STUDIES OF THE MORPHOLOGICAL ACTIVITY OF RIVERS AS ILLUSTRATED BY THE RIVER FYRIS

INAUGURAL DISSERTATION

BY

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BY DUE PERMISSION OF THE PHILOSOPHICAL FACULTY OF UPSALA, TO BE PUBLICLY DISCUSSED IN LECTURE ROOM OF THE GEOGRAPHICAL INSTITUTION, MAY 23th, 1935, AT 10 O'CLOCK A. M., FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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CHAPTER III.

Erosion, transportation and deposition.

Erosion.

The force with which a running fluid influences its bedding, causing erosion and transport of any material there, depends upon the qualities and velocity of the fluid. For any certain specified fluid the *velocity* decides the erosion and transportation, other conditions being similar in the various instances. A great number of investigations have also been made in order to fix quantitative laws for the influence of the velocity. However, the results arrived at generally do not agree with each other very well. According to the various writers the same material is eroded at differing velocities. It is thus not surprising that, within the technic for solving practical hydraulic construction problems, the principle of using the velocity as an independent variable is being relinquished. An additional reason for doing this is that the conception velocity is somewhat indefinite, as surface-, bottom-, or average velocity may be referred to but not specifically mentioned. In the technic, one has instead endeavoured to make a change to more easily definable factors, such as slope, depth or hydraulic radius. French and German hydraulic construction technicists have introduced the conception »force d'entrainement», »Schleppkraft» or »Stosskraft», tractive force, in which these units are included, a conception which has been used with success in many cases. — But, not even by using that conception has it been possible to make any considerable approach to solving the problem of the bed-load movement. From a survey of most of the published data on bed-movement O'Brien and Rindlaub (1934) have concluded that none of the equations for critical tractive force or rate of bed-movement is, however, sufficiently reliable to be used for design.

From a geological point of view the introduction of the conception tractive force means no advantage, rather the contrary. The problem of explaining a certain series of layers or stratification would certainly be desirably facilitated by a knowledge of the depth, and such information

may sometimes be obtained by the aid of geological methods. But the slope and the hydraulic radius of the stream that has transported the material concerned are generally of no great interest, and can but seldom be determined. From a geological and geographical point of view it would appear more advantageous to relate the qualities of the sediments to velocity rather than to a product of other factors; this also from a mechanical point of view.

Relationship between erosion velocity and grain size.

Even though the principle of expressing the force of erosion and traction as a function of the velocity has, to a certain degree, been considered antiquated and out-of-date, investigations have been made during the last few years to make clear this relationship. In the following there are mentioned some points of view on the erosion, transportation and deposition of bedload based on old and new investigations. They are mainly caused by an endeavour to give a graphical picture of the relationship between the kind of material and the minimum erosion velocity, and would appear to be confirmed by the writer's observations in the field and in laboratory.

In order to express the relationship mentioned it is necessary first to more clearly define the variables, the velocity and kind of material. As far as the speed is concerned it would certainly, to obtain an exact result, be necessary to have a whole curve or formula stating the variation of the velocity according to the height above the bottom. As such a diagram is never obtainable it would certainly be preferable to use the bottom-velocity. But this is only stated in a limited number of cases, and is more difficult to decide than the surface- and average velocity. For these reasons the *average velocity* has been made use of, it being presumed that this is 40 % greater than the bottom velocity. This percentage depends inter alia upon the depth, but it has been presumed that this exceeds one meter. In shallower water the velocities stated here will be somewhat less, roughly about 10—20 cm./sec. less. — Greater demands as to exactitude cannot be satisfied at present.

The kind of material is, of course, characterized by the specific gravity, the shape and grain size of the particles. The last mentioned quality is undoubtedly of paramount importance, seeing that the shape has no very great effect, which is shown by experience, and the specific gravity is subject to but slight variations, 2.6—2.7. As indicated by modern investigations, for inst. Gilbert's in 1914, Schaffernak's in 1922 and Kramer's in 1932, the composition of the material as to grain size is of very great importance. For different relative relations of quantity between the grain sizes in various materials the corresponding erosion velocities will vary, also in cases when these sizes are the same. It may be this complicated influence of the composition of the material that causes the results of all investigations of the relation between the velocity of erosion, transport and the grain size to become so inconsistent, as mentioned above. For a graphical picture the least complex case has been selected, i. e. when the material is uniform, monodispersed. But also in such cases varying results were obtained. A body is put in motion at varying velocities, dependent upon whether it is on a rough or a smooth bedding. A severely defined and practical starting point is obtained by presuming that *a uniform material moves over a bedding of loose material of the same grain size*. Table 7 gives the values stated in the literature to correspond to erosion, i. e. a spontaneous starting of quiescent material under these conditions. This velocity will in the following be called *erosion velocity*.

The difficulties encountered when making such a comparison are firstly that it is not always possible definitely to decide whether the erosion velocity in question under the conditions stated really is that concerned, and secondly that the statements of velocity, depth and grain size occasion certain questions. The information selected and contained in the table is not all equally reliable, LAPPARENT's might be questioned seeing that in his observations the eroded material was not always moved over a bedding consisting of the same material. The same lack is the most common cause for other observations having to be excluded, and it mostly occurs when studies of natural rivers have been made. On the other hand laboratory tests must also be excluded for highly dispersed systems such as clays, as the stratification may have been changed due to silting. In cases where the surface-velocity has been stated, it has been reduced 20 % to obtain the average velocity, and — as already mentioned — the bottom-velocity has been increased by 40 %.

The question of varying velocities is connected with that of varying depths. The difference between the bottom-velocity, important with regard to erosion, and the average velocity used in practice, is increased with the depth. THRUPP (1908) has made a graph of the scouring power in relation to velocity and depth, Fig. 16 being an extract showing the course of a curve. It is, however, reproduced very reluctantly as it appears to be founded upon a rather limited amount of observation material, and as it is not for uniform material. Generally speaking, it might, however, be said to give a correct idea of the conditions, at least for limited depths when the material for observation is more comprehensive. The curve in the figure states the velocity for which coarse sand is moved. — The velocities given in Table 7 and in Figures 17 and 18 are for slightly varying depths, but in most cases a correction has been inserted when the figures stated have been for such limited depths as 1 foot by adding 0.2 m./sec. The

Table 7.

Erosion velocities for a monodisperse material on a bed of loose material of the same size of particles.

Author:	Characteristics of the material (by the resp. author)	Size of particles	Erosion velocity
		mm	cm/sec.
Etcheverry (Fortier and Scobey p. 951)	Stiff clay soil	(0.0015)	137
Fortier and Scobey	Stiff clay (very colloidal)	(0.0015)	130
» » »	Alluvial silts, when colloidal	(0.005)	130
» » »	» » » non-colloidal	(0.005)	76
Umpfenbach (Penck, 1894, p. 283)	Feiner Lehm und Schlamm	(0.05—0.1)	26
Etcheverry (Fortier and Scobey)	Very light pure sand of quicksand character	(0.13)	27
Gilbert, 1914, p. 69	Grade B	0.38	24
Lapparent (Schoklitsch, 1914, p. 25)	Schlamm, grob	0.40	15.0
Telford (» , » , » »)	Feiner sand	(0.45)	15.2
Gilbert, 1914, p. 69	Grade C	0.51	28
Lapparent (Schoklitsch, 1914, p. 25)	Sand, fein	0.70	20
Gilbert, 1914, p. 69	Grade D	0.79	34.1
» , » , » 70	Grade E	1.71	34.4
Etcheverry (Fortier and Scobey, p. 951)	Coarse sand	(2)	45 à 60
Schaffernak, 1922, p. 14		2	25
Sainjon (Schoklitsch, 1914, p. 24)	Kiesel	2.50	50
Gilbert, 1914, p. 70	Grade F	3.17	54
Schaffernak, 1922, p. 14		4	49
Gilbert, 1914,	Grade G	4.94	64
Schaffernak, 1922, p. 14		6	61
Gilbert, 1914, p. 70	Grade H	7.01	85
Schaffernak, 1922, p. 14		8	81
» ,		10	104
» ,		12	120—125
» ,		14	125—150
» ,		16	130—180
» ,		20	189—197
» ,		25	203—210
» ,		30	218—221
» ,		50	238
» ,		70	266—280

velocity statements may thus be said to cover depths of at least one meter. FORTIER and SCOBEY (1926) state this correction to be suitable. But the greatest difficulties have been encountered when the size of particle should be defined. The literature often contains such very indefinite statements as for inst. »large stones«. The Table has therefore been made to include both the information supplied by the writer in question and the numerical value of the size of particle stated in the Diagram. This has been put in brackets in Column 4 in case it was not given in the original. The valuation then made has, of course, occasioned a certain subjectivity due to the existing confusion in the terminology in this sphere.

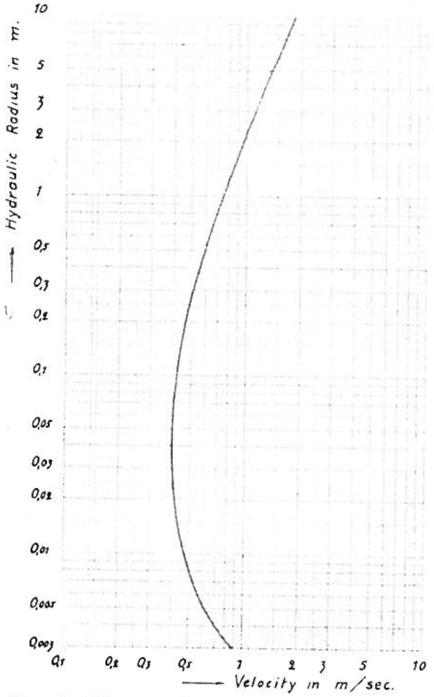

Fig. 16. The scouring power for coarse sand in relation to velocity and depth, according to THRUPP (1908).

These are the reasons why the curve in Figures 17 and 18 has not been shaped as a simple curve but as a zone. It must of course only be considered as an endeavour to make a preliminary comparison of the results obtained up to date, and may later be replaced by a more exact relation. But this will require much additional work.

In Figures 17 and 18 the values of the Table have been made the basis of a graph. (Note the upper curve.) The Figures show the same thing, but in Figure 18 the values have been dotted in a logarithmic scale in order to more clearly illustrate the interesting conditions connected with a small size of particle in a better manner than is possible in an ordinary scale.

The most noticeable deviations of the erosion curve in these illustrations from older accounts, for inst. SCHAFFERNAK'S (1922, p. 14) and S. A. ANDERSEN'S (1931, p. 33), is that it has a minimum and does not go down to the origin of the coordinate system. The minimum is not at the size of particle 0 but within the range 0.1—0.5 mm. This thus indicates that loose, fine sand, for inst. of quicksand character is the easiest to erode, whereas silty loam and clay as well as coarser sand and gravel, etc. demand greater velocities.

The great resistance of the clay to erosion was first strongly empha-

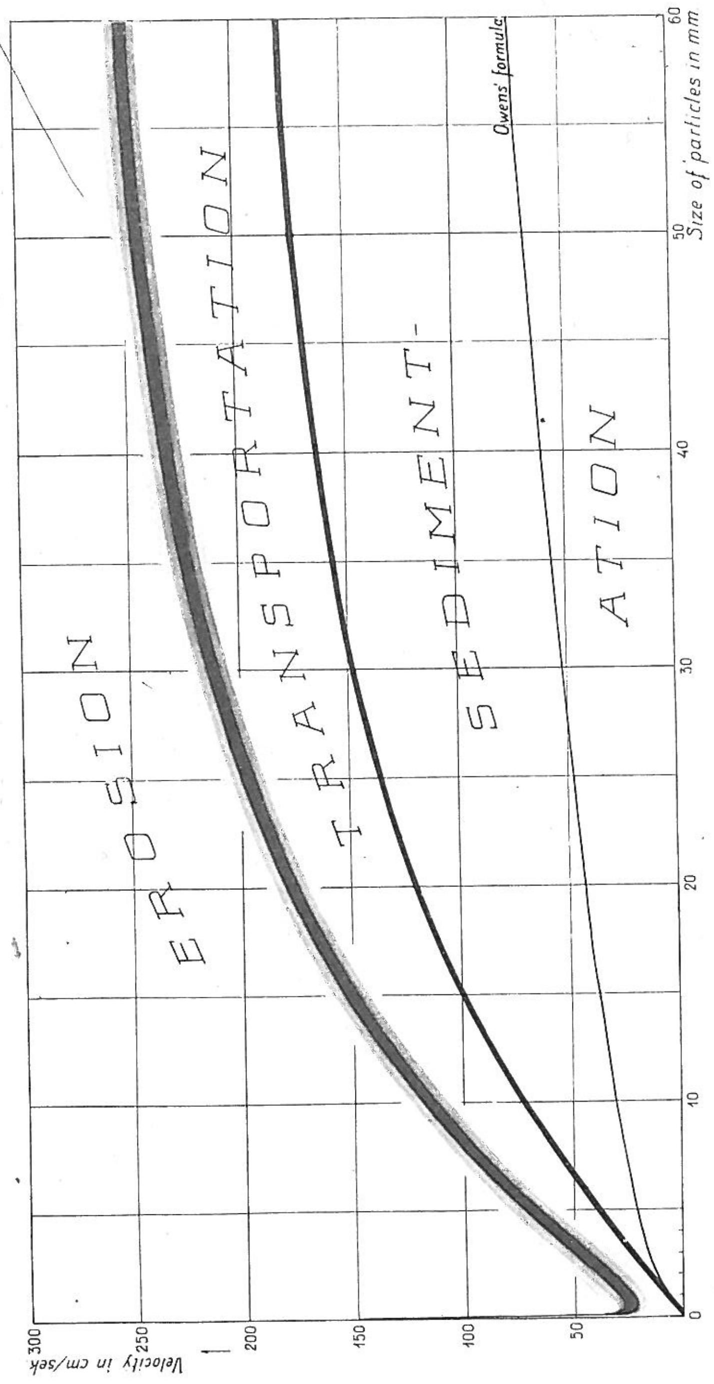

Fig. 17. The curves for erosion and deposition of a uniform material.

