(\upsilon;\theta\right)=f\left(\upsilon|u;\theta\right)f\left(u;\theta\right) \quad (4.475)$$

If the variables u and v are independent, $f(u\,|\,\upsilon;\theta) = f(u;\theta)$. Consequently,

$$f\left(u,\upsilon;\theta\right)=f\left(\upsilon;\theta\right)f\left(u;\theta\right) \quad (4.476)$$

4.10.4.2 Marginal functions

The density function and the distribution of one of the variables, independently of the values of the other, are known as marginal functions. They are obtained by

$$f\left(u;\theta\right)=\int_c^e f\left(u,\upsilon;\theta\right)d\upsilon;\quad F\left(u;\theta\right)=\int_a^u ds\int_c^e f\left(s,t;\theta\right)dt \quad (4.477)$$

$$f\left(\upsilon;\theta\right)=\int_a^b f\left(u,\upsilon;\theta\right)du;\quad F\left(\upsilon;\theta\right)=\int_c^\upsilon ds\int_a^b f\left(s,t;\theta\right)dt \quad (4.478)$$

4.10.4.3 Expected conditioned value and conditioned variance

If U and V are discrete random variables with a joint density function $p(u,\upsilon;\theta)$, the mean value of U, conditioned to a given value of V, is

$$E\left[U|V=\upsilon_j\right]=\sum_{i=1}^n u_i p\left(u_i|\upsilon_j;\theta\right) \quad (4.479)$$

where $p(u_i\,|\,\upsilon_j)$ is the conditioned probability of the value u_i, knowing that the value υ_j has occurred. Similarly,

$$E\left[V\middle|U=u_j\right]=\sum_{i=1}^{n}\upsilon_i p\left(\upsilon_i\middle|u_j;\theta\right) \tag{4.480}$$

The expected value of the conditioned mean is the following:

$$E\left[E\left[U\middle|V\right]\right]=\sum_{i=1}^{n}E\left[U\middle|V=\upsilon_j\right]p\left(\upsilon_j;\theta\right) \tag{4.481}$$

If U and V are continuous variables, the conditioned mean value of U, given that $V=\upsilon$, is

$$E\left[U\middle|V=\upsilon\right]=\int_a^b uf\left(u\middle|\upsilon;\theta\right)du \tag{4.482}$$

Similarly,

$$E\left[V\middle|U=u\right]=\int_c^e \upsilon f\left(\upsilon\middle|u;\theta\right)d\upsilon \tag{4.483}$$

The conditional variance of U, given that $V=\upsilon$, is the following:

$$Var\left[U\middle|V\right]=E\left[U^2\middle|V\right]-\left\{E\left[U\middle|V\right]\right\}^2 \tag{4.484}$$

4.10.4.4 Covariance and correlation

The covariance of two random variables, U and V, $Cov[U,V]$, is the expected value of the products of their deviations with respect to their means,

$$Cov\left[UV\right]=E\left[\left(U-E\left[U\right]\right)\left(V-E\left[V\right]\right)\right]=E\left[UV\right]-E\left[U\right]E\left[V\right] \tag{4.485}$$

The correlation coefficient or normalized covariance, $\rho_{U,V}$, is the following:

$$\rho_{U,V}=\frac{E\left[\left(U-E\left[U\right]\right)\left(V-E\left[V\right]\right)\right]}{\sqrt{Var\left[U\right]Var\left[V\right]}} \tag{4.486}$$

If $\rho_{U,V}=\pm1$, the linear relation between the two variables is perfect. When the pairs of variables are plotted on a Cartesian system of coordinates, U and V, all of them are above a straight line. If $\rho_{U,V}=0$, there is no correlation or linear relation between the two variables. This does not mean that the two variables are statistically independent. On the contrary, if two variables are statistically independent, their correlation coefficient is zero. The correlation coefficient provides information regarding the degree of association between two variables, but it does not give any information regarding its cause-effect relation.

Note. Once the random variables and the probability functions are defined, the following objective is the description and application of various models that can be used in the description of project factors, the evaluation of the failure probability of a mode, and the joint failure probability. The following section analyzes the probability, density, and distribution functions that are necessary to model the behavior of a given population. The distinction is made between discrete-variable models and continuous-variable models.

4.10.5 Discrete-variable models

This section describes statistical distributions, density function, and distribution function, which should be used to model the behavior of discrete variables. These include the Bernoulli, binomial, negative binomial, multinomial, and Poisson distributions. These models are used to evaluate the joint failure probability of a subset of a structure in a given time interval. All of them are based on the Bernoulli experiment, also known as the Bernoulli process.

4.10.6 Bernoulli process

The simplest statistical experiment is that in which there is only one trial, such as when a coin is tossed in the air, which comes out either heads or tails. In the same way, when the state of a breakwater is evaluated, the trial has two possible outcomes. Either the breakwater has failed or it has not failed. Both events, heads or tails, failure or success, are mutually exclusive, and are a complete set of all the possible results of the statistical trial. The result of this type of experiment is a Bernoulli random variable, which can only have two values. The distribution function of this result is known as the Bernoulli distribution.

When, instead of one trial, the experiment is composed of many (n) trials, the series of *n* trials is a Bernoulli process. The term *process* can be applied to an experiment carried out to analyze the behavior of a system in space as well as time.

Note. Based on the previous explanation, the year can be regarded as a statistical trial. When the performance of a breakwater section is observed in a year, two outcomes can be obtained: the failure of the section or the success of the section. The useful life (V years) of the breakwater can be regarded as a statistical trial made up of a sequence of V tests, one per year, which together form a Bernoulli process. Each of the trials (years) has two possible outcomes, failure or success. Similarly, the breakwater can be divided into subsets, and each subset analyzed to see if it has failed or not. In this case, each subset is a trial, and the study of the failure or success in each subset can make up a sequence of trials (experiment), known as a Bernoulli process. For the trial and the experiment to be considered a Bernoulli process, they should fulfill the general criteria described in the following sections.

4.10.6.1 Application Criteria for a Bernoulli process

The criteria governing a Bernoulli process are the following:

1. Each trial has only two possible results, $s = 2$: failure or success (non-failure).
2. Failure or success makes up a complete collection of events (outcomes).
3. Both outcomes are mutually exclusive.
4. The probability of occurrence of the failure event is constant, p.
5. For the events to be a complete set of mutually exclusive outcomes, the probability of the non-failure or success event is constant, and equal to $1 - p$.
6. Each of the *n* trials of the experiment is independent. The outcome of any of the trails does not depend on the result of the other trails.

These criteria can be used to define the principal probability models of discrete random variables. Thus, special attention should be paid to the identification of the random variable and the distribution parameters.

Note. *In maritime engineering, it is essential to evaluate the failure probability of the subsets of a structure. For this purpose, the description of probability models in these Recommendations is focused on the failure event, and the one associated with the probability of occurrence p.*

4.10.6.2 Bernoulli distribution, $B(n = 1;p)$

The characteristics of this distribution are:

1. The experiment is made up of only one trial, $n = 1$, the determined value.
2. The random variable X is the outcome of the trial, which can only be failure or success $(0 \text{ or } 1)$.

4.10.6.2.1 Density function

The Bernoulli density function is

$$
\begin{aligned}
p_X(x) &= \Pr[X = x; p] = p^x (1-p)^{(1-x)} \\
x &= 0.1 \quad 0 \le p \le 1 \\
p_X(x) &= 0, \text{ another case}
\end{aligned}
\tag{4.487}
$$

4.10.6.2.2 Statistical descriptors

- Mean value of the variable: $E[X] = p$
- Variance of the variable: $Var[X] = p(1-p)$

Example. The evaluation of breakwater stability in the laboratory is carried out by subjecting the breakwater section to an experiment consisting of one trial. The result obtained is either that the section has failed or that it has not failed. If the probability that the section will fail in the experiment is p, the experimental outcome follows a Bernoulli distribution. Generally speaking, the probability p is generally considered to be the probability of occurrence of the predominant agent that causes the failure. Consequently, when this agent occurs, it is certain that the breakwater section will fail.

4.10.6.3 Binomial distribution, $BN(n;p)$

Besides having the properties of a Bernoulli process, a binomial distribution also has the following characteristics:

1. The experiment consists of n trials; n is a given value.
2. The random variable X is the number of failures in the n trials (the experiment).
3. The failure probability in each trial is constant, and is equal to p.

4.10.6.3.1 Distribution and density functions

The density function of the binomial probability is

$$
\begin{aligned}
p_X(x) &= \Pr[X = x; n; p] = \binom{n}{x} p^x (1-p)^{n-x} \\
x &= 0,1,\ldots,n \quad 0 \le p \le 1 \quad \binom{n}{x} = \frac{n!}{x!(n-x)!} \\
p_X(x) &= 0, \text{ another case}
\end{aligned}
\tag{4.488}
$$

The distribution function is

$$
F_X(n) = \sum_{k=0}^{k=n} \binom{n}{k} p^k (1-p)^{n-k}
\tag{4.489}
$$

The probability function has $n+1$ terns that represent the probabilities that no failure ($x_n = 0$) will occur, or that one, two … n failures ($x_k = n$) will occur.

4.10.6.3.2 Statistical descriptors

- The mean value of the number of failures in n trials:

$$
E[X] = np
\tag{4.490}
$$

- Variance of the number of failures in n trials:

$$Var[X] = np(1-p) \tag{4.491}$$

If X_1 and X_2 are two binomial variables of parameters (n_1, p) and (n_2, p), respectively. Thus, $Y = X_1 + X_2$ is a binomial random variable of parameters (n = $n_1 + n_2, p$).

Example. *In the previously described case regarding the outcome of the trial, it is assumed that the section is subjected to n equal trials. In other words, each breakwater section is reconstructed at the end of each trial, and is then subjected to the action of the same storm event. Supposing that in the n trials, there are m trials in which the breakwater failed, and* n – m *trials in which the breakwater did not fail, the random variable,* X *number of failures, is then said to follow a binomial distribution.*

The result of the trial is a compound event that can occur in different combinations. If $n = 4$ *and* $m = 1$*, the section can fail in the first trial or in the second, third, or fourth. Consequently, the failure probability of the compound event, one failure in four trials, is the sum of the probabilities of each, in other words:*

$$\Pr[X; n=4, m=1] = p(1-p)^3 + (1-p)p(1-p)^2 + (1-p)^2 p(1-p) + (1-p)^3 p = \binom{4}{1} p^1 (1-p)^{4-1} \tag{4.492}$$

If the predominant agent occurs, the probability that the breakwater section will fail is $p = 0.1$*; the probability of a failure in one of the four trials is* $\Pr[X;n = 4, m = 1] \simeq 0.29$*; and the probability of failure in all four trials is* $\Pr[X;n = 4, m = 4] \simeq 10^{-4}$*. If* $p = 0.85$*, then,* $\Pr[X;n = 4, m = 1] \simeq 0.01$ *and* $\Pr[X;n = 4, m = 4] \simeq 0.52$*. These results indicate that there is a greater probability that there will be more trials with a failure outcome, and that there is less probability that the experiment will end in only a few failures. If* $p = 0.5$*, the most probable outcomes are two or three failures in four trails with the associated density function,* $\Pr[X;n = 4, m = 1] \simeq 0.31$*. The mean number of failures and the variance of the experiment are*

$$E[X, n=4, p=0.5] = np = 2 \tag{4.493}$$

$$Var[X, n=4, p=0.5] = E[X^2] - (E[X])^2 = np(1-p) = 1 \tag{4.494}$$

4.10.6.4 Geometric distribution, $G(r = 1;p)$

The characteristics of a geometric distribution are the following:

1. The experiment consists of an x number of trials.
2. The experiment continues until the occurrence of the first failure, $r = 1$, a given value.
3. The random variable, X, is the number of trials performed until the occurrence of the first failure.

4.10.6.4.1 Distribution and density functions

The density function of the geometric probability is the following:

$$\begin{aligned} p_X(x) &= \Pr[X = x; r = 1; p] = p(1-p)^{x-1} \\ x &= 1,2,\ldots; 0 \le p \le 1 \\ p_X(x) &= 0, \text{ another case} \end{aligned} \tag{4.495}$$

The distribution function is

$$F_X(n) = \sum_{k=1}^{x} p(1-p)^{k-1} \tag{4.496}$$

4.10.6.4.2 STATISTICAL DESCRIPTORS

- Mean value of an x number of trials that gives the first failure,

$$E[X] = \frac{1}{p} \tag{4.497}$$

- Variance of an x number of trials that gives the first failure,

$$Var[X] = \frac{1-p}{p^2} \tag{4.498}$$

Note. *The geometric model defines the return period, which generally is the mean number of years that should pass before the first failure occurs in the structure.*
Let us suppose that a breakwater is calculated to resist, without failure, a maximum sea state, whose probability of occurrence taken from the extreme sea state or storm regime is p. This regime is based on the maximum sea state in each year.
In this way, the year is a Bernoulli trial. If a sea state occurs with a significant wave height greater than that of the design wave height, the breakwater will fail. If it is less than the design wave height, then the breakwater will not fail. In order to know how many years should pass before the first failure in the breakwater, the geometric model can be used. The year can be taken as trial, and the question asked how many trials should be performed before the first failure outcome is obtained.
The expected value of the number of trials is a first approximation to the answer. Given that each trial is a year, the mean number of years that should pass until the first failure is $E[X] = 1/p = T_R$. If, for example, the probability associated with the design sea state is $p = 0.05$, the return period is $T_R = 20$ years.
It is interesting to compare this result with the probability that at least one failure during the useful life of the structure will occur. If the useful life is $V = 25$ years, the probability of occurrence of at least one failure is $p_f = 1 - (1-p)^V = 0.7226$. Both calculations indicate that it is very probable that the structure will fail at least once during the useful life of the structure since p_f is very high, and $T_R < V$.
Another frequent question is the probability that n years will pass before a failure occurs. The value of the random variable or number of trials (years) until the first failure (in binomial form) is the probability of there being one failure in two trials of the experiment. In other words, $n = 2$, $X = x = 1$,

$$F_X(2) = \sum_{x=1}^{2} \binom{2}{1} p^1 (1-p)^{2-1} = p + p(1-p) \simeq 0.14 \tag{4.499}$$

and the probability that in at least two years the first failure will not occur is the complementary value, namely, $1 - F_X(2) \simeq 0.86$
Occasionally, one wishes to know the probability that there will be at least one failure during the useful life of the structure. If the failure probability is $p_{f,ELU}$, the probability of at least one failure in V years can be calculated, based on the probability of the complementary event of the event sequence, in other words, the probability that there will be one, two … v failures in the $n = V$ years, as shown in the following:

$$p_{f,ELU} = 1 - \sum_{x=1}^{x=V} \binom{V}{x} p^x (1-p)^{V-x} \tag{4.500a}$$

This involves the calculation of $n = V$ binomial terms. Another more direct approximation can be obtained by considering the probability of the complementary event, namely, that no failure occurs $(x = 0)$ in $n = V$ years.

$$p_{f,ELU} = 1 - \binom{V}{x} p^x (1-p)^{V-x} = 1 - \binom{V}{0} p^0 (1-p)^{V-0} = 1 - \frac{V!}{0!(V-0)!} p^0 (1-p)^{V-0} = 1 - (1-p)^V \tag{4.500b}$$

4.10.6.5 Negative binomial distribution, $BNn(n, r \geq 1; p)$

The negative binomial distribution has the following characteristics:

1. The experiment consists of an x number of trials.
2. The experiment continues until the occurrence of $r \geq 1$ failures where r is a previously given value.
3. The random variable X is the number of trials performed until the occurrence of r failures.