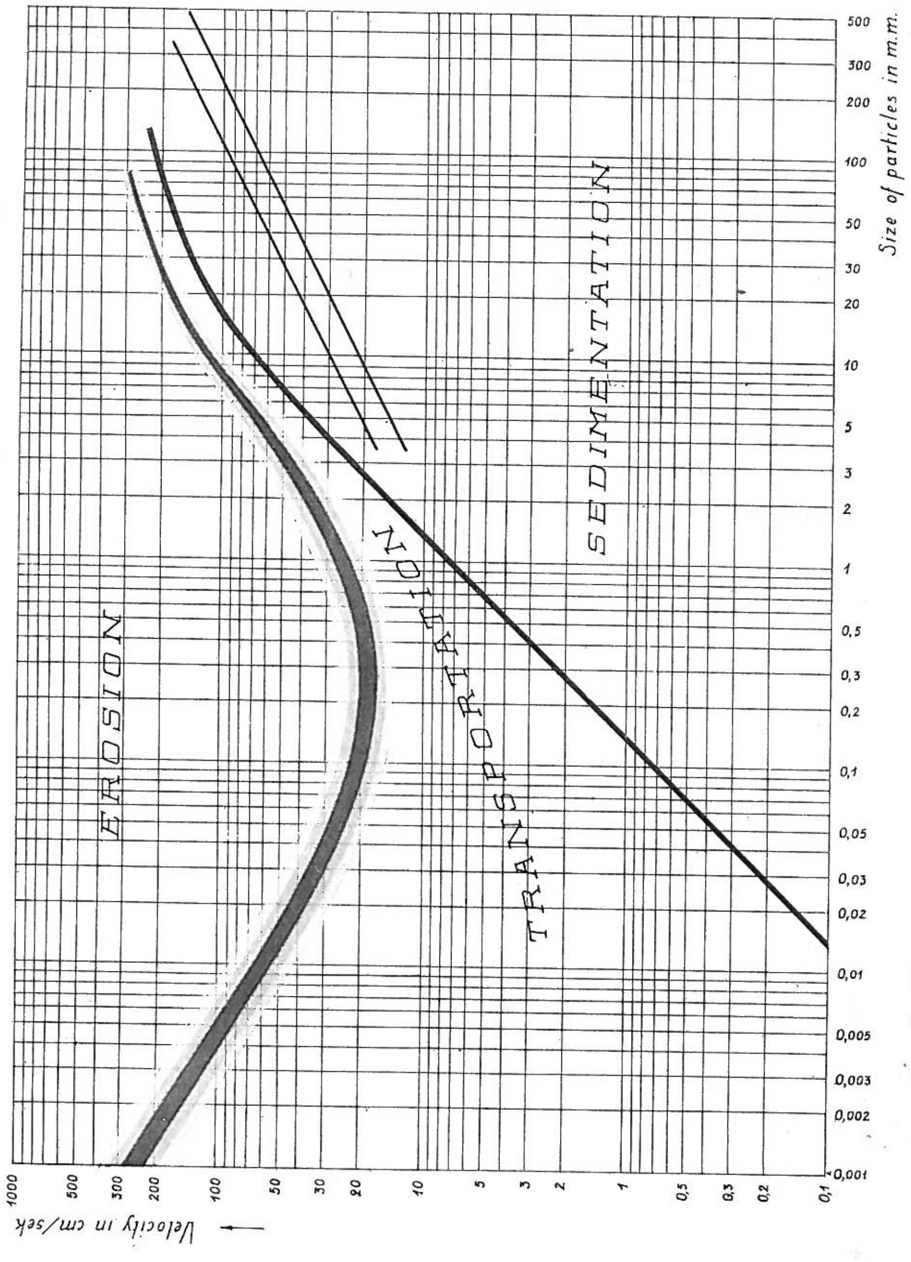

Fig. 18. The curves for erosion and deposition of a uniform material. (Logaritmic scale).

sized by the American hydraulic engineers FORTIER and SCOBEY in a paper on permissible canal velocities 1926.

This quality in clay of course depends upon the influence of cohesion and adhesion, which powers tend to unite the particles. The effect increases in line with an increased degree of dispersion, inter alia due to the number of contact points between the particles of a certain weight quantity thus being increased. It is therefore only when the size of particle is small that they become noticeable in the erosion. See for inst. DENSCH in Handbuch d. Bodenlehre, Vol. VI and BRENNER 1931.

In hydraulics it must certainly be considered of great importance to be able to calculate with these great erosion velocities for clay, for consequently the cross-section of for inst. a projected irrigation canal may of course be made correspondingly smaller and the cost of construction be reduced. Even though this condition was but recently pointed out, it is an old observation. FORBES (1857, p. 475) gives the following result of experiments tried inter alia on brick-clay from Portobello: »The brick clay in its natural moist state, had a specific gravity of 2.05; and water passing over it for half an hour at a rate of 128 feet in the minute, which was the greatest velocity I could conveniently obtain, made no visible impression on the clay. When this clay was mixed with water, and allowed to settle for half an hour, it required a velocity of fifteen feet in the minute to disturb it. This mud sank in water at a rate of 0.566 feet in one minute, but the very fine particles were very much longer in subsiding.» In a work by ETCHEVERRY[1], not obtainable in Sweden, quoted by FORTIER and SCOBEY, it is also mentioned that stiff clay soil and ordinary gravel soil have the same maximum mean velocity safe against erosion, namely 4.00—5.00 feet per second (121.9—152.4 cm/sec.). When engaged on engineering work for irrigation in India already in 1874—75, also KENNEDY (quoted from GIBSON 1919, p. 345), when publishing his oft-used formula for the critical velocity at which a long canal will maintain its channel in silty equilibrium, stated that this velocity is greater for loam and silt than for light or coarse sandy soil.

Finally, also CHATLEY (1921) in his silt-studies in China arrived at the conclusion that silty and clay beds will bear very much higher velocity than sandy beds. He has made the formula

$$v = \frac{0.02}{d} \text{ centimetres per second,}$$

v being = the erosion velocity and d = the size of particle. It is valid for grains held in place only by mutual cohesion, thus for the ascending branch in Figures 17 and 18, and agrees very well with these. CHATLEY

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[1] ETCHEVERRY: Irrigation Practice and Engineering. Vol. II: The Conveyance of Water. New York 1916.

states that the actual limits of velocity (200 cm/sec.) and grain-size (0.0001 cm.) in the Huangpu and Yangtze also agree with the formula. A qualitative graph of the variation of eroding velocity with size of particles also exists, in the main corresponding with the curves made by the writer.

Mixed materials.

As already mentioned these reflections are valid for uniform material. Certainly such material is not unusual, but generally, the tractional load of a natural stream includes particles with great range in size. Erosion velocity in this case means the velocity for which also the greatest size, that a *great* number of particles attain (i. e. *normal maximum* acc. to NELSON 1910, p. 21), is loosened from the bedding and removed. In his monumental work on the transportation of débris by running water (1914) GILBERT has shown how the erosion velocity[1] varies with mixed grades of the particles. He composed inter alia mixtures of two different grain-sizes with differing mutual relations of the weight quantities. The results of his experiments show that the mixture is easier eroded when an addition of fine material is made to coarser, and most easy when the mixture contains an average of 75 % of the finer sand. The tractability of the mixture then decreases to the value valid for the finer sort.

GILBERT (1914, p. 178) points out that when a finer grade of débris is added to a coarser the finer grains occupy interspaces between the coarser and thereby make the surface of the stream bed smoother. One of the coarser grains resting on a surface composed of its fellows, may sink so far into a hollow as not to be easily dislodged by the current, but when such hollows are partly filled by the smaller grains its position is higher and it can withstand less force of current. The larger particles are moved more rapidly than the smaller, a condition which the writer has always found correct when the velocity is not too violent and the particles roll or slide over the bottom. The traction then usually occurs in the shape of small stream ripples or dunes (»Transportkörper» acc. to AHLMANN, 1914 a). When the velocity is increased saltation and suspension are added. In these kinds of transportation the velocity of the coarser particles is less than that of the finer ones — a condition which may thus occur occasionally but not generally as RUBEY (1933 b, p. 498) appears to have interpreted DAUBRÉE's and GILBERT's statements. In the Karlsruhe laboratory the writer observed in 1931 in a testing channel, whose bottom consisted of natural sand from the Rhine transported in the shape of stream ripples, the following approximate velocities for the movement of the sand particles when the velocity of the water was about 50 cm/sec.:

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[1] Certainly, GILBERTS investigations concern the lowest transportation velocity, but the accordance with the results of other writers shows that they are valid also for the erosion velocity.

5 mm.'s grain size 30 cm/sec.
2 » » » 25 »
1 » » » 15 »

When the velocity is increased so that the water can transport also larger particles these will roll faster than the others, if once started. Nor do the larger particles stop now and then but roll incessantly. The small particles have a comparatively greater friction to surmount in the crowd of particles of the same size and will have a short moment of rest now and then, calculated in the velocity figures above.

According to GILBERT (1914, p. 173) the amount of increase in the transportation of the coarser débris appears to be greater as the contrast in fineness of components is greater, and in the extreme case the transportation of the coarser is multiplied by 3.5.

KRAMER's investigation (1932) of erosion and traction of three different kinds of sand completely confirm these observations. Sand composed of grain sizes 0.385 to 5.0 mm. is eroded at a lower velocity than sand of 0 to 5.0 mm.'s grain size. An addition of fine-grained material to a certain mixture increases the coarser material's tractability in the beginning, but at last a condition is arrived at when the added material cements the mixture and prevents the transport of the originally loose but now cemented grains. The reduction of the pore volume decreases the tractability of the material.

SCHAFFERNAK's curves (1922) also show the same thing. They clearly indicate the increased transportation due to a limited addition of fine material (Mischungstype II) compared with the reduction when an ample quantity of the fine material is added (Mischungstype IV) (see his Figs 10 and 12).

According to the works of various writers quoted above there is thus a tendency to decrease the tractability if a mixture of a sufficient quantity of the finer material is added, and this even if it is not so fine that cohesion and adhesion may be considered of great importance. The particles being very fine, their influence will of course be great also with a fairly limited concentration. As stated by FORTIER and SCOBEY the greatest effect is exercised by the finest particles, the colloids.

To this must be added that also the nature of the water affects the erosion velocity. Material carried along with the water has a purely mechanical effect, which may cause the bed-load to be stirred and thus make it an easy prey to erosion. FORTIER and SCOBEY report interresting experiences also in this respect. With the aid of all sources at their disposal, inter alia an inquiry to all hydraulic technicists, they have made a comparison of the maximum permissible mean canal velocities for varying nature of water, which Table is reproduced below (Table 8). The

Table 8.

Maximum permissible mean canal velocities. (From FORTIER and SCOBEY 1926).

Original material excavated for canal	Velocity in feet per second after aging of canals carrying:		
	Clear water, no detritus	Water transporting colloidal silts	Water transporting non colloidal silts, sand, gravel or rock fragments
Fine loam (non-colloidal)	1.50	2.50	1.50
Sandy » (» - »)	1.75	2.50	2.00
Silt » (» - »)	2.00	3.00	2.00
Alluvial silts when non-coll.	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when non-colloidal	3.75	5.00	5.00
Alluvial silts, when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, coll.	4.00	5.50	5.00
Coarse gravel (non-colloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hard-pans	6.00	6.00	5.00

velocities are valid for straight courses. »At sinous alignement a reduction of about 25 % is recommended. Likewise the figures are for depths of 3 feet or less. For greater depths a mean velocity greater by 0.5 feet per sec. may be allowed.» It is seen from the table that water transporting colloidal silts may generally be allowed to have a much greater velocity than clear water and water transporting non-colloidal silts, sand, gravel or rock fragments. The colloids »will make the bed all the more tough and tenacious, increasing its resistance to erosion.» FORTIER and SCOBEY also point out that »all experienced canal operators know the trick of holding muddy water above one chick structure after another until the mud has painted over the sides and bottom of a new canal, reducing seepage losses and making the bed of the canal less susceptible to scour.» In the discussion in the paper quoted, R. H. HART states (p. 961) that an important consideration is the position of the ground-water table with respect to the water surface of the canal. As long as the latter is higher, seepage is out of the canal, and there is a tendency for the finer materials to be carried into the interstices between coarser particles, thereby permitting a silting-up process. On the other hand, if the ground-water table is higher, as frequently happens, seepage into the canal takes place and the whole process is reversed.

It is also evident from the Table that the difference between erosion velocity for water transporting non colloidal silts, sand, gravel or rock fragments and clear water with no detritus is not so great except for coarse gravel, cobbles and shingles. The first-mentioned water in this case fills the interstices and the erosion velocity is increased. As regards finer material it has been observed that in the case that it is colloidal, it is less able to resist water with detritus than clear water. Conditions will be reversed for non-colloidal material.

These accounts show how complex natural erosion really is. It is not to be wondered at that the determinations of the erosion velocity have given such varying results. The deviations from the erosion curve in Figures 17, and 18 for non-monodispersed material may, however, be expressed in such a manner that values lying above the curve depend upon cementation with fine material, whereas values below the curve denote a less comprehensive mixture of finer components which smooth the surface.

Of the formulas that have been made and which do not agree very well, there is one by OWEN (1908, p. 418), which when re-expressed to be valid for a specific gravity of 2.7 and for cm. as a unit of length, reads

$$d = 0.0011\, v^2,$$

d being = the diameter of the particle in cm., and
v » = the erosion velocity in a special case, for the transport of coarser material over fine sand or clay.

This formula agrees remarkably well with the one obtained by JEFFREYS (1929) in a theoretical manner, see p. 268, which with the same designations as above reads

$$d = 0.0010\, v^2.$$

In the curves, Figs 17 and 18, OWEN's formula is graphically expressed as a fine line. The velocity in the formula in question is for the surface-velocity of a stream with a depth of water of 2.5 to 152 cm. It may be presumed to be approximately equal to the average velocity of a greater depth.

In this connection an interesting observation by W. W. RUBEY (1933 a) is worth mentioning, namely, that the current required to move a particle along the bottom of a stream (after OWEN's formula) is approximately the same as the settling velocity of the same particle in still water.

The description of the velocity as given here is certainly very approximate. It is not the average velocity that is decisive for the erosion but the velocity in the bottom-layer, where the increase in velocity in proportion to the height over the bottom is great, and where particles of different sizes are thus affected by different velocities. In a laboratory investigation M. Welikanoff (1932) aimed at a physical expansion of the erosion theory. He investigated the connection between velocity and grain size, Airy's law, and found that the said law is not fully correct. The grain size must not be put proportional to the square of the velocity. The formula should read

$$\frac{v^2}{g} = \alpha \cdot d + \beta$$

where g = the acceleration due to gravity and α and β are constants, β being dependent upon the depth. Welikanoff also found that for a small grain size other conditions occur, so that from a grain size of from 0.4 or 0.5 mm. the constants α and β have other values. According to Welikanoff the »$\frac{1}{n}$ potential function» cannot be made the basis of a more exact theory.

Though it is thus impossible to mathematically formulate a theory explaining the particulars of erosion the process would, however, appear to be fairly well explained in its main features. The active powers are the pressure of the water in the direction of movement and further the hydrodynamic upthrust and the effect of turbulence. The latter affects the water's direction of movement which becomes greatly variable. The vertical velocities of the turbulence become also of importance. When observing the movements of the individual particles the question soon arises as to what degree of effect may be attributed to the turbulence in this respect. The grains of sand appear to be lifted; this is also seen from the film made at the Karlsruhe River Hydraulics Laboratory. See also Schaffernak's (1922, p. 12—13) expressive description.

The pulsations of the water will be of very great importance for the erosion. When the velocity fluctuates the erosion will be by fits and starts. In addition to the value for the average velocity the force and frequency of the fluctuations are also of very great importance (see page 252).

The material loosened by the erosion is easily transported when once in motion. The coarsest material which the stream can transport is tracted as bed-load and the finer particles are carried in suspension. Saltation is a transition state between these two modes of transportation.

Erosion may occur when the water with constant velocity comes across material that the stream is capable of eroding. It may also occur due to increased velocity. If the eroded material cannot be transported in suspension it will in the former case only result in an increase of the bed-load. In the latter case, it may, on the other hand happen that part of the bed-load is put into suspension. The process of this change of new-eroded material or of bed-load to suspension has been treated above.

Erosion by running water in solid rocks.

The mechanical erosion by running water in solid rocks is generally assumed to take place

1) by »evorsion», that is to say by the wearing of excavations by eddying water with or without the help of stones, and

2) by the direct wearing of the solid rocks by silt-laden water.

The forms caused by the first-mentioned erosion process are especially characteristic pot holes, while, of course, the direct wearing process through its own nature becomes less noticeable from a morphological point of view. Maurice Lugeon (1914—1915) has shown that under certain circumstances, a marked »striage» is called forth in those parts of the bottom and the shores which are especially exposed to the current. They obtain an appearance which makes them deceptively similar to those wind polished rock-formations which are found in deserts.

The relative importance of the two types of mechanical erosion is difficult to estimate and certainly varies to a high degree with the consistency of the solid rock, with the velocity of the flowing water, with the mass of transported silt, etc. Chamberlain and Salisbury (1906, p. 140) describe pot holes as »a peculiar rather than important erosion feature», which certainly implies an underestimation. The importance of this type of erosion has especially been emphasized by J. Brunhes, who used the name »érosion tourbillonnière», also by Ahlmann (1914) and Ljungner (1930). There can be no doubt as to the great importance of the mechanical erosion caused by transported solid particles in eddying or directly flowing water. K. G. Gilbert (1875, p. 73) even goes so far as to say: »It is to be doubted whether pure water, or water with no mineral matter in mechanical suspension, has any appreciable erosive power. In the beds of streams of clear water, disintegration, if not due entirely to solution, at least depends so largely upon it, that the surfaces of calcareous pebbles are covered by spony films marking the depth to which the removal of the most soluble matter has extended».

In the following, the writer will point out or attempt to show that another, hitherto unobserved, mode of mechanical erosion by means of running water also exists. In the case of this type of erosion the presence of suspended matter is not necessary; it can only appear, however, with very high velocities, and occurs by means of corrosion and corrasion in connection with cavitation.

Briefly, cavitation implies that, with high velocities, hollows are formed in the water which are able to collapse with great violence, as a »implosion», while developing a very high pressure. Marked erosion of solid matter close to the place of »implosion» is thereby caused.

Cavitation or the formation of hollows is a phenomenon based on simple physical laws and has been known for a long time. It has been treated only very slightly in the field of physics, but it has been highly observed within the field of technics after its destructive influence on metal surfaces which are exposed to rapidly flowing water, as, for example, ship's propellors and turbines, and its lowering of the efficiency in hydraulic machinery was discovered in England in 1894. During the last 20 years cavitation has been the object of intense study in the field of engineering. Many important points have not as yet been made clear as is found from the following account of the most important papers about cavitation research, mainly ACHERET (1930, 1931 and 1932), WEINIG (1931), FÖTTIGER (1926 and 1932) and COOK (1928).

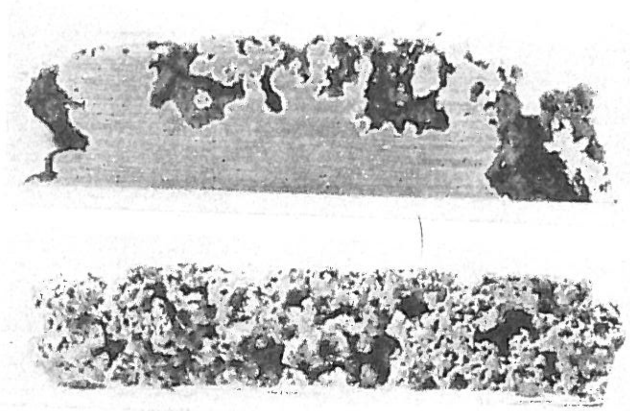

Fig. 19. Cast iron destroyed by cavitation-erosion.

Origin of cavitation.

Cavitation — the formation of hollows — presupposes low pressures within the liquid. Now the question arises as to how such pressure-reductions can be brought about in a flowing liquid.

BERNOULLI's equation for stationary potential flowing without friction or eddy-formations which, for further discussion, may be considered as approximately valid in this case. According to this formula the energy of the liquid is constant, that is to say, the sum of the kinetic, potential, and pressure energy does not change with time and has the same value at all points along the flowing-line (see for instance W. KAUFMANN, I, 1931, p. 48 or LAMB, p. 19):

$$p = \text{constant} - \varrho \cdot g \cdot z - \frac{\varrho}{2} c^2$$

where p = the pressure
ϱ = the density
c = the velocity
z = the geometric height over a certain zero-plane.
g = the acceleration of gravity.

As is seen the pressure becomes low if the sum of the potential and kinetic energy is large. Thus when the velocity is increased the pressure is decreased.

In natural water courses are always found gases, dissolved in larger or smaller quantities, above all gases of the air, especially oxygen, nitrogen, argon and carbonic acid. Their volume relation in solution is not the same as in atmospheric air. It should be especially observed that the volume relation between oxygen and nitrogen in water-absorbed air is 1 : 2, while in atmospheric air it is about 1 : 4. (KRÜMMEL, I, p. 293). Oxygen's absorption-coefficient is, namely, twice as large as nitrogen's. Thus air dissolved in water contains twice as much oxygen as atmospheric air. According to HENRY's law the amount of gas absorbed by the water is proportional to the pressure of the gas. If the pressure in the liquid is below the pressure which corresponds to the saturation at the temperature and the gas-content in question, the gas surplus is liberated in the form of bubbles. In water which at 760 mm's atmospheric pressure contains a volume percent of a gas, $\frac{p - p_d}{760}$ of the previous absorbed air volume will remain at the absolute pressure of p mm. Hg. Every volume of water has therefore delivered the gas volume

$$\frac{a}{100} \cdot \left(1 - \frac{p - p_d}{760}\right) \cdot \frac{760}{p - p_d}$$

measured at the existing pressure. This gas volume is indeed rather small; — water contains about 3 volume-percent of the air's gases — but can be imagined as playing quite an important role. — It ought at the same time to be pointed out that the conditions are somewhat complicated by the evaporation from the liquid by these gas bubbles.

This separation of gases which can often be observed in swift streams (air bubbles behind stones) causes no erosion. The actual cavitation appears only when the absolute pressure p on a water particle sinks nearly to or below the waters vapour tension p_d, at the temperature present, that is to say:

$$p \leq p_d.$$

The vapour tension p_d depends on the composition of the liquid and on the temperature, t. For water p_d is:

t°	0°	5°	10°	15°	20°	25°	30°
p_d	0.0063	0.0089	0.0125	0.0173	0.0236	0.0320	0.0429 kg. per cm².

As soon as the pressure sinks below these values hollows and cavities are formed in the water.

Cavitation always appears at those points in the liquid which have the lowest pressure, that is to say, close to the restricting walls, above all at cross-section reductions, where the velocity is great. When eddies are formed the lowest pressure prevails at the centre of the eddy, and their cavitation appears often in connection with the forming of whirl-pools.

The water is no longer a continuous medium — possibly with solitary air bubbles — but is composed of a gas- and a waterface. It appears as if the water is boiling in an open vessel at a medium temperature (OSBORNE REYNOLDS, 1894).

It is easy to calculate at which velocity this state appears by means of BERNOULLI's formula.

If one disregards the effect of gravity and assumes that the working pressure powers are formed only by the air pressure B, one will find that cavitation in a gas-free liquid appears, when the velocity reaches the value of

$$c = \sqrt{\frac{2(B-p_d)}{\varrho}}.$$

From this it can be calculated that for a pressure of 760 mm. and a temperature of 0 the *velocity* becomes *14.3 m. per sec.* — that is to say, a very high value.

However, this velocity decreases with the air pressure, that is to say, with the height over the sea. The following Table 9 and the Fig. 20 founded on it may give an idea of the velocities in question.

In column 2 the height above sea-level has been given which the selected barometer-pressures correspond to fairly well (according to HANN—KNOCH). It is found that the decrease of the boundary velocity with height is at first almost constant, 0.8 m. per 1000 m., but that it afterwards diminishes. Even at the height of 6000 m. the boundary velocity's 9.8 m. per sec., that is to say, ²/₃ of its amount at sea-level. The temperature's influence by the changing of the water's vapour tension and density is noticeable only at high atmospheric pressure; however, this factor vanishes completely when compared with the atmospheric pressure.

Table 9.

Limiting velocity for beginning of cavitation at different temperature and pressure.

B mm.	Height above sea-level	Velocity for beginning of cavitation 0°	10°	20°	30°
760	0 m.	14.3	14.3	14.2	14.1
714	500	13.9	13.9	13.9	13.9
673	1.000	13.5	13.5	13.5	13.5
632	1.500	13.1	13.1	13.1	13.1
594	2.000	12.7	12.7	12.7	12.7
557	2.500	12.3	12.3	12.3	12.3
523	3.000	11.9	11.9	11.9	11.9
459	4.000	11.1	11.1	11.1	11.1
402	5.000	10.4	10.4	10.4	10.4
351	6.000	9.8	9.8	9.8	9.8

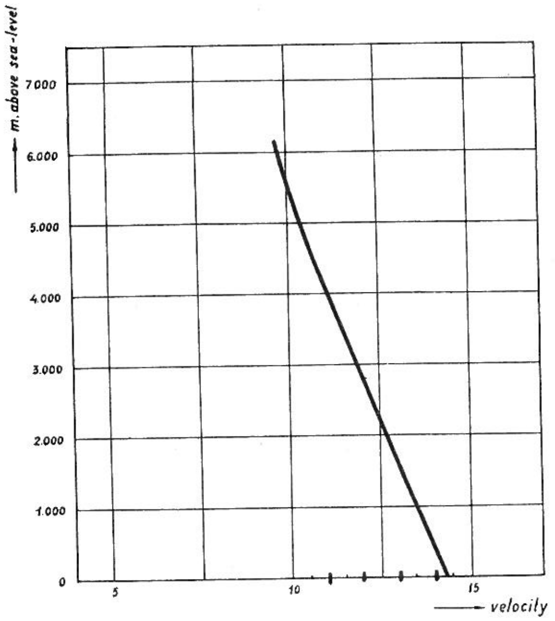

Fig. 20. Velocity for beginning of cavitation.

These values for the velocity are valid only if a liquid is eddy-free and gas-free. However, for turbulent streams with which we are here concerned, the pressure is changed very rapidly in an irregular matter. Therefore, if one calculates with the average for these pressures which characterize the flowing, one does not obtain the absolutely lowest pressures which are deciding for cavitation, but *higher,* as ACKERET (1930) has pointed out. The values given above for the velocity are therefore too high. It may, however, be assumed that the error is probably lower than 10 %. The previously mentioned gas percentage in the water also contributes to the lowering of the boundary velocity. This may therefore be set at about *12 m. per sec.*

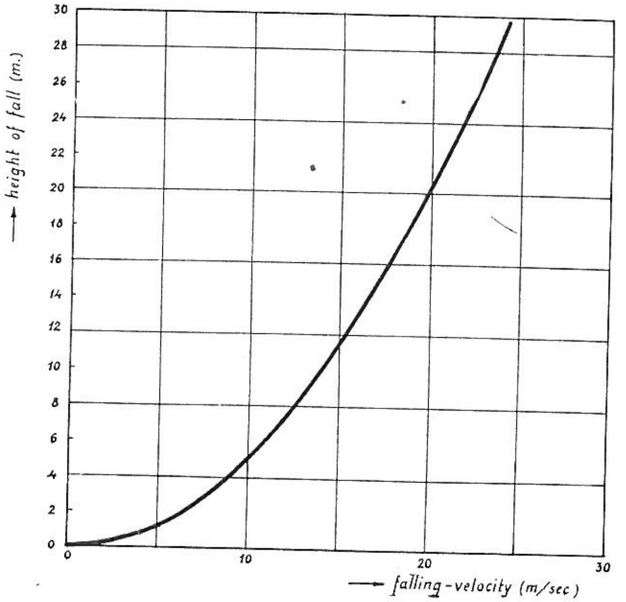

Fig. 21. The falling velocity.

It may be objected to that even after these reductions the velocity remains so high that it is encountered only very seldom in nature. Velocities of 12 m. per sec. are probably not very common. The average velocity in currents remains within rather narrow limits and attains in large rivers seldom more than 3 m. per sec., while in the wild mountain streams it rises to 5 or 6 m. per sec. (PENCK, 1894, Band I, p. 269). JOSEF FISCHER (1913) has given this matter a special investigation in which it is pointed out that the mountain rivers in Southern Bavaria *as a rule* attain velocities of 3—5 m. per sec. at high water (»hohen und höchsten Wasserständen»). It is therefore by no means unjustifiable to assume velocities above the cavitation boundary within these parts of a river course — at a not too low water depth — where the gradient is the greatest. In rapids and water-falls the velocities in question should appear

rather frequently. However, any direct measurements of the velocity have not been published. In a water-fall with a freely falling water-jet a fallingheight, h meters, is, however, required according to the elementary laws for uniformly accelerated motion without air-resistance (TORRICELLI'S theorem)

$$h = \frac{v^2}{2g}$$

in order to obtain the falling-velocity, v m. per sec. ($g = 9.81$). Fig. 21 shows graphically the relation between falling-height and falling-velocity. It is then found that at a falling-height of 11.5 m. the falling-velocity is 15 m. per sec., at 20. 4 m. the falling-velocity is 20 m. per sec., at 100 m., 44.3 m. per sec. etc. These falling-velocities are transformed, in the case of a horizontal surface, into horizontal velocities, and according to the results obtained within technics this transformation takes place without great loss of energy.

From this analysis it is found, without further explanation, that the velocities required for cavitation appear in water-falls quite often. Even in rapids they may occur when the water-mass is pressed together between obstacles on the river bed. But in rapids may also occur the previously mentioned (p. 257) maximum velocity which the water cannot surpass.

Disappearing of cavitation.

It is, however, the collapse of the formed holes, which in this connection is of the greatest interest.

If now no increase in pressure occurs, the hollows and bubbles con tinue their course until they vanish at the surface. But when the velocit decreases, the pressure is increased and the hollows collapse. Fig. 22 This collapse takes place with the production of very sharp noises an violent impacts because of the almost complete absence of elastic buffe influence. The increase in pressure always takes place very rapidly, (»Ve dichtungsstoss», FÖTTIGER 1926 p. 20 and following, ACKERET 1932, 234 and following, 1931, p. 468 and following). The pressure-condition which arise by contraction in a tube are shown by Fig. 23 (according t ACKERET, 1931). Cavitation at the narrowest portion of the tube whe the velocity is the greatest is apparent by the formation of a whi non-transparent foam. The pressure has there decreased to the vapou tension of the water at the prevailing temperature. When then, due an increase of the cross section (or some sort of dam~m~ing-up) the veloci is decreased, the pressure increases and cavitation ceases at a sharp marked line. Fig. 23 shows this sudden pressure-increase in the liqui

This sudden pressure-increase in a liquid at the increase of the area of the cross-section corresponds, in a natural stream, to the earlier mentioned (pp. 255—256) hydraulic jump (Wassersprung) which often appears at the transition from streaming to shooting state of motion.

The absolute amount of the increase in pressure is not great — according to Ackeret (1930, 1931) at its highest:

$$\frac{1}{4}\cdot\varrho_0\cdot v_0$$

where ϱ_0 = the density of the water
v_0 = the velocity of the water at the smallest cross-section of the tube.