4.10.6.5.1 Distribution and density functions

The density function of the negative binomial probability is the following:

$$\begin{aligned} p_X(x) &= \Pr[X = x; r; p] = \binom{x-1}{r-1} p^r (1-p)^{x-r} \\ x &= r, r+1, \ldots; \quad 0 \leq p \leq 1 \\ p_X(x) &= 0, \ \text{another case} \end{aligned} \tag{4.501}$$

The distribution function is

$$F_X(x) = \sum_{k=r}^{x} \binom{k-1}{r-1} p^r (1-p)^{k-r} \tag{4.502}$$

4.10.6.5.2 Statistical descriptor

- Mean value of an x number of trials to obtain r failures,

$$E[X] = \frac{r}{p} \tag{4.503}$$

- Variance of an x number of trials to obtain r failures,

$$Var[X] = \frac{r(1-p)}{p^2} \tag{4.504}$$

This probability model is a generalization of the geometric model in which the random variable is the number of trials necessary for a failure to occur $(r = 1)$. The term, *negative binomial function,* is frequently used to designate the probability model whose random variable is the number of failures Y until r non-failures occur. According to the previously described terminology, this variable is $Y = X - r$, and when $1 - p$ is the probability of failure. Consequently,

$$\begin{aligned} p_Y(y) &= \Pr[Y = y; r; p] = \binom{r+y-1}{y} p^r (1-p)^y = \\ &= (-1)^y \binom{-r}{y} p^r (1-p)^y, \quad y = 0,1,2,\ldots \\ p_Y(y) &= 0, \ \text{another case} \end{aligned} \tag{4.505}$$

If $r = 1$, then the number of failures is one. The negative binomial distribution is the equal to the geometric distribution.

4.10.6.6 Logarithmic series distribution, $SL(r = 1;p)$

The geometric distribution has the property of conserving the probability associated with x trials until the occurrence of the first failure although in the first k trials of the experiment, there were k failures. In order to provide the probability model with memory, such that it can consider the results obtained in all the trials, a logarithmic series distribution can be used. This distribution has the following characteristics:

1. The experiment consists of x trials.
2. The experiment continues until the occurrence of the first failure, $r = 1$, which is a given value.
3. The random variable X is the number of trials performed until the occurrence of the first failure.

4.10.6.6.1 DISTRIBUTION AND DENSITY FUNCTIONS

The logarithmic series probability density function is

$$p_X(x) = \Pr[X = x; r = 1; p] = -\frac{(1-p)^x}{x\ln(p)}$$
$$x = 1,2,\ldots, 0 \le p \le 1 \tag{4.506}$$
$$p_X(x) = 0, \text{ another case}$$

The distribution function is

$$F_X(x) = \sum_{k=1}^{x}\left(-\frac{(1-p)^k}{k\ln(p)}\right) \tag{4.507}$$

4.10.6.6.2 STATISTICAL DESCRIPTORS

- Mean value of the x number of trials that gives the first failure,

$$E[X] = \frac{1}{\ln(p)}\frac{1-p}{p} \tag{4.508}$$

- Variance of the x number of trials that gives the first failure

$$Var[X] = \frac{E[X]}{p} - \{E[X]\}^2 \tag{4.509}$$

Note. In the geometric distribution, the probability of occurrence of the first failure after $x - 1$ trials without a failure and when performing trial x, proportionally decreases to $1 - p$, which is a constant and independent value of x. On the contrary, the logarithmic series distribution decreases, $(1 - p)^{x-1/x}$, which depends on x. When x is small, in other words, for the first trials, the correction factor is also small.

4.10.6.7 Multinomial distribution, $MN(n;x_i;p_i;i = 1, \ldots, m)$

The binomial distribution can be expanded and applied to a more general case in which the sample space of the experiment is divided into m mutually exclusive events with probabilities of occurrence, $p_1, p_2, \ldots, p_m$, and the number of failures by each of the m partitions, $x_1, x_2, \ldots, x_m$. The characteristics of this distribution are:

1. The experiment consists of n trials, where n is a given value.
2. The results are grouped in m partitions, where m is a given value.
3. The failure probability in each partition is $p_1, p_2, \ldots, p_m$, such that $p_1 + p_2 + \ldots + p_m = 1$.
4. The random variable, X_i, $i = 1, \ldots, m$, is the number of failures in each partition, and the assigned values satisfy the condition, $x_1 + x_2 + \ldots + x_m = n$.
5. Each partition has a binomial marginal distribution function, $B(n,p_i)$.

4.10.6.7.1 DISTRIBUTION AND DENSITY FUNCTIONS

The multinomial probability density is

$$p_{Xi=1,m}\left(x_{i=1,m}\right) = \Pr\left[X_1 = x_i;\ldots X_m = x_m; n; p_1,\ldots p_m\right] = \frac{n!}{x_1!x_2!\ldots x_m!}p_1^{x_1}p_2^{x_2}\ldots p_m^{x_m}, \quad 1 = p_1 + p_2 + \ldots + p_m \tag{4.510}$$

4.10.6.7.2 STATISTICAL DESCRIPTORS

- Mean value of the number of failures in each partition in n trials,

$$E\left[X_i\right] = np_i \tag{4.511}$$

- Variance of the number of failures in each partition in n trials,

$$Var\left[X_i\right] = np_i\left(1 - p_i\right) \tag{4.512}$$

Example. This distribution can be applied to estimate the failure probability of the entire breakwater by considering that its principal failure modes form a complete set of mutually exclusive elements. Let us suppose that the breakwater has five failure modes, $n = 5$, whose failure probabilities in V years are $p_1, p_2, \ldots, p_5$. If this process is normalized, this gives the following:

$$p_{f,ELU} = p_1 + p_2 + \ldots + p_5 \tag{4.513}$$

$$p_m^* = \frac{p_m}{p_{f,ELU}}, m = 0,1,2\ldots,5 \tag{4.514}$$

where the experiment consists of $n = 5$ responses. For each failure mode there is a random variable X_i, which defines the number of responses, corresponding to each failure mode. Each mode follows a binomial distribution, based on a Bernoulli trial, $BN(n = 5; p_m^*)$, with a mean value and variance,

$$E\left[X_m\right] = np_m^* \tag{4.515}$$

$$Var\left[X_m\right] = np_m^*\left(1 - p_m^*\right) \tag{4.516}$$

The probability that a mode only fails five times is

$$p\left[X_1\right] = 5, X_2 = 0,\ldots, X_5 = 0, p_1^*,\ldots, p_5^* = \frac{5!}{5!0!\ldots,0!}\left(p_1^*\right)^5 = \left(p_1^*\right)^5 \tag{4.517}$$

and the probability that the section will fail once with each mode is

$$p\left[X_1 = 1, X_2 = 1,\ldots, X_5 = 1, p_1^*,\ldots, p_5^*\right] = \frac{5!}{1!1!\ldots,1!}\left(p_1^*\right)\left(p_2^*\right)\left(p_3^*\right)\left(p_4^*\right)\left(p_5^*\right) \tag{4.518}$$

If probabilities are equally distributed in the five modes, then both probabilities are equal.
This same scheme can be applied to calculate the probability that a breakwater with five independent subsets will fail in the same subset or once in each subset.
If all subsets have been calculated with the same probability because each subset has the same nature, then all probabilities will be equal.

4.10.6.8 Poisson distribution, $P(v)$

When n, the number of trials in the experiment is very large and the probability of failure is very small, but all the Bernoulli process criteria are maintained, it is possible to define the parameter v or Poisson parameter, $v = np$, which represents the mean number of failures that occurred in the experiment of n trials. And the binomial function can be approximated by the Poisson probability function. The characteristics of this distribution are the following:

1. The experiment is formed by n trials, and n should be a large number.
2. The random variable X is the number of failures in the n trials, $X \geq 0$.
3. The failure probability is p, which should be a small number.
4. The Poisson parameter is, $\nu = np$.

4.10.6.8.1 Distribution and density functions

The Poisson probability density function is

$$\begin{aligned} p_X(x) &= \Pr[X = x; \nu] = \frac{\nu^x e^{-\nu}}{x!} \\ x &= 0,1,\ldots,n, \quad \nu = np, \quad 0 \leq p \leq 1 \\ p_X(x) &= 0, \text{ another case} \end{aligned} \tag{4.519}$$

The distribution function is

$$F_X(k) = \sum_{k=0}^{x} \frac{\nu^k e^{-\nu}}{k!}, \quad x = 0,1,\ldots,n \tag{4.520}$$

4.10.6.8.2 Statistical descriptor

- Mean number of failures: $E[X] = \nu$
- Variance of the number of failures: $Var[X] = \nu$

4.10.6.9 Truncated Poisson distribution, $P(\nu; X \geq 1)$

When $x = 0$, in other words, no failure event in the n trials, and the other conditions required for the Poisson distribution are conserved, it is possible to define the truncated Poisson distribution by eliminating the result, $x = 0$, from the event space, and by calculating the probability conditioned to the event, $X \geq 1$.

The characteristics of this distribution are the following:

1. The experiment consists of n trials, and n should be a large number.
2. The random variable, X, is the number of failures in the n trials, and it is truncated, $X \geq 1$.
3. The failure probability is p, which should be a small number.
4. The Poisson parameter is $\nu = np$.

4.10.6.9.1 Distribution and density functions

The truncated probability density function is

$$\begin{aligned} p_X(x) &= \Pr[X = x | X \geq 1; \nu] = \frac{\nu^x}{x!(e^{\nu} - 1)} \\ x &= 1,2,\ldots,n, \quad \nu = np, \quad 0 \leq p \leq 1 \\ p_X(x) &= 0, \text{ another case} \end{aligned} \tag{4.521}$$

The distribution function is

$$F_X(k) = \sum_{k=1}^{x} \frac{\nu^x}{k!(e^{\nu} - 1)}, \quad x = 1,2,\ldots,n \tag{4.522}$$

4.10.6.9.2 STATISTICAL DESCRIPTORS

- Mean value of the x number of trials that gives r failures,

$$E[X] = \frac{\nu e^{\nu}}{(e^{\nu} - 1)} \tag{4.523}$$

- Variance of the x number of trials that gives r failures,

$$Var[X] = \frac{\nu e^{2\nu} - \nu(\nu + 1)e^{\nu}}{(e^{\nu} - 1)^2} \tag{4.524}$$

4.10.7 Poisson processes

The sequence of trials that fulfill the requirements of the Poisson distribution, regardless of whether it is truncated, is said to be a Poisson process. The process is applicable to the time domain as well as to the space domain. If the number of trials or experiments is equal to the number of years t_p, it is then possible to define a subinterval of time, $\Delta t = t_p/n$, which fulfills the following conditions,

1. The probability that more than one event will occur in the time subinterval is zero.
2. Each time interval behaves like a Bernoulli process.
3. The failure occurrence in any of the subintervals is independent of the occurrences in other subintervals.
1. The probability that the failure event will occur in a subinterval is constant.

According to these conditions, the number or rate of failure occurrences per time unit is defined by $\lambda = \nu/t_p$. Based on the definition of the subinterval Δt and of the Poisson parameter, $\nu = np$, it is possible to obtain $\lambda\Delta t = p$, in other words, that the probability of the failure event in a subinterval is p. A Poisson process is a randomly constructed experiment, based on the preceding conditions.

If the Poisson process is applied to a spatial domain s_p, it is also possible to similarly define the number or rate of failures per unit of space (length, width, etc.), $\lambda = \nu/s_p$, where the Poisson parameter, $\nu = np$, represents the mean number of failures in the space s_p. Dividing the space into subspaces, $\Delta s = s_p/n$, the probability of a failure occurring in a subinterval is $\lambda\Delta s = p$, when the rest of the conditions described for the time domain are fulfilled.

Note. In reference to space, the application domain is composed of n mutually exclusive subsets, each of which is a Bernoulli process with probability p that a failure will occur. In order to apply the Poisson approximation, it is necessary for p to be small and n to be large.

4.10.7.1 Non-stationary, non-homogeneous Poisson processes

In the case of time as well as space, it is possible to define non-stationary or non-homogeneous Poisson processes in which the rate of occurrence depends on the space or time, namely, $\lambda(s)$ or $\lambda(t)$, and the results of the process, number of failures in the n trials, depend on the space, $X(s)$ or on the time, $X(t)$.

The random variable of the failure increase in the space or time interval $X(t_1) - X(t_2)$, gives the number of failures in the interval, (t_1,t_2). This variable follows a Poisson process of parameter ν, mean number of failures in a spatial interval (s_1,s_2) or time interval (t_1,t_2), defined by

$$\nu = \int_{s_1}^{s_2} \lambda(s)\,ds \quad \text{and} \quad \nu = \int_{t_1}^{t_2} \lambda(t)\,dt \tag{4.525}$$

The failure increase is a sequence of independent random variables, such that the joint probability of two of them is equal to the product of their probabilities.

Note. *A storm (or loading cycle) is defined by a sequence of sea states, whose significant wave height continuously exceeds a certain threshold value, which for the Cantabrian Sea can be $H_s \geq 3(m)$. The number of storms that can occur in the Port of Gijon throughout the year is a non-stationary Poisson process, since during the winter and spring, there are usually a greater number of storms than in the summer or fall. The year can thus be divided into four time intervals. The first interval includes the months of September, October, and November, and lasts three months. The second interval is from December to March, and lasts four months. The third interval from April to June is three months long, and finally, the fourth and last interval includes the months of July and August, and is two months long. For each interval, it is possible to define a rate of storm event occurrence, $\lambda_i(t)$, $i = 1,2,3$ and 4, which can be constant or vary with t. The Poisson parameter for any of the intervals is obtained by integration in the duration of the interval, $v_i = \int_i \lambda_i(t)dt$. The Poisson parameter can also be obtained for more than one time interval, $v_{1,2} = \int_{i=1,2} \lambda_i(t)dt$, or for the entire year, $v = \int_{year} \lambda_i(t)dt$.*

4.10.8 Derived functions and associated variables

On many occasions during the general calculation procedure, it is necessary to obtain the probability of a variable that is functionally related to other variables, e.g. the amplitude of the meteorological tide at a point on the coast that functionally depends on the atmospheric pressure gradient and the wind speed. The terms of a verification equation can be obtained by functional relations between project factors. Thus, for example, the equation to verify the weight *W* of the pieces of a sloping breakwater is

$$S = W - \gamma_w R \Psi H^3 = X_1 - X_2 \tag{4.526}$$

where S is the safety margin, $R = \gamma_w/(\gamma_w/\gamma_s - 1)^3$, γ_w and γ_s are the specific weights of the water and material; Ψ is the stability function; and *h* is the wave height In certain very general conditions, the height of the waves in a sea state is a random variable with a Rayleigh-function probability model. From this perspective of the verification, it is interesting to know the probability model of S. Thus, one also needs to know the probability model of the two terms, $W = X_1$, and, $\gamma_w R \Psi H^3 = X_2$. It seems logical that if the probability models of the intervening project factors are known, it will be possible to obtain the probability model of the terms X_1 and X_2, and use them to obtain the probability model of $S = X_1 - X_2$. The models obtained in that way are known as derived probability models and random variables, as associated variables. The following section describes how to obtain these models, beginning with the case of functions that have only one variable.

4.10.8.1 Functions of a variable

Supposing that one knows the density function $f_X(x)$ of a random variable, X, which has a functional relation with another variable Y, such that $Y = \varphi(X)$, where *h* should be an increasing or decreasing monotonic function (e. g. $Y = exp(aX)$, for a value of X there is only one value of Y), which has the inverse function,

$$X = 1/a.\ln(Y) = \psi(Y), a \neq 0 \tag{4.527}$$

Given that X is a random variable, Y is also a random variable. The density function $f_Y(y)$ of the random variable Y is obtained by

$$f_Y(y) = |J| f_X[x = \psi(y)] \tag{4.528}$$

where $|J| = d\psi(y)/dy$ is the Jacobian of the transformation of X in Y, and in the case of only one variable, it is equal to the derivative of the inverse function ψ.

Example. *In the previously mentioned example, $|J| = d\psi(y)/dy = (1/a)(1/y)$, which means the density function of y is*

$$f_Y(y) = \frac{1}{a}\frac{1}{y} f_X\left[\frac{1}{a}\ln(y)\right], \quad y > 0 \tag{4.529}$$

This example anticipates the main difficulty in obtaining the derived probability function, which is the domain of the inverse function, its derivative, and its evaluation. On many occasions, it is impossible to analytically evaluate the functions, and when possible, it is necessary to resort to numerical methods.

4.10.8.2 Functions of two or more variables

Supposing that the multidimensional random variable, $X_1, X_2, \ldots, X_r$, has the density function,

$$f_{X_1,X_2,\ldots X_r}\left(x_1,x_2,\ldots,x_r\right) \tag{4.530}$$

and that $Y_1, Y_2, \ldots, Y_s$ is another variable, defined by the biunivocal transformation,

$$Y_j = \varphi_j\left(X_1,X_2,\ldots,X_r\right), j = 1,\ldots s \tag{4.531}$$

which has an inverse function, and is continuous with continuous first derivatives,

$$X_i = \psi_i\left(Y_1,Y_2,\ldots,Y_s\right), i = 1,\ldots r, \exists \frac{\partial x_i}{\partial y_j}, \forall i,j \tag{4.532}$$

The density function of the multidimensional variable Y is

$$f_{Y_1,Y_2,\ldots,Y_s}\left(y_1,y_2,\ldots,y_s\right) = \left|J\right| f_{X_1,X_2,\ldots,S_r}\left(x_1,x_2,\ldots,x_r\right) \tag{4.533}$$

where $\left|J\right|$ is the Jacobian of the transformation, which should be non-zero, given by

$$\left|J\right| = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \cdots & \frac{\partial x_1}{\partial y_s} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} & \cdots & \frac{\partial x_2}{\partial y_s} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x_r}{\partial y_1} & \frac{\partial x_r}{\partial y_2} & \cdots & \frac{\partial x_r}{\partial y_s} \end{vmatrix} \tag{4.534}$$

4.10.8.3 Expectation and moments of the associated variables

If $Y = \varphi(X_1, X_2, \ldots, X_r)$ is a variable associated with the multidimensional variable, $X_1, X_2, \ldots, X_r$, the n-order value of the variable Y is

$$E\left[Y^n\right] = E\left[\left\{\varphi\left(X_1,X_2,\ldots,X_r\right)\right\}^n\right] = \tag{4.535}$$

$$\int_{a_1}^{b_1}\cdots\int_{a_r}^{b_r}\left\{\varphi\left(X_1,X_2,\ldots,X_r\right)\right\}^n f_{X_1,X_2,\ldots,X_r}\left(x_1,x_2,\ldots,x_r\right)dx_1\ldots dx_r \tag{4.536}$$

where $a_1, a_2, \ldots, a_r, b_r$, define the domains of the multidimensional variable, and the expected or mean value is obtained by making $n = 1$. The variance of Y can be obtained from the definition,

$$Var\left[Y\right] = E\left[\left\{\varphi\left(X_1,X_2,\ldots,X_r\right)\right\}^2\right] - \left\{E\left[\varphi\left(X_1,X_2,\ldots,X_r\right)\right]\right\}^2 \tag{4.537}$$

4.10.8.4 Case studies

In many cases, the associated variable only depends on a bidimensional random variable, X_1, X_2, and the relation between them are simple algebraic operations: sum, $Y = X_1 + X_2$; subtraction, $Y = X_1 - X_2$; product, $Y = X_1X_2$; and quotient, $Y = X_1/X_2$. The first two cases can be treated together:

Case (1): $Y = X_1 \pm X_2$. This case can be applied to the verification equation in the form of the safety margin. The distribution function of Y is

$$F_Y(y) = \Pr[Y \le y] = \int_{a_1}^{b_1} dx_1 \int_{a_2}^{y-x_1} f_{X_1,X_2}(x_1,x_2)\,dx_2 \tag{4.538}$$

and the density function of *y* is obtained by the derivation of the distribution function,

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \int_{a_1}^{b_1} f_{X_1,Y}(X_1, y \mp x_1)\,dx_1 \tag{4.539}$$

and if X_1 and X_2 are statistically independent,

$$f_Y(y) = \int_{a_1}^{b_1} f_{X_1}(x_1)\, f_{X_2}(y \mp x_1)\,dx_1 \tag{4.540}$$

where two random variables, X_1 and X_2, can be interchanged. The expected value and the variance of Y are

$$E[Y] = E[X_1] \pm E[X_2] \tag{4.541}$$

$$Var[Y] = Var[X_1] + Var[X_2] \pm 2\rho_{X_1X_2} Cov[X_1, X_2] \tag{4.542}$$

Case (2): $Y = X_1X_2$. As in the previous example, the first step is to obtain the distribution function of the variable Y, by integration in the domain defined by $x_1x_2 \le y$ and by defining the auxiliary variable, $u = x_1x_2$.

$$F_Y(y) = \Pr[Y \le y] = \int\int_{x_1x_2 \le y} f_{X_1,X_2}(x_1,x_2)\,dx_1dx_2 = \int_{a_1}^{y} \left\{ \int_{a_2}^{b_2} \frac{1}{|x_1|} f_{X_1,X_2}\left(x_1, \frac{u}{x_1}\right) dx_1 \right\} du \tag{4.543}$$

and deriving the distribution function with respect to the variable y,

$$f_Y(y) = \int_{a_1}^{b_1} \frac{1}{|x_1|} f_{X_1,X_2}\left(x_1, \frac{y}{x_1}\right) dx_1 = \int_{a_2}^{b_2} \frac{1}{|x_2|} f_{X_1,X_2}\left(x_1, \frac{y}{x_2}\right) dx_2 \tag{4.544}$$

If X_1 and X_2 are statistically independent, then,

$$f_Y(y) = \int_{a_1}^{y} \left\{ \int_{a_2}^{b_2} \frac{1}{|x_1|} f_{X_1}(x_1)\, f_{X_2}\left(\frac{u}{x_1}\right) dx_1 \right\} du \tag{4.545}$$

The mean and the variance are

$$E[Y] = E[X_1]E[X_2] + Cov[X_1, X_2] \tag{4.546}$$

$$Var[Y] = E\left[(X_1X_2)^2\right] - \{E[X_1X_2]\}^2 \tag{4.547}$$

where,

$$X_1X_2 = E[X_1]E[X_2] + E[X_2](X_1 - E[X_1]) + E[X_1](X_2 - E[X_2]) + (X_1 - E[X_1])(X_2 - E[X_2]) \tag{4.548}$$

If X_1 and X_2 are independent, the previous expressions are simplified,

$$E[Y] = E[X_1]E[X_2] \tag{4.549}$$

$$Var[Y] = Var[X_1]Var[X_2] + Var[X_1]\{E[X_2]\}^2 + Var[X_2]\{E[X_1]\}^2 \tag{4.550}$$

Case 3: $Y = X_1/X_2$. This is the case of the verification equations that have the form of a safety coefficient. The work method is similar to that used in the two previous examples.

4.10.9 Statistical analysis of extreme values

Generally speaking, the term, *extreme value,* is assigned to the largest value that a random variable can take in a given number of observations. The largest value is itself a random variable that depends on the size of the sample.

4.10.9.1 Limit distribution function of extreme values

According to extreme value theory, the largest value of a set of identically distributed independent random values is fit to an asymptotic distribution that only depends on the upper tail of the distribution function of the initial variable. Let $X_1, X_2, \ldots, X_n$ be a set of independent random variables (initial variable) with the same distribution function, where x is an observed value and n is the number of equally spaced data in a given time interval. This set can be ordered from the smallest value to the largest, $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$, where $X_{(1)}$ is the smallest and $X_{(n)}$ is the largest. The distribution of $X_{max} = X_{(n)}$ is

$$G_{X_{máx}}(x) = \left[F_X(x)\right]^n \tag{4.551}$$

When n increases indefinitely, $G_{X_{max}}(x)$ approximates zero for all (finite) values of x, belonging to the domain of X. Consequently, to obtain the limit distribution, it is necessary to change coordinates and define a new random variable Y_n (standardized variable), such that,

$$Y_n = \frac{X_{(n)} - b_n}{a_n} \tag{4.552}$$

where $a_n > 0$ is a scale parameter and *bn* is a location parameter. The limit distribution function of the new random variable can only be one of the following three types (see Table 4.4.15)

Table 4.4.15. Limit distribution functions of the reduced random variable

Type	$F_Y(y) = Pr[Y \leq y]$	$-\infty < y < \infty$
I. EV1 or Gumbel Distribution	$exp(-e^{-y})$	$-\infty < y < \infty$
II. EV2 or Frechet Distribution	$exp(-y^{-\gamma})$ 0	$y > 0$ $y \leq 0$
III. EV3 or Weibull Distribution	$exp[-(-y^{\gamma})]$ 0	$y < 0$ $y \geq 0$

where $\gamma > 0$ is a constant, and Y is the asymptotic value of Y_n, in other words, the value taken when $n \Rightarrow \infty$. The condition sufficient for convergence is that $F_X(x)$ be strictly monotonic and continuous in the application domain.

This result is based on the stability postulate which establishes that if X has an extreme distribution function, the maximum value of n independent observations of X has the same distribution, but different location and scale parameters. Consequently, the solution of the equation

$$G_{X_{max}}(x) = \left[F_X(x)\right]^n = F_X\left(\frac{x - b_n}{a_n}\right) \tag{4.553}$$

provides all possible shapes of the upper tail of $F_X(x)$ when $n \Rightarrow \infty$, where a_n and b_n are functions of n.

Distributions with the $EV1$ (type I asymptote of maxima) as their asymptotic distribution function include the exponential, gamma, Rayleigh, Weibull of minima, normal, lognormal and the $EV1$ distributions. The Pareto, Student, Cauchy, log-gamma, and $EV2$ distributions have the $EV2$ as their asymptotic distribution function. Finally, the uniform, beta, and $EV3$ distributions, among others, have the $EV3$ as their asymptotic distribution function.

4.10.9.2 Generalized distribution function of the extreme value in a series of n values

Let there be a time series of independent, random values that have the same probability model in a time interval, the distribution function of the maximum value of the data series of size n, when $n \Rightarrow \infty$ asymptotically tends to the generalized extreme value function, $GEV(\sigma_E, \delta_E, \mu_E)$. If X is the random variable (maximum value when $n \Rightarrow \infty$), then its distribution function is

$$G_{X_{max}}(x) = \Pr[X \le x] = \exp\left\{-\left[1+\delta_E \frac{(x-\mu_E)}{\sigma_E}\right]^{-\frac{1}{\delta_E}}\right\} \quad (4.554)$$

where the three distribution parameters are:

- μ_E location parameter
- σ_E $(\sigma_E > 0)$ scale parameter
- δ_E shape parameter

The Weibull $(\delta_E < 0)$ and Gumbel $(\delta_E > 0)$ distributions are limit distributions, and appear as particular cases of the GEV. The mean and variance of the GEV function are, respectively,

$$\mu_{X_{max}} = E\left[X_{max}\right] = \mu_E + \frac{\sigma_E}{\delta_E}\left[1-\Gamma\left(1+\delta_E\right)\right], \quad \delta_E > -1 \quad (4.555)$$

$$\sigma^2_{X_{max}} = Var\left[X_{max}\right] = \left(\frac{\sigma_E}{\delta_E}\right)^2 \left\{\Gamma\left(1+2\delta_E\right) - \Gamma^2\left(1+\delta_E\right)\right\}$$
$$\delta_E > -0.5 \quad (4.556)$$

where Γ represents the gamma function. The skewness coefficient is calculated by

$$\gamma_{1,X_{max}} = sign\left(\delta_E\right)\frac{-\Gamma\left(1+3\delta_E\right)+3\Gamma\left(1+\delta_E\right)\Gamma\left(1+2\delta_E\right)-2\Gamma^3\left(1+\delta_E\right)}{\left\{\Gamma\left(1+2\delta_E\right)-\Gamma^2\left(1+\delta_E\right)\right\}^{\frac{3}{2}}}, \quad \delta_E > -\frac{1}{3} \quad (4.557)$$

The exceedance of the value, x_P, whose probability is P, namely,

$$\Pr\left[X > x_p\right] = P \quad (4.558)$$

$$x_P = \mu + \sigma_E \left\{\frac{\left[\ln(1-P)^{\delta_E} + 1\right]}{\delta_E}\right\} \quad (4.559)$$

Generally speaking, the return period, $T_R = 1/P$, can be defined as the number of observation time units that, on average, should occur before the occurrence of the first exceedance, $X = x_P$.

4.10.9.3 Fit of the GEV distribution

If the first three statistical moments of X_{max} exist and are known, it is possible to determine the values of the three distribution parameters $(\sigma_E, \delta_E, \mu_E)$, based on the mean, variance and skewness of the data. Since the skewness only depends on δ_E, this parameter should be first obtained, depending on the skewness of the sample. This can be done by using the expression given in the previous section, and by replacing $\gamma_{1,X_{max}}$ with the sample value. After that, the scale parameter can be obtained,

$$\sigma_E = \frac{\delta_E \sigma_m}{\sqrt{\Gamma\left(1+2\delta_E\right)-\Gamma^2\left(1+\delta_E\right)}} \quad (4.560)$$

where σ_m^2 is the variance of the sample. Finally, the location parameter is obtained by

$$\mu_E = \mu_m - \frac{\sigma_E}{\delta_E}\left[1 - \Gamma\left(1 + \delta_E\right)\right] \tag{4.561}$$

where μ_m is the mean of the sample.

4.10.10 Statistical inference, estimator, and estimate

This section describes the process by which it is possible to obtain the probable behavior of a population from a sample. The probabilistic model obtained is a distribution function or probabilistic model, $F(u;\vartheta)$, where F is a family selected from distribution functions, and ϑ is an estimated value of the parameter θ of the function, F. Given a sample of values, it is possible to use different techniques or methods to estimate the parameter or parameters of the distribution function.

4.10.10.1 Sample distribution function

An experiment provides a sample of size n from which a value ϑ can be inferred from the parameter θ of the distribution function, F. If the experiment is repeated m times, m samples of size n. From each one, a different value of ϑ is inferred. The distribution function of the sample models and quantifies this variability. The sample variable is generally represented by T, and the sample values by $t_{i=1,\ldots,m}$.

4.10.10.2 Properties of the estimators

Regarding estimator properties, it should be underlined that any function of observed or sample values that is quantifiable and which does not contain any unknown parameter is known as a statistic. A statistic is a random variable that allows us to make estimates. An estimator is the method used or way in which the estimates are made. For example, the mean of the sample is a point estimator of the mean population. An estimate is the value that the estimator provides when it is applied to a specific case, ϑ.

It is advisable to work with estimators (methods for making estimates), which when iteratively applied, provide a statistical sample of values that follow a probability model, whose mean value coincides with the desired parameter value, The estimators that fulfill the condition, $E(\vartheta) = \theta$ do not have skewness.

An estimator is said to be consistent when it produces statistics that converge to the parameter value in terms of probability, namely, $lim n\to\infty \Pr[\,|\,\vartheta_n - \theta\,|\le \varepsilon] = 1$, where ε is any positive number, and ϑ_n is based on an n sample size.

4.10.10.3 Likelihood of the sample

Given a sample of values u_i of the project factor U, the likelihood of the sample is $L(\theta) = \Pi_{i=1}^{n} f(u_i;\theta)$. The quantity of available information in a sample to infer the value of the distribution function parameter is defined by the following expected value, $I_\theta = -E[\partial^2 ln\{L(\theta)\}/\partial\theta^2]$, which linearly varies with size n of the sample. The larger the sample, the more information there is available to estimate θ.

Example. The exponential density function is,

$$\begin{aligned} f(u;\lambda) &= \lambda \exp(-\lambda x), \quad x \ge 0; \lambda > 0 \\ f(u;\lambda) &= 0, \quad another\ case \end{aligned} \tag{4.562}$$

where $\theta = \lambda$ is the distribution parameter. The mean μ and the variance σ^2 of the exponential model are $1/\lambda$ and $1/\lambda^2$, respectively. The likelihood function is

$$L(\lambda) = \prod_{i=1}^{n} f(u_i;\lambda) = \lambda^n \exp\left\{-\lambda \sum_{i=1}^{n} u_i\right\} \qquad (4.563)$$

and the second derivative of the logarithm of $L(\lambda)$ is

$$\frac{\partial^2 \ln\{L(\lambda)\}}{\partial \lambda^2} = \frac{n}{\lambda^2} \qquad (4.564)$$

Accordingly, the expected information is $I_\lambda = -n/\lambda^2$, which is a measure of information available in a sample of size n to estimate the value of parameter λ.