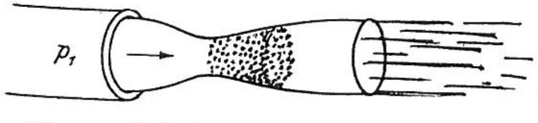

Fig. 22. Cavitation at a contraction of a tube.

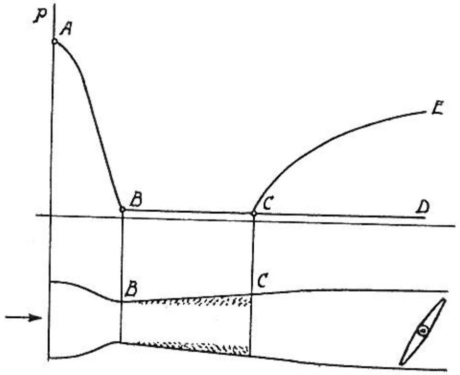

Fig. 23. The variations of pressure at a contraction of a tube.

A bubble, containing water-vapour and possibly even some of the water's dissolved gases, which passes, with great velocity, the place at which the pressure suddenly increases, is compressed very rapidly. Rapid instantaneous photographs show that the bubbles are thereby pressed together along the longitudinal line so that the back wall bounces against the front wall. It is at this blow that the above-mentioned high pressures arise. A calculation of their size is very difficult to make without a closer knowledge of the physical process. Parson and Cook (Cook, 1928) calculate the pressure to 10,000 atmospheres while assuming a vacuum in the hollows. Föttiger (1926), assuming isothermal compression, also attains high values, about 1,400 atmospheres. However, it is necessary to take into consideration the gas contained in the bubbles and also the increase in temperature. The hollows contain water-vapour and — to a smaller extent — other gases. Water may, at 0° contain 30 ccm. air per liter. At the sudden compression of the bubbles, an increase in temperature takes place which may be quite important. The increase in temperature causes, in its turn, the water to evaporate and the steam percentage in the bubbles to increase.

Ackeret (1930) has made a thorough calculation of the appearing pressure while giving special regard to the gas content and the increase in temperature, and has obtained values from several hundreds to thousands of atmospheres. But he points out that there is little probability that by means of theoretical procedure one could proceed further without experiments.

Direct measurements have not as yet been successfully carried out. However, also water-drops which move in the air with great velocity, for example, 40 m. per sec., have an influence similar to that of these bubbles in the water. The pressure, which is produced when a drop bounces against a solid surface and attacks it (see p. 229 above), has been successfully measured. P. de Haller (1933) has, by means of a pietzo-quartz-cell, shown that the arising pressures have a size of some hundred atmospheres. Because of the absolutely similar influence of corrasion it may be assumed that, at the ceasing of cavitation, pressures of the same size appear.

For the arising temperatures no measurements have been obtained. But H. Schröter's (1934) observation seems to be of importance in this regard, namely that rubber, which was exposed to cavitation, already after 3 min. was so heated that it partially melted away and was deformed. Since this is the fact with flowing water at room-temperature (or below room-temperature), the temperature must, of course, be very considerable at those points where, for very short moments, it is increased with lightning-speed. The heat conduction power is also small.

Finally, it should be pointed out that other than the above-mentioned pressure forces are at work, namely the capillary and electric forces. The bubbles become electrically charged by the presence of an electric dubble-layer. These capillary- and electric forces, which are also highly dependant upon each other, disappear at the collapse of a bubble, and contribute to the increase in temperature. The ozone odour which can be noticed near water-falls, indicates the presence of strong electric fields.

The destructing influence of cavitation.

These pressure-blows similar to hammer strokes cause a corrosive influence which is extremely feared within technics. Metal surfaces are eaten away with unparallelled speed. Above all, propellors, turbines and pumps are exposed to this type of corrosion. The time in which these attacked parts are practically unfit varies in certain cases between 1—2 hours' and a month's activity. (Föttiger, 1926, p. 27.)

Parts of a metal surface corroded by cavitation show a rather typical appearance (see Fig. 19). Thus is formed a highly gorged, porous surface which has the appearance of having been fretted; it is in rather sharp contrast to the polished appearance which is caused by the wearing away by sand. If the water contains sand, the surface may, of course, thereby be polished and evened.

Now it must be noticed that these corrosions are formed only at those places where the hollows collapse but not where they are formed or on the surface which they pass before collapsing. This has been interpreted as an evidence of the correctness of the conception that a purely mechanical phenomenon causes corrasion. The strong and continuous hammering of the surface must naturally have a destructive influence even if the force of these pressure blows lies below the hardness of the matter, »Ermüdungsfestigkeit». This may moreover be completely different in river water as compared to air. The corrasion of glass, the dependance of high velocities in the water etc. — all these experiences indicate, according to the general conception, that the mechanical influence has a deciding importance. (Weinig, 1931, p. 928.) But the physical-chemical influence should not be overlooked and merely regarded as secondary. In the hollows gases are also present. At high pressures and high temperature a strong active chemical influence must be considered, especially that of oxygen. Furthermore, a disintegration of carbonic acid gas into carbonid oxide and oxygen may be assumed; oxygen may combine with nitrogen and form oxygen-forming oxides etc. The mixture of these chemically active gases now reachs a higher pressure and a higher temperature in the same degree as the compression becomes more violent, which usually depends on the velocity of the flowing water. At high velocities the chemical activity is highly increased — the reaction velocity is increased rapidly with temperature. Therefore, the influence of the increase in velocity can hardly be interpreted as evidence of the correctness of a purely mechanical explanation of the phenomenon. The weakening of the matter caused by the pressure blows becomes apparent by, among other things, the formation of microscopically small cracks in which the gases, of course, find an extended action-field.

An estimation of the relative relation between the mechanical and the chemical influence cannot be made, but the facts which have hitherto been published do not seem to justify a simply mechanical interpretation of the destructing influence of cavitation.

If the flowing liquid contains solid particles in suspension their corrasive influence will be increased enormously. The collapse of the hollows takes place at high velocity. A particle which is close to the collapsing liquid-wall receives an acceleration which is of very short duration, it is true, but which is, on the other hand, very strong, against the solid surface which it hits with great velocity. In accordance to an above quoted calculation of the pressure, according to Parson and Cook (Cook, 1928), a wall of a collapsing hollow should, in their example, have at a certain moment a velocity of 730 m. per sec., that is to say, double the velocity of a rifle bullet. It is certainly too high a value, but some ten m. per sec. may be considered to correspond to the force of the measured pressure blows. It is, therefore, evident that the particles in suspension are accelerated up to very considerable velocities just at the moment of their being thrown against the solid surface and must contribute to the hammering of the solid surface in an effective manner.

The importance of the material. What power of resistance do the different types of matter have against the influence of cavitation?

This question has been the object of an especially intensive study within industry and technics of what concerns the permanence of different types of metal. According to Springorum (Hydraulische Probleme, Seite 231) it has been shown that the structure is of deciding importance. A rough cristalline alloy, for example, is attacked violently in a short time even if it is extremely hard, while a structure, as far as possible fine-grained, with a velvety appearance resists all attacks incomparably better.

No investigation of the resistance-power of different types of rock against erosion in connection with cavitation has been made either directly or relatively in relation to metals. There is no doubt that solid rocks are corroded as an example exists of the corrosion of quartz. According to Föttiger (1932) quartz used in a membrane for under-water signalling was rapidly destroyed. This has in this case, of course, taken place under very special conditions, but the phenomenon is quite the same.

The above mentioned example is the only one the writer has been able to find mentioned in connection with cavitation-erosion of a mineral, and as this is applicable just to hard quartz one is justified in assuming that erosion can take place much more markedly in the case of other minerals and even hard rocks.

What forms may now be expected to appear through the above described corrosion-influence? The literature on cavitation offers no information on this question which is important from the geographic and geologic point of view. The conditions for its appearance indicate that a great many varying forms are to be expected.

The collapse of the bubbles at the sudden increase in pressure can, naturally, occur directly in the liquid without any influence on the solid surfaces which confine the current — this may be expected to be the most frequently occuring case. In order that any corrosion and corrasion may occur, the local flowing conditions must be such as to force the bubbles very close to a solid surface at the instant of the collapse. The occuring erosion form, therefore, depends on the appearance of the section between the limiting solid surface and the surface — or rather the disk — at which the collapse takes place. But the section between two surfaces is, of course, a line and one might therefore expect a groove, straight or curved. If the limiting surface has the form of a rotation-surface, the surface of contact may assume the shape of all kinds of conic sections, as, for instance, ellipses, hyperbolas, parabolas, or straight lines. In the experimental investigations it has been shown that the limit at which the gas bubbles disappear does not have an altogether constant position but oscillates about an average position. In laboratory tests with small water-masses the oscillating remains within 1 cm. In natural water courses it may be expected to be larger. Moreover, the limiting surface may change its position and be displaced forwards or backwards and change its gradient with variations in the velocity of the water. The corrosion therefore takes place along a zone, the breadth of which varies with the angle between the back limiting surface of the cavitation field and the solid limiting surface. The breadth of the zone also increases with the length of the time of the cavitation as SCHRÖTER (1933) has convincingly shown. Experiments by SCHRÖTER show that erosion by cavitation may also work along a surface (op. cit. Fig. 22).

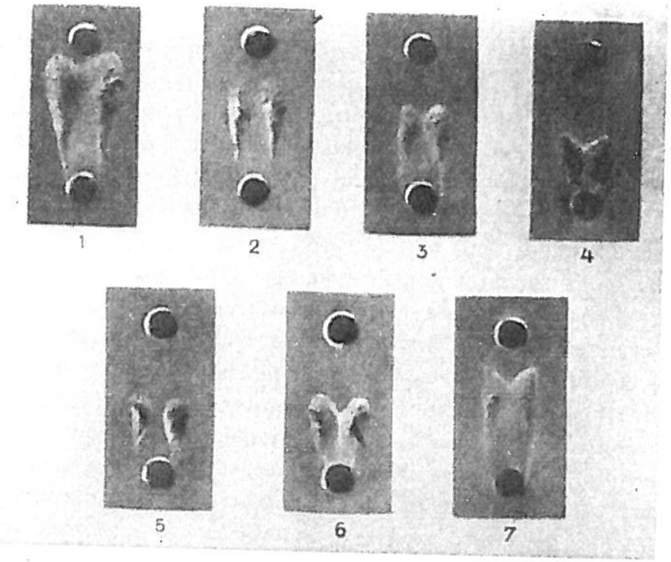

Fig. 24. Forms caused by cavitation-erosion. The direction of the water is upward. According to P. DE HALLER.

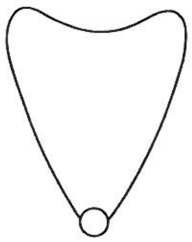

Fig. 25. A usual form caused by cavitation-erosion.

A very common form of cavitation-erosion influence is shown in Fig. 24 (P. DE HALLER, 1933, Abb. II, p. 260). It shows the appearance of a metal plate which has been exposed to cavitation in a special test arrangement, consisting of a rectangular tube with rapidly flowing water. Cavitation is produced by a bolt fastened in the lower hole, and the water flows, in the picture, from below upwards. The bolt mentioned, which thus rises above the plate, probably causes cross movements in the water which influence the appearing forms. These are therefore shallow hollows in the metal surface and have forms which in their appearance vary greatly. But they may all be imagined as included in the heart-shaped figure which is shown in Fig. 25. It is drawn after the pictures given by ACKERET, SCHRÖTER and DE HALLER.

SCHRÖTER also shows how a hole in a metal surface (1933, Fig. 19) causes cavitation-erosion in the shape of a ribbon in the direction of the stream.

Erosion of solid rocks by cavitation.

In a natural current, cavitation may be thought of as occuring at some obstacle. There the currents are forced together, the velocity increased, and the pressure decreased so that cavities in the water appear. The bubbles continue with the current, and when the velocity decreases, they collapse.

Glacio-fluvial erosion by cavitation.

The most favourable conditions for the occurance of cavitation are found where the greatest velocities are found. These, in their turn, appear, as has been mentioned, in water-falls and in water under hydrostatic pressure, for instance under a glacier. When the great ice-caps during the quaternary glaciation reached the period in their recessive state when their temperature was no longer too low for the existence of running water in their interior, a very high hydrostatic pressure must have been prevalent there. Water was then flowing along the bottom, at a very high velocity, more or less concentrated to tunnels. That the hydrographic static pressure must have been very important is apparent from the fact that the flowing direction of the water is independant of topographical irregularities (LJUNGNER, 1930, p. 399). The velocity of the water has certainly at many points been above the cavitation-velocity.

Do then forms of erosion within the ice-capped areas appear which may be taken as having been caused by *cavitation-erosion?*

In the most detailed description which exists of glacial and fluvioglacial forms of erosion in solid rock, namely LJUNGNER's (1930), is found a thorough description of several such forms. The most important of these is the peculiar formation which LJUNGNER describes under the name »Sichelwanne» (the sickle-trough). As Fig. 26 shows[1] it consists of a hollow in the solid rock of a groove- or bow-shaped appearance.[2] As is seen it is

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[1] The photograph has been kindly put at my disposal by Dr. LJUNGNER.

[2] The writer has observed them in the fjord of Oslo and in Bohuslän.

highly similar to those forms obtained through erosion by cavitation, and it is easy to assume that it has been formed in the manner mentioned. The similarity is quite striking in spite of the somewhat different conditions during its formation. The erosion marks, shown in Fig. 24 and Fig. 25 have, as has been mentioned before, been formed by cavitation behind an obstacle rising somewhat from the corroded surface which was not the case with the sickle-trough (»Sichelwanne»). This may, on the other hand, be thought of as having been formed, for example, behind a morain-block

Foto *Erik Ljungner.*

Fig. 26. The »Sichelwanne», according to LJUNGNER.

fastened to the under surface of the ice, which is in contact with the solid rock surface. LJUNGNER mentions the presence of »Sichelwannnen», drawn out in the longitudinal direction. The obstacle which caused the cavitation can here be thought of as having been removed, following the movement of the ice. The formations mentioned often appear in groups; LJUNGNER's example of their spreading in the flowing direction of the water is especially interesting. Several blocks, one after the other, may be thought of as having caused cavitation — or the obstacle may have been transported. Here may perhaps lie a possibility of determining the movement of a block in the lowest layer of the ice. A closer knowledge of the influence of cavitation here may perhaps lead to important results.

Erosion by cavitation in natural streams.

The writer has not been able to make investigations into the appearance of the erosion forms in question. In the Fyris the conditions for their appearance are not present, and consequently the forms do not exist there.

However, in the foregoing it has been pointed out that the necessary and sufficient conditions are present in many water courses. The knowledge of the nature of the thus formed erosion forms is, however, as yet so negligible that a pointing out of the influence of cavitation-erosion at the formation of present forms may cause difficulties. Furthermore, the following condition should be taken into consideration. The forms caused by the type of erosion in question have little resistance against the attack of other kinds of erosion. If, therefore, an erosion form is caused mostly by cavitation-erosion, a further small erosion, for example, wearing away by transported mineral matter, may alter its appearance so that the impression is obtained of the latter erosion type being the only type prevailing.