4.10.10.4 Sample distribution of minimum variance

In the same way as for any other random variable, the expected value or mean of t, $E[T]$ and the variance of T, $Var[T]$ are defined. Between this and the expected value of the information I_θ, there is the following relation: $Var[T] \geq 1/I_\theta$.

The sample functions for which the previous relation is an equation are known as minimum variance functions. When the previous inequality is converted into an equality, the estimator is said to be a bounded minimum variance estimator.

4.10.10.5 Standard error of the sample function

The standard error of the sample function is defined by $s_e(\vartheta) = Var[T_\vartheta]$. The estimated value of any parameter should be accompanied by the standard error $\vartheta \pm s_e(\vartheta)$. When the project factor depends on various parameters (T_i, T_j, etc), which generically represent this dependence by a function $g(\theta)$, the estimate of g should be bounded with the standard error. In a similar way, this is defined by $s_e(g) = \sqrt{Var[g]}$

$$Var(g) = \sum_i \sum_j \left(\frac{\partial g}{\partial T_i}\right)\left(\frac{\partial g}{\partial T_j}\right) Cov(T_i, T_j) \qquad (4.565)$$

where Cov represents the elements of the covariance matrix of the estimators.

Example. The Gumbel density function is generally used as the probability model of maximum sea states or storm regimes. This model has two parameters, a and b,

$$f(u;a,b) = \frac{1}{b}\exp\left[-\frac{u-a}{b}\right]\exp\left[-\exp\left\{-\frac{u-a}{b}\right\}\right]; \; b > 0; -\infty < u \text{ and } a < \infty \qquad (4.566)$$

$$F(u;a,b) = \exp\left[-\exp\left\{-\frac{u-a}{b}\right\}\right] \qquad (4.567)$$

Let us suppose that the calculation value selected is the value associated with the 95% quantile $u_{0.95}$, a value that can be expressed in terms of the two distribution parameters,

$$u_{0.95} = a - b\left[\ln\left\{\ln\left(\frac{1}{0.95}\right)\right\}\right] = g(a,b) \qquad (4.568)$$

When the function g is derived with respect to a and b, this gives $\partial g/\partial a = 1$, and $\partial g/\partial b = -\ln\{\ln(1/0.95)\} = -2.97$. And when the expression of the variance of g is applied, and its square root calculated, the standard error of the 95% of the quantile value is obtained,

$$se(u_{0.95}) = \sqrt{Var(T_a) + 8.82 Var(T_b) + 5.94 Cov(T_a, T_b)} \qquad (4.569)$$

where T_a and T_b are the estimators of the a and b parameters, respectively.

4.10.10.6 Standard error of a measured function

When the project factor depends on an n-size sample of k measured variables, $U_{s=1,\ldots,k}$, the approximate standard error of the project factor can be obtained, based on the covariance matrix of the value adopted.

4.10.10.7 Optimal estimators

Optimal estimators are those that are unbiased [(24)], and whose variance is minimum.

4.10.11 Probability fitting methods

Probability fitting models are used when the information in a sample is analyzed to obtain conclusions about the population, and more specifically, to obtain the value of parameter θ. The methods described in this section are known as *point methods* so as to differentiate them from those methods that locate the parameter value within a certain interval. For all purposes, these methods assume that the distribution function of the population is known, and that the parameter values can be obtained from the sample, which is also assumed to be randomly taken. The most frequently applied methods are the moments method and the minimum square method.

4.10.11.1 Maximum likelihood method

The maximum likelihood method is a universal method for determining the estimator ϑ of an unknown parameter θ, such that the likelihood function u_i of a given sample is maximized, in other words, $|\ \partial L(\theta)/\partial\theta|_{\vartheta} = 0$ [(25)]. The maximum likelihood estimators have three important properties for the decision: (i) they are not biased; (ii) they are of the bounded minimum variance type; (iii) the sample functions are Gaussian.

4.10.12 Methods for estimating confidence intervals

In some cases instead of obtaining only one estimate ϑ of the parameter θ can be necessary to define a range of values $(\vartheta_i, \vartheta_s)$, which contains the value of θ with a guarantee or previously established statistical confidence.

Before carrying out an experiment or a measurement, the estimator T of the parameter θ is a sample random variable with the density function, $f(t)$. An interval, (t_1, t_2), such that $\Pr[t_1 \le T \le t_1] = 1 - \alpha$. If the condition is imposed that the probability assigned to each tail of the density function is equal to $\alpha/2$, then there is only one interval, (t_1, t_2). The extremes of the interval, t_1 and t_2, are the quantiles, $\alpha/2$ and , of the sample density function $f(t)$, respectively. The random event $[t_1 \le T \le t_1]$ can be transformed in the random interval,

$$\left[l_1(T) \le \theta \le l_2(T)\right] = 1 - \alpha, \qquad (4.570)$$

which includes the parameter, θ, with the same probability. Once there is a sample, the interval $[l_1(T),\ l_2(T)]$ is known: $[l_1(t),\ l_2(t)]$. This interval is the confidence interval, and the value $(1-\alpha)$ is the statistical confidence of the interval. Since each sample provides different interval values, the percentage of intervals containing the unknown value of θ and approximates $(1-\alpha)$, when there is a sufficiently large number of samples. Thus, $(1-\alpha)$ is a measurement of the statistical confidence that a certain interval will contain the parameter θ. In order to determine the confidence interval, one must know the sample density function $f(t)$. When this is not known, it is necessary to apply approximate methods.

---

(24) When an estimator calculates from various samples, and on average, gives the value of the desire parameter, it is said to be an unbiaased estimator. A measurement system is also said to be unbiased, when after various measurements of the same object, it gives, on average, the measurements of the object.

(25) There are functions for wwhich the estimator $\vartheta(U)$ has an explicit expression. However, for other functions, the value of ϑ is obtained as the toots of an equation.

4.10.13 Significance test

Many times it is not necessary to find an estimator or a confidence interval of the parameter θ, but rather it is sufficient to determine if a given value of the parameter characterizes the observed or analyzed project factor. From the sample information, it can be inferred whether the parameter value is reasonable or not, and if the data significantly deviates from the distribution tails. This work method is known as a test.

There are two types of test: (i) hypothesis test; (ii) graphic test. Besides those tests described here, it is also possible to apply the Chi-Square test and the Kolmogorov-Smirnov test.

4.10.13.1 Hypothesis test

When there is a family of distribution functions F and their parameters have been estimated, based on a sample U, it is advisable to verify the fit of the estimated probability data, $F(u;\vartheta)$, to the data. This means carrying out the null hypothesis significance test, $\{H_0 : F_0(u;\vartheta)\}$ on the basis of the same sample used to make the estimate. There are various methods that can be used to measure the discrepancy between the estimated model and the distribution of the sample.

4.10.13.2 Anderson-Darling Test.

This test was designed to give greater weight to the distribution tails by the division of the difference of the empirical distribution function F_n and the theoretical distribution function F_0 by $\sqrt{F_0(1-F_0)}$. Anderson and Darling showed that this operation is equivalent to

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln(w_i) + \ln(1-w_{n-i+1})\right] \tag{4.571}$$

where w_i is the theoretical distribution function, $F_0(u;\theta)$, evaluated in the order statistic $u_{(i)}$, and, for example, the value of the maximum likelihood estimator ϑ. For samples of a sufficient size, $n > 10$, the values of A^2 corresponding to the 0.05 and 0.01 significance level are 2.492 and 3.857, respectively. The null hypothesis that the experimental function and the theoretical function have the same distribution should be rejected for high values of A^2.

4.10.13.3 Graphic test

The previous method is not capable of detecting whether the data significantly deviate from the distribution tails. One way of observing this deviation is to graphically represent the estimated function distribution $F(u_{(i)};\vartheta)$, and the pointed data with the point probability of the sample, $p(u_{(i)})$. A suitable point probability is $p_i = (i-0.3)/(n+0.4)$, where i indicates the position of the order statistic and n is the size of the sample.

Whenever possible, the distribution function should be transformed, such that its representation is a straight line. Although the graphic system offers the possibility of estimating the distribution function parameters directly from the drawing, it should be remembered that the point probability is subjective. It is thus best in all cases to use the maximum likelihood method for this purpose.

In those cases in which the samples are incomplete and there are censored data, it is recommended to apply the Kaplan-Meier reliability function to obtain the point probability.

$$p_i = 1 - \frac{n+0.7}{n+0.4}\prod_{j=1}^{i}\frac{n-q_j+0.7}{n-q_j+1.7} \tag{4.572}$$

where q_j is the order number of the observation in the total sample, including censored data.

4.10.13.4 Probability paper

A probability paper is an XY graph, in which the axis scales have been changed so that the distribution functions of a certain probability model are a straight line. Let $F(x;\ \theta_1,\ \theta_2)$ be a distribution function of a probability model, where θ_1 and θ_2 are the parameters of the model. A transformation is then sought, which is defined by $\xi = g(x)$ and $\eta = h(y)$, such that the family of curves, $y = F(x;\ \theta_1,\ \theta_2)$ is transformed into a family of straight lines, namely,

$$h(y) = h\left[F(x;\theta_1,\theta_2)\right] = \theta_1 g(x) + \theta_2 \qquad (4.573)$$

$$\eta = \theta_1 \xi + \theta_2 \qquad (4.574)$$

where the variable η is a reduced variable.

4.10.13.4.1 Normal probability paper

If the function F is the Gaussian distribution function, $F(x;\ \theta_1,\ \theta_2) = \Phi(x - \mu/\sigma)$, where μ and σ are the mean and typical deviation, respectively, and Φ is the distribution of the normal variable $N(0,1)$. In this case, $\xi = x$ y $\eta = h(y) = \Phi^{-1}(y)$, which means that $\theta_1 = 1/\sigma$ and $\theta_2 = -\mu/\sigma$, and the family of straight lines is $\eta = \xi/\sigma - \mu/\sigma$. The parameters of the distribution function can be estimated by taking two ordinates on the straight line, $\eta = 0$ and $\eta = 1$, and thus obtaining a system of two equations in μ and σ : $\xi = \mu$ and $\xi = \mu + \sigma$.

For other probability models, the same procedure is followed. More specifically, the transformation of variables is defined, and the distribution parameters are estimated, depending on the reduced variables. The following sections describe the transformation functions and the equations of the parameters for some of the previously mentioned probability models.

4.10.13.4.2 Log-normal probability paper

$$F(x;\theta_1,\theta_2) = \Phi\left(\frac{\log(x) - \mu}{\sigma}\right) \qquad (4.575)$$

$$\xi = g(x) = \log(x) \qquad (4.576)$$

$$\eta = h(y) = \Phi^{-1}(y) \qquad (4.577)$$

$$\theta_1 = 1/\sigma; \quad \theta_2 = -\mu/\sigma \qquad (4.578)$$

$$\eta = \xi/\sigma - \mu/\sigma \qquad (4.579)$$

$$\eta = 0; \quad \xi = \mu; \quad \eta = 1; \quad \xi = \mu + \sigma \qquad (4.580)$$

Recommendations. In extreme value problems, there is a great difference between the fact that the sample comes from a given distribution and the fact that it only asymptotically tends towards the distribution. Since the right or left tail is the only part of the distribution function that intervenes in the behavior of the extremes (maximum or minimums, respectively), the rest of the information pertaining to the distribution function is unnecessary. Thus, two distributions with identical values in the tail of interest have exactly the same extreme distribution even when the rest of the original distributions are very different. For this reason, the use of the other original data only hinders the analysis.

For this reason when the distribution is not known, it is necessary to weight the data before beginning to fit it. These weights can be zero. In other words, certain points can be excluded, and the rest of data can be assigned different weights.

In the selection of weights, two crucial elements should be taken into account. On the one hand, the estimated percentile in the tail of interest has greater variances, and on the other, they are those that most closely approximate the theoretical values of the limit distribution. Consequently, it is necessary to establish a balance between both. However, it is evident that the values of the other tail should have very small or zero weights.

4.11 SAMPLE ELEMENT AND SPACE

During its useful life, a subset of a structure can reach a state which produces one or various failure modes related to a given ultimate, serviceability or operational limit state. The useful life can be regarded as an experiment, and the states are the realizations of this experiment. In some of them, the structure can fail or lose its operationality. The recommended procedure in the ROM 0.0 to verify a project includes the limit state method and the assigned failure or stoppage modes. The theory of sample spaces and of event spaces is the foundation used to calculate the joint probability of failure or stoppage of the subset of a structure in a time interval.

The set of failure modes $\{s_0, s_1, \ldots, s_8\}$, is a collection of possible states of the structure, also known as sample elements, where s_0 is the sample element that represents the non-occurrence of failure modes. The term, *sample*, is used to take into account the uncertainty associated with the occurrence of each sample element.

4.11.1 Mutually exclusive elements and the complete collection

When each sample element is an exclusive mechanism or failure mode, it excludes the participation of any other sample element. (For example, the failure mode of the main layer only refers to the main layer and not to the berm.) Such elements in a sample are said to be mutually exclusive.

The set that contains all possible modes is the sample space of the failure modes of the section, and is represented by S_p. This space is a complete or exhaustive collection of sample elements.

4.11.2 Graphic representation of the sample space

The sample elements and the sample space can be graphically represented by means of a Venn diagram. If the elements in the sample are mutually exclusive and collectively exhaustive, the representative areas of the various points do not intersect. The set of all of them represent the sample space S_p.

4.11.3 Events and event space

An event can consist of a sample element or a combination of sample elements: It represents a project manifestation or state. The sample elements are the simplest events that permit the description of the set of possible events.

4.11.3.1 Null event and complementary event

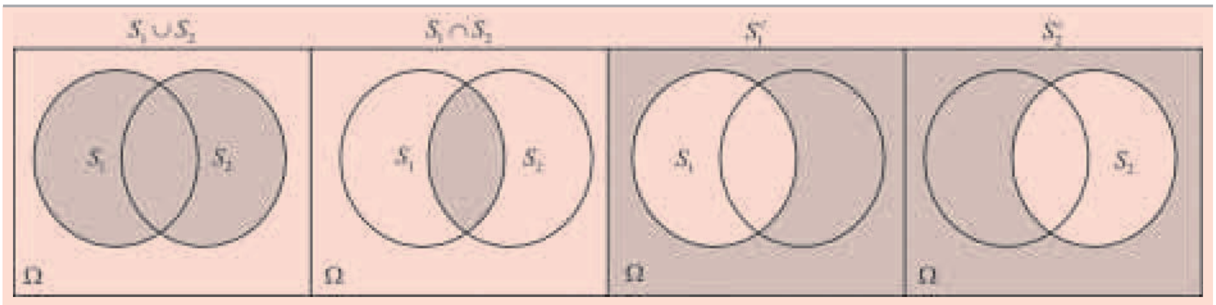
One of the possible results of an experiment is the non-occurrence of any sample element. This null event does not contain any of the sample elements, and is represented by the Greek letter ϑ. An event that is complementary to another event is one that does not contain any of its sample elements, and is represented by $S_i[s_i^c] \equiv S_i^c$.

4.11.3.2 Combination of events: union and intersection

The union of two events, S_i and S_j, which is represented by $S_i \cup S_j$, is also known as *combination o*, and contains all the shared and unshared sample elements of S_i and S_j. The intersection of two events, S_i and S_j, which is represented by $S_i \cap S_j \equiv S_iS_j$ is also known as *combination y*, and only contains two sample points shared by S_i and S_i.

Example. *Figure 4.4.25 shows various Venn diagrams, which identify the union and intersection of two events, S_1 and S_2, as well as their complementary events, S_1^c, S_2^c, the event space and the complementary events of the union $\{S_1 \cup S_2\}^c$ of the intersection, $\{S_1 S_2\}^c$ and the intersection of the complementary events, $S_1^c S_2^c$, known as a null event or non-failure event.*

Figure 4.4.26. Venn diagram



4.11.3.3 Mutually exclusive events and empty event

In the same way as the sample elements, events in which one event impedes the occurrence of another are said to be mutually exclusive. In a Venn diagram, these events are not in the same area, in other words, they do not intersect. This intersection can be regarded as the empty event ϕ, or the event that cannot exist or which is impossible.

4.11.3.4 Complete collection of events

The set of all possible events is the event space. Thus, events are said to form a complete collection Ψ. This collection can consist of different combinations of events. Each collection that describes all of the space of possible events is said to be complete. An event and its complement are a complete collection because their union totally describes the even space.

It is possible to have various complete collections of event. If, furthermore, the complete collection is made up of events whose intersection is the empty set, then the complete collection is said to be of mutually exclusive events. Each event and its complement make up a complete collection of mutually exclusive events.

Example. *A space composed of events, S_1, S_2, and its complementary events. $\{S_1 \cup S_1^c\}$, $\{S_2 \cup S_2^c\}$, $\{S_1 \cup S_2, (S_1 \cup S_2)^c\}$, $\{S_1 \cup S_2, (S_1^c S_2^c)\}$ are various ways of constructing a complete space. All of them are composed of mutually exclusive events. The space can be constructed by means of various combinations of events $\{S_1, S_2, S_1^c, S_2^c, S_1 S_2, S_1^c S_2^c\}$, which is complete, but the events are not mutually exclusive.*
The importance that the space of events be constructed of mutually exclusive events is evident when one studies the probability of the events. If the space of the events is complete, the algebraic sum of the probabilities of events that compose it should be the unit. If the events are mutually exclusive, their probabilities always intervene with a positive sign.
If there are events that are not mutually exclusive, in order to obtain the probability of the complete space, some terms will be positive and others will be negative. If there are events that are not mutually exclusive, in order to obtain the probability of the complete space, some terms will be positive and others will be negative.

4.11.3.5 Rules for combining events

In order to combine events, it is necessary to define basic rules. These are the following:

$$
\begin{aligned}
S_i \cup S_i &= S_i; \; S_i \cap S_i = S_i; \; S_i \cup \phi = S_i; \; S_i \cap \phi = \phi \\
S_i \cup \Psi &= \Psi; \; S_i \cap \Psi = S_i; \; S_i \cup S_i^c = \Psi; \; S_i \cap S_i^c = \phi \\
\left\{S_i^c\right\}^c &= S_i
\end{aligned}
\quad (4.581)
$$

4.11.3.5.1 COMMUTATIVE, ASSOCIATIVE, AND DISTRIBUTIVE PROPERTIES

The same as other algebraic operations, event combination, union, and intersection individually have the commutative, associative and distributive property of one with respect to the other. These properties are expressed in the following way:

- Commutative property: $S_i \cup S_j = S_j \cup S_i$ and $S_iS_j = S_jS_i$
- Associative property: $(S_i \cup S_j) \cup S_k = S_i \cup (S_j \cup S_k)$, and $(S_iS_j)S_k = S_i(S_jS_k)$
- Distributive property (of the union regarding the intersection): $(S_i \cup S_j)\, S_k = S_iS_k \cup S_jS_k$

Thanks to these properties, it is easy to verify that: (1) two events, S_i and S_j, are mutually exclusive if their intersection is the empty event, i.e. $S_iS_j = \phi$; (2) the sample elements shared by two events, S_i and S_j, belong to its intersection, S_iS_j; (3) the union of two events contains the sample elements of both events, those that are shared as well as those that are not. This can be expressed as $S_i \cup S_j = S_i + S_j$.

4.11.3.6 Creation of an event space

The event space is the set of all possible events, and can be constructed by means of the union and the combination of a complete collection of sample elements. In the calculation of the failure probability, the events should be a complete collection, but they should also be mutually exclusive.

4.11.4 Probability of events

Once defined the space of possible events Ψ, it is necessary to evaluate the probability of occurrence of each of them in the time interval. The occurrence of an event can be regarded as the result of an experiment to which the value of a random variable X is assigned. It is necessary to determine the probability function of this variable. For this purpose, the following axioms are adopted as a foundation for an event probability theory.

4.11.4.1 Probability axioms

If the probability of an event is expressed by $\Pr[S]$,

- Axiom I: The probability of an event is a magnitude that verifies $0 \leq \Pr[S_i] \leq 1$ for all $S_i \in \Psi_S$.
- Axiom II: The probability of a certain event or of the sample space Ω is equal to the unit, $\Pr[\Omega] = 1{,}0$.
- Axiom III: The probability of the occurrence of the union of two events, S_i and S_j, which are mutually exclusive and belong to the event space Ψ_S is equal to the sum of the probabilities of occurrence of the individual events, $\Pr[S_i \cup S_j] = \Pr[S_i] + \Pr[S_j]$

4.11.4.2 Probability of the union of complementary events

Based on these axioms, and by applying the operations of combination of events, the following results can be obtained,

$$\Pr\left[S_i \cup S_i^c\right] = \Pr\left[S_i\right] + \Pr\left[S_i^c\right] = 1.0 \quad (4.582)$$

$$\Pr\left[S_i^c\right] = 1.0 - \Pr\left[S_i\right] \tag{4.583}$$

Note. *It can be observed that the probability of occurrence of the null event, ϑ_S is the probability that the subset of the structure will not fail in the time interval. In the case of a vertical breakwater with three failure modes (i.e. sliding, rigid overturning, and toe berm erosion) considered as three mutual exclusive sample events, the probability that at least one failure mode (sliding or overturning or berm erosion) will occur in the section is*

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = 1.0 - \Pr\left[\vartheta_S\right] \tag{4.584}$$

where $\Pr[\vartheta_S]$ is the probability of occurrence of the null event, $\vartheta_S = S_1^c S_2^c S_3^c$,
In the case that the sample event contains n mutually exclusive event $S_i (i = 1, \dots, n) \in \Psi_S$, the generalization of axiom III gives

$$\Pr\left[S_1 \cup S_2 \cup \dots \cup S_n\right] = \Pr\left[S_1\right] + \Pr\left[S_2\right] + \dots + \Pr\left[S_n\right] \tag{4.585}$$

4.11.4.3 Probability of the union and intersection of mutually exclusive events

If two events, S_i and $S_j \in \Psi_S$ are mutually exclusive, their intersection is the impossible or empty event ϕ and consequently, the probability of occurrence of the intersection event is zero, $\Pr[\phi] = \Pr[S_i S_j] = 0$. If the events are not mutually exclusive, $S_i S_j \neq \phi$, and generally, $\Pr[S_i S_j] \neq \phi$. It is possible to obtain an expression of this probability based on the definition of the complementary event and the probability of the union of two events,

$$\Pr\left[S_1 \cup S_j\right] = \Pr\left[S_i \cup S_i^c S_j\right] = \Pr\left[S_i\right] + \Pr\left[S_i^c S_j\right] \tag{4.586}$$

Furthermore, the event S_j is the union of the events $S_i^c S_j$ and $S_i S_j$, and consequently,

$$\Pr\left[S_j\right] = \Pr\left[S_i^c S_j\right] + \Pr\left[S_i S_j\right] \tag{4.587}$$

and as a result,

$$\Pr\left[S_i^c S_j\right] = \Pr\left[S_j\right] - \Pr\left[S_i S_j\right] \tag{4.588}$$

By substituting in the first equation, the following is obtained:

$$\Pr\left[S_i \cup S_j\right] = \Pr\left[S_i + S_j\right] = \Pr\left[S_i\right] + \Pr\left[S_j\right] - \Pr\left[S_i S_j\right] \tag{4.589}$$

which relates the probability of the occurrence of the union of two events that are not mutually exclusive to the probability of their intersection.

Note. *In a section of a vertical breakwater with two failure modes, sliding and toe berm erosion, and assuming that they are not mutually exclusive, the probability that at least one of them will occur is equal to the sum of their individual probabilities of occurrence after the joint probability of occurrence has been subtracted.*

4.11.4.4 Probability of the union of n non-mutually exclusive events

When the event space is composed of n events, the equation to evaluate the probability of the union significantly expands the number of terms. For example, for three events, S_1, S_2 and S_3.

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = \Pr\left[S_1\right] + \Pr\left[S_2\right] + \Pr\left[S_3\right] - \Pr\left[S_1 S_2\right] - \Pr\left[S_1 S_3\right] - \Pr\left[S_2 S_3\right] + \Pr\left[S_1 S_2 S_3\right] \tag{4.590}$$

In some cases, the evaluation of one of the terms on the right can be complicated. The term on the left can be written, depending on the intersection of the complementary events,

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = 1 - \Pr\left[\left\{S_1 \cup S_2 \cup S_3\right\}^c\right] \tag{4.591}$$

which can be expressed in terms of the intersection of complementary events,

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = 1 - \Pr\left[S_1^c S_2^c S_3^c\right] \tag{4.592}$$

which in many occasions is easier to evaluate. This expression can be generalized for n events,

$$\Pr\left[S_1 \cup S_2 \cup \ldots \cup S_n\right] = 1 - \Pr\left[\left\{S_1^c S_2^c \ldots S_n^c\right\}\right] \tag{4.593}$$

If, in addition, the events are mutually exclusive, then,

$$\Pr\left[S_1 \cup S_2 \cup \ldots \cup S_n\right] = \sum_{i=1}^{n} \Pr\left[S_i\right] \tag{4.594}$$

Note. *Consider the case of the vertical breakwater when the three principal failure modes are mutually exclusive. If the probabilities of occurrence of the event S_1 (sliding), S_2 (overturning), and S_3 (toe berm erosion) in the time interval are $\Pr[S_1] = p_1$, $\Pr[S_2] = p_2$ y $\Pr[S_3] = p_3$, the probability that at least one of the failure modes will occur is*

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = \sum_{i=1}^{3} \Pr\left[S_i\right] = p_1 + p_2 + p_3 \tag{4.595}$$

However, if, for example, the events, toe berm erosion and sliding are not mutually exclusive, the probability of the intersection event is

$$\Pr\left[S_1 S_3\right] = p_{13} \neq 0 \tag{4.596}$$

Then, the probability that at least one of the failure modes will occur is

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = \Pr\left[S_1\right] + \Pr\left[S_2\right] + \Pr\left[S_3\right] - \Pr\left[S_1 S_3\right] = p_1 + p_2 + p_3 - p_{13} \tag{4.597}$$

4.11.4.5 Other properties of event probability

The probability axioms can be used to demonstrate the properties of the probability of events, described in the following sections.

4.11.4.5.1 PROBABILITY OF THE EMPTY SET

The probability of event ϕ, which is an empty or impossible event is zero, $\Pr[\phi] = 0$.

4.11.4.5.2 PROBABILITY OF THE EVENT CONTAINED IN ANOTHER EVENT

If the event $S_i \subset S_j$, (S_i is contained in S_j), then it holds that $\Pr[S_i] \leq \Pr[S_j]$.

4.11.4.5.3 BOOLE'S INEQUALITY

The probability of the union of n events does not exceed the sum of their individual probabilities,

$$\Pr\left[S_1 \cup S_2 \cup \ldots \cup S_n\right] \leq \Pr\left[S_1\right] + \Pr\left[S_2\right] + \ldots + \Pr\left[S_n\right] \tag{4.598}$$

Note. *This inequality gives a conservative estimate of the failure probability of a subset of the structure. If, furthermore, the events are mutually exclusive, the probability of event intersection is zero, and thus, the probability of the occurrence of at least one of the failure modes (union of events) is equal to the sum of the individual probabilities of each one.*

4.11.4.6 Conditioned probability of events

It is often necessary to determine the probability of occurrence of events that are not mutually exclusive, and this means knowing the probability of their intersection. If S_i and S_j are two events, the probability of the intersection event, S_iS_j can be obtained from the probability that the event S_i will occur, knowing that the event S_j had occurred (conditioned probability),

$$\Pr\left[S_iS_j\right]=\Pr\left[S_i\left|S_j\right.\right]P\,r\left[S_j\right]=\Pr\left[S_j\left|S_i\right.\right]P\,r\left[S_i\right] \tag{4.599}$$

where the terms, $\Pr[S_i\,|\,S_j]$, (or $\Pr[S_j\,|\,S_i]$), represent the probability of occurrence of the event S_i, (or S_j), conditioned to the occurrence of the event S_j, (or S_i), and $\Pr[S_j]$, (or $\Pr[S_i]$), represents the probability that the event S_j, (or S_i). will occur.

The conditioned probability of the occurrence of various events can be calculated by means of the following expressions.

An event and its complement,

$$\Pr\left[S_i^c\left|S_j\right.\right]=1-\Pr\left[S_i\left|S_j\right.\right] \tag{4.600}$$

Union of two events conditioned to a third event,

$$\Pr\left[S_1\cup S_2\left|S_3\right.\right]=\Pr\left[S_1\left|S_3\right.\right]+\Pr\left[S_2\left|S_3\right.\right]-\Pr\left[S_1S_2\left|S_3\right.\right] \tag{4.601}$$

Intersection of two events conditioned to a third event of non-zero probability,

$$\Pr\left[S_1S_2\left|S_3\right.\right]=\frac{\Pr\left[S_1S_2S_3\right]}{\Pr\left[S_3\right]} \tag{4.602}$$

Generalization of the intersection of *n* events,

$$\Pr\left[S_1S_2\ldots S_n\right]=\Pr\left[S_1\left|S_2S_3\ldots S_n\right.\right]\Pr\left[S_2\left|S_3S_4\ldots S_n\right.\right]\ldots\Pr\left[S_{n-1}\left|S_n\right.\right]\Pr\left[\left|S_n\right.\right] \tag{4.603}$$

4.11.4.7 Statistically independent events

When the probability of occurrence of an event is not conditioned to the occurrence of another event, though they are not mutually exclusive, these events are said to be statistically independent. The conditioned probability is equal to the product of their individual probabilities,

$$\Pr\left[S_iS_j\right]=\Pr\left[S_i\right]\Pr\left[S_j\right]=\Pr\left[S_j\right]\Pr\left[S_i\right] \tag{4.604}$$

4.11.4.8 Intersection of n statistically independent events

The application of these results gives

$$\Pr\left[S_1S_2\ldots S_n\right]=\Pr\left[S_1\right]\Pr\left[S_2\right]\ldots\Pr\left[S_{n-1}\right]\Pr\left[S_n\right]=\prod_{i=1}^{n}\Pr\left[S_i\right] \tag{4.605}$$

Example. In the case of the vertical breakwater, the events, sliding and toe berm erosion, were assumed not to be mutually exclusive. Consequently, the probability of the intersection is $\Pr[S_1S_3]=p_{13}\neq 0$. *According to this section, this probability can be expressed as*

$$\Pr\left[S_1S_3\right]=\Pr\left[S_1\left|S_3\right.\right]\Pr\left[S_3\right]=\Pr\left[S_3\left|S_1\right.\right]\Pr\left[S_1\right] \tag{4.606}$$

The joint probability of failure is expressed by

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = \Pr\left[S_1\right] + \Pr\left[S_2\right] + \Pr\left[S_3\right] - \Pr\left[S_1 | S_3\right]\Pr\left[S_3\right] \quad (4.607)$$

This equation clarifies the main difficult of the problem to be solved, namely, the calculation of the conditioned probability, $\Pr[S_1 \mid S_3]$, *(oR* $\Pr[S_3 \mid S_1]$*) that sliding of the breakwater will occur when there is toe berm erosion. Generally, experience and available information concerning the occurrence of conditioned failure modes are slight. Assuming that the two failure modes, sliding and toe berm erosion are statistically independent, then,*

$$\Pr\left[S_1 | S_3\right] = \Pr\left[S_1\right] = p_1 \quad (4.608)$$

and substituting in the previous expression,

$$\Pr\left[S_1 \cup S_2 \cup S_3\right] = p_1 + p_2 + p_3 - \left(p_1 p_3\right) \quad (4.609)$$

In some cases, it might be of interest to know the probability of sliding, but without the occurrence of toe berm erosion, in other words, of the event, $S_1 S_3^c$*. When the definition of conditioned probability is applies, this gives*

$$\Pr\left[S_1 S_3^c\right] = \Pr\left[S_3^c | S_1\right]\Pr\left[S_1\right] = \left(1 - \Pr\left[S_3 | S_1\right]\right)\Pr\left[S_1\right] \quad (4.610)$$

4.11.4.9 Importance of working with mutually exclusive events

The previous section described how the event space can be constructed with different combinations of events that make up a complete collection. However, some of these collections are composed of mutually exclusive events, whereas others are not. From the perspective of the calculation of the joint failure probability, the best solution is to work with a complete collection of mutually exclusive events. Nevertheless, this is often not possible, and there is no choice but to work with events whose intersection is not an empty event.