If a river passes through a large lake in which all its matter is deposited and thereafter flows over a water-fall or a rapid, no erosion should take place there other than that by direct solution of the rooks, according to the current theory as it has been formulated, for example, by GILBERT (see p. 305 above).

The complicated appearance of the erosion forms in a natural stream running through solid rock indicates a rather varied influence on the solid rock. JEAN BRUNHES (1902) has in his treatise: Le travail des eaux courantes: La tactique des tourbillons, taken his examples partly from granite islands in the Nile at the first cataract, and partly from the north slope of the Swiss Alps. He presents interesting pictures from both territories. In these pictures phenomena are found which might easily be interpreted as marks from cavitation-erosion.

An erosion form at which cavitation-erosion may have been at work is shown in the facetted surfaces which GILBERT (1875) pictures in WHEELER's Report. No acceptable explanation of these phenomena has been presented. However, in order to be able to present complete evidence it would be necessary to make an experimental investigation.

From the geographic and geologic view-point it is to be regretted that in earlier laboratory investigations attention has not been directed to these appearing forms. However, from what has been done in this field, for example by SCHRÖTER, DE HALLER and ACKERET, it has been found that the erosion forms have a rather varied appearance. But they all consist of hollows and cavities.

These may possibly serve as the first beginning of pot-holes. In natural streams a cooperation always exists between the three types of erosion; erosion by cavitation, eversion and wearing by transported mineral matter. — However, the large principal difference which exists between the first mentioned type of erosion and the other two should be observed: the former does not require the presence of mineral matter in the water and works very effectively without it. On the other hand, the last two types can hardly come into action if such is the case.

The relative effectivity of the three processes certainly varies greatly with different types of rock. But it must be regarded as being probable that erosion by cavitation plays a rather important part in the forming of canyons.

Deposition.

It has long been known that the velocity for which a certain transported material is deposited is another — and lower — one than the erosion velocity. This is evident from SUCHIER's surveys as early as 1883. PENCK (1894, p. 284) has on that basis arrived at the approximate value 1.4 for the relation between the erosion velocity and the lowest velocity for transport only, which latter velocity coincides with the velocity when deposition begins. This border velocity will in the following be called the *lowest transportation velocity* to avoid the expression sedimentation velocity which is used to denote the velocity with which a body falls in water.

In the literature there is considerably less information to assist in determining the lowest transportation velocity than for the erosion velocity. The very severe condition that the deposition is to be on a material of the same kind as the deposit itself can scarcely be fulfilled by any other than SCHAFFERNAK's investigation. In that investigation the relation between the two velocities was found to be rather near 1.50. The lower transportation velocity is thus 2/3 of the erosion velocity. In Figures 17 and 18 this velocity curve has been inserted after SCHAFFERNAK's Figure p. 14.

This curve differs from the erosion curve in the manner that it makes no bend upwards for the smallest grain sizes but falls to very low velocity values for the grain size 0, (WELIKANOFF 1932), indicating that fine material is deposited at these low velocities. But here the curve is extrapolated; SCHAFFERNAK's smallest particles had a diameter of 5 mm. One is inclined to presume that the curve in this instance approaches OWEN's curve. That is nearly identical to JEFFREY's formula for the upthrust. It is that power that is decisive for the deposition of these fine particles. The turbulence has generally discontinued at these low velocities; the particles have sedimented to the bottom layer and it is the value of the upthrust that decides whether they are to remain inert on the bottom or not.

The lowest transportation velocities for mixed materials are difficult to define, as there are very few direct measurements to decide the matter.

Table 10.

Difference between erosion velocity and lowest transportation velocity in percent of the erosion velocity. (After KRAMER.)

Material	Slope	Depth (cm.)	Velocity-difference in %
Sand I. 0—5 mm.	1: 800 1: 1.000	3.57—4.12 4.73—5.30	14 30
Sand II. 0—1.77 mm.	1: 400 1: 600 1: 800	1.60—1.64 2.94—2.50 5.53—2.86	26 28 30
Sand III. 0.385—5 mm.	1: 600 1: 800 1: 1.000	2.88—2.93 4.22—3.75 5.20—4.83	9 5 1

KRAMER has made observations »bei fallenden Wasserständen — um das Aufhören der Geschiebebewegung festzulegen». Unfortunately it is not quite clear whether his lower limit for the starting of the transportation due to increased water velocity corresponds to the conception of erosion velocity used here, which is also necessary if comparisons are to be made. But presumably that is the case. From KRAMER's tables has been calculated the proportional amount of the necessary decrease from erosion velocity to the lower transportation velocity, for which deposition is commenced. The decrease varies for varying slope or depth, as will be seen from Table 10 below. Sand I there denotes a mixture of well-polished quartz-particles of the sizes 0—5 mm. Similar to the following, however, it contained no colloids; according to the curve no grains of less than 0.1 mm. In sand II the coarser components were removed (grain sizes 0—1.77 mm.), and in sand III the finer ones (grain sizes 0.385—5 mm.).

As will be seen from the table the difference in velocity is the least for sand III, from which the finer grains were removed, and the greatest for sand II, consisting of fine particles. This probably depends upon the great increase of the erosion velocity required for the finer material due to cohesion. Sand III covers a more comprehensive range of sizes of the sand particles; the result may be that a limited decrease of the velocity may cause the largest particles to stop. Sand I, containing all the grain sizes, has a value for the difference in velocity which is between these two values but nearer that for the finer sand.

According to other investigators (KRAPF and KREUTER) the tractive force causing the bed-load to move is 30 % greater than that for which deposition occurs (KRAMER 1932). This would correspond to an approximate difference in velocity of about 12 % in the case as above. This is, however, disputed by SCHOKLITSCH's (1914, p. 34) unauthenticated statement: »Eigene Messungen im Versuchsgerinne zeigten dass dieser Unterschied vom Gehalt der Sohle an feinem Zwischenmaterial abhängt, und dass er für feines gleichmässiges, rein gewaschenes Geschiebe ohne Zwischenmaterial nahezu gleich Null ist.»

Keeping to published measurements it would, however, appear that for a uniform material af the same composition as that of the bottom the velocity may be reduced to the curve in Figures 17 and 18, as per SCHAFFERNAK, without deposition occurring. For a mixed material the deposition begins for greatly varying velocity-decreases, dependent upon the quantity of fine material and upon the size of the largest particles in the mixture. In this case there is probably a very complicated co-activity between several factors. Consequently, the results differ so greatly that according to different investigators the velocity may be reduced from the figure for erosion by 54.5 % (acc. to DUBUAT) or only 1 % (acc. to SCHOKLITSCH and KRAMER), before deposition commences. Another cause that contributes to the varying statements is that the erosion velocity may have been calculated in different manners, i. e. either as the velocity causing the finest material to start moving or, as the velocity when not only the finest material is transported but for which all grain sizes up to the normal-maximum are put in motion. However, it is generally the latter conception that has been intended.

OWEN's fixation of the erosion velocity for comparatively large particles on a smooth surface lacks parallels for the lower transportation velocities, excepting SUCHIER's results above mentioned. According to them the relation between the two velocities was 1.4. But SUCHIER's erosion velocity is rather much above OWEN's; it agrees very well with the erosion-curve for a uniform material as shown in Figures 17 and 18. In this case the difference undoubtedly depends upon a whole layer of loose material moving over a smooth surface, whereas in OWEN's experiment single stones were placed on the bottom. In the former case, there was a friction against the neighbouring particles too, the friction thus possibly attaining almost five times the value as compared with the latter case. A spherical particle may have seven contact-points in the former case and but one in the latter.

Better values not being available, we shall use SUCHIER's 1.4 for the velocity-quotient, corresponding to a proportional decrease of the velocity from erosion to deposition of 29 %, which is also a good average value. This value being introduced into OWEN's formula, it is found, seing that $(1.4)^2$ approximately equals 2, that *in the velocity for which a certain grain size is eroded deposition occurs of grains of twice that grain size.* This thus holds good, approximately, for the transportation of coarse particles (> 7 mm.)

over a smooth sandy bedding, but according to Figures 17 and 18 also approximately for a uniform material moving over a bottom of the same material on condition that the grain size of the material is between 8 and 30 mm. It would thus appear to be a fairly common condition. The above term is independent of the validity of OWEN's formula, and only provides for the grain size being in proportion to the square of the velocity (acc. to NEWTON, AIRY, and others; see SCHOKLITSCH 1914, p. 200, 40—45), and that the relation between erosion velocity and lower transportation velocity is 1.4.

Problems concerning the stratigraphy of the deposit.

The viewpoints that have been expressed here concerning erosion and deposition may assist to explain some of the peculiar stratification conditions and irregularities often found in for inst. glacifluvial strata. One remarkable observation is that coarse glacio-fluvial material and whole eskers, rest on clay. See for inst. NELSON, 1910, p. 145 and 146, HÖRNER, 1927, and SANDEGREN, 1929, with a discussion by ASKLUND, CLAESSON and RUDEBERG. Considering the old conception that the erosion velocity depended upon the size of the particles it appears peculiar that the clay was not eroded away by the considerable velocity that must have occurred when the superposed coarser material was transported.

A look at Figures 17 and 18, however, will show that such a transportation and super-deposition is rather easy to explain from a dynamic point of view. The question as to how the increase in velocity, noticeable from the character of the sediments, can have occurred involves a special series of problems, which must be solved separately.

The transportation of a certain kind of material over a bedding of loose material of another mechanical composition is of rather great interest.

The deposition pre-supposes transportation of the material concerned. And this transportation in turn must be the work of a certain water-velocity. In the event of the material not being too small-grained, this velocity may be so great that there is a risk of the bedding being eroded. In other cases this will not be the case.

Let us first examine the case of erosion. The loosening of the material as expressed by *erosion* causes a mixture of the coarser transported material with the finer, eroded material. This mixed material now acquires another — and usually greater — mobility than the materia originally transported. GILBERT's experiments, mentioned on page 300—30 above, show that the mobility of a mixture increases up to the point wher the finer material (i. e. what has been eroded, in this case) amounts t approx. 75 %. The material is thus rapidly removed and the bedding is quickly eroded — as long as the composition remains the same. Unless the layer affected by the erosion is very considerable it will thus be removed rather quickly. We can, therefore, not expect very often to strike deposits of the coarser material on a finer bedding of such a kind as may easily be eroded. When this occurs, there will be a transition zone, containing a mixture of the two kinds of material and no clearly defined contact-surface. The thickness of the transition zone may be expected to increase somewhat downstream.

In the cases when *no* erosion occurs a closer study reveals that there are two differing series of combination possibilities. A given material, with a grain size of for inst. 1 cm., may first of all be transported and deposited over a bottom consisting of loose material from blocks down to a certain smaller grain than the material itself. On the other hand a bottom consisting of smaller grain would be eroded. Not until the grains are considerably smaller than 0.1 to 0.5 mm., for inst. clay, may transportation without erosion again be possible, this material not being so easily eroded.

Fig. 27 illustrates and defines these conditions. The Figure is a schedule on the basis of Fig. 18, *a* denotes the erosion curve, *b* the curve for the lowest transportation velocity, both for a uniform material, *c* is the curve for the lowest transportation velocities of coarser grains on a smooth bedding. We examine which material a grain size characterized by the abscissa 1 can be transported without erosion. The ordinate 1—2 is drawn till it reaches the curve *b* at 2. The lowest velocity has then been determined for which transportation may still occur and deposition just begins on a bottom consisting of loose material of the same grain size. Transportation and deposition of the given material may of course take place on a coarser bedding but also on the same material, seeing that this is not eroded until the velocity is greater, as denoted by point 3, and even on finer material. For the velocity for which deposition commences, erosion of a considerably finer grained material takes place; the size of the particles for this material is obtained by drawing the line 2—4 parallel to the axis of the abscissa. It is thus denoted by grain size 5. According to what has previously (p. 322) been explicated this is approximately half of grain size 1. Here it may, however, be pointed out that in this case it will undoubtedly be somewhat smaller, the transportation being easier on the slightly finer material. Curve *b* should be a little lower. In the lower part of the figure a coarse striation denotes the range within which transportation and deposition of material 1 may occur without eroding the bedding.

Another similar range is also shown by the figure in the smallest grain size groups. Transportation and deposition of fairly coarse material perhaps also over silty loam and clay. Here again, as has been mentioned, we find a higher erosion velocity due to cohesion forces. The transportation of coarser material over a bed consisting of clay must however be looked upon as if it were over a smooth surface. To decide the lower transportation velocity we must therefore not start from curve *b* as in the former case but from curve *c*. Point 6 denotes that velocity. This is the erosion velocity for the grain size denoted by abscissa 8. Smaller sizes of particles have a greater erosion velocity. The transported material mentioned may thus be transported and deposited on a fine material of a grain size up to the value denoted by abscissa 8.

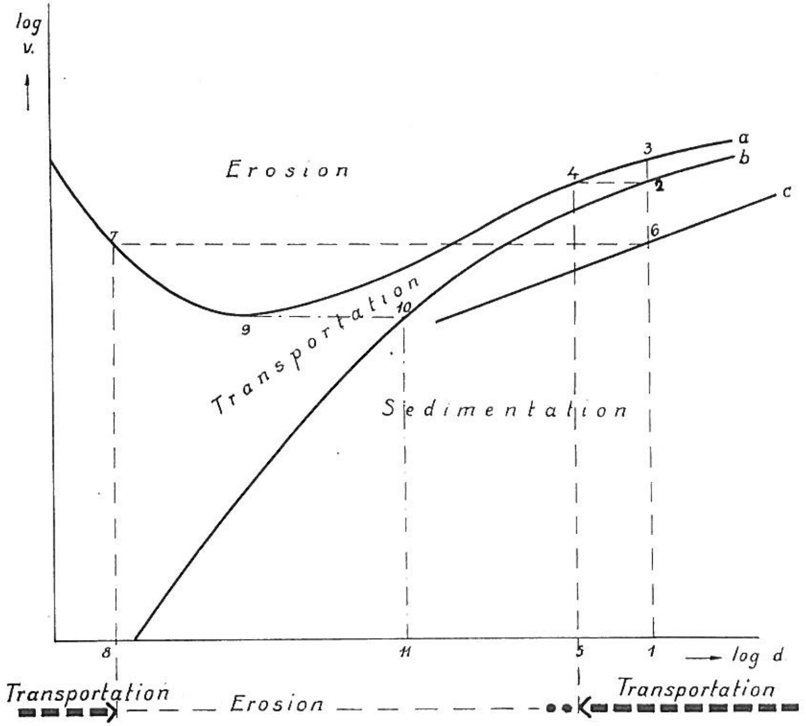

Fig. 27. Figure illustrating the deductions about the transportation and deposition of bed-load.