Let us suppose the case of a vertical breakwater with only two failure modes: (i) S_1, sliding; (ii) S_3, erosion of the toe berm. A complete space of events is made up of $\Psi = \{S_1, S_3, S_1^c S_3^c\}$, where the intersection event of the complementary events is the zero or non-failure event. This space is complete (i.e. constructed by the union of the events, $\Psi = S_1 \cup S_3 \cup S_1^c S_3^c$, but it is not generally composed of mutually exclusive events. The two failure modes can occur simultaneously, and $S_1 S_3 \neq \phi$.

In the Venn diagram, this space is represented by a rectangle, inside of which there are two intersecting rectangles (see Figure 4.4.27).

The probability of the complete collection or certain event is equal to the unit, in other words,

$$\Pr\left[S_1 \cup S_3\right] + \Pr\left[\left(S_1 \cup S_3\right)^c\right] = \Pr\left[S_1 \cup S_3\right] + \Pr\left[S_1^c S_3^c\right] = 1 \quad (4.611)$$

It can be observed that the fulfillment of probability axiom II is not expressed in terms of the events in the collection, but rather as combinations of them. The probability of the event that the breakwater will fail because of sliding or erosion of the toe berm is

$$\Pr\left[S_1 \cup S_3\right] = \Pr\left[S_1\right] + \Pr\left[S_3\right] - \Pr\left[S_1 S_3\right] \quad (4.612)$$

The Venn diagram represents a complete collection of mutually exclusive events, made up of combinations of events, S_1 and S_3, (see Figure 4.4.28).

This space is $\Psi = \{S_1 S_3, S_1 S_3^c, S_1^c S_3, S_1^c S_3^c\}$, and in the same way as in the previous case, the union of the events forms the event space,

Figure 4.4.27. **Venn diagram of the space of two mutually exclusive events**

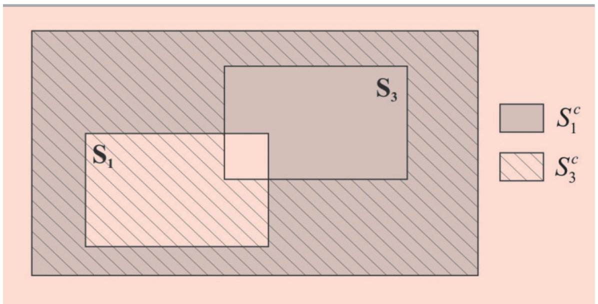
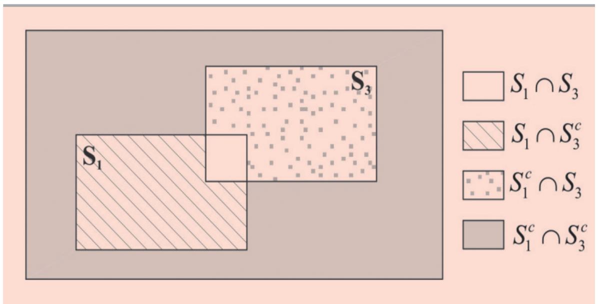


Figure 4.4.28. **Venn diagram of a complete collection of mutually exxclusive events**



$$\Psi = S_1S_3 \cup S_1S_3^c \cup S_1^cS_3 \cup S_1^cS_3^c \tag{4.613}$$

Consequently,

$$\Pr\left[S_1S_3 \cup S_1S_3^c \cup S_1^cS_3 \cup S_1^cS_3^c\right] = \Pr\left[S_1S_3\right] + \Pr\left[S_1S_3^c\right] + \Pr\left[S_1^cS_3\right] + \Pr\left[S_1^cS_3^c\right] = 1 \tag{4.614}$$

The definitions of union and intersection result in the following,

$$\Pr\left[S_1 \cup S_3\right] = 1 - \Pr\left[S_1^cS_3^c\right] = \Pr\left[S_1S_3\right] + \Pr\left[S_1S_3^c\right] + \Pr\left[S_1^cS_3\right] \tag{4.615}$$

To calculate the probability of the union of the two failure modes, all the terms are positive, though all of them are made up of the intersection of events.

4.12 TOTAL PROBABILITY THEOREM

One of the project requirements is that a subset of the structure in the project design phase be safe with a high level of reliability or upper tail of the joint probability of non-failure. Accordingly, it should be verified that the joint probability of failure in the project phase in terms of all the failure modes assigned to the ultimate limit states are lower than the value proposed in Chapter 2, depending on the social and environmental index of the subset (see Table 2.2.3, Chapter 2).

The occurrence of failure modes is associated with the occurrence of different combinations of agents. To evaluate the probability of occurrence of a failure mode, the total probability theorem is applied in a new event space Ψ_T, composed of the scalar product of the event subspace of failure modes and the subspace of agents.

4.12.1 Sample space of failure modes and project factors

If the failure mode event S_j depends on the occurrence of agents, $A_1, \ldots, A_m$, and these form a complete and mutually exclusive collection, such that $\Pr[A_i] > 0$, $i = 1,2,\ldots, m$, then,

$$S_j = S_j \cap \left(A_1 \cup A_2 \cup \ldots \cup A_m\right) = \left(S_j \cap A_1\right) \cup \left(S_j \cap A_2\right) \cup \ldots \cup \left(S_j \cap A_m\right) = \left(S_jA_1\right) \cup \ldots \cup \left(S_jA_m\right) \tag{4.616}$$

Applying event probability axiom III,

$$\Pr\left[S_j\right] = \Pr\left[S_jA_1\right] + \Pr\left[S_jA_2\right] + \ldots + \Pr\left[S_jA_m\right] = \sum_{i=1}^{m} \Pr\left[S_jA_i\right] \tag{4.617}$$

and considering the conditioned probability,

$$\Pr\left[S_j\right] = \Pr\left[S_j|A_1\right]\Pr\left[A_1\right] + \Pr\left[S_j|A_2\right]\Pr\left[A_2\right] + \ldots + \Pr\left[S_j|A_m\right]\Pr\left[A_m\right] = \sum_{i=1}^{m} \Pr\left[S_j|A_i\right]\Pr\left[A_i\right] \tag{4.618}$$

This expression is known as the *total probability theorem*. This theorem can only be applied if the agents A_i of the sample are mutually exclusive.

4.12.1.1 Probability of a landslide

The study of the probability of occurrence of an earthquake and high sea waves at the vertical breakwater shows that A_2A_3, $A_2A_3^c$, $A_2^cA_3$, ϑ are mutually exclusive. In this case, it is possible to apply the total probability theorem, which results in the following:

$$\Pr\left[S_1\right]=\Pr\left[S_1\left|A_2A_3\right.\right]\Pr\left[A_2A_3\right]+\Pr\left[S_1\left|A_2A_3^c\right.\right]\Pr\left[A_2A_3^c\right]+ \\ \Pr\left[S_1\left|A_2^cA_3\right.\right]\Pr\left[A_2^cA_3\right]+\Pr\left[S_1\left|A_2^cA_3^c\right.\right]\Pr\left[A_2^cA_3^c\right] \tag{4.619}$$

Note. As shown in the example of the vertical breakwater, this theorem is crucial in the methodology that permits the evaluation of the joint failure probability of a subset of the structure as a consequence of its interference with the project factors.
As will be explained, they also form a sample space of mutually exclusive elements, whose combination allows the construction of the event space (occurrence of project factors). By means of the total probability theorem, it is possible to evaluate the probability of occurrence of a supposed failure event, supposing that a complete, mutually exclusive collection of project factors has occurred.

4.12.1.2 Bayes Theorem

The Bayes theorem formulates the inverse problem to that which is resolved with the total probability theorem. Once the subset of the structure has failed because of overturning S_1, it is necessary to calculate the probability that the failure was caused by occurrence of the agent, wave action A_2. Based on the previous definitions, if S_1 is a non-zero probability event, this probability can be calculated by

$$\Pr\left[A_2\left|S_1\right.\right]=\frac{\Pr\left[S_1\left|A_2\right.\right]\Pr\left[A_2\right]}{\Pr\left[S_1\right]} \tag{4.620}$$

and generalizing this for any of the $m-1$ agents, and since the $A_1,\ldots,A_m$ is a complete, mutually exclusive collection of non-zero probability agents, $\Pr[A_i]>0,\ i=1,2,\ldots,m$,

$$\Pr\left[A_i\left|S_1\right.\right]=\frac{\Pr\left[S_1\left|A_i\right.\right]\Pr\left[A_i\right]}{\sum_{j=1}^{m}\Pr\left[S_i\left|A_j\right.\right]\Pr\left[A_j\right]} \tag{4.621}$$

4.13 FAILURE PROBABILITY: LEVELS II AND III

The verification equation used to determine if the structure is safe against a given failure mode is composed of terms, which are also functions of the project factors. Let us consider the case of a failure mode in which n project factors intervene, as defined by the *n*-dimensional vector, $X(X_1, X_2, \ldots, X_n)$, whose joint density function is $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)$. Let the verification equation be $S=G(X_1, X_2, \ldots, X_n)$, which defines a critical hypersurface in the *n*-dimensional space, such that $S=G>0$ is the safety domain, and $S=G\le 0$ defines the failure domain.

As described in the previous section, in the time interval, the failure probability p_f, and the reliability r_f of the structure against the failure mode is expressed by the following *n*-multiple integrals:

$$p_f=\int_{G(X_1,X_2,\ldots X_n)\le 0}\cdots\int f_{X_1,X_2,\ldots X_n}\left(x_1,x_2,\ldots x_n\right)dx_1dx_2\ldots dx_n \tag{4.622}$$

$$r_f=1-p_f \tag{4.623}$$

The integral or the failure probability can be evaluated by direct integration, which is rarely possible, or by simulation, for example, by the Monte Carlo Method (Level III) or by the transformation of the integrand (Level II) in order to work with independent Gaussian variables.

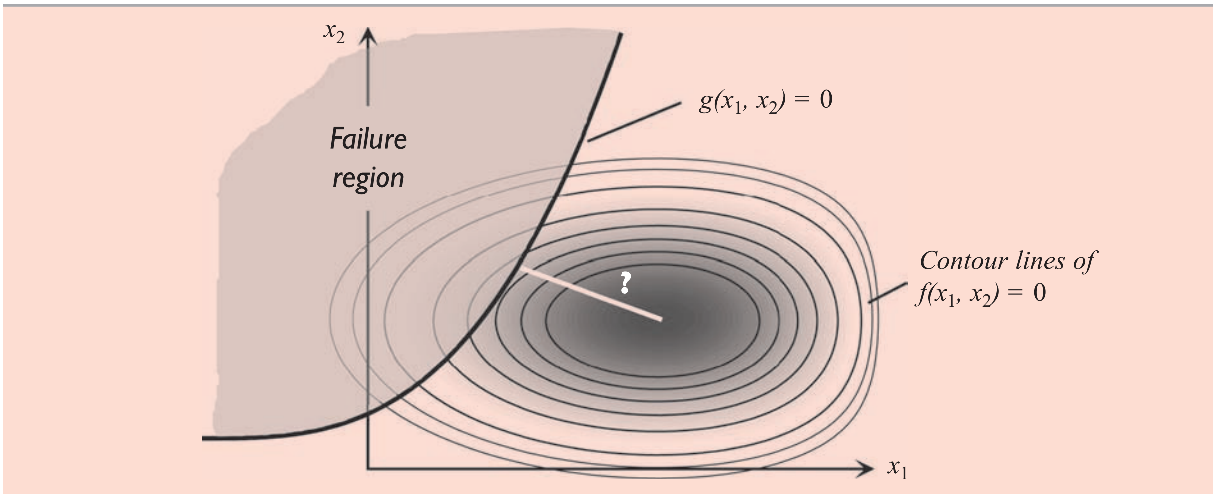
4.13.1 Verification equation with two terms

Let us assume a failure mode whose verification equation, $G(X_1, X_2)$, is expressed with a favorable term X_1 and an unfavorable term X_1. The safety margin is thus defined as $S = G(X_1, X_2)$. According to this definition, the structure is safe and reliable, and will not fail because of the failure mode when $S > 0$. This means that it will fail when $S \leq 0$. The value of the variable, $S = 0$, separates the failure domain D_f from the safety or reliability domain D_r.

The curve, $G(X_1, X_2)$ is the interphase of the domains, D_f and D_r. It is the geometric location of the values of the terms, X_1 and X_1, of the equation that strictly fulfill the critical condition or limit state of the structure against the failure mode. Since the terms X_1 and X_1 are random variables, the safety margin S is also a random variable.

In a given time interval, the failure probability p_f of the structure, because of the failure mode, is the probability of all occurrences, $S \leq 0$, in the time interval, namely, $p_f = \Pr[S \leq 0]$. The replacement of S by its expression gives the following: $p_f = \Pr[G(X_1, X_2) \leq 0]$ (see Figure 4.4.29).

Figure 4.4.29. ***Failure region and criterion, and contour lines of the probability of two random variables***



Let it be supposed that the equation terms, X_1 and X_2, are random variables, defined in a given bidimensional domain D with a joint density function, $f_{X_1X_2}(x_1x_2)$. The domain D has two subdomains. The first subdomain is the failure domain D_f, which contains all value pairs $(X_1, X_2) \in D$ that satisfy the inequality, $S = G(X_1, X_2) \leq 0$.

The second is the safety or reliability domain D_r, which contains all value pairs, $(X_1, X_2) \in D$ that satisfy the inequality, $S = G(X_1, X_2) > 0$. The failure probability is obtained by integrating the density function in the failure domain D_f

$$p_f = \Pr\left[G\left(X_1, X_2\right)\right] = \iint_{D_f} f_{X_1X_2}\left(x_1, x_2\right) dx_1 dx_2 \quad (4.624)$$

Generally, three procedures can be used to resolve this integral: (1) direct integration by analytical methods, which is only possible in certain special cases; (2) resolution by numerical methods (e.g. the Monte Carlo method); (3) avoiding integration by transforming the integrand in a density function of Gaussian variables. The second option is known as a Level III procedure, whereas the third procedure involving the transformation of the integrand is known as a Level II procedure.

4.13.1.1 Analytical evaluation of the integral

In certain cases, the integral has an analytical solution. Let us consider two non-correlated random variables, whose marginal density functions, f_{X_1} and f_{X_2}, and distribution functions, F_{X_1} and F_{X_2}, are a Gaussian mean function and a Gaussian standard deviation function, μ_{X_1}, σ_{X_1}, and μ_{X_2}, σ_{X_2}, respectively. Thus, the failure probability can be expressed as

$$p_f = \iint_{D_f} f_{X_1}(x_1) f_{X_2}(x_2) dx_1 dx_2 \tag{4.625}$$

where D_f is the failure domain. When the definition of the distribution function is considered,

$$F_{X_2}(x_1) = \int_{-\infty}^{x_1} f_{X_2}(u)\, du = \Pr[X_2 \le x_1] \tag{4.626}$$

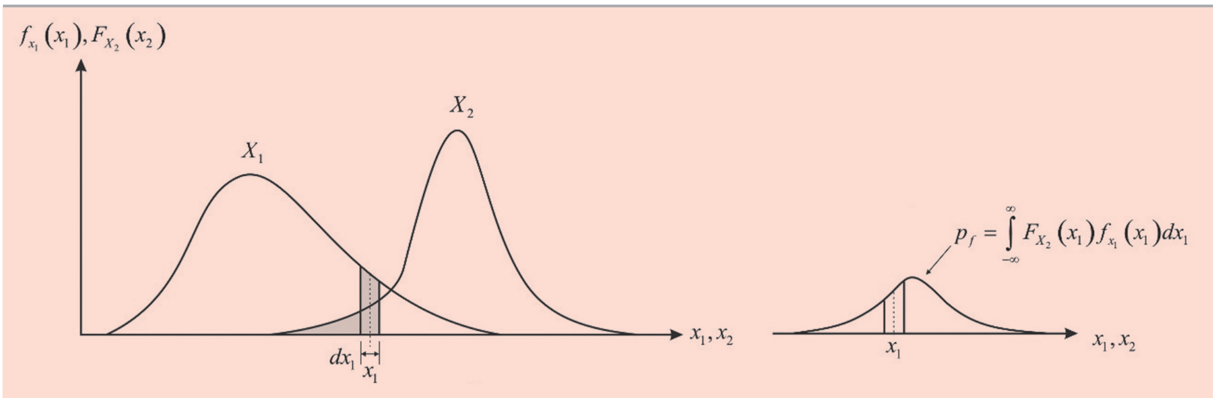
which, when substituted in the previous expression, gives the failure probability by means of a simple integral,

$$p_f = \int_{-\infty}^{\infty} F_{X_2}(x_1) f_{x_1}(x_1) dx_1 \tag{4.627}$$

also known as the convolution integral, where $F_{X_2}(x_1)$ represents the probability that the variable X_2 will take values that are smaller than or equal to x_1. The marginal density function, $f_{X_1}(x_1)dx_1$, represents the probability that the variable X_1 will take values in the interval, $(x_1, x_1 + dx_1)$.

By considering all the possible values that can be taken by the variable X_1, it is possible to obtain the failure probability p_f. In the previous definition, the integration domain for both variables is the set of real numbers. These results are shown in the Figure 4.4.30. It can be observed that the probability of failure against the failure mode is not equal to the intersection area of the density functions, f_{X_1} and f_{X_2}.

Figure 4.4.30. Failure probability of the two variables, X_1 and X_2



4.13.1.2 Evaluation by numerical simulation

The Monte Carlo simulation method can be used to evaluate a defined integral. As an introduction, this section describes a simple case. Let us consider the evaluation of the defined integral,

$$I = \int_a^b G(X_1) dx_1 \tag{4.628}$$

where $G(X_1) > 0$ in the interval $[a,b]$. The value of the integral is the area under the curve in the interval $[a,b]$. A rectangle of height h is defined, which contains the curve G, in other words, $h \ge G(X_1)$. The area of the rectangle is $h(b - a)$. Also defined are two independent random variables, u and v, uniformly distributed in the intervals, $a \le u \le b$ and $0 \le v \le b$, respectively. By generating N value pairs $[u,v]$, it is possible to count those cases in which $v \leq G(u)$. If this number is supposed to be n, for a sufficiently large N, the value of the integral can be approximated by

$$I \simeq h(b-a)\frac{n}{N} \tag{4.629}$$

This example shows how a statistical experiment can be used to evaluate a non-statistical problem. This procedure is the basis of the Monte Carlo simulation method included in Level III.

4.13.1.3 Density function of the safety margin

If the verification equation terms are independent Gaussian variables, there is a functional relation between the reliability index and the failure probability. This relation is exploited in the development of Level II methods. For example, in the case of a linear verification equation, $S = G(Y_1, Y_2) = Y_1 - Y_2$, the difference of two non-correlated Gaussian random variables, Y_1 and Y_2, the failure probability can be evaluated by transforming the integrand. The first-order statistical moments, the mean μ_S, and the second-order moments, the variance σ_S of the random safety margin variable S, depending on the first- and second-order moments of Y_1 and Y_2 are

$$\mu_S = \mu_{Y_1} - \mu_{Y_2} \tag{4.630}$$

$$\sigma_S^2 = \sigma_{Y_1}^2 + \sigma_{Y_2}^2 \tag{4.631}$$

where the variable S is a Gaussian variable $N(\mu_S, \sigma_S)$. The reduced variable, $S_* = (S - \mu_S) / \sigma_S$, is a Gaussian variable of zero mean and unit variance, $N(0,1)$. For the sake of clarity, the Gaussian random variables are represented by Y, whereas any other random variable is represented by X.

Given that the critical condition of failure is when $S = 0$, a failure occurs whenever the value of the safety margin is in the domain, $S \leq 0$. Taking into account the definition of the distribution function, the failure probability is expressed by

$$p_f = \Pr[S \leq 0] = \Phi\left[\frac{0-\mu_S}{\sigma_s}\right] = \Phi\left[-\frac{\mu_S}{\sigma_s}\right] = \Phi\left(-\frac{\mu_{Y_1} - \mu_{Y_2}}{\sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2}}\right) \tag{4.632}$$

where Φ is the function of the standard distribution.

The previous analysis can be immediately applied to the case in which the variables, X and Y, are correlated Gaussian random variables with a known correlation coefficient, $\rho_{Y_1Y_2} \leq |1|$. For the previous case study, $S = G = Y_1 - Y_2$, this gives

$$\mu_S = \mu_{Y_1} - \mu_{Y_2} \tag{4.633}$$

$$\sigma_S^2 = \sigma_{Y_1}^2 + \sigma_{Y_2}^2 - 2\rho_{Y_1Y_2}\sigma_{Y_1}\sigma_{Y_2} \tag{4.634}$$

This means that it is sufficient to replace the variance σ_S with its value, according to the variances of the variables and correlation coefficient, $\rho_{Y_1Y_2}$, in the equation of p_f.

When the reliability index, $\beta = \mu_S/\sigma_S$, is defined by the quotient of the mean and the standard deviation of the safety margin, the failure probability is obtained by the value of the Gaussian distribution function for the reliability index,

$$p_f = \Phi(-\beta) \tag{4.635}$$

It can be observed that, due to the symmetry of the Gaussian function, $\Phi(\beta) = 1 - p_f = r_f$, which is the reliability or probability that the structure will not suffer the failure mode.

4.13.1.4 Evaluation by the optimization of the distance to the origin

If the verification equation, $S = G(Y_1, Y_2) = Y_1 - Y_2$, is linear and both terms, Y_1 and Y_2, are independent Gaussian variables, it is possible to define reduced Gaussian variables, Y_1^* and Y_2^*

$$Y_1^* = \frac{Y_1 - \mu_{Y_1}}{\sigma_{Y_1}} \tag{4.636}$$

$$Y_2^* = \frac{Y_2 - \mu_{Y_2}}{\sigma_{Y_2}} \tag{4.637}$$

which, substituted in the verification equation for the critical safety condition, gives the following:

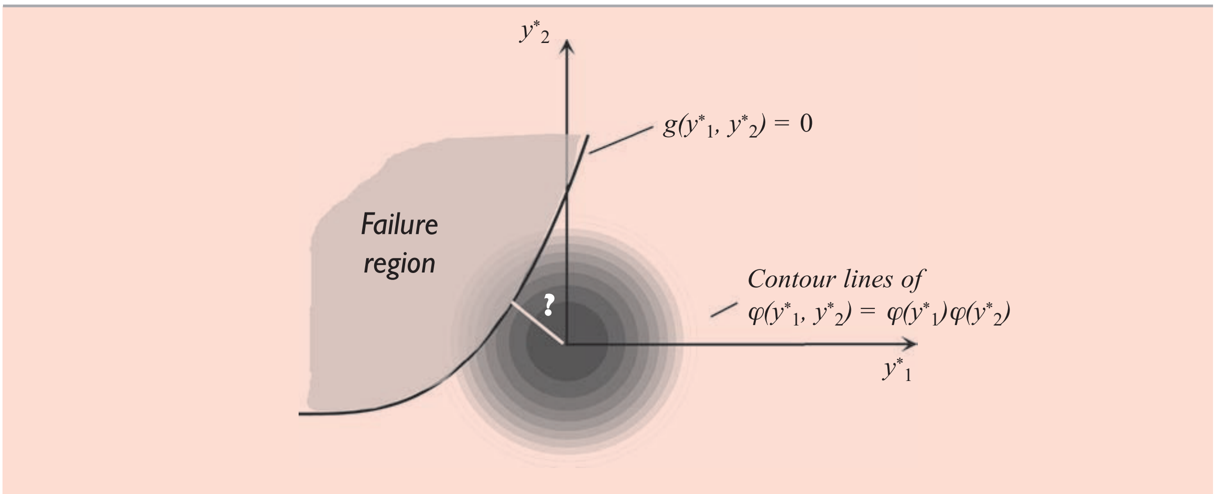
$$G(Y_1, Y_2) = g(Y_1^*, Y_2^*) = \sigma_{Y_1} Y_1^* - \sigma_{Y_2} Y_2^* + (\mu_{Y_1} - \mu_{Y_1}) = aY_1^* + bY_2^* + c = 0 \tag{4.638}$$

which is the equation of a straight line with $a = \sigma_{Y_1}$, $b = -\sigma_{Y_2}$, and, $c = -\mu_{Y_1} - \mu_{Y_2}$. In order to avoid confusion, the verification equation in reduced coordinates, Y_1^*, Y_2^*, is represented by the lowercase letter, g. The safety domain D_r is the set of points in the plane that are located on one side of the straight line, such that $S = g(Y_1^*, Y_2^*) > 0$. The failure domain, D_f, is the set of points in the plane that are located on the other side of the failure line, such that $S = g(Y_1^*, Y_2^*) \leq 0$. The distance Δ,of the origin of the coordinates $O(0, 0)$ to this straight line is

$$\Delta = \frac{c}{\sqrt{a^2 + b^2}} = \frac{\mu_{Y_1} - \mu_{Y_2}}{\sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2}} = \beta \tag{4.639}$$

The shortest distance from the origin of the coordinates to the straight line of the failure or interphase of the safety and failure domains is the reliability index. The distance of this straight line, expressed in reduced coordinates (Y_1^*, Y_2^*), from the origin is equal to the numerical value of its reliability index. When reduced coordinates are used to express the cut-off or critical point by $P_c(y_{1c}^*, y_{2c}^*)$, then, $\beta = (y_{1c}^{*2} + y_{2c}^{*2})^{1/2}$ (see Figure 4.4.31).

Figure 4.4.31. Failure region and criterion, and contour lines at two reduced random variables



This result is only valid for non-correlated Gaussian variables and a linear verification equation. If the verification equation is not a linear function of reduced variables, but is strictly monotonic, then the reliability index is the shortest distance from the origin to the curve, $g(Y_1^*, Y_2^*)$. With these assumptions, the failure probability, $p_f = \Phi(-\beta)$, is obtained by calculating the reliability index, $\beta = min(\Delta)$, where Δ is the distance from the origin to the curve defined by the verification equation, $g(Y_1^*, Y_2^*) = 0$, expressed in reduced coordinates. In this way, the evaluation of the failure probability of the structure, against a failure mode, is transformed in the resolution of a problem of extreme values with constraints expressed by

$$\begin{aligned} \beta &= \min(\Delta) = \min\left(Y_1^{*2} + Y_2^{*2}\right)^{\frac{1}{2}} \\ g\left(Y_1^*, Y_2^*\right) &= 0 \qquad \text{restriction} \end{aligned} \tag{4.640}$$

Once obtained the value of the reliability index, the failure probability is obtained as explained in the previous subsection.

4.13.1.5 Relation between the critical point and the reliability index

From the perspective of the calculation procedure, it is advisable to establish a relation between the critical or verification point (y_{1c}^*, y_{2c}^*) and the reliability index, β. For this purpose, one first obtains the components of the normal, exterior to the curve. $g(Y_1^*, Y_2^*)$, by the critical point,

$$\alpha_{Y_1^*} = \frac{\dfrac{\partial g}{\partial Y_1^*}}{l} = \frac{\sigma_{Y_1^*}}{l} \tag{4.641}$$

$$\alpha_{Y_2^*} = \frac{\dfrac{\partial g}{\partial Y_2^*}}{l} = \frac{\sigma_{Y_2^*}}{l} \tag{4.642}$$

$$l = \sqrt{\left(\frac{\partial g}{\partial Y_1^*}\right)^2 + \left(\frac{\partial g}{\partial Y_2^*}\right)^2} \tag{4.643}$$

where $\alpha_{Y_1^*}$, $\alpha_{Y_2^*}$ are the director cosines or components of the normal exterior to the curve, and l is the length of the normal. The coordinates of the critical point (y_{1c}^*, y_{2c}^*) can be expressed, according to the components of the outer normal and the reliability index.

$$y_{1c}^* = -\alpha_{Y_1^*}\beta \tag{4.644}$$

$$y_{2c}^* = -\alpha_{Y_2^*}\beta \tag{4.645}$$

and the interphase equation, $g(Y_1^*, Y_2^*) = 0$, can be written in terms of $\alpha_{Y_1^*}$, $\alpha_{Y_2^*}$ and β,

$$g\left(Y_1^*, Y_2^*\right) = \beta + \alpha_{Y_1^*} Y_1^* + \alpha_{Y_2^*} Y_2^* = 0 \tag{4.646}$$

and undoing the change of variables,

$$G\left(Y_1, Y_2\right) = b_0 + b_{Y_1} Y_1 + b_{Y_2} Y_2 \tag{4.647}$$

$$b_0 = \beta - b_{Y_1}\mu_{Y_1} - b_{Y_2}\mu_{Y_2} \tag{4.648}$$

$$b_{Y_1} = \frac{\alpha_{Y_2^*}}{\sigma_{Y_1}} \tag{4.649}$$

$$b_{Y_2} = \frac{\alpha_{Y_2^*}}{\sigma_{Y_2}} \tag{4.650}$$

one obtains the verification equation G, expressed in the system of coordinates, Y_1 and Y_2, according to the reliability index and the first-order and second-order statistical moments of the equation.

The reliability index is obtained from the verification equation, $g(Y_1^*, Y_2^*) = 0$ in the critical point, (y_{1c}^*, y_{2c}^*),

$$\beta = -\alpha_{Y_1^*} y_{1c}^* - \alpha_{Y_2^*} y_{2c}^* \tag{4.651}$$

and the mean and variance of the variance of the safety margin random variable, S, are obtained by calculating the mean and variance of the function G,

$$\mu_S = E\left[G(Y_1, Y_2)\right] = E\left[b_0 + b_{Y_1} Y_1 + b_{Y_2} Y_2\right] = b_0 + b_{Y_1}\mu_{Y_1} + b_{Y_2}\mu_{Y_2} \tag{4.652}$$

$$\sigma_S^2 = E\left[(S - \mu_S)^2\right] = b_{Y_1}^2 \sigma_{Y_1}^2 + b_{Y_2}^2 \sigma_{Y_2}^2 = 1 \tag{4.653}$$

As a result, $\beta = \mu_S = \mu_g = \mu_G$. In other words, the value of the reliability index does not depend on the coordinate system used. This result is only valid if the verification equation is linear.

4.13.1.5.1 SENSITIVITY INDEXES

The components, $\alpha_{Y_1^*}, \alpha_{Y_2^*}$, are important elements in the application of this procedures. Since β is calculated as the sum of the products of the component by the reduced coordinate of the critical point, the contribution to the summatory depends on the value of the component. Consequently, this value is an indication of the importance that the variability of that coordinate in the value of β, and thus, the failure probability. The variable, whose component is very small, only slightly influences the final result. Consequently, there is no need to determine its variability with a great deal of precision. In other words, for all practical purposes, that variable can be regarded as deterministic. Evidently, this argument can also be applied in the opposite sense, and the value of the component used to quantify the effect in the value of β, when the corresponding variable is regarded as deterministic instead of random.

As a result, the components are known as sensitivity indexes, omission indexes or negligible indexes, depending on whether they are employed to convert a random variable into a deterministic one, or to evaluate the effect of regarding a random variable as a deterministic variable.

4.13.2 Development of the Monte Carlo method

The section on the evaluation of the failure probability provides a justification of the Monte Carlo simulation method to obtain the value of a defined integral. This section describes this simulation procedure as well as different methods that can be used to generate random numbers.

The application of the Monte Carlo method permits the evaluation of the integral that defines the failure probability in the failure domain D_f by

$$p_f = \int_{D_f} f_{X_1}(x_1)\, dx_1 \tag{4.654}$$

which can be written in the following way,

$$p_f = \int_D J\left[G(X_1) \le 0\right] * f_{X_1}(x_1)\, dx_1 \tag{4.655}$$

where D is the domain defining the verification equation, and J [] is an index function that takes the unit value, if what is inside the brackets is true. This happens if the failure condition is fulfilled, and if it takes the zero value in the event that it is not fulfilled, in other words, if the structure is safe. If X_1 represents random observations of the variable whose density function is f_{X_1}, then the index function J, which only takes two values, 0 or 1, is also a random variable. The previous integral can be expressed by

$$p_{f,J} \approx \frac{1}{N}\sum_{i=1}^{N} J\left[G(X_{1,i}) \le 0\right] \tag{4.656}$$

where $p_{f,J}$ is an unbiased estimator of p_f. For the simulation to provide reliable results, a series of requirements need to be fulfilled, which are related to the size of the sample, the number of samples, the transformation methods of random variables, etc. This section justifies some of the recommendations concerning their use in the verification and calculation of the structure against a mode.