But this grain size cannot yet be decided with any great degree of exactness, curve *c* being so indefinite as to position. The quantity of material transported is probably of a certain importance. SUCHIER's values even exceed those expressed by curve *b*, whereas in OWEN's tests the lower transportation velocities must be below curve *c*. In the former case a whole stratum was transported, in the latter single particles. Judging from this we might be inclined to believe that the velocity (points 6 and 7) would be higher, and grain size 8 displaced towards lower values. Another factor with the same effect is the material contents of the water. The coarse material transported must, of course, endeavour to erode the clay. This effect is also evident from Table 8. FORTIER and SCOBEY there state that the erosion velocity for water transporting non-colloidal silts, sand, gravel or rock fragments as compared with that of clear water without detritus is subject to a decrease from 114 to 91 cm./sec. The decrease is thus not very considerable but the curve branch to the left of the minimum, point 9, is lowered a little and point 8 is somewhat displaced towards a smaller grain size. We must also presume that the coarser material easily sticks in the clay, which will, so to say, be paved and will thus be protected from erosion.

From point 8 up towards point 5 there is, however, a series of grain sizes over which the material mentioned cannot be transported by running water without erosion occurring. That interval encompasses the most easily eroded material, fine sand and similar material, which is removed at these velocities.

The curves for the lower transportation velocities being examined to find the grain sizes that correspond to the minimum velocity for erosion of a uniform material (draw a line from point 9 parallel to the axis of the abscissa until it intersects curve *b* at 10 and then find the corresponding grain size 11), we obtain the maximum grain size transportable without erosion and depositable over every kind of material. A comparison with Fig. 17 and 18 indicates that this grain size is 2 to 3 mm.

Very coarse sand or fine granule can thus, as all finer material, be transported without erosion over every kind of other material, finer as well as coarser. A fluvially transported sediment of particles coarser than fine gravel can, however, not be transported without erosion over a material of a mechanical composition of grain sizes from clay up to a figure approximately expressed in the following Table, the last column. It is therefore seldom found as a super-stratum on such material.

Table 11.

Composition of deposited material	Composition of bottom material	
	may be	is but seldom
10 cm. particles	silt clay, 3—4 cm. and greater	0.0001—3—4 cm.
5 » »	» » , 2 » » »	0.0001—2 cm.
3 » »	» » , 1.5 » » »	0.0001—1.5 »
2 » »	» » , 1 » » »	0.0001—1 »
1 » »	» » , 0.5 » » »	0.0001—0.5 »
0.5 » »	» » , 0.2 » » »	0.0001—0.2 »
0.2 » and smaller	all kinds of material	

When the grain size of the suspended material is increased there will simultaneously be an increase in the grain size group of the lying material that is eroded when the suspended material is transported.

The values stated in Table 11 have been taken from Fig. 17 and 18. They are not more exact than are those figures, and are likewise valid for material of a uniform composition.

Conclusions.

Finding in a deposit material on top of a finer one it is unusual for their respective grain sizes to appear in the combination shown in columns 1 and 3, Table 11. Should this be the case, 2 alternatives appear.

1. The coarser material was not brought there by running water. In this case there may be a clearly defined contact surface.
2. The coarser material was brought there by a stream which eroded the finer material, but had no time to erode it away entirely before the velocity of the water diminished and deposition occurred. There is a border zone where the two kinds of material are mixed.

It would be of interest to get these conclusions, based on empirical data, verified by direct tests.

Transportation.

The area above the erosion curve in Figures 17 and 18 indicates erosion, and the area below the curve for the lowest transportation velocity deposition. Between these two curves there is evidently a field indicating transportation. For bigger grain sizes this field is of a considerable width as also for the very smallest sizes (see Figures 17 and 18). Thus for a uniform material of these sizes there is rather a wide range of velocities, within which a material in motion is transported further before deposition occurs. Under the supposition mentioned there is then a kind of equilibrium without either erosion or sedimentation. The total quantity of matarial transported neither increases nor decreases. For a mixed material containing all grain sizes up to the largest which the river is able to carry, circumstances are more complicated to the above. The erosion velocity meaning the velocity for which all grain sizes up to the normal maximum are put in motion, and the lowest transportation velocity the velocity at which the material starts to deposit, the transportation interval still becomes shorter than for a uniform material. If the velocity is kept constant to any certain value within the transportation interval mentioned, there exists neither erosion nor sedimentation; we have a kind of equilibrium. If the velocity changes to another lower value within the transportation field, there is no change in the total quantity of material transported, but the relation between the bed-load and the suspended material is displaced. Part of the latter material is deposited in the bed-load. If on the contrary the velocity increases, the quantity of the suspended material increases and at the same time an erosion of finer particles of the bed may take place.

The motion of the bottom-layer.

The current- and other physical conditions prevailing in the lowest zone of a river are extremely complicated, and have defied every effort to make a fairly exact description. They are presumably very variable.

This boundary-layer where the transportation of the bed-load takes place, gets a high specific gravity due to the pressure of solid material, this of course influencing the current. The turbulence is of course strongly affected thereby, and it must be impeded in the same manner as when stable stratification occurs in the atmosphere, for inst. at temperature inversion. Water with a heavy load of sediment, and therefore heavy, should be raised and substituted by lighter water thanks to the exchange-process. If the increase of the silt percentage is sufficient, the turbulence will, therefore, have to cease or become very diminished. At least, in certain cases there is, therefore, along the bottom a *boundary-layer* with a *laminary* or a *slightly turbulent* motion.

This problem is analogous to the problem of the stability of a fluid in which the density and velocity vary with height above the ground, a problem studied in hydrodynamics and meteorology. It has been treated by inter alia: LEWIS F. RICHARDSON (1920) PRANDTL (for inst. 1932), TAYLOR (1931), and GOLDSTEIN (1931). As a criterium for the appearance of turbulence the following expression has been stated:

$$-\frac{\frac{g}{\varrho}\cdot\frac{d\varrho}{dz}}{\left(\frac{du}{dz}\right)^2}$$

where $\varrho =$ the density $\left(= \frac{1}{g}\cdot(1 + 0.63\ S)\right.$, where $S =$ the sediment contents, if the specific gravity $= 2.7\Big)$

$u =$ the velocity
$g =$ the acceleration of gravity
$z =$ the height above the bottom.

If this expression is < 1 the motion is turbulent according to RICHARDSON; if it is > 1 the motion is laminary. But »a theory like this one, which supposes the mean velocities to be horizontal straight lines, can only fit in with observations at a height above the ground which is large compared with the irregularities of the surface.» (RICHARDSON op. cit. p. 365).

If the expression $\frac{g}{\varrho}\cdot\frac{d\varrho}{dz}$ is greater than the square of the change of the velocity the motion is laminary. Consequently, the decrease of the silt percentage towards the surface is, as mentioned above, of fundamental importance. The decrease must not be too inconsiderable if a laminary motion is to prevail. The decrease has its greatest value in erosion, when the bottom-layer is enriched and the equilibrium distribution of the silt percentage has not yet occurred.

However, erosion must not necessarily occur exactly at the place intended; the main object is that the lower layers be greatly enriched with silt compared with higher layers. This may also be the case below an erosion place in the river, which, however, must not be so far below that the silt distribution has already been stabilized. It is of decisive importance that an increase in the quantity of material suspended and in the saltation zone takes place. The transportation of material in contact with the bottom has no, or in any case minor importance; a deposition of this material may even occur. Therefore, it is not easy to relate the presence of this laminating bottom-layer to the conditions on the spot. But generally the following rules may be stated. The prospects are *greatest* for the appearance of a bottom-layer with laminary motion when there is a *rising water-level* accompanied by erosion. The water-level *falling*, accompanied by deposition, the decrease upwards of the silt percentage has its lowest value, and the prospects for the appearance of such a bottom-layer are the *smallest*. To obtain a starting point in order to judge the conditions when there is only transportation without erosion or sedimentation, the values previously mentioned for the variations of the velocity and the silt percentage with the height above the bottom have been inserted in the above expression, by way of trial (pp. 272 and 273). After derivation one finally gets the condition for laminary motion:

$$1 < \frac{0.63\,g\,.\,c\,.\,p}{A_1\,.\,u_1^2}\cdot\frac{S}{1 + 0.63\,.\,S}\cdot\frac{z^{p-1}}{p}.$$

A_1 here indicates the Austausch-coefficient at a height of 1 cm. above the bottom, which would cause the silt distribution present if the motion was turbulent.

If $c = 6.6$ — equivalent to a grain size of 1 mm. at + 15° C, $p = 8$,

$g = 981$, $A_1 = 10$, and $u_1 = 5$, it is found that in order to get a boundary-layer of 10 cm. density, a silt percentage exceeding 109 gr./liter is necessary there.[1] This must be considered an extremely high value which should be rare at the low values indicated for turbulence and velocity. If we instead estimate the density of the boundary-layer this is found to be 7 mm., the silt percentage being put at 10 gr./liter. Consequently, the layer would be extremely thin — so thin that it may be completely disregarded. And then the formula would scarcely be valid.

The example chosen refers to a river running rather slowly with limited turbulence, where the silt percentage strongly decreases upwards, and where there are thus great prospects for the formation of boundary-layers. A boundary-layer nevertheless not appearing here, this indicates that no laminating boundary-layers occur or are at least very rare in a state of stability without erosion or sedimentation.

Thus, laminating boundary-layers along the bottom of the river are principally found in connection with *erosion*.

However, this cannot be anticipated in all rivers. The first condition is, that a solid material really is transported in a greatly enriched layer along the bottom, the bed-load. Such a layer does not appear in all rivers. If the bed consists of fine clay and erosion occurs, the small particles are immediately brought in suspension and spread in the water. This is the case in the Fyris, the writer thus having had no opportunity to study these bottom-layers. — Nor is it probable that they will appear in rapid mountain streams with great turbulence. On the other hand, rivers with a sand bottom and a slow calm course afford great possibilities for the formation of such bottom-layers, especially if the grain size of the bottom material is not too limited.

What may the effect be of such a bottom-layer?

First of all, the erosion must of course decrease, when the water above the bottom has a laminary motion or diminished turbulence. This need not stop the erosion but it is less active, as already explained (Chapter II).

Another important effect is that the velocity changes. For a turbulent current, the increase in the velocity and the velocity itself close to the bottom are greater than were the current laminary, other conditions being similar. See formulae p. 231 and 246 and Figure 28. Higher above the bottom the conditions are reversed; the velocity is greater for a laminary motion than for a turbulent one. The effect of a boundary-layer appears in an increase in the velocity, which is greater the thicker the laminating boundary-layer is. The turbulent velocity-curve now does not start from the bottom but from a velocity-value already existing, namely from the velocity prevailing in the contact surface between the laminary boundary-layer and the turbulent current above it. The laminary boundary-layer serves as a lubricant between the bottom and the water. The friction decreases and the hydraulic formulae for the velocity as a function of hydraulic radius and slope or the like loses its validity. The formulae of KUTTER, BAZIN and MANNING pertain to this.

As mentioned above no observations could be made of this laminary boundary-layer, since transportation of bedload is non-existent in the Fyris. However, there is an interesting investigation from the Nile made by A. B. BUCKLEY (1923, see also the discussion which is not less interesting). It is based on material which is partly incomplete and difficult to interpret; for instance the investigation at Beleida, 1921, shows only a general influence on the velocity by the silt transportation. From the description it is not possible to definitely ascertain when erosion and when sedimentation or transportation prevails. BUCKLEY has also expressed the silt percentage as a factor influencing the velocity and not the criterium mentioned. The examinations at Menufia give a better illustration; the rapid decrease in slope, the silt percentage rising, signifies a decrease in the friction against the bottom (10—27 August 1919). However, these measurements are not completely conclusive owing to the lack of observations regarding the changes in the river-bed.

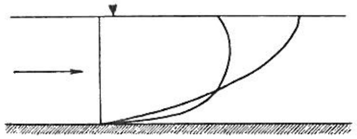

Fig. 28. Distribution of velocity for laminar and turbulent flow. The curve upwards inclined to the left is valid for turbulent, the other, parabolic curve for laminar motion.

The study of the formation and effect of the laminary boundary-layer would be a profitable task for a laboratory investigation.

Modes of transportation.

A factor that renders it more difficult to understand the conditions in the bottom-layer and also the velocity distribution as well as the whole transportation of solid material, is the imperfect knowledge of transportation mechanics.

The writer has previously made a difference between the transportation of bed-load and suspended material, and in Chapter II transition stage, saltation, has been mentioned. The transportation of the bed-load may, however, be effected in still more ways. GILBERT (1914) has further explained these ways in his admirable book on transportation of débris by

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[1] All the formulae presuppose cm., gr., and sec. as units.

running water. He at first makes a difference between movement of individual particles and collective movement. In the movement of individual particles »sliding» is a negligible factor. The roughness of the bed causes particles that retain contact to roll. — Rolling is the mere prelude to saltation». (op. cit. p. 26). Saltation or jumping was caused by the hydrodynamic upthrust, but of course the vertical velocities of the turbulence are also rather important, at least in the toplayer of the saltation zone, where a transition to suspended matter exists.

Individual particles in the bed-load thus move in one of the following ways:

1. sliding;
2. rolling;
3. saltation.

Transportation by rolling may easily be effected without saltation, especially of mixed débris. On the other hand saltation would not appear usual otherwise than in connection with the transition state, rolling.

There is perhaps more of a graduation difference than a species difference between the transportation states mentioned. And also when sliding and rolling the grains are forced to lose contact with the bed for very short distances; these little jumps increasing in length, we get a transition to saltation.

When several individual particles move forwards over a river-bed in one of the above mentioned ways, their movements sometimes are arranged in an extraordinary manner, by which the morphology in miniature gets its structure at the bottom.

Gilbert writes: »In another experiment a bed of sand was first prepared with the surface level and smooth. Over this a deep stream of water was run with a current so gentle that the bed was not disturbed. The strength of current was gradually increased until a few grains of sand began to move and then was kept steady. Soon it was seen that the feeble traction did not effect the whole bed, but only certain tracts, and after a time a regular pattern developed and the bed exhibited a system of waves and hollows. As the waves grew the amount of transportation increased, showing that, under the given conditions, the undulating surface was better adopted to traction than the plane». (op. cit. p. 30). The velocity being increased a state gradually developes, when the dune[1] motion ceases, and the sand surface becomes comparatively even, »although somewhat ruffled in the run immediately following the disappearance of dunes». The velocity being further increased there developes another kind of motion, characterized by antidunes which travel against the current instead of with it. »Their downstream slopes are eroded and their upstream slopes receive deposit. They travel much faster than the dunes, and their profiles are more symmetric» (op. cit. p. 31).

Thus Gilbert also distinguishes between three different kinds of collective movement, in which the bed is characterized by different appearances:

1. the dune mode of traction;
2. the traction without waves uniformly over a plane bed;
3. the antidune mode of traction.

The two first of these three ways are most usual while the third one does not appear so often.

Perhaps to these three different kinds of movement there may be added one kind more. This sort of movement resembles the dune mode of traction but the forms of transportation are not exactly the same. Instead of parallel transverse dunes there appear, when the state of movement of the water i *shooting*, a pattern of tongue-like waves of sand, separated by furrows. This kind of movement has been investigated by H. Blasius (1910). Any detailed explanation of these forms is, however, not yet set forth.

There is a very extensive literature about these dunes or current-marks. No details as to their appearance or occurrence will be given here, (see Ahlmann (1914), and Henning Kaufmann (1929) with valuable bibliography). Only some points of view will be expressed here.

The origin of the dune mode of traction.