4.13.2.1 Required number of samples

The value of $p_{f,J}$ is obtained as the sum of independent values. Consequently, the central limit theorem for a sufficiently large sample, $N \to \infty$, the random variable $p_{f,J}$ approximates a Gaussian variable, whose mean and variance values can be calculated by

$$E\left(p_{f,J}\right) = \frac{1}{N}\sum_{i=1}^{N}\left[G\left(X_{1,i}\right) \le 0\right] = E\left[G\left(X_{1,i}\right) \le 0\right] \tag{4.657}$$

$$\sigma^2_{p_{f,J}} = \frac{1}{N^2}\sum_{i=1}^{N}\left[G\left(X_{1,i}\right) \le 0\right]^2 = \frac{\sigma_{G\left(X_{1,i}\right)} \le 0}{N} \tag{4.658}$$

It can be observed that the variance of the failure probability estimate directly varies with the variance of the index function, and inversely with $N^{-1/2}$. In other words, the error in the failure probability estimate decreases with $N^{-1/2}$. This result can be used to determine the number of simulations required to evaluate the failure probability with a certain confidence level. When simulation is used to evaluate the probability p that a certain event will occur (e.g. that the verification equation, $S \le 0$, will not be fulfilled, and where N is the size of the sample or total number of simulations, and *n* is the number of times that $S \le 0$), an estimator of p, such as p', is the quotient, n/N. If each of the successive simulations is independent, *n* is a binomial random variable of parameters N and p, and the standard error σ_p, of the estimate of p' is

$$\sigma_{p'} = \sqrt{\frac{p(1-p)}{N}} \tag{4.659}$$

For large values of N, $(N > 30)$, and Np, $(Np > 5)$, the sample distribution function approximates a Gaussian mean function, $\mu_p = Np$, and the variance, $\sigma_p = Np(1-p)$. It is possible to define an error, ε, $(0 \le \varepsilon \le 1)$, as the relative difference between estimated value of p' and its mean value. In other words,

$$\varepsilon = \frac{p' - Np}{Np} \tag{4.660}$$

such that the estimate of the probability p has an error smaller than or equal to ε, with a confidence level of $100(1 - \alpha)$ per cent. The two extremes of the interval can be calculated by

$$p' - z_{\frac{\alpha}{2}}\sqrt{\frac{p'(1-p')}{N}};\ p' + z_{\frac{\alpha}{2}}\sqrt{\frac{p'(1-p')}{N}} \tag{4.661}$$

Where $z_{\alpha/2}$ is the value of a reduced normal variable that is exceeded with probability $\alpha/2$. Once selected a confidence level α and an error ε, the minimum sample size necessary is obtained by limiting the deviation. In other words,

$$z_{\frac{\alpha}{2}}\sqrt{\frac{p'(1-p')}{N}} \le \varepsilon p' \tag{4.662}$$

and solving for *n* and replacing the sample value p' with p results in the following,

$$N \ge \frac{z^2_{\alpha/2}(1-p)}{\varepsilon^2 p} \tag{4.663}$$

Since N is a function of p, which is unknown before performing the simulation, p should be estimated in advance. For the case of $\alpha = 95\%$, $\varepsilon = 0.05$, and $p = 0{,}1$, $N > 13400$.

4.13.2.2 Sample size and convergence

Frequently, instead of initially evaluating the sample size that is necessary to attain a given level of precision in the calculation of the failure probability, it is best to draw the successive evaluations of p_f and the estimate of its variance, depending on the size of the sample N. Generally speaking, this curve is usually a decreasing curve, and the oscillations (i.e. the calculation stability) usually are reduced as the sample size increases.

4.13.2.3 Improvement of the accuracy

Evidently, the accuracy of the Monte Carlo simulation method is related to the size, N, of the samples generated, as can be observed in the expression of the variance. Nonetheless, there are methods capable of reducing the variance without increasing the size of the sample. One such method is the antithetic variates method. It involves the generation of a sequence of standard uniform variables u_i, obtaining an estimator $X_{1,i}$, and using the sequence, $1 - u_i$ to obtain another estimator $X_{2,i}$, which is negatively correlated with $X_{1,i}$. Consequently, the estimator, $X_i^* = (X_{1,i} + X_{2,i})/2$, has a variance lower than $(var[X_1] + var[X_2])/4$.

In other cases, as can occur when two typologies are compared in the same work conditions, the difference of the structural responses simulated with the same sequence of uniform variables $(0,1)$ has a lower variance than the individual variance of each.

Other techniques used to improve the accuracy of the simulation (i.e. variance reduction techniques) use previously acquired information regarding the problem to be resolved, and particularly, that which allows the bounding of the failure region.

4.13.2.4 Generation by means of the integral transformation of the probability

As described in the previous section, the application of the Monte Carlo simulation requires working with random numbers with a uniform distribution function. However, in the majority of cases, the random variables have a distribution function that is different from the uniform one. The integral transformation of the probability provides the theoretical basis for the generation of random number based on any distribution function.

If a random variable X follows a continuous distribution $F_X(x)$, the transformation, $u = F_X(x)$, produces a random variable U with a uniform distribution $(0,1)$. And the reciprocal is also true. If U is a variable with a uniform distribution $(0,1)$, then, $X = \Psi(U)$ is a random variable with a distribution function $F_X(x)$, as long as there is a correspondence between the u and the smallest x, such that $F_X(x) \geq u$.

4.13.2.4.1 Justification of the method

This result is can be immediately inferred, when one takes into account that the value of both distribution functions should be the same for the values of u and x, such that $u = F_X(x)$ with $x = \Psi(U)$,

$$F_U(u) = \Pr[U \le u] = \Pr[F_X(x) \le u] \tag{4.664}$$

$$F_X(x) = \Pr[X \le \Psi(u)] \tag{4.665}$$

$$F_X(x) = F_U(u) \Rightarrow F_X[\Psi(u)] = u \tag{4.666}$$

It is usually supposed that the wave heights in a sea state follow a Rayleigh distribution of parameter λ, whose density and distribution functions are

$$f_X(x) = \frac{x}{\lambda^2}\exp\left[-\frac{\left(\frac{x}{\lambda}\right)^2}{2}\right] \tag{4.667}$$

$$F_X(x) = 1 - \exp\left[- - \frac{\left(\frac{x}{\lambda}\right)^2}{2}\right] \tag{4.668}$$

By making $u = F_X(x)$ and inverting this equation, a value of x is obtained, according to the variable u, which is a uniform variable $(0,1)$,

$$x = \lambda\left[-2\ln(1-u)\right]^{\frac{1}{2}} \tag{4.669}$$

The verification equation establishes a relation between the agents and the project parameters and the response of the structure or of one of its parts. Certain agents and parameters can be random variables with a distribution function that is known, but which is not uniform. Accordingly, the derivation of the distribution function of the response of the structure, which is needed to evaluate the failure probability, can be fairly complex. The numerical simulation can be used to generate a sequence of agent and parameter values, and the verification equation applied to obtain a sequence of responses of the structure. These can be used to derive statistical properties.

4.13.2.4.2 FIT OF THE DISTRIBUTION FUNCTION, $F_X(x)$

In order to estimate the failure probability, it is necessary to estimate the range of values for which the failure condition is fulfilled, namely, $G(x) \leq 0$. The estimate of p_f can be improved by fitting the distribution function only to the points x, for which the failure condition is fulfilled. Generally speaking, the failure is produced for values of the variable that are located in the tail of the distribution, which is where the fit is usually more complex because of the lack of data and because of the behavior of the random variable in the distribution tail. This is particularly important when one is working with extreme values, which because of the fit of the tails, becomes a critical element in the use of the numerical simulation method. In such cases, it is advisable to fit various distributions to the sample points and optimize parameters in order to obtain a curve with a better fit.

4.13.2.5 Generation of random numbers in other suppositions

This section describes the generation of random numbers for factors and discrete, mutually dependent terms.

4.13.2.5.1 GENERATION WITH DISCRETE VARIABLES

The inverse transformation method can also be applied to generate random numbers of a discrete distribution function, $F_X(x_i)$, using random numbers u with uniform distribution, $(0,1)$. For this purpose, the condition to obtain a value of x is that the value of u be located in the interval, $F_X(x_{i-1}) < u \leq F_X(x_i)$, which means that the value of the random number is x_i. This method is slow because each time that a number u is generated, it is necessary to evaluate the interval in which u is located to obtain x_i.

4.13.2.5.2 GENERATION WITH MUTUALLY DEPENDENT VARIABLES

In most cases, the verification equation is made up of a set of random terms, made up of agents and project parameters, $(X_1, X_2, \ldots, X_n)$. When these terms are statistically independent, the generation of random numbers can be carried out independently for each variable. When the *n* terms are mutually dependent, the random numbers can be generated in cascade, taking into account that the joint probability can be expressed as the product of the marginal probability of one of its variables and the conditional probabilities,

$$F_{X_1,\dots X_n}(x_1,\dots,x_n) = F_{X_1}(x_1)\,F_{X_2|X_1}(x_2|x_1)\ldots\ldots F_{X_n|X_1,\dots X_{n-1}}(x_n|x_1,\dots,x_{n-1}) \quad (4.670)$$

Thus, the generation procedure is the following: the value x_1 is obtained as the quantile u_i of the marginal distribution function of X_1. With this value of x_1, the distribution function of X_2, given X_1, is only a function of x_2. Thus, it is possible to obtain a value x_2, as the quantile u_2 of the distribution function of X_2, given X_1. Using the two values, x_1, x_2, the value of x_3 is obtained as the quantile u_3 of the distribution function of X_3, given X_2 and X_3. This process is repeated until a set of values, $(x_1, \ldots, x_n)$ of n dependent random variables has been obtained from a set of values $(u_i, \ldots, u_n)$ of n independent uniform variables.

4.13.2.6 Other generation methods with continuous variables

When it is difficult to obtain the inverse function, $x = \Psi(u)$, of a distribution function $u = F_X(x)$, and it cannot be derived by analytical methods, one must then use techniques such as the Box-Muller method, the rejection method, and the decomposition method. This is the case of normal, log-normal, beta, and gamma distributions.

4.13.2.6.1 Box-Muller method

This method permits the user to obtain value pair sequences of independent Gaussian variables, Y_1, Y_2, from the generation of two independent, standard uniform variables, U_1, U_2, by means of the following expressions:

$$Y_1 = \left(-2\ln U_1\right)^{\frac{1}{2}} \cos\left(2\pi U_2\right) \quad (4.671)$$

$$Y_2 = \left(-2\ln U_1\right)^{\frac{1}{2}} \sin\left(2\pi U_2\right) \quad (4.672)$$

4.13.2.6.2 Decomposition method

The total probability theorem can be used to express the density function, $f_X(x)$, of a random variable X by means of the weighted sum of a set of other density functions,

$$f_X(x) = \sum_{i=1}^{m} f_{X_i}(x)\,p_i \quad (4.673)$$

where $f_{X_i}(x) = f_X(X|B_i)$, $i = 1, \ldots, m$ are the density function components, and $p_i = \Pr[B_i]$ is the probability or relative weight associated with $f_{X_i}(x)$ for the component, B_i. In this way, a complex density function can be decomposed into a combination of density functions, whose distribution functions can be analytically reversed. In order to begin using this method, it is first necessary to generate *m* random numbers for each of the *m* weights, p_i, $i = 1, \ldots, m$, associated with each of the density functions $f_{X_i}(x)$.

4.13.2.6.3 Rejection method

The rejection method can be used to generate values of a random variable with a known density function $f_X(x)$, which can be evaluated, in contrast to its distribution function, which cannot be evaluated. This method first entails defining a function $g(x) \geq f_X(x)$ for all possible values of x of area *A*. The second step is to generate values of a random variable X from the function $g(x)/A$. The resulting random variable is accepted if, once a uniform variable u is generated, it is verified that $u < f_X(x)/g(x)$. In this way, the accepted values of x are uniformly distributed in the acceptance area, which is the desired distribution.

4.13.2.7 Numerical simulation applications

This section describes four cases in which the Monte Carlo simulation method was applied.

4.13.2.7.1 Simulation of the distribution functions of equation terms

The verification equation usually consists of the sum and difference of terms, which can be expressed by more or less complicated functional relations of the project factors, parameters, agents, and actions. In certain cases, the project factors are deterministic. However, in others, they are or should be regarded as random variables because of the uncertainties associated with their modeling, parametrization, etc. In such cases, to evaluate the failure probability, it may be necessary to derive the distribution functions of the project factors. In those cases where this is possible, the derivation can be performed analytically. However, this type of derivation is frequently not possible. In such contexts, the simulation can be used to derive the distribution function of the term.

4.13.2.7.2 Statistical sample

In Level I methods, the failure probability of a breakwater due to wave action can be approximated by means of the probability of exceedance of the calculation value of the agent, which is determined by a quantile of the distribution function of the extreme sea states or storm event regimes. This can be obtained from an *N*-size sample, which can be evaluated, according to the standard error. The consideration of prediction confidence bands can increase the reliability of the calculation. These statistics depend on the probabilistic model applied to the data, on the method used to estimate parameters and on the sample size N. In many cases, these calculations cannot be performed analytically or the data contain systematic errors. In such contexts, numerical simulation is a possible alternative to determine the properties of the statistical estimators by applying fit techniques between the different magnitudes.

In the case of the quantile of the extreme value distribution, it is possible to use numerical simulation to obtain a relation between the standard error of the quantile estimate with the probability of no-exceedance, the sample size, and parameter values for a given distribution function and a parameter estimate method. The result is an empirical formal equal to the formula obtained in the laboratory or in field studies.

4.13.2.7.3 Standard error in the estimators of the quantiles of an extreme distribution

Let us assume that it is necessary to evaluate the standard error of the estimate of the *q*-th quantile of the Gumbel Maximum, which has been estimated by the moments method. Consequently, the following procedure is followed:

M samples of size N are generated by each random number $x_{i,j}$ of the Gumbel distribution of parameters, β and α,

$$x_{i,j} = \beta - \alpha \ln\left(-\ln u_{i,j}\right), j = 1,\ldots,M; i = 1,\ldots,N \tag{4.674}$$

and where the $u_{i,j}$ are a sequence of independent, uniform random numbers $(0,1)$. For each of the M samples of N values, distribution parameters, α_j', β_j', are estimated by the moments method, and they are used to obtain the *q*-th quantile of the Gumbel extreme distribution function,

$$\xi'_{q,j} = \beta'_j - \alpha'_j \ln\left(-\ln q\right), j = 1,\ldots,M \tag{4.675}$$

The variance of the quantile estimate is

$$\mathrm{var}\left[\xi'_q\right] = \left\langle \xi'^2_{q,j} \right\rangle - \left\langle \xi'_{q,j} \right\rangle^2 \tag{4.676}$$

where the symbols, $\langle \ \rangle$ indicate the arithmetic mean calculated with the M samples. Similarly, it is possible to estimate the bias of the quantile estimate and the square mean error. The confidence limits of the quantile estimates can be obtained by supposing that the estimate is a normal random variable, $N(\xi'_q, \mathrm{var}[\xi'_q])$, and obtaining for each exceedance level the confidence interval of width, $100(1-\alpha_b)\%$, where $\alpha_b/2$ is the ordinate that limits the upper and lower area of the density function, which is outside of the confidence level.

4.13.2.7.4 Design of alternatives and optimal design

The Monte Carlo simulation method can be used for the design of alternatives and the selection of the optimal design. For this purpose, it is necessary to consider the economic analysis in the simulation by evaluating the costs of the initial construction, maintenance, and repairs of the structure during its useful life, and comparing this with the updated benefit obtained from its exploitation.