To understand the conditions of existence of these current marks the writer would consider it essential to study their first appearance on a smooth surface. This is easily done in a laboratory-channel with a sand bottom. In one case the writer observed in such a channel how the first indication of the dunes on a smooth sand surface *suddenly* occurred. The water poured down on the sand from a model building of a veir-construction with a height of about 30 cm. Due to the erosion below the fall an excavation was formed and the material was transported downstream. The erosion only took place at the excavation and thus the transportation occurred over a material of the same composition. At first the movement was approximately uniform on the smooth surface, but suddenly there were formed, almost momentarily, one or two current marks shortly below the excavation. This sudden forming must give the observer the impression

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[1] In the following the writer, in agreement with Gilbert, will often use the term dunes for these small submerged ribbons of sand on the bed of streams, arranged transversally to the direction of the stream. Cornish (1914) employs the term current-marks. They must not be confused with the great continental dunes, which in many respects follow other laws.

that it has something to do with the pulsations of the water. The motion of the sand grains does not take place with a quite constant velocity but with minor variations around an average value — in the same way as the water. The grains also make short stops owing to the roughness. It seemed as if the formation of the current marks in the smooth surface took place due to an especially strong increase in the momentary velocity value, an extremely strong pulsation. On the occasion in question the sand grains were swept over the bottom with considerably greater velocity than on the average. It is difficult to observe if superficial inequalities play any greater part in the localisation of these first current marks. They may surely form a core for them but they are not necessary. Even if the bottom is quite smooth current marks may anyhow be formed. Henning Kaufmann's (1929 p. 9) theory of an accelerating effect of small obstacles on the formation of current marks seem to be plausible.

The observation showed that the appearance of these current marks is connected with the pulsations of the water, an observation which must seem both clear and unavoidable to every examiner. In his observations on transportation of bed-load in the Indalsälven Ahlmann (1914 p. 24) has arrived at the same result. »Das Transportdelta bildet sich auch auf einer ungestörten, ganz ebenen und homogenen Oberfläche durch die Pulsationen des Wassers». Ahlmann seems to be the first to mention this fact. To a certain degree it also agrees with H. Jeffrey's statement[1], that »the beginning of any sand-wave seems to depend on turbulence». The effect of the pulsations may, however, occur in several ways. One might possibly expect, that it simply consists of a direct sand aggregate caused by pulsation following each other. When the first formed dunes have moved on a bit, a new pulsation comes sweeping sand forward to a new dune. In this case, the distance between the crests of the dunes divided by the rate of advance of the dunes should indicate the time-interval between two pulsations of such a force as to form dunes in the sand bottom of the stream. However, this theory is contradicted by the observation that pulsations of equal force appear much more often. They have, however, not the same effect until earlier formed dunes have had time to move on a short distance. Not until then is a new dune formed on the spot where the previous one was formed. Further, it is difficult to understand how several dunes of this type can simultaneously appear when the erosion suddenly begins on a whole surface, as in the above cited experiment by Gilbert. It would appear reasonable to expect the movable sand to be transported in an even layer over the bottom, almost in the same way as a carpet is dragged over a floor.

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[1] Additional notes (pp. 121—159) by Harold Jeffrey's to *Vaughan Cornish: Ocean waves and kindred geophysical phenomena*. Cambridge 1934.

This not being the case there must be some other factor making the motion rhythmical and creating the extraordinary morphological phenomenon. If there is in the motion itself any tendency towards non-uniformity, this must further be accentuated by the pulsations. For instance, if there is a tendency towards wave formation a powerful pulsation will result in it, so to say, being conserved in a smooth surface consisting of movable material. The current marks then formed move on owing to the general transportation, and new pulsations only influence the transportation but do not change the general appearance of the bottom.

It is evident from the lack of correspondence in dimension as well as sometimes in the moving direction too, that the waves of the surface are of no decisive importance — at least not generally. The waves of the surface generally have a much greater wave-length than the dunes, usually 5 to 20 times greater. However, they are able to influence the pulsations at a low depth of water.

Several scientists V. Cornish, O. Baschin (1899), de Candolle, Solger, Mayer (1928) and others, have expressed the opinion that a wave formation of the kind that appears in the boundary-surface between two mediae of differing densities and moving conditions (Helmholtz-waves) might have a certain influence. F. Exner (1920) also refers to some kind of fluctuations in the boundary-surface. As the question of the possible occurrence of a wave formation in the boundary-surface, movable sand-flowing water, does not seem to have been the subject of a quantitative investigation but in Exner's special case, an examination of the applicability of the theory might not be unjustified. In the following, G. I. Taylor's investigation of the effect of variation in density on the stability of superposed streams of fluid (1931) as well as a similar investigation by B. Haurwitz (1931) are used tentatively as a starting point.

Transportation by rolling. Taylor points out that the wave system and stability of the surface of separation of two superposed homogeneous fluids which move with relative velocity are well known. Such a system is always unstable, but with a given relative velocity $U_2 - U_1$ and a given ratio of densities $\frac{\varrho_2}{\varrho_1}$ only waves whose wavelength is less than

$$\frac{2\pi \cdot \varrho_1 \cdot \varrho_2 (U_2 - U_1)^2}{g(\varrho_1^2 - \varrho_2^2)}$$

are unstable. Longer waves are propagated at a constant speed and with constant amplitude.

If one tries to apply this on an inhomogeneous liquid such as sand in water the density should apply to the whole mixture. At first let us consider the case of the velocity slowly increasing up to the value when the finer particles are put in motion. There is then a layer of sand rolling over the bottom, and above that pure water. In the boundary-layer between these two layers there is then a tendency towards wave formation which seems to increase due to the influence of the pulsations. If the density of the water is considered = 1, and in the movable sand layer = 1.1, and the difference in speed between the latter and the former (i. e. between the sand and the water above it) equal to 20 cm./sec. we find that waves whose wave-length is less than 13 cm. are unstable, while longer wave-lengths subsist. The stabilising effect of the density distribution is also at very small relative speeds of the two fluids insufficient to stabilise short-length waves which have the greatest effect on the motion. It might be expected that out of the waves for which the motion is stable the shortest ones should be of the greatest importance with regard to the influence on the bottom. A tendency towards wave formation being at hand, the shortest possible waves should in this case first of all be observed, i. e. the limiting wave-length.

Consequently, the decisive factor for this wave-length is the velocity difference in the boundary-layer as well as the density inside the two layers. The limiting wave-length increases with the square of the velocity difference. If in our example $U_2 - U_1 = 25$ cm., the wave-length is = 21 cm. The rules for this velocity difference are not known however. (See p. 301). No systematic study of the motion velocity of the sand grains exists. But Penck (1894 p. 281) cites an investigation made by T. E. Blackwell which gives some illustrations. The velocity of the grains increases rapidly when the speed of the water is increased over the value at which the grains are put in motion. When the velocity has reached a certain value, the increase is less rapid. After that it finally makes a new rapid increase, upon transition to saltation or suspension. However, this is only the case for coarser grains, minimum weight 42 grams. In all these cases, the velocity difference was greater than 40 cm./sec. and often considerably higher (except for light pieces of coal with a specific gravity of 1.26). However, it does not appear under which circumstances Blackwell's tests were performed. This is really a pity considering the great importance of the bedding. In a homogeneous material the velocity difference and thus also the wave-length is, of course, greater than after the addition of a fine-grained material. — P. Hahmann (1912) has, however, by experiment found that the distance between the crests of the dunes is directly proportional to the velocity of the water. This should mean that the velocity of the sand, U_1, is connected with the water's, U_2, by the relation:

$$U_1 = U_2 - \sqrt{U_2 \cdot \frac{k_1}{k_2}}$$

where $k_1 = \dfrac{2\pi \cdot \varrho_1 \cdot \varrho_2}{9(\varrho_1^2 - \varrho_2^2)}$

k_2 = the proportionality factor in a relation between wave-length λ and velocity of water, U_2,

$$\lambda = k_2 \cdot U_2.$$

k_2 increases with the grain size and decreases with rising viscosity of the water; probably it also depends on other factors. — However, it would be a kind of argument in a circle to use this relation to decide the limiting wave-length. So we return to our example.

Erosion causing more material to be brought to the bottom-layer, the density of same is increased; the stream grows more stable and shorter wave-lengths can now appear. If the density should increase from 1.1 to 1.5, the limiting wave-length is decreased from 21 cm. to about 5 cm. (for $U_2 - U_1 = 25$ cm./sec.).

Since erosion generally occurs more easily in a finer sand than in a coarser one, this agrees with the observation that the distance between the dunes increases with the grain-size. — The suppositions regarding the density are, of course, very hypothetical and not easy to check by observations. However, they seem to be very plausible in consideration of the fact that the specific gravity of the sand generally is about 2.6 to 2.7.

However, it has followed from the example that these estimated values, especially when a minimum density difference between the two layers is assumed, very well agree with the values observed. The theory of a wave-formation according to Helmholtz would, therefore, seem successfully usable, though not strictly valid near a boundary.

Transportation by rolling and saltation.

In this case there are three layers, as a transition layer has been formed, which brings about the transition of the water and the rolling bottom material.

The case of three superposed fluids has also been solved and one special case been worked out by Taylor. He says: »There is a difference in regard to the type of condition of stability between fluids in which the velocity and density vary continuously and those in which both of them are discontinuous». If the velocity and density of the layer of rolling material is U_1 and ϱ_1, of the water U_2 and ϱ_2 as before, and of the intermediate transition layer (height h) resp. U_3 and ϱ_3, in Taylor's example $\varrho_1 : \varrho_3 : \varrho_2 = 1 : 1.5 : 2$; thus very high concentrations in the lower layers, indicating erosion and small grain sizes. In the transition layer the velocity increases linearly from U_1 to U_2. »It appears that for any given value of $U_2 - U_1$ there is always a range of wave-lengths in which the flow is unstable, but as the wave-length diminishes the effect of the transition layer is to make the flow stable again for very short waves, which in the discontinuous case become more and more unstable the shorter the wave-length». (TAYLOR p. 500). With the prevailing indication system the wave-length is $\frac{2\pi}{\varkappa}$, and $\varkappa$ thus the number of wave-lengths on the distance 2π units of length (cm.). The thickness of the layer of transition has an important effect on the limiting wave-length. If h is low, so that $\varkappa \cdot h$ can be $= 0$, the limiting wave-length is $\frac{4\pi}{g} \cdot \left(\frac{V}{1.75}\right)^2$, for $\varkappa \cdot h = 1$ it will be $\frac{4\pi}{g} \cdot \left(\frac{V}{1.53}\right)^2$, for $\varkappa \cdot h = 1.6$: $\frac{4\pi}{g} \cdot \left(\frac{V}{1.47}\right)^2$, for $\varkappa \cdot h = 3.0$: $\frac{4\pi}{h} \cdot \left(\frac{V}{1.3}\right)^2$, and for very high $\varkappa \cdot h$ $\frac{4\pi}{g} \cdot \left(\frac{V}{1.67}\right)^2$. If the velocity difference $V (= U_2 - U_1)$ is put $= 30$ cm. the limiting wave-lengths will be respectively 3.8, 4.9, 5.3, 6.8, and 8.4 cm. Greater wave-lengths may thus occur, but not shorter ones, without causing the movement to become turbulent. Only if the thickness of the layer exceeds a value corresponding to $\varkappa h$ between 1.0 and 1.6 a further series of wave-lengths is possible, namely, very short waves. (For $U_2 - U_1 = 30$ cm./sec. and $\varkappa \cdot h = 1.6$ the upper limit will be only 1.1 cm., and for $\varkappa h = 3.0$ it will be 3.2 cm.).

Owing to the strong concentration chosen for the example, which concentration will not be so usual in nature, the wave-lengths estimated are shorter than those usually seen. As far as the magnitude is concerned the result is, however, correct, and an estimate of another example with less density downwards, will give the wave-length a higher value, in better agreement with reality. It is thus evident that the theory is applicable also in this case.

The example calculated by TAYLOR shows, amongst other things, the influence of the thickness of the saltation-zone. When this increases the wave-length grows at the same time. The thickness of the saltation-zone in its turn depends upon the velocity of the water; it grows as the velocity increases.

The actual work of a river as shown by the forms.

The special manner of transportation caused by the dune mode of traction is to a very high degree rhythmic. The movement is rhythmic not only in time, in that the particles have a certain changing between rest and movement, but also in space, in that the mass of transported matter changes in a regular manner in the dune.

For a closer analysis[1] of transportation of this type, for example a series of stream ripples, according to Fig. 29, the following simple reasoning which, however, does not seem to have been used earlier, is taken as the starting-point: *If the form of the dunes is to be maintained unchanged the same amount of matter must be carried away per time unit from every surface unit of the windward sides of the dunes.* If this is not the case a change in the form of the dune takes place immediately.

Transportation by rolling. First of all the case will be examined when transportation takes place exclusively through rolling, and saltation and suspension are not present as forms of transportation. In the case as shown by Fig. 29 the mass of rolled matter must increase from 0 at the point *a* up to its maximum value at the point *b*. The longitudinal section, Fig. 29, can therefore at the same time be regarded as a diagram, where the ordinate gives the intensity of the transportation and the abscissa the distance. For the distal declivity this description is highly approximative; here the eddies, described by many authors, are at work to the leeward of the crests of the dunes.

If the dunes maintain their form unchanged, this kind of transportation must imply equilibrium. Neither erosion nor deposition takes place, only transportation. Erosion would imply an increase in the mass of transported matter. With the maintainance of the distance between the crests an increase of the intensity of the transportation must go hand in hand with an increase of the height of the dune. An increase of the travelling velocity instead of an increase of the volume should in its turn imply an increase of the distance between the crests of the dunes within the area where the *acceleration* takes place. Only if the erosion takes place above the place observed where the dune mode of traction exists, and only if the acceleration takes place simultaneously over the whole of this area, can an increase of the mass of the transported matter take place without changing the appearance of the dunes. Therefore, within the area considered, only transportation takes place. In the above-mentioned laboratory observations it was always found that movement in form of dunes occured below the places of erosion, the excavations, but never at the places where the erosion took place. There the matter moved with the same intensity over the whole area as a uniformly thick layer.

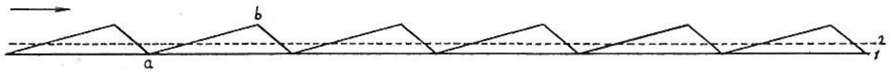

Fig. 29. Current-marks.

Deposition of a portion of the transported matter would, provided that the form of the dunes is maintained unchanged, cause a decrease of their height. A decrease in velocity must cause a compression of the dunes.

Another form of dunes corresponding to a smaller transport of matter also exists. In Fig. 29 all the matter above line 1 takes part in the move-

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[1] The examination was caused by the attempts to find a method for the determination of bed-load.

ment, in that the whole sand-mass is transported. Erosion implies a lowering of this level line and deposition, a heightening. If the above-mentioned line is raised to level 2 and the surface in the meanwhile is levelled out, it may happen that the dunes become separated and move independantly of each other. This type of dune for a smaller transport of matter appears when the bottom layer has a different quality than the transported matter, and is solid or less moveable than this. In a laboratory it can easily be produced, for instance, by means of placing a sand-layer on a smooth metallic surface. By means of the effect of flowing water this dune-formation can appear, and if the existing mass of matter is not very large, the dunes become separated in the manner indicated above. This form of movement is, however, rather unstable. A very slight difference in the height of the dunes causes a change in their travelling velocity. According to EXNER's excellent paper on his studies of dunes on »Kurische Nehrung» (1928) the travelling velocity is reversely proportional to the height of the crest; therefore, the small dunes overtake the larger ones and combine with them. Moreover, these dunes most often have the form of barchans, that is to say, their sides are curved in the direction of the movement.

From these observations it is found, therefore, that the dune mode of traction signifies a state of equilibrium if the form of the dune remains unchanged; there is neither erosion nor deposition.

This rule is a corroboration, under previously stated conditions, of AHLMANN's more extensive proposition: »dass nähmlich die Furche die Form ist, in welcher das Geschiebe transportiert wird, und ihre Bewegung die Art, in welcher der Transport vorsichgeht» (1914 a, p. 22).

An assumption for the propositions above is, that the matter is present in abundance — preferably the whole river bed should have the same consistency. AHLMANN (op. cit. p. 31) makes the following important observation:

»In dem obenerwähnten Arpojokibach war dies mit grosser Deutlichkeit zu beobachten: Der Bach floss nähmlich erst über Moränenboden, wo alles feine Material wegerodiert, und der Boden gleichsam mit mittelgrossen Steinen gepflastert war; die kleine hier vorkommende Sandquantität wurde kontinuierlich einen langen Weg entlang geführt. An einer Stelle berührte das Wasser aber einen Sandrücken, der die Sandlast sehr vermehrte, was zu Folge hatte, dass sofort Transportdeltas entstande, obschon sowohl die Form des Bodens wie die Stromgeschwindigkeit dieselbe war. *Die Vorwärtsschaffung des Geschiebes mittels Transportkörper setzt somit eine gewisse minimale Materialquantität voraus.*»

Transportation by rolling and saltation.

When the velocity increases so that the finer ingredients of a mixed matter are transportated through saltation, the movement becomes more complicated. The dunes still remain, but above is found a layer with sand in saltation. The exchange between the layer of the rolling matter and the zone of saltation varies naturally with the velocity and the composition of the matter. If both of these remain unchanged the mass of matter which is transported in each of these modes of transportation is also constant. Then the form of the dunes is constant as well.

If erosion appears because of an increase in velocity, new matter passes at the same time, over into saltation or suspension. The form of the dune can not thereby be held constant; the dunes are swept away and the transportation occurs in a uniform layer. The opposite process, deposition, can hardly either happen without changes in the form of the dunes.

Transportation through rolling and saltation follows mainly the same laws as transportation through rolling alone. The kind of movement in question is a transition form to transportation of the moving bed-load as a uniformly extended layer.

Transportation of the bed-load as a uniform layer.

As has been mentioned earlier the coming into existence of the dune mode of traction is always connected with the influence of pulsations. They occur always, almost without exception, in running water, which, on the contrary, is not the case with the dune mode of traction of the bed-load. Why do then, under certain conditions, dunes appear, while in other cases, the bed-load moves as a uniform layer?

The answer is that a certain connection must exist between the pulsations and the nature of the matter, in order that the formation of these small submerged dunes may appear. If this condition is not filled, the movement takes place in a uniform layer.

At the formation of the dunes the sand was set in rapid motion over the bottom, by means of a pulsation. A gathering of sand took place at certain places with regular intervals, according to the waves in the boundary layer between the sand in movement and the water. A condition for the gathering together of the sand is, however, that the particles in the bed-load have resting-pauses in their movement. A pulsation increases the movement of the matter by means of an increase in its own velocity; when the velocity then decreases under its average value shortly afterwards, the grains must stop or else a dune will not be formed at all. If this is not the case the movement continues the whole time and will take place in the form of a uniform layer. It is easy to understand that existing obstacles can have an accelerated influence on the formation of these dunes.

If the resting pauses in question are to appear, the following conditions must be fulfilled:

1) The matter must not be too light, or else it follows the movements

of the water. Because of this, these dunes do not appear in running water over a bottom of clay.

2) The matter must not be too heavy either, or else it continues its movement because of its own momentum of inertia. Dunes of very coarse matter are not often to be found.

Thus it is essential for the appearance of dunes that the mean velocity is low enough so that the lowest instantaneous values fall below the lowest transportation velocity. The range of velocities between the value where the particles are set in motion by water and by other particles in motion, and deposition, must not be too great. It must not surpass the fluctuations in the velocity caused by the pulsations. It is shown by Fig. 17 that the velocity-interval in question is smallest for the grain-dimension groups 0.1 mm. It is also with this matter that the dune mode of transportation is most common.

According to KRAMER (1932, p. 27) KREY has in a hitherto unpublished investigation of the Elbe found that »die Reffelbildung sowohl von der Gleichmässigkeit der Korngrössen (Mischungsverhältnis) als auch von der absoluten Grösse der stärkeren Sandkörner abhängt». According to the observations of the writer the existance of rougher grains has a levelling effect upon the bottom. At the earlier mentioned (p. 301) observations of the velocity of the sand grains it was found that the rougher grains rolled continuously without pauses. They were influenced by the water even in lee of the crests of the dunes. But it was shown that it was not necessary to increase the velocity much over the velocity of the erosion in order that the dunes should disappear. The rougher matter put the finer matter in the crests in movement and the dunes were corroded away. In this way a zone was effected in the middle of the hydraulic channel with transportation of the material in a uniform layer, while on both sides the dune mode of traction occured. However, it was found that the bottom in the middle got lower and lower; *the bed was eroded at that part of the channel where the transportation in a uniform layer occured.* The other parts of the channel retained the same level; erosion was not present there. If stones or other obstacles which the water could not displace were placed on the bottom within the eroded area, some of the material was deposited behind these obstacles. There occurred, therefore, a longitudinal dune. At the sides and in front of the obstacles excavation occurred. All the forms of the bed were elongated in the direction of the stream. It is not always easy to decide if they are formed by accumulation or erosion. However, when the velocity is great, it seems as if these longitudinal forms were more often the result of erosion than of accumulation. The writer has made the same observations concerning the form-system of snow.

Similar observations can be made in natural streams when the water is sufficiently shallow or transparent. If the velocity is increased in an

artificial way it is found that the dunes disappear, the matter passed over more and more into suspension, and the bed-load soon moves in a uniform layer. Pl. VI shows dunes in fine sand (0.2—0.4 mm.) formed in the small delta which the »Prostgårdsälven» builds where it empties into the Borssjön at Molkom, Värmland. The distance between the crests was on the average 15 cm., the water's velocity 30 cm. per sec. and that of the dunes about 3 cm. per minute. By damming up a little outlet-branch of the stream, the water-mass could be increased, whereby the velocity rose to about 70 cm. per sec. At this velocity erosion appeared at that place, illustrated by Pl. VI, the dunes vanished away, the sand was transported in a uniform layer and the bottom was lowered.[1]

These and other similar observations seem to the writer to justify the following conclusion:

At places where the water is eroding its bed the material is transported in a uniform layer.

The word »*uniform*» signifies the opposite to the dune mode of transportation, where the transportation has rhythmic variations according to the form of the dunes; however, it should not be taken too literally. In the boundary surface is even here found a tendency for wave-bilding, which, however, only expresses itself in variations with the thickness of the moveable layer. Illustrating figures are offered by BUTCHER (1919, p. 1933). This thickness can, indeed, be quite considerable.

This wave-formation in the saltation zone's upper boundary surface can be backward-moving (see TAYLOR, 1931, p. 500 and 516). Consequently the *anti-dune* motion appears, as described by GILBERT, CORNISH and others.

Conclusions.

A summary of the conclusions of the preceding pages may be formulated in the following manner.

The dune mode of traction with unaltered form of the dunes signifies equilibrium, without erosion or deposition. At places where the water is eroding its bed the material is transported in a uniform layer. The longitudinal forms seem to indicate erosion; they are — at greater velocities — caused by erosion and no such forms characterizing transportation exists.

If the mechanical composition of the material is uniform and the size of the particles is 0.5—4 mm. the presence of transversal dunes generally signifies a state of equilibrium without erosion or sedimentation; transportation of the sand as a uniform layer signifies erosion. — The process of deposition has no other form of its own than that of a delta.

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[1] The figure is interesting because the surface-waves of the water are also visible. The dunes are, however, not caused by these waves, as they appear even when the water is calm, and exist even in the other outlet-branch, not exposed to wind or waves. (The system of white lines in the foreground is caused by refraction of sun-light.)

The capacity of a stream.

When a river deposites a part of its matter, this depends upon the fact that the actual velocity sinks below the lowest transportation velocity. If the material consists of particles of uniform size, all the quantity of material carried is deposited. But if there are particles of a wide range of sizes only a part of the load (load = »the actual quantity of material carried at any one time», Twenhofel, p. 42) may be deposited. This part then is most often the group of greatest grain-dimension; only in certain rare cases may the finer particles be deposited at first, while the greater particles still roll over the smooth surface.

One sometimes meets the more or less clearly formulated supposition that the deposition depends upon the fact that the stream is *saturated* (»gesättigt»). Energy is spent for transportation. In a river with a maximum load all the available energy is consumed for the transportation of mineral matter.

The writer has earlier pointed out (Hjulström, 1932, p. 252) that the expression »saturated» must be regarded as very unsuitable; this leads to the conception that the stream should be saturated with mineral matter in the same manner as a solution of salt can be saturated. Thus, it should not be able to carry a greater quantity of mineral matter. However, this is erroneous especially for the matter in suspension. A supply of matter lowers the velocity somewhat, it is true, (see Jakuschoff, 1932 b, p. 18) and causes a sedimentation of the coarsest particles. We assume that these are found in a small quantity, while the matter on the whole is very fine-grained. As a result, an increase of the load may then be obtained. And if only a sufficiently fine-grained matter is selected, this increase in weight may be imagined to be of any magnitude whatever. It seems to be difficult to indicate an upper limit. At the same time, the water becomes more and more sluggish-moving. In nature all transitions between silt-carrying streams and mud flow (»Erdfliessen») should occur. Raymond C. Pierce (1916) mentiones, from the San Juan River, that during a sudden heavy flood 75 per cent of the original volume of a sample was silt and red sand. But whilst in such cases water is mixed with silt and sand, the mud flow has from the beginning an opposite origin: that is to say, the soil has absorbed water.

According to Gluschkoff (quoted from Jakuschoff, 1932 b, p. 18) the river Murgab in Turkestan uses only 1 to 3 % of its total power of work for the transportation of suspended matter.

W. W. Rubey, (1933 b) calculates that the energy consumed in supporting débris is closely (measured in ergs):

$$\frac{L(\varrho s - 1)}{\varrho s} \cdot g \cdot c \cdot t^2$$

where L = mass of debris passing a cross-section per unit time (in gr. per sec.).
g = acceleration of gravity.
ϱ = density of débris-particles.
t = time in sec.
c = the settling-velocities of both large and small particles.

Rubey calculates that in 22 experiments by Gilbert, 97.5 %, on the average, of the total stream-energy appears to have been lost in friction and only 2.5 % spent in transporting débris. Unpublished data for the Colorado River indicate values of the same magnitude (Rubey, op. cit. p. 503). The three cases for which calculations exist thus show unanimously an especially low energy consumption for silt-transportation.

A saturation of the stream should indicate that, because of a high percentage of transported matter, it has becomme so sluggish that its velocity approaches 0. Any such phenomenon has not been mentioned in geological or geographical literature. — When rivers carry very small quantities of matter and in spite of this do not erode, it is due to the fact that the velocity is too low in comparison to the size of the particles in the matter.

In connection with the term »saturated» there is also another term, namely, the *capacity* of a stream. By this term is meant the maximum load a stream can carry. According to the above this is a term which does not hold in case one imagines that the river has access to all kinds of grain-dimensions.

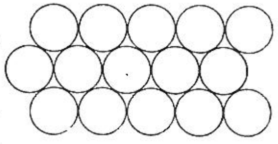
Fig. 30. Arrangement of spheric particles.

The term »capacity» has been brought forward by K. G. Gilbert in connection with his investigations concerning the transportation of débris by running water. These investigations concerned almost exclusively the transportation of the bed-load. It should now be noticed that for this kind of transportation — especially of coarse matter — a maximum load should exist. If the transportation consists only of rolling, the water can work upon the top-layer only, and only this layer can then move. A movement in layers lying one above the other as Du Boys assumes, (see for instance Schoklitsch, 1914) should be possible only in fine material, where a sheet of water round the particles plays some rôle. The existance of a movement of the indicated type has moreover been disproved by Schoklitsch (1914, p. 16—17) concerning sand.

Under the condition that the particles are spheric $\left(\text{diameter} = d,\ \text{weight}\ \frac{\pi}{6} \cdot d^3 \cdot \varrho\right)$ and arranged as in fig. 30, the maximum matter-transport per length unit of the dross-section becomes:

$$0.6 \cdot \varrho \cdot d \cdot U_1$$

where U_1 = the velocity of the grains.

If, besides, the matter exists in saltation it becomes more difficult to explain the occurrence of a maximum limit for transportation. But according to GILBERT's observations the velocity at the bottom is greatly lowered at the transportation of bed-load and, perhaps, because of this only a certain maximum mass may exist within the zone of saltation: this maximum mass increases with the size of the particles (GILBERT, p. 150—154).

Conclusion.

According to the above it appears that a river which has access to matter of all grain-dimensions down to clay and which possesses such a large velocity that even the clay may be eroded, lacks a maximum limit for its capacity of transportation. However, a stream which transports rolling matter without access to anything else can only transport a certain mass of this matter.

CHAPTER IV.

The degradation of the Fyris river basin.

Extent and plan of the investigation.

The previous chapters have treated the question of erosion and transportation from mainly a theoretical standpoint. The above-mentioned processes have, as a rule, been placed in connection with the velocity of the water. The velocity has been considered as the most important trait of character as to erosion, transportation and sedimentation of a river. The treatise has, however, shown the impossibility of making direct calculations of the erosion of a river without direct measurements. Important general laws can be laid down, and the physical explanation of several of the observed phenomena can be given. But the velocity depends upon many indeterminable factors, most important the water-mass which, in its turn, is determined by the hydrographical and climatological elements. Even if the velocity is presumed to be known, many unknown links in the chain of calculations for the obtaining of the extent of erosion remain. Among these incalculable factors the nature of the material should be mentioned. In Ch. III the difficulty of calculating the velocity of erosion for a given material was pointed out. If this is not altogether uniform, with the same size of the particles, the velocity of erosion cannot be regarded as known. Even after a determining of the velocity of erosion the difficulty still remains of determining how much material the river will transport. As has been pointed out in the preceeding pages this calculation cannot be carried out under certain conditions.

The knowledge of the eroding influence on the earth's surface of running water, must, therefore, be based on direct measurements according to one of the two methods — or both — which have been described on p. 225. Such measurements must be made for as many rivers as possible of different types and in different parts of the world. Every river has its own individuality in certain respects, and experiences concerning erosion in a river basin may be used as a basis for conclusions concerning other river systems only after very careful consideration. Furthermore, such measurements ought to be stretched over as long a time as possible in order that fluctuations in the climate will not have too great an effect